

Mumps4py: Python interface for the MUMPS solver

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1 Introduction

This report describes seven Python scripts demonstrating the MUMPS solver [2, 1] via the `mumps4py` <https://github.com/imadki/mumps4py> interface. Each example highlights different functionalities, such as centralized and distributed matrix assembly, sparse RHS solving, and elemental matrix formats. The scripts use MPI for parallelism and NumPy for numerical operations. Full Python code with added comments is included for clarity.

2 Example 1: Centralized COO Matrix with Dense RHS

This example solves a linear system $Ax = b$ using a centralized sparse matrix in COO format with a dense RHS.

2.1 Matrix Data

- `irn` (0-based): `[0, 1, 2, 3]`
- `jcn` (0-based): `[0, 1, 2, 3]`
- `a`: `[1.0, 2.0, 3.0, 4.0]`

2.2 Code

```
1  #!/usr/bin/env python3
2  # -*- coding: utf-8 -*-
3  """
4  Created on Thu Feb 27 14:08:37 2025
5
6  @author: kissami
7  """
8  import numpy as np
9  from mpi4py import MPI
10 from scipy.sparse import coo_matrix
11 from mumps4py.mumps_solver import MumpsSolver
```

```

12
13 # Get MPI rank and size for parallel execution
14 rank = MPI.COMM_WORLD.Get_rank()
15 size = MPI.COMM_WORLD.Get_size()
16
17 # Set system type to double precision
18 system = "double"
19 dtype = np.float32 if system == "single" else np.float64
20 if system in ["complex64", "complex128"]:
21     dtype = np.complex64 if system == "complex64" else np.complex128
22
23 # Initialize MUMPS solver with double precision and no verbose output
24 solver = MumpsSolver(verbose=False, system=system)
25
26 # Define a 4x4 diagonal matrix in COO format
27 A = coo_matrix(([1.0, 2.0, 3.0, 4.0], ([0, 1, 2, 3], [0, 1, 2, 3])), shape=(4, 4))
28
29 # Set the centralized COO matrix in the solver
30 solver.set_coo_centralized(A)
31
32 # Define the right-hand side vector
33 rhs = np.array([1.0, 2.0, 3.0, 4.0])
34 solver.set_rhs_centralized(rhs)
35
36 # Perform analysis phase (symbolic factorization)
37 solver.analyze()
38 # Perform numerical factorization
39 solver.factorize()
40 # Solve the system, overwriting rhs with the solution
41 solver.solve()
42
43 # Print the solution on all ranks
44 print("Solution is:", rhs)

```

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Expected solution: $x = [1, 1, 1, 1]$.

3 Example 2: Distributed COO Matrix with Dense RHS

This example uses a distributed COO matrix with a dense RHS.

3.1 Matrix Data

- `irn` (0-based): $[0, 1, 3, 4, 1, 0, 4, 2, 1, 2, 0, 2]$
- `jcn` (0-based): $[1, 2, 2, 4, 0, 0, 1, 3, 4, 1, 2, 2]$
- `a`: $[3.0, -3.0, 2.0, 1.0, 3.0, 2.0, 4.0, 2.0, 6.0, -1.0, 4.0, 1.0]$

