Aggregation of the distortion models induced by the KL divergence and Euclidean distance

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Overview

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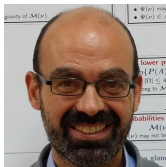
Distortion models

S.Destercke   E.Miranda   P.R.Alonso

- Unifying neighbourhood and distortion models: Part I. Montes, Miranda, Destercke. IJGS 2020.

- Unifying neighbourhood and distortion models: Part II. Montes, Miranda, Destercke. IJGS 2020.

- Processing distortion models: a comparative study. Destercke, Montes, Miranda. IJAR