Proceedings of the 34th Chinese Control Conference
July 28-30, 2015, Hangzhou, China

A multi-threshold image segmentation approach using state transition algorithm

HAN Jie, ZHOU Xiaojun*, YANG Chunhua, GUI Weihua
School of Information Science and Engineering, Central South University, Changsha 410083, P. R. China
E-mail: michael.x.zhou@csu.edu.cn

Abstract: Thresholding is an important approach for image segmentation and analysis. In this study, the combination of normal distribution functions is used to fit the normalized histogram of the original image since each normal distribution function represents a pixel class. On the other hand, state transition algorithm (STA) is a promising method for solving complex optimization problems. By transforming the fitting problem into an optimization problem, the STA is used to select the optimal parameters in the fitting function. Experimental results of several images show that the proposed approach is efficient and effective for multilevel thresholding problems. Comparisons with OTSU, PSO and GA also demonstrate that STA not only outperforms computationally efficient but also provides competitive thresholding results.

Key Words: Image segmentation; State Transition Algorithm; Multilevel thresholding

1 Introduction

Image segmentation is the process of subdividing a digital image into its constituent regions. The level to which the subdivision is carried depends on the problem being solved. The accuracy of image segmentation determines the eventual success of computerized analysis procedures [1]. Successful image segmentation can simplify the representation of an image into something that is more meaningful and easier to analyze. Commonly, intensity values belonging to a region are substantially the same whereas the intensity values belonging to adjacent regions are significantly different. Among all possible techniques, thresholding enjoys a central position in applications of image segmentation. It can be used in several classics such as document image analysis, image enhancement [2, 3], characters extraction, map analysis [4], industrial inspection, and medical image processing.

In general, thresholding segmentation techniques are classified into two main categories: single thresholding and multiple thresholding. The difference between single and multiple thresholding is the number of the threshold levels. In the former technique, a single threshold level is chosen to divide the entire image into two classes: one representing the background and another one standing for the object. The multiple thresholding technique uses multiple threshold levels to subdivide distinct objects in an image, and each threshold level should be adapted to the respective segmentation [5].

Thresholding segmentation problem has been studied by many researchers for several years, and a variety of thresholding methods have been proposed, including traditional techniques [6] and intelligent techniques [7]. Thresholding approaches can be applied to segmentation, but it may create some difficulties. On the one hand, there is no rules to distinguish one object from others if their intensity values are close. On the other hand, to find multiple threshold levels may not only cost too much computation time, but also lead to slow convergence results.

In order to simplifying the process of segmentation, several probability density functions of a normal distribution can be combined to fit the probability distribution function of the original image. The associated parameters in the combination function are unknown and the estimation of parameters is typically assumed to be a non-linear optimization problem. Considering the parameters obtained by state transition algorithm (STA) [8–12] can effectively make the combination function adapt to the normalized histogram, a multi-threshold image segmentation approach using STA is proposed in this paper.

Inspired by the concepts of state and state transition, a new heuristic search algorithm named STA is proposed, which consists of four essential state transformation operators: rotation, translation, expansion and axesion. The performance of STA has been compared to other heuristic algorithms such as Genetic Algorithms (GA), Differential Evolution (DE) and Particle Swarm Optimization (PSO). The results showed that STA is a promising algorithm due to its better global search capability and convergence property by contrast with other methods for several optimization problems. Such features have propelled the use of STA to solve different sorts of problems within different fields.

The remainder of the paper is organized as follows. Section 2 presents the normal distribution combination function to fit the normalized histogram of the original image. Section 3 discusses on the STA while Section 4 provides the processing of threshold determination. Experimental results for the proposed method are demonstrated in Section 5. Finally, the paper is summarized in Section 6.