3.2 Code

```
1  #!/usr/bin/env python3
2  # -*- coding: utf-8 -*-
3  import numpy as np
4  from mpi4py import MPI
5  from mumps4py.mumps_solver import MumpsSolver
6
7  # MPI setup
8  rank = MPI.COMM_WORLD.Get_rank()
9  size = MPI.COMM_WORLD.Get_size()
10
11 # Set double precision
12 system = "double"
13 dtype = np.float32 if system == "single" else np.float64
14 if system in ["complex64", "complex128"]:
15     dtype = np.complex64 if system == "complex64" else np.complex128
16
17 # Initialize solver
18 solver = MumpsSolver(verbose=False, system=system)
19
20 # Define matrix size on rank 0 only
21 if rank == 0:
22     n = 5
23 else:
24     n = None
25
26 # Define COO matrix data (0-based indices)
27 irn = np.array([0,1,3,4,1,0,4,2,1,2,0,2], dtype=np.int32)
28 jcn = np.array([1,2,2,4,0,0,1,3,4,1,2,2], dtype=np.int32)
29 a = np.array([3.0,-3.0,2.0,1.0,3.0,2.0,4.0,2.0,6.0,-1.0,4.0,1.0], dtype=dtype)
30 b = np.array([20.0,24.0,9.0,6.0,13.0],[20.0,24.0,9.0,6.0,13.0], dtype=dtype)
31
32 # Split matrix entries across MPI processes
33 indices = np.arange(len(irn))
34 split_indices = np.array_split(indices, size)
35 local_indices = split_indices[rank]
36
37 # Extract local portions of the matrix
38 local_irn = irn[local_indices]
39 local_jcn = jcn[local_indices]
40 local_a = a[local_indices]
41
42 # Set distributed matrix (convert to 1-based indices for MUMPS)
43 solver.set_rcd_distributed(local_irn+1, local_jcn+1, local_a, n)
44 solver.set_icntl(18,3) # Enable distributed assembly
45
46 # Analyze and factorize the matrix
47 solver.analyze()
48 solver.factorize()
49
50 # Set RHS on rank 0 only
51 if MPI.COMM_WORLD.Get_rank() == 0:
52     solver.set_rhs_centralized(b)
53
54 # Solve the system
55 solver.solve()
56 if rank == 0:
57     print("Solution 3", b)
```

$$A = \begin{bmatrix} 2 & 3 & 4 & 0 & 0 \\ 3 & -1 & -3 & 0 & 6 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4 Example 3: Distributed COO Matrix with Distributed Solution

This example extends Example 2 with a distributed solution.

4.1 Matrix Data

- `irn`: [0, 1, 3, 4, 1, 0, 4, 2, 1, 2, 0, 2]
- `jcn`: [1, 2, 2, 4, 0, 0, 1, 3, 4, 1, 2, 2]
- `a`: [3.0, -3.0, 2.0, 1.0, 3.0, 2.0, 4.0, 2.0, 6.0, -1.0, 4.0, 1.0]

4.2 Code

```

1  #!/usr/bin/env python3
2  # -*- coding: utf-8 -*-
3  """
4  Created on Thu Feb 27 14:23:15 2025
5
6  @author: kissami
7  """
8  from mumps4py.mumps_solver import MumpsSolver
9  import numpy as np
10 from mpi4py import MPI
11
12 # MPI initialization
13 comm = MPI.COMM_WORLD
14 rank = comm.Get_rank()
15 size = comm.Get_size()
16
17 # Set double precision
18 system = "double"
19 dtype = np.float32 if system == "single" else np.float64
20 if system in ["complex64", "complex128"]:
21     dtype = np.complex64 if system == "complex64" else np.complex128
22
23 # Initialize solver
24 solver = MumpsSolver(verbose=False, system=system)
25
26 # Define matrix size
27 n = 5
28 # Define COO matrix (1-based indices for MUMPS)
29 irn = np.array([0, 1, 3, 4, 1, 0, 4, 2, 1, 2, 0, 2], dtype=np.int32) + 1
30 jcn = np.array([1, 2, 2, 4, 0, 0, 1, 3, 4, 1, 2, 2], dtype=np.int32) + 1
31 a = np.array([3.0, -3.0, 2.0, 1.0, 3.0, 2.0, 4.0, 2.0, 6.0, -1.0, 4.0, 1.0], dtype=dtype)
32 b = np.array([20.0, 24.0, 9.0, 6.0, 13.0], dtype=dtype)
33
34 # Distribute matrix entries across processes

```

```

35 indices =np.arange(len(irn))
36 split_indices =np.array_split(indices, size)
37 local_indices =split_indices[rank]
38
39 # Local matrix portions
40 local_irn =irn[local_indices]
41 local_jcn =jcn[local_indices]
42 local_a =a[local_indices]
43
44 # Configure solver for distributed assembly and solution
45 solver.set_icntl(18, 3) # Distributed matrix input
46 solver.set_rcd_distributed(local_irn, local_jcn, local_a, n)
47 solver.set_icntl(21, 1) # Distributed solution output
48
49 # Analyze and factorize
50 solver.analyze()
51 solver.factorize()
52
53 # Set RHS on rank 0
54 if rank ==0:
55     solver.set_rhs_centralized(b)
56
57 # Enable and compute distributed solution
58 solver.enable_distributed_solution(1)
59 solver.solve()
60
61 # Retrieve distributed solution
62 shape =b.shape
63 dtype =np.float64
64 distributed_solution =solver.pointer_to_numpy(solver.struct.sol_loc, dtype, shape)
65 print("Distributed solution :", distributed_solution)
66
67 # Get solution indices
68 isol_indices =solver.pointer_to_numpy(solver.struct.isol_loc, np.int32, b.shape)
69 print("Solution indices :", isol_indices)
70
71 # Reconstruct full solution
72 final_solution =np.zeros(n, dtype=dtype)
73 final_solution[isol_indices -1] =distributed_solution # Adjust for 0-based indexing
74 print("Final solution :", final_solution)

