Kernel-based density estimation in tests of symmetry

Marina Iturrate-Bobes[1] and Raúl Pérez-Fernández[1]

[1]Deptartment of Statistics and O.R. and M.D., University of Oviedo, Spain

Symmetry is a fundamental and often desirable property in many statistical contexts. For instance, many tables for symmetric distributions —such as the normal distribution— are greatly simplified, as cumulative probabilities in the right tail can be easily derived from those in the left tail. As another example, symmetry is a key assumption that simplifies the understanding of statistical tests such as Wilcoxon's signed rank test.

The earliest efforts to quantify the lack of symmetry of a random variable using what is now known as a skewness coefficient are commonly attributed to Pearson in the 19th century, although a rigorous axiomatic framework for these coefficients was developed much later with notable contributions from van Zwet [4] and Oja [3]. Numerous skewness coefficients have been proposed, ranging from classical ones such as those by Fisher, Pearson, and Bowley to more modern alternatives like the medcouple. Every skewness coefficient has an associated sample version that, under certain conditions, follows an asymptotically normal distribution centered at the population value. In particular, for symmetric distributions, the asymptotic distribution is always centered at zero but the asymptotic variance varies from one symmetric distribution to another. This is an issue that has been faced by different authors when developing tests of symmetry. In order to circumvent such a problem, some authors have considered the bootstrap [2] and some others have opted for either fixing a reference distribution or estimating the asymptotic variance directly [1].

In the present work, we study the influence of using kernel-density estimation techniques in tests of symmetry based on different skewness coefficients and, in particular, present a comparative power analysis across different symmetric and asymmetric distributions for different choices of kernel and bandwidth. The results show that the choice of skewness coefficient plays a much bigger role than the choices of kernel and bandwidth.

Keywords: Test of symmetry; Skewnesss coefficient; Kernel density estimation.

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