2 Normal distribution fitting

The histogram of a digital image with L total possible intensity levels in the range $[0, L-1]$ is defined as the discrete function

$$h(r_j) = n_j \tag{1}$$

where, r_j is the jth intensity level in the interval $[0, L-1]$ and n_j is the number of pixels in the image whose intensity level is r_j. Often, it is useful to work with normalized histograms, which is obtained simply by dividing all elements of $h(r_j)$ by the total number of pixels in the image, which we denote

*Corresponding author of this paper. This work is supported by National Key Technology R&D Program of China (Grant No. 2012BAF03B05), the National Science Fund for Distinguished Young Scholars of China (Grant No. 61025015), and the Scientific Research Foundation of Cental South University.

by n:

$$p(r_j) = \frac{h(r_j)}{n} = \frac{n_j}{n} \tag{2}$$

for $j = 1, 2, \ldots, L-1$. The normalized histogram can be obtained by a combination of normal distribution functions of the form:

$$N(\boldsymbol{x}) = \sum_{i=1}^{K} \phi_i \cdot n_i(\boldsymbol{x}) = \sum_{i=1}^{K} \frac{\phi_i}{\sqrt{2\pi}\sigma_i} \exp\left[-\frac{(\boldsymbol{x}-\mu_i)^2}{2\sigma_i^2}\right] \tag{3}$$

$$\sum_{i=1}^{K} \phi_i = 1 \tag{4}$$

where K is the number of thresholding levels, $n_i(\boldsymbol{x})$ is the ith normal distribution function, ϕ_i represents the priori probability of mode i, μ_i denotes the mean and σ_i^2 is the variance of the i-th part. The combination function must be fitted to the normalized histogram data, typically by the minimum mean-square error approach in order to locate the optimal thresholds. With the purpose of finding the set of parameters, $\Omega = \{\phi_i, \mu_i, \sigma_i; i = 1, 2, \ldots, K\}$, the problem can be defined as follows [1]:

$$\begin{aligned} Minimize\ E = &\frac{1}{L-1} \sum_{j=1}^{L-1} [N(\boldsymbol{x}_j) - p(\boldsymbol{x}_j)]^2 + \\ &+ \omega \cdot [(\sum_{i=1}^{K} \phi_i) - 1]^2 \end{aligned} \tag{5}$$

where E is the objective function, ω is the penalty related to the constraint $\sum_{i=1}^{K} \phi_i = 1$.

Generally speaking, the methods used for solving such optimization problems can be divided into two categories: deterministic and stochastic. The traditional deterministic algorithms are always based on the gradient informations and they are more appropriate for solving unimodal problems. Contrary to deterministic algorithms, the stochastic algorithms such as GA, DE, PSO and STA are faster and more stable. According to the performance of stochastic algorithms, the STA is adopted for the sake of finding the parameters and their relevant threshold levels.

3 State transition algorithm

In recent years, a novel optimization method named state transition algorithm (STA) is introduced to solve the problems. Based on the concepts of state transition and state space representation in control theory, a solution to optimization problems can be regarded as a state, and the process of optimization algorithms can be treated as a state transition. In general, the form of state transition can be defined as follows:

$$\begin{cases} \boldsymbol{x}_{k+1} = A_k \boldsymbol{x}_k + B_k \boldsymbol{u}_k \\ y_{k+1} = f(\boldsymbol{x}_{k+1}) \end{cases}, \tag{6}$$

where $\boldsymbol{x}_k \in \mathbb{R}^n$ is a state, corresponding to a solution of an optimization problem; A_k and B_k are state transition matrices with appropriate dimensions, which mean transformation operators; $\boldsymbol{u}_k$ is a function of $\boldsymbol{x}_k$ and history states; f is the objective function or evaluation function.

By consulting various types of space transformation for references, four special state transformation operators are designed.

(1) Rotation transformation

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \alpha \frac{1}{n\|\boldsymbol{x}_k\|_2} R_r \boldsymbol{x}_k, \tag{7}$$

where α is a positive constant, named rotation factor; $R_r \in \mathbb{R}^{n\times n}$ is a random matrix with its elements belonging to the range of [-1, 1] and $\|\cdot\|_2$ is the 2-norm of a vector.