```

5 Example 4: Elemental Matrix Format

This example uses an elemental matrix format for a complex-valued system.

5.1 Matrix Data

- eltptr (1-based): [1, 4, 7]
- eltvar (1-based): [1, 2, 3, 3, 4, 5]
- a_elt: [-1, 2, 1, 2, 1, 1, 3, 1, 1, 2, 1, 3, -1, 2, 2, 3, -1, 1]

5.2 Code

```

1 #!/usr/bin/env python3
2 # -*- coding: utf-8 -*-
3 import numpy as np

```

```

4 from mpi4py import MPI
5 from mumps4py.mumps_solver import MumpsSolver
6
7 # MPI setup
8 rank = MPI.COMM_WORLD.Get_rank()
9 size = MPI.COMM_WORLD.Get_size()
10
11 # Set complex128 precision
12 system = "complex128"
13 dtype = np.float32 if system == "single" else np.float64
14 if system in ["complex64", "complex128"]:
15     dtype = np.complex64 if system == "complex64" else np.complex128
16
17 # Initialize solver
18 solver = MumpsSolver(verbose=False, system=system)
19
20 # Define matrix parameters
21 n = 5 # Matrix order
22 nelt = 2 # Number of elements
23
24 # Elemental matrix data (1-based)
25 eltptr = np.array([1, 4, 7], dtype=np.int32) # Element pointers
26 eltvar = np.array([1, 2, 3, 3, 4, 5], dtype=np.int32) # Variable indices
27 a_elt = np.array([-1, 2, 1, 2, 1, 1, 3, 1, 1, 2, 1, 3, -1, 2, 2, 3, -1, 1], dtype=dtype) # Values
28 bb = np.array([1, 20, 3, 4, 5], dtype=dtype) # RHS
29 rhs = bb.copy() # Copy for verification
30
31 # Configure solver for elemental format
32 solver.set_icntl(5, 1) # Use elemental matrix format
33 solver.set_icntl(18, 0) # Centralized assembly
34
35 # Set matrix and RHS
36 solver.set_elemental_matrix(eltptr, eltvar, a_elt, n, nelt)
37 solver.set_rhs_centralized(bb)
38
39 # Solve the system
40 solver.analyze()
41 solver.factorize()
42 solver.solve()
43
44 # Print solution on rank 0
45 if rank == 0:
46     print("Solution:", bb)
47
48 # Define A for verification (not in original code)
49 A = np.array([[-1, 2, 3, 0, 0], [2, 1, 1, 0, 0], [1, 1, 3, -1, 3], [0, 0, 1, 2, -1], [0, 0, 3, 2, 1]], dtype=dtype)
50 print("Check solution:", A.dot(bb) - rhs)

```

$$A = \begin{bmatrix} -1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & 3 & -1 & 3 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix}$$

6 Example 5: Centralized COO Matrix with Manual Job Calls and Reused Factorization

This example uses manual MUMPS job calls to solve a 5×5 system and illustrates that if the matrix structure (irn, jcn) and values (a) remain unchanged, the factorization step can

be reused for multiple RHS vectors, avoiding redundant computation.

