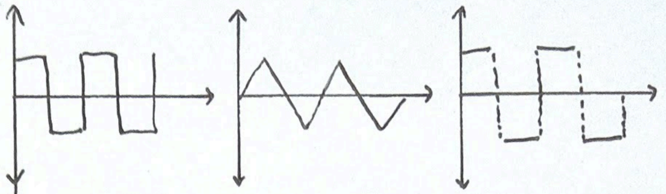
ALTERNATING CURRENT

(Current which changes its direction)



AVG. VALUE OF CURRENT

i_{avg} or $\bar{i}$ or $\langle i \rangle$

$$\boxed{i_{avg} = \frac{\int_{t_1}^{t_2} i\,dt}{t_2 - t_1}}$$

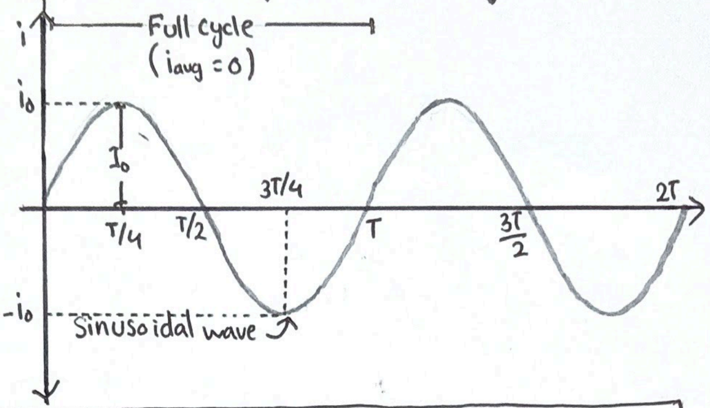
POWER

$$\boxed{P_{avg} = \frac{\int_{t_1}^{t_2} P\,dt}{t_2 - t_1}}$$

RMS VALUE

$$\boxed{i_{rms} = \sqrt{\overline{i^2}}}$$

$$\boxed{i_{rms} = i_0/\sqrt{2} = 0.707\, i_0}$$



$$\boxed{i(a.c) = i_0 \sin\omega t = i_0 \sin 2\pi f t = i_0 \sin \frac{2\pi}{T} t}$$

AMPLITUDE

Max or peak value
Denoted by 'i_0'

TIME PERIOD

$$\boxed{T = \frac{1}{f} = \frac{2\pi}{\omega}}$$

Unit : seconds

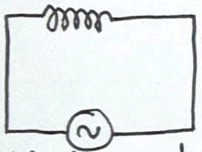
FREQUENCY

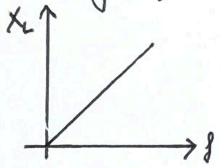
$$\boxed{f = \frac{1}{T} = \frac{\omega}{2\pi}}$$

Unit : Hertz (Hz)

(1) PURE INDUCTIVE CIRCUIT

(Current trails EMF by 90°)





$V = V_0 \sin\omega t$

$i = -i_0 \cos\omega t = i_0 \sin(\omega t - \pi/2)$

Phase diff. $= \phi = \pi/2$

Inductive Reactance (X_L) :

$\boxed{X_L = \omega L}$

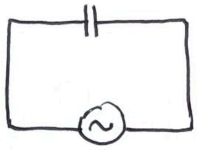
$\boxed{X_L = 2\pi f L}$

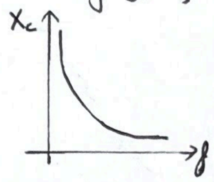
Unit : Ω (ohm)

$\boxed{i_0 = \frac{V_0}{X_L}}$ $\boxed{i_{rms} = \frac{V_{rms}}{X_L}}$

(2) PURE CAPACITIVE CIRCUIT

(Current leads EMF by 90°)





