Differentiation

Differentiability, Derivatives and Affine Approximations

Differentiability in R A function $f : R \to R$ being differentiable at some $a \in R$ implies that there exists a *good* strai

where for all a, $y = f(a) - f'(a)a$ and $L : R \to R = f'(a)x$.

Recall that

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Affine Maps A function $T : R^n \to R^m$ being affine means that there exists a y_0 such that for all $x \in R^n$

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In $T : R \to R$ this sis of the form $y = mx + b$.

A function $f : R \to R$ is differentiable if there is a good affine approximation to f of the form

In this context good implies that $f'(x)$ is defined in the usual manner and exists.

Differentiability in $R^n \to R^n$

A function $f : \Omega \subset R^n \to R^m$ is differentiable for some $a \in \Omega$ if there exists a linear map $L : Rn \to R^m$ such that

Notation: the matrix of the linear map L, the derivative of f at a is denoted by $D_a f$.

Delta Epsilon Definition of Differentiability A function $f : \Omega \subset R \to R^m$ is differentiable on $a \in \Omega$ if there is a linea

Clairaut's Theorem / Mixed Derivative Theorem Suppose $f, \frac{\partial f}{\partial x_i}, \frac{\partial f}{\partial x_j}, \frac{\partial^2 f}{\partial x_i \partial x_j}, \frac{\partial^2 f}{\partial x_j \partial x_i}$ all exist and are continuous on

That is, the partial derivatives commute.

Differentiability and Continuity Differentiability implies continuity. However, continuity does not imply differentiab

Gradients, Affine Approximations and Matrices

Jacobian Matrices