(2) Translation transformation

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \beta R_t \frac{\boldsymbol{x}_k - \boldsymbol{x}_{k-1}}{\|\boldsymbol{x}_k - \boldsymbol{x}_{k-1}\|_2}, \tag{8}$$

where β is a positive constant, named translation factor; $R_t \in \mathbb{R}$ is a random variable with its elements belonging to the range of [0,1].

(3) Expansion transformation

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \gamma R_e \boldsymbol{x}_k, \tag{9}$$

where γ is a positive constant, named expansion factor; $R_e \in \mathbb{R}^{n\times n}$ is a random diagonal matrix with its elements obeying the Gaussian distribution.

(4) Axesion transformation

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \delta R_a \boldsymbol{x}_k \tag{10}$$

where δ is a positive constant, named axesion factor; $R_a \in \mathbb{R}^{n\times n}$ is a random diagonal matrix with its elements obeying the Gaussian distribution and only one random position having nonzero value.

The procedure of the continuous STA can be outlined in the following pseudocode.

repeat
    if $\alpha < \alpha_{\min}$ **then**
        $\alpha \leftarrow \alpha_{\max}$
    end if
    Best $\leftarrow$ expansion(funfcn,Best,SE,β,γ)
    Best $\leftarrow$ rotation(funfcn,Best,SE,α,β)
    Best $\leftarrow$ axesion(funfcn,Best,SE,β,δ)
    $\alpha \leftarrow \frac{\alpha}{fc}$
until the specified termination criterion is met

where *SE* is called search enforcement, representing the times of transformation by a certain operator, and the "greedy criterion" is used to adopt a new best solution. By the way, translation operator is only executed when a better solution can be found by other transformation operators.

In the STA, the rotation transformation has the function of local search, because it can search in a hypersphere with a given radius α. The rotation factor α is reducing from a maximum value $\alpha_{\max}$ to a minimum value $\alpha_{\min}$ in an exponential way with base *fc*, which is called lessening coefficient. The expansion transformation is for global search, which aims to search in the whole space with probability. The translation transformation is designed for a line search and the axesion transformation is proposed to strength the single dimensional search.

4 Thresholding value selection strategy

After fitting the normalized histogram of the original image by STA, the optimal thresholdings can be determined by minimizing the estimate of the overall fitting error for two adjacent normal distributions functions. It is given by

$$E(T_i) = \phi_i \cdot E_1(T_i) + \phi_i \cdot E_2(T_i),\\ i = 1, 2, \ldots, k-1 \tag{11}$$

where k is the number of normal distribution function and ϕ_i represents the priori probability within the combined function. T_i is the thresholding level between the ith and the $(i+1)$th parts. In this function, the detailed form is:

$$E_1 T_i = \int_{-\infty}^{T_i} n_{i+1}(\boldsymbol{x}) d\boldsymbol{x} \tag{12}$$

and

$$E_2(T_i) = \int_{T_i}^{\infty} n_i(\boldsymbol{x}) d\boldsymbol{x} \tag{13}$$

$E_1(T_i)$ presents the probability of erroneously classifying the pixels in the $(i+1)$th part belonging to the hth part, where $E_1(T_i)$ is the probability of classifying the pixels in the hth part belonging to the $(h+1)$th part by mistake. When the error $E(T_i)$ is minimized, the T_i value will be chosen. According to relate knowledge of differential and integral calculus, it is possible to use the following equation to define the optimum thresholding level T_i:

$$AT_i^2 + BT_i + C = 0 \tag{14}$$

considering

$$\begin{aligned} A =&\ \sigma_i^2 - \sigma_{i+1}^2, \\ B =&\ 2 \cdot (\mu_i \sigma_{i+1}^2 - \mu_{i+1} \sigma_i^2), \\ C =&\ (\sigma_i \mu_{i+1})^2 - (\sigma_{i+1} \mu_i)^2 + 2 \cdot \\ &\cdot (\mu_i \mu_{i+1})^2 \cdot In(\frac{\sigma_{i+1} \phi_i}{\sigma_i \phi_{i+1}}) \end{aligned} \tag{15}$$

Although the above quadratic equation has two possible solutions, only the one which is positive and inside the interval can be adopted [13].