$V = V_0 \sin\omega t$

$i = i_0 \cos\omega t = i_0 \sin(\omega t + \pi/2)$

Phase diff. $= \phi = \pi/2$

Capacitive reactance (X_C) :

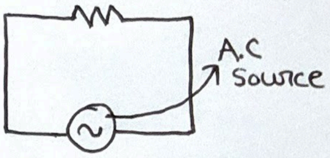
$\boxed{X_C = 1/\omega C}$

$\boxed{X_C = 1/2\pi f C}$

Unit : Ω (ohm)

$\boxed{i_0 = \frac{V_0}{X_C}}$ $\boxed{i_{rms} = \frac{V_{rms}}{X_C}}$

(3) PURE RESISTIVE



$V = V_0 \sin\omega t$

$i = i_0 \sin\omega t$

$\boxed{i_0 = \frac{V_0}{R}}$

Phase diff. $= \phi = \omega T - \omega T$
$= \underline{\underline{0}}$

(4) SERIES L-R CIRCUIT

(Current trails Voltage by ϕ)

$\boxed{V_0 = i_0 \sqrt{R^2 + X_L^2}}$

$\boxed{\text{Impedence, } Z = \sqrt{R^2 + X_L^2}}$

Unit : Ω (ohm)

$\therefore \boxed{i_0 = \frac{V_0}{Z}}$ or, $\boxed{V_0 = i_0 Z}$

$\boxed{\tan\phi = \frac{X_L}{R}}$

(5) SERIES R-C CIRCUIT

(Current leads Voltage by ϕ)

$\boxed{V_0 = i_0 \sqrt{R^2 + X_C^2}}$

$\boxed{\text{Impedence, } Z = \sqrt{R^2 + X_C^2}}$

Unit : Ω (ohm)

$\therefore \boxed{i_0 = \frac{V_0}{Z}}$ or, $\boxed{V_0 = i_0 Z}$

$\boxed{\tan\phi = \frac{X_C}{R}}$

⑥ POWER

$$\boxed{P_{inst.}(t) = V_0 i_0 \sin\omega t \sin(\omega t + \phi)}$$

$$\boxed{P_{avg.} = V_{rms} i_{rms} \cos\phi}$$

$$\boxed{\text{Power factor } (\cos\phi) = R/Z}$$

Unit : Watt

where,
ϕ = Phase diff.
$\cos\phi$ = Power factor

$$\boxed{\text{Wattless current} = i_{rms} \sin\phi}$$
(Powerless current)

Power factor for different circuits

CIRCUIT	ϕ	POWER FACTOR ($\cos\phi$)
1. Pure resistive	0°	$\cos 0^\circ = 1$ $P_{avg} = V_{rms} i_{rms}$
2. Pure inductive	90°	$\cos 90^\circ = 0$, $P_{avg} = 0$
3. Pure capacitive	90°	$\cos 90^\circ = 0$, $P_{avg} = 0$
4. L-R	same value	$\cos\phi = R/Z$ $P_{avg} = V_{rms} i_{rms} (R/Z)$
5. R-C	same value	$\cos\phi = R/Z$ $P_{avg} = V_{rms} i_{rms} (R/Z)$

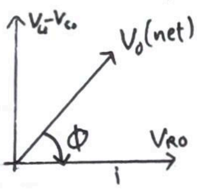
⑦ SERIES LCR CIRCUIT

CASE I

(When $V_{L0} > V_{C0}$ or, $X_L > X_C$)

$$\boxed{V = V_0 \sin\omega t}$$

$$\boxed{i = i_0 \sin(\omega t - \phi)}$$



$$\boxed{V_0 = i_0 \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$$

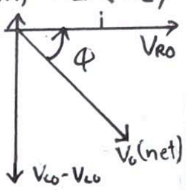
$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

