# Higher Several Variable Calculus
# Math2111 UNSW

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2022T1

## Contents

# 1 Curves and Surfaces

## 1.1 Curves

**Curves** A curve in $\mathbb{R}^n$ is a vector function

$$\mathbf{c} : I \to \mathbb{R}^n,$$

where $I$ is an interval in $\mathbb{R}$.

**Forms / Notations** Curves may be defined in the following ways:

- **Parametrically** by $c(t) = (x_1(t), x_2(t), \ldots, x_n(t))$
- **Cartesian** by eliminating the $t$ variable to get $y$ in terms of $x$
- **Implicitly** As $F(x, y) = 0$.

# 2 Analysis

## 2.1 Assumed

**Assumed Concepts from Real Single-Variable Calculus**

- limits
- continuity
- differentiability
- integrability

**Assumed Theorems**

- Min/ Max Theorem
- Intermediate Value Theorem
- Mean Value Theorem

## 2.2 Limits

Recall that $\lim_{x \to a} f(x) = L$ requires that for all $\epsilon > 0$, there exists a $\delta > 0$ such that if $|x - a| < \delta$ then

$$|f(x) - L| < \delta.$$

## 2.3 Metrics

We have metrics (distance functions) as

$$m : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$$

satisfying the following 3 axioms.

- **Positive Definite** such that for all $x, y \in \mathbb{R}^n$, $m(x, y) > 0$ and, $m(x, y) = 0 \iff x = y$.
- **Symmetric** $m(x, y) = m(y, x)$.z
- **Triangle Inequality** such that for all $x, y, z \in \mathbb{R}^n$, $m(x, y) + m(y, z) \leq m(x, z)$.

**Euclidian Distance** We allow the Euclidian distance to be defined as

$$d_n(x, y) := ||x - y|| = \sqrt{\sum_{i=1}^{n} (x_i - y_y)^2}$$

We often allow $d$ to be $d_2$.

**Norms** Norms will be revisited in the Fourier Series section. They can be thought of as the length of an element in vectors space.

**Equivalent Metrics** Two metrics $d$ and $\delta$ are considered equal if there exists constants $0 < c < C < \infty$ such that

$$c\delta(x, y) \leq d(x, y) \leq C\delta(x, y).$$

## 2.4 Limits of Sequences

//soon$^{\mathrm{tm}}$