Introduction to Probabilistic Graphical Models
Homework 1

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Instructions: (please read carefully)

1. This homework can be done in groups of **maximum 3** people. I personally encourage group work, try to form groups.
2. Prepare your report in English by using LaTeXor an ipython (jupyter) notebook. Do not submit scanned papers.
3. Put all your files (code and report) in a zip file: *surname_name_hw1.zip* and upload it to moodle before the deadline (check moodle for the deadline). Late submissions will not be accepted.

Question 1 (Bayes rule)

Let $\lambda, \mathcal{D} \in \mathbb{R}$ be two random variables. Let $p(\lambda) = \mathcal{N}(\lambda; 0, \sigma_\lambda^2)$, and $p(\mathcal{D}|\lambda) = \mathcal{N}(\mathcal{D}; \lambda, \sigma_\mathcal{D}^2)$, where $\mathcal{N}(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$. Derive analytically $p(\lambda|\mathcal{D})$ (find its form, do not attempt to take the derivative!).

Question 2 (Generative modeling)

We want to model a domain where we want to model a troubleshooter for a printer. A printer can print successfully a page or not. There are possible reasons for failure. The driver is corrupt, the printer is not plugged to the computer, the printer may be out of paper, there might be a problem with the network software. Another possibility is that there is no power. If there is no power the lights in the room are also off.

1. Carefully define the appropriate random variables to represent this scenario.
2. Draw the directed graphical model.

Question 3 (Graphical models)

A distribution factorizes according to the following factorization

$$p(A, B, D, F, T, L, M, X) = p(F|T, L)p(M)p(T|A)p(B|M)p(X|F)p(L|M)p(D|F, B)p(A)$$

1. Draw the corresponding directed graphical model
2. Draw an equivalent factor graph and undirected graphical model
3. If all the variables have N states, compute the space to store the model specification.
4. Verify the following conditional independence statements using d-separation. State if they are true or false and explain why.

- $A \perp\!\!\!\perp M|\emptyset$
- $A \perp\!\!\!\perp M|X$
- $T \perp\!\!\!\perp L|X$
- $X \perp\!\!\!\perp L|F$
- $X \perp\!\!\!\perp L|D$

Question 4 (Generalized gamma distribution)

The Generalized gamma distribution is a three parameter family defined as follows:

$$\mathcal{GG}(v; \alpha, \beta, c) = \frac{|c|}{\Gamma(\alpha)\beta^{c\alpha}} v^{c\alpha-1} \exp(-(v/\beta)^c)$$

Here, α is the shape, β is the scale and c is the power parameter.

1. Is the Generalized Gamma distribution an exponential family? If so, give the canonical parameters and the sufficient statistics.
2. Which famous distributions can be obtained as a special case of this distribution? Name at least two of them and describe how they appear as a special case.