CASE II

(When $V_{L0} < V_{C0}$ or, $X_L < X_C$)

$$\boxed{V = V_0 \sin\omega t}$$

$$\boxed{i = i_0 \sin(\omega t + \phi)}$$



$$\boxed{V_0 = i_0 \sqrt{R^2 + (X_C - X_L)^2}}$$

$$\boxed{Z = \sqrt{R^2 + (X_C - X_L)^2}}$$

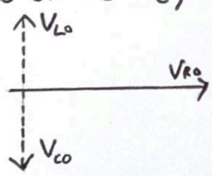
$$\boxed{\tan\phi = \frac{X_C - X_L}{R}}$$

CASE III

(When $V_{L0} = V_{C0}$ or $X_L = X_C$)

$$\boxed{V = V_0 \sin\omega t}$$

$$\boxed{i = i_0 \sin\omega t}$$



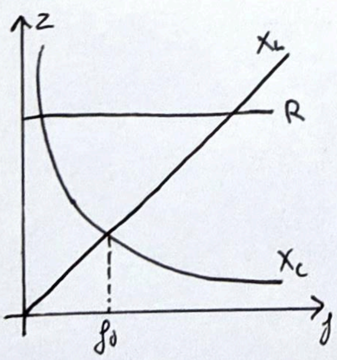
$$\boxed{V_R = V_{R0} \sin\omega t}$$

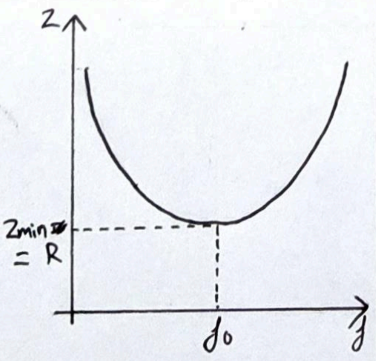
$$\boxed{V_L = V_{L0} \sin\left(\omega t - \frac{\pi}{2}\right)}$$

$$\boxed{V_C = V_{C0} \sin\left(\omega t - \frac{\pi}{2}\right)}$$

$$\boxed{Z = R} \quad \boxed{\phi = 0^\circ}$$

GRAPH FOR R, X_L, X_C and Z vs. f

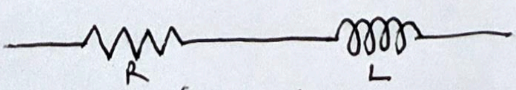




⑧ RESONANCE

- Current is maximum, $\boxed{(i_0)_{max} = E_0/R}$
- Impedence → minimum $\boxed{(Z)_{min} = R}$
- Phase diff (ϕ) = 0
- $\boxed{X_L = X_C}$
- Resonating frequency, $\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$
- Angular frequency, $\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$
- Current
  → independent of values of L and C
  → depends only on Resistance
- $\cos\phi = 1$, Power consumed → maximum

⑨ CHOKE COIL (Series ~~LCR~~ L-R circuit with Large 'L' and Small 'R')



(R < L)

Power factor, $\cos\phi = \frac{R}{Z}$ ← small R, ← ~~tg~~ large Z

$P_{avg} = V_{rms} i_{rms} \cos\phi$
→ nearly zero

⑩ TUNING OF A RADIO RECIEVER (The radio is a series LCR CIRCUIT.)

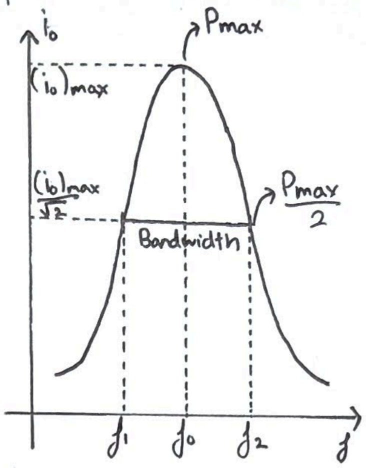
Small bandwidth → sharp tuning (sharp resonance)

$$\boxed{\text{Bandwidth} = 2\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}}$$