5 Computational results and analysis

By computing the parameters contained in the fitness function (Eq.(5)) after applying the STA, several experiments are set to evaluate the performance of the proposed algorithm. These examples are all from the Segmentation evaluation database [14]. In the same time, the performances of the OTSU method, genetic algorithm (GA) and particle swarm optimization algorithm (PSO) in MATLAB are investigated for comparison. Table 1 shows the maximum and minimum values of intensity levels in all images.

Table 1:Search ranges of all three test images

Images	Minimum value	Maximum value
ship	0	255
flower	0	255
building	0	252

All of these methods are run under the MATLAB(Version R2010b) software platform. And in all experiments, each candidate solution includes nine dimensions, yielding:

$$I_i = \{\phi_1^i, \phi_2^i, \phi_3^i, \mu_1^i, \mu_2^i, \mu_3^i, \sigma_1^i, \sigma_2^i, \sigma_3^i\} \tag{16}$$

with i representing the individual's number. The parameter settings of STA are give as follows: $\alpha_{\max} = 1, \alpha_{\min} = 1e-4, \beta = 1, \gamma = 1, \delta = 1, fc = 2, SE = 30$. For the parameters in fitness function, $\omega = 10$ is used. The maximum number of iterations is 1000. For fairness, the population size is 30, and the maximum number of generations is 1000 in PSO and GA. The goal is to segment the image into three different classes and separate objects from the background. In order to estimating the results, two aspects are considered: (1) the definition of the object segmentations; (2) the speed and precision of the algorithm.

Figure 1(a) and figure 1(f) show the original image and the normalized histogram of ship image respectively. Figure 1(b) to figure 1(e) illustrate the segmentation image by OTSU, STA, PSO, and GA severally. Both OTSU and STA can identify the ships, but the sky is perfectly identified in figure 1(c) (*i.e.*,obtained by STA). While in figure 1(d) and figure 1(e), the results by PSO and GA are unable to identify the ships. Figure 1(g) to figure 1(i) show that the fitting curves obtained by STA and GA nicely fit the normalized histogram of original image with very small mean square error 7.2514e-4 and 7.6227e-4, respectively. However, the fitted curve obtained by PSO is markedly different with initial normalized histogram according to figure 1(h).

The original image and normalized histogram of flower image are shown in figure 2(a) and figure 2(f), correspondingly. Figure 2(b) to figure 2(e) show the thresholding results of the flower image obtained by OTSU, STA, PSO and GA. Notice that, due to the initial normalized histogram is a smooth curve, the original image with low contrast is difficult to segment accurately. Compared with the figure 2(a), the white part of figures 2(b) and 2(c) are closer to the left flower while the gray part of figure 2(e) is closer to the right flower, besides, the black part of figure 2(c) is closer to the background. It means that contrast-enhancing is important to some special picture before image segmentation. Figure 2(g) to figure 2(i) show that the STA and GA also nicely fit the normalized histogram with very small mean square error of merely 5.7361e-4 and 5.8808e-4, respectively. Similar to ship image, the fitted curve obtained by PSO is also different with the normalized histogram according to figure 2(h).

Figure 3 summarizes the thresholds results of the last image. Figures 3(a) and 3(f) show the original image and related intensity-level histogram of building image. It is obvious that STA performs better than OTSU, PSO and GA according to the figure 3(b), figure 3(d) and figure 3(e). The two buildings are perfectly separated in figure 3(c). Nevertheless, the figure 3(a) and 3(e) obtained by OTSU and GA severally have some noises. In figure 3(d), the result by PSO is unable to distinguish the two buildings. Figure 3(g) to figure 3(i) show that the fitting curves obtained by STA and GA nicely fit the initial normalized histogram with very small mean square error 5.0457e-4 and 5.1132e-4, respectively. However, the fitted curve obtained by PSO is markedly different with the normalized histogram according to figure 3(h).

