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Hands-On 6.

Q3) Mathematically derive the average runtime complexity of the non-random pivot version of quicksort.

Sol) For array of size n,

recurrence relation for $T(n)$

$$\Rightarrow T(n) = T(k) + T(n-k-1) + O(n)$$

k - Number of elements in left sub-array

$O(n)$ - Time partition of array.

$$\Rightarrow T(n) = \frac{1}{n} \sum_{k=0}^{n-1} T(k) + T(n-k-1) + O(n)$$

Now, Since left & right part follow symmetry

$$\Rightarrow T(n) = \frac{2}{n} \sum_{k=0}^{n-1} T(k) + O(n)$$

Let's assume $S(n) = \sum_{k=0}^{n-1} T(k)$

$$\therefore S(n) = \int_0^n n \cdot \log n \cdot dn = \frac{n^2 \log n}{2} - \frac{n^2}{4}$$

$$= \frac{n^2 \log n}{2}$$

$$\therefore T(n) = \frac{2}{n} \cdot n^2 \frac{\log n}{2} + \Theta(n)$$

$$\simeq n \log n + \Theta(n)$$

$$\Rightarrow T(n) = \Theta(n \log n)$$