Quality factor (Q-factor)

$$\boxed{\text{Q-factor} = \frac{\text{Resonating frequency}}{\text{Bandwidth}}}$$

$$\boxed{Q = \frac{\omega_0}{\omega_2-\omega_1} = \frac{1}{R}\sqrt{\frac{L}{C}}}$$



Half-power frequencies (frequencies corresponding to $\frac{(i_0)_{max}}{\sqrt{2}}$)

$$\boxed{\omega_1 = \omega_0 - \frac{R}{2L}} \quad \boxed{\omega_2 = \omega_0 + \frac{R}{2L}}$$

Power output of circuit is maximum for $(i_0)_{max}$

$$\boxed{P_{max} = \left(\frac{(i_0)_{max}}{\sqrt{2}}\right)^2 R = \frac{(i_0)^2_{max} R}{2}}$$

⑪ L-C OSCILLATIONS

Energy oscillates between Electric field of the capacitor and Magnetic field of the inductor.

If there is no loss of energy (Ideal situation)

$$\boxed{U_{Total} = \frac{q^2}{2C} + \frac{1}{2}Li^2}$$

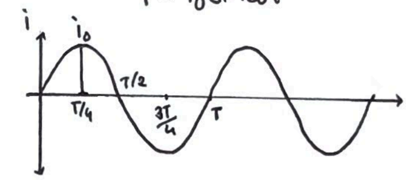
The Charge on each plate of Capacitor Oscillates between

$$\begin{array}{ccccccccc} +q_0 & \longrightarrow & +q & \longrightarrow & 0 & \longrightarrow & -q & \longrightarrow & -q_0 \\ \uparrow & & & & & & & & \downarrow \\ +q & & \longleftarrow & & 0 & & \longleftarrow & & -q \end{array}$$

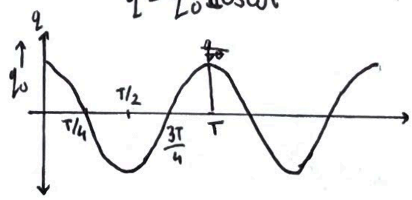
General Formula =

$$\boxed{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0}$$

$i = i_0 \sin\omega t$



$q = q_0 \cos\omega t$



Oscillating Electrical Energy

$$\boxed{U_E = \frac{q^2}{2C} = \frac{q_0^2}{2C}\cos^2\omega t}$$

Oscillating Magnetic Energy

$$\boxed{U_B = \frac{1}{2}Li^2 = \frac{Li_0^2}{2}\sin^2\omega t}$$

⑫ TRANSFORMER

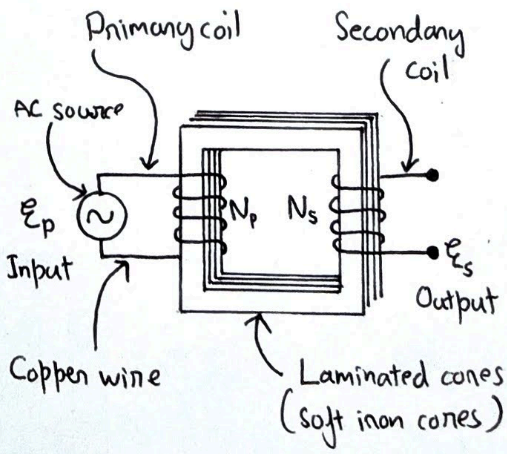
Voltage input =

$$\boxed{\mathcal{E}_p = -N_p\frac{d\phi}{dt}}$$

Voltage output =

$$\boxed{\mathcal{E}_s = -N_s\frac{d\phi}{dt}}$$

$$\boxed{\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s} = k}$$



For an ideal transformer, (0% energy loss and 100% energy transferred)

$$\boxed{\frac{\mathcal{E}_s}{\mathcal{E}_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s} = K}$$

Transfer ratio