6 Conclusion and future work

For general image segmentation problem, an optimization algorithm associated with the normal distribution fitting approach is proposed in this paper. The combination of normal

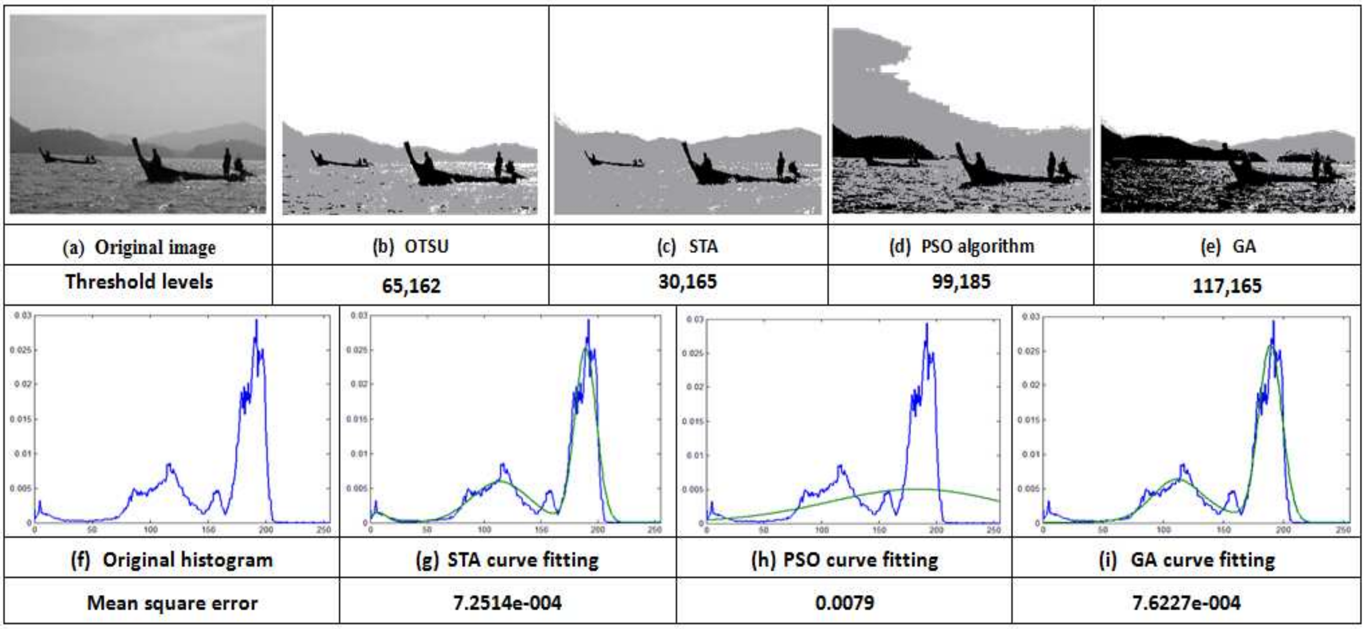

Fig. 1: Experimental results for ship image

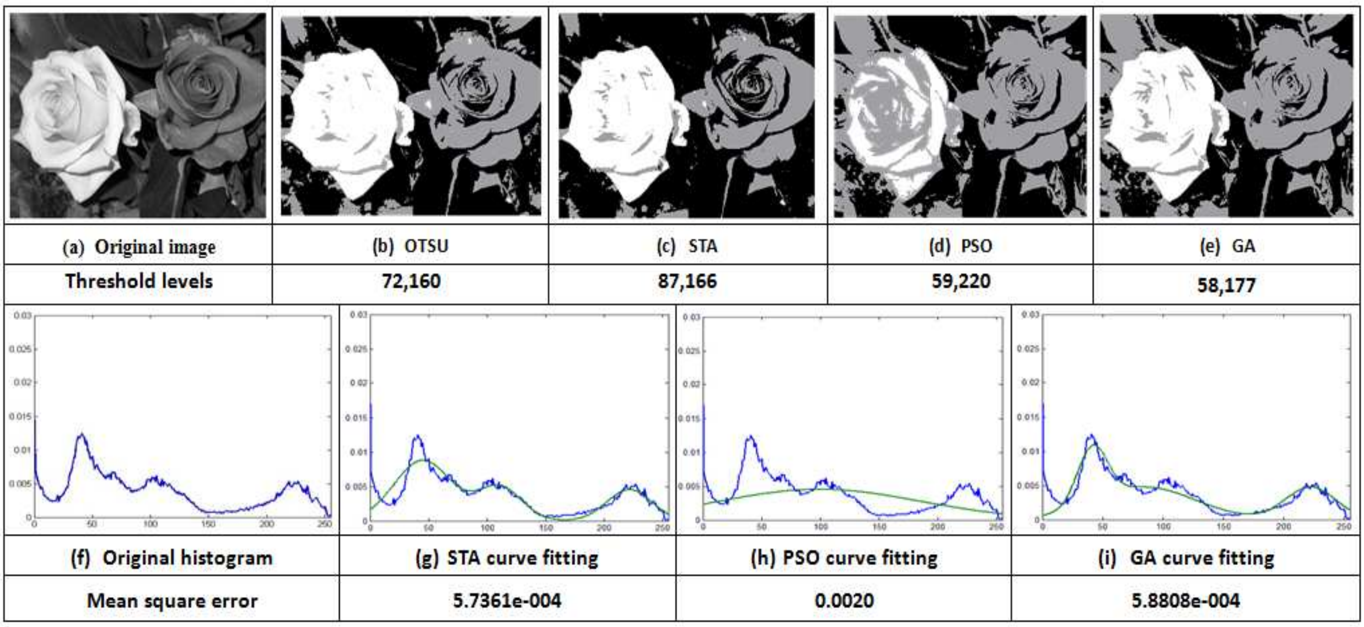

Fig. 2: Experimental results for flower image

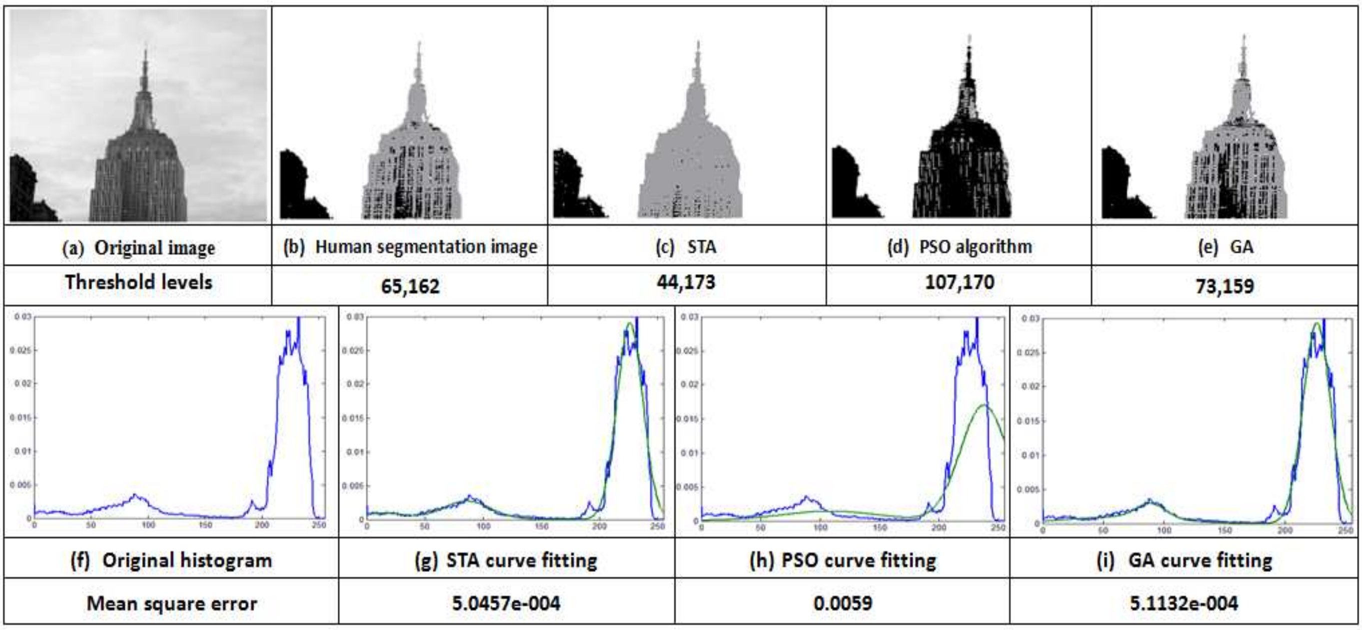

Fig. 3: Experimental results for building image

distribution functions is considered as an objective function. The process of optimization is based on state transition algorithm (STA). Experiments have shown that STA has better results than OTSU, PSO and GA in terms of both solution precision and final effect, which has testified the effectiveness of the proposed method.

However, the goal in this paper is to segment the image into only three different classes, and future work may include more classifications of image segmentation. In addition, with the experience gained in the study, an appropriate thresholding value selection strategy should be further studied to complete optimization process.