6.1 Matrix Data

- `irn`: [0, 1, 3, 4, 1, 0, 4, 2, 1, 2, 0, 2]
- `jcn`: [1, 2, 2, 4, 0, 0, 1, 3, 4, 1, 2, 2]
- `a`: [3.0, -3.0, 2.0, 1.0, 3.0, 2.0, 4.0, 2.0, 6.0, -1.0, 4.0, 1.0]

6.2 Code

```
1  #!/usr/bin/env python3
2  # -*- coding: utf-8 -*-
3  import numpy as np
4  from mpi4py import MPI
5  from mumps4py.mumps_solver import MumpsSolver
6
7  # MPI setup
8  rank = MPI.COMM_WORLD.Get_rank()
9  size = MPI.COMM_WORLD.Get_size()
10
11 # Set double precision
12 system = "double"
13 dtype = np.float32 if system == "single" else np.float64
14 if system in ["complex64", "complex128"]:
15     dtype = np.complex64 if system == "complex64" else np.complex128
16
17 # Initialize solver
18 solver = MumpsSolver(verbose=False, system=system)
19
20 # Define matrix and two different RHS vectors
21 n = 5
22 irn = np.array([0,1,3,4,1,0,4,2,1,2,0,2], dtype=np.int32)
23 jcn = np.array([1,2,2,4,0,0,1,3,4,1,2,2], dtype=np.int32)
24 a = np.array([3.0,-3.0,2.0,1.0,3.0,2.0,4.0,2.0,6.0,-1.0,4.0,1.0], dtype=dtype)
25 b1 = np.array([20.0, 24.0, 9.0, 6.0, 13.0], dtype=dtype) # First RHS
26 b2 = np.array([1.0, 2.0, 3.0, 4.0, 5.0], dtype=dtype) # Second RHS
27
28 n = len(b1) # Matrix size from RHS
29
30 # Start timing
31 ts = MPI.Wtime()
32
33 # Set centralized matrix (1-based indices)
34 solver.set_rcd_centralized(irn+1, jcn+1, a, n)
35 solver._mumps_call(job=1) # Analysis phase (symbolic factorization)
36
37 # Factorize once (numerical factorization)
38 solver._mumps_call(job=2) # Factorization phase, done only once
39
40 # Solve for first RHS
41 rhs1 = b1.copy() # Copy to preserve original
42 solver.set_rhs_centralized(rhs1)
43 solver._mumps_call(3) # Solve phase
44 if rank == 0:
45     print("Solution for b1:", rhs1)
46
47 # Solve for second RHS reusing factorization
48 rhs2 = b2.copy() # Copy to preserve original
49 solver.set_rhs_centralized(rhs2)
50 solver._mumps_call(3) # Solve phase, no need to re-factorize
```

```

51 if rank ==0:
52     print("Solution for b2:", rhs2)
53
54 # Print total CPU time
55 if rank ==0:
56     print("CPU time is ", MPI.Wtime() -ts)

```

6.3 Explanation

The matrix A is defined once, and its factorization (`job=2`) is performed only once after analysis (`job=1`). Two different RHS vectors, $b_1 = [20, 24, 9, 6, 13]$ and $b_2 = [1, 2, 3, 4, 5]$, are solved using the same factorization by calling only the solve phase (`job=3`) for each. This demonstrates that as long as `irn`, `jcn`, and `a` do not change, re-factorization is unnecessary, optimizing performance.

7 Example 6: Updating Centralized Matrix Values

This example updates matrix values without re-analysis.

7.1 Matrix Data

- `irn`: $[0, 1, 3, 4, 1, 0, 4, 2, 1, 2, 0, 2]$
- `jcn`: $[1, 2, 2, 4, 0, 0, 1, 3, 4, 1, 2, 2]$
- Initial `a`: $[3.0, -3.0, 2.0, 1.0, 3.0, 2.0, 4.0, 2.0, 6.0, -1.0, 4.0, 1.0]$
- Updated `a`: $[6.0, -33.0, 2.0, 1.0, 33.0, 22.0, 41.0, 2.0, 66.0, -11.0, 4.0, 1.0]$

7.2 Code

```

1  #!/usr/bin/env python3
2  # -*- coding: utf-8 -*-
3  """
4  Created on Wed Mar 5 11:23:56 2025
5
6  @author: kissami
7  """
8  from scipy.sparse import coo_matrix
9  from mumps4py.mumps_solver import MumpsSolver
10 import numpy as np
11 from mpi4py import MPI
12
13 # MPI setup
14 rank = MPI.COMM_WORLD.Get_rank()
15 size = MPI.COMM_WORLD.Get_size()
16
17 # Set double precision
18 system = "double"
19 dtype = np.float32 if system == "single" else np.float64
20 if system in ["complex64", "complex128"]:
21     dtype = np.complex64 if system == "complex64" else np.complex128
22