References

[1] R.C.Gonzalez, R.E.Woods, Digital Image Processing, 2nd Edition, Prentic-Hall Inc., Upper Saddle River, NJ, 2002.

[2] J.Pan, C.Zhang, Q.Guo, Image enhancement based on the shearlet transform, ICIC Express Letters, 3(3): 621–626, 2009.

[3] Y.Zhang, C.Zhang, J,Chi, R.Zhang, An algorithm for enlarged image enhancement,ICIC Express Letters, 3(3): 669–674, 2009.

[4] O.D.Trier, A.K.Jain, Goal-directed evaluation of binarization methods, IEEE Transactions on Pattern Analysis and Machine Intelligence, 17(12): 1191–1201, 1995.

[5] L.Chih-Chih, A novel image segmentation approach based on particle swarm optimization, IEICE Transactions on Fundamentals, 89(1): 324–327, 2006.

[6] R.Guo, S.M.Pandit, Automatic threshold selection based on histogram modes and discriminant criterion, Machine Vision and Applications, 10: 331–338, 1998.

[7] S.Chen, M.Wang, Seeking multi-thresholds directly from support vectors for image segmentation, Neurocomputing, 67(4): 335–344, 2005.

[8] X.J.Zhou, C.H.Yang, W.H.Gui, Initial version of state transition algorithm, International conference on digital manufacturing and automation (ICDMA), 2011: 644–647.

[9] X.J.Zhou, C.H.Yang, W.H.Gui, A new transformation into state transition algorithm for finding the global minimum, International conference on intelligent control and information processing(ICICIP), 2011: 674–678.

[10] X.J.Zhou, C.H.Yang, W.H.Gui, State transition algorithm, Journal of industrial and management optimization, 8(4): 1039–1056, 2012.

[11] X.J.Zhou, D.Y.Gao, C.H.Yang, A Comparative study of state transition algorithm with harmony search and artificial bee colony, Advances in intelligent Systems and computing, 212: 651–659, 2013.

[12] X.J.Zhou, C.H.Yang, W.H.Gui, Nonlinear system identification and control using state transition algorithm, Applied mathematics and computation 226: 169–179, 2014.

[13] E.Cuevas, D.Zaldivar, M.PerezPerez-Cisneros, A novel multi-threshold segmentation approach based on differential evolution optimization, Expert Systems with Applications, 37(7): 5265–5271, 2010.

[14] S.Alpert, M.Galun, R.Basri, A.Brandt, Image Segmentation by Probabilistic Bottom-Up Aggregation and Cue Integration, Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, 2007.