```



```

23 # Initialize solver
24 solver =MumpsSolver(verbose=False, system=system)
25
26 # Define initial matrix and RHS
27 n = 5
28 irn = np.array([0,1,3,4,1,0,4,2,1,2,0,2], dtype=np.int32)
29 jcn = np.array([1,2,2,4,0,0,1,3,4,1,2,2], dtype=np.int32)
30 a = np.array([3.0,-3.0,2.0,1.0,3.0,2.0,4.0,2.0,6.0,-1.0,4.0,1.0], dtype=dtype)
31 b = np.array([20.0,24.0,9.0,6.0,13.0], dtype=dtype)
32
33 # Set initial matrix (1-based)
34 solver.set_rcd_centralized(irn+1, jcn+1, a, n)
35 solver._mumps_call(job=1) # Analyze structure
36
37 # Solve with initial values
38 rhs = b.copy()
39 solver._mumps_call(job=2) # Factorize
40 solver.set_rhs_centralized(rhs)
41 solver._mumps_call(3) # Solve
42 print("Solution:", rhs)
43
44 # Update matrix values
45 a = np.array([6.0,-33.0,2.0,1.0,33.0,22.0,41.0,2.0,66.0,-11.0,4.0,1.0], dtype=dtype)
46 solver.set_data_centralized(a, n) # Update values only
47
48 # Solve with updated values
49 rhs = b.copy()
50 solver._mumps_call(job=2) # Re-factorize
51 solver.set_rhs_centralized(rhs)
52 solver._mumps_call(3) # Re-solve
53 print("Solution:", rhs)
54
55 # Verify solution
56 n = 5
57 A = coo_matrix((a, (irn, jcn)), shape=(n, n))
58 print("check the solution:", A.dot(rhs), b)

```

8 Example 7: Sparse RHS with Centralized COO Matrix

This example solves a system with a sparse RHS.

8.1 Matrix Data

- `irn`: [0, 0, 1, 1, 1, 2, 2, 3, 3]
- `jcn`: [0, 1, 0, 1, 2, 1, 2, 2, 3]
- `a`: [4.0, -1.0, -1.0, 4.0, -1.0, -1.0, 4.0, -1.0, 3.0]

8.2 Sparse RHS Data

- `rhs_values`: [1.1, 2.2, 3.1, 4.1, 3.2]
- `rhs_row_indices` (1-based): [1, 3, 4, 2, 3]
- `rhs_col_ptr` (1-based): [1, 4, 6]

8.3 Code

```
1  #!/usr/bin/env python3
2  # -*- coding: utf-8 -*-
3  """
4  Created on Thu Feb 27 14:08:37 2025
5
6  @author: kissami
7  """
8  import numpy as np
9  from mpi4py import MPI
10 from mumps4py.mumps_solver import MumpsSolver
11
12 # MPI setup
13 rank = MPI.COMM_WORLD.Get_rank()
14 size = MPI.COMM_WORLD.Get_size()
15
16 # Set double precision
17 system = "double"
18 dtype = np.float32 if system == "single" else np.float64
19 if system in ["complex64", "complex128"]:
20 dtype = np.complex64 if system == "complex64" else np.complex128
21
22 # Initialize solver
23 solver = MumpsSolver(verbose=False, system=system)
24
25 # Define 4x4 matrix
26 n = 4
27 nnz = 9
28 irn = np.array([0, 0, 1, 1, 1, 2, 2, 3, 3], dtype=np.int32)
29 jcn = np.array([0, 1, 0, 1, 2, 1, 2, 2, 3], dtype=np.int32)
30 a = np.array([4.0, -1.0, -1.0, 4.0, -1.0, -1.0, 4.0, -1.0, 3.0], dtype=dtype)
31
32 # Set centralized matrix (1-based)
33 solver.set_rcd_centralized(irn+1, jcn+1, a, n)
34 solver.analyze()
35 solver.factorize()
36
37 # Configure for sparse RHS
38 solver.set_icntl(20, 1) # Enable sparse RHS
39
40 # Define sparse RHS parameters
41 nz_rhs = 5 # Number of non-zeros
42 nrhs = 2 # Number of RHS columns
43 rhs_values = np.array([1.1, 2.2, 3.1, 4.1, 3.2], dtype=dtype) # Non-zero values
44 rhs_row_indices = np.array([1, 3, 4, 2, 3], dtype=np.int32) # Row indices (1-based)
45 rhs_col_ptr = np.array([1, 4, 6], dtype=np.int32) # Column pointers (1-based)
46
47 # Initialize dense RHS storage on rank 0
48 if rank == 0:
49 rhs = np.zeros((2,4))
50 solver.set_rhs_centralized(rhs)
51
52 # Set sparse RHS
53 solver.set_rhs_sparse(rhs_values, rhs_row_indices, rhs_col_ptr, nrhs)
54 solver.solve()
55
56 # Print solution
57 print("Solution is :", rhs)
```

$$A = \begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 3 \end{bmatrix}$$

9 Benchmarks: Python vs C

In this section, we compare the runtime of Python and C (through Cython wrapper) using MUMPS-5.3.5 (pt-scotch, without OpenMP) for various matrix sizes obtained from Finite Volume discretization using Manapy [3]. We attempt to determine if the Cython-based interface imposes any extra computational cost over the pure C implementation.

Figures 1 and 3 show the performance of the 2D and 3D problems, respectively. The x-axis is the configuration (matrix size and number of processes), and the y-axis (log scale) is the execution time in seconds for analysis, factorization, and solution phases.

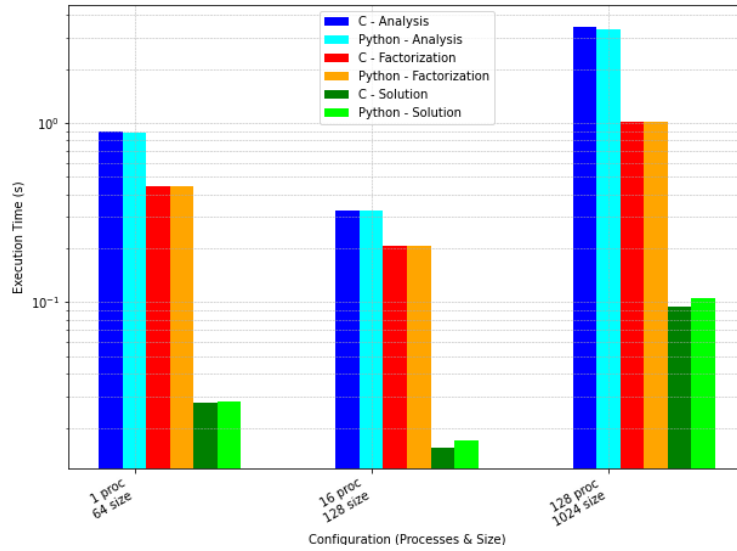


Figure 1: Timing (s) using MUMPS for all steps through C vs Python for 2D matrices

Key Observations:

- All configurations tested have Python (through Cython) performing as well as C.
- There is no extra CPU cost of calling the Cython wrapper, i.e., the Python function calls are just as efficient as direct C function calls. This validates that Cython removes Python overhead by compiling Python code to C and providing almost native call speed in calling MUMPS.

Figures 2 and 4 illustrate the speedup achieved when using the Python-based MUMPS wrapper compared to the native C implementation for 2D and 3D matrices, respectively.

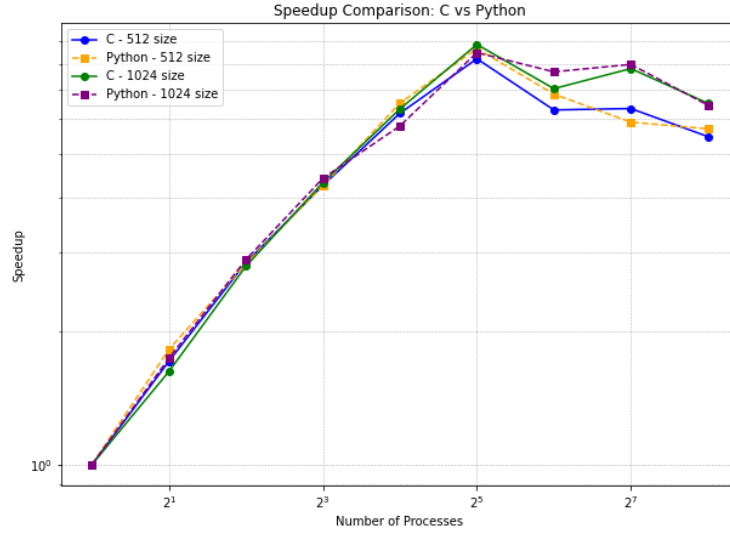


Figure 2: Speedup using MUMPS for all steps through C vs Python for 2D matrices

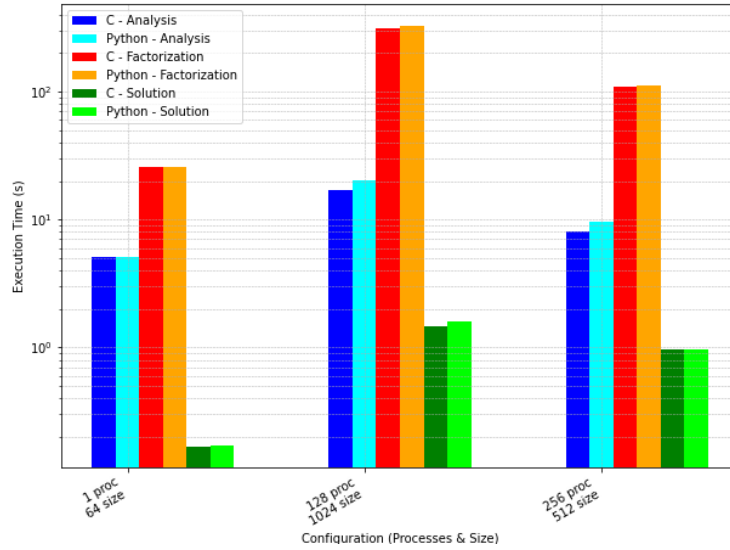


Figure 3: Timing (s) using MUMPS for all steps through C vs Python for 3D matrices

10 Conclusion

This report has illustrated the application of MUMPS through the mumps4py interface to solve sparse linear systems in parallel computing. Various examples have been presented, spanning various matrix assembly methods (centralized and distributed), right-hand side (RHS) management (dense and sparse), and factor reuse for enhancing performance.

The benchmarking outcomes indicate that the Python MUMPS interface (through Cython)

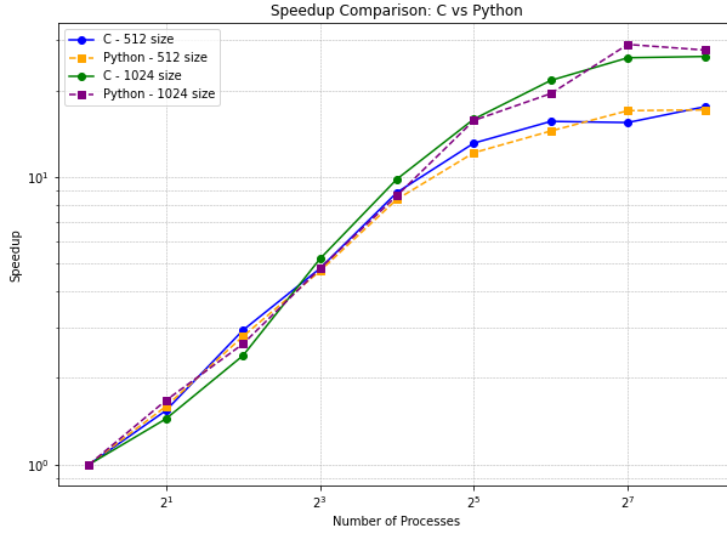


Figure 4: Speedup using MUMPS for all steps through C vs Python for 3D matrices

is equally fast as the native C code with little or no overhead. This confirms the effectiveness of the Python wrapper and its suitability for high-performance computing (HPC) applications. The tests also demonstrate the scalability of MUMPS along with its efficiency in handling big sparse matrices.

In summary, mumps4py offers an easy-to-use and flexible interface to call MUMPS from Python and is a great software package for scientists and engineers dealing with large-scale scientific computations. GPU acceleration and more optimizations on heterogeneous computing architectures can be investigated in future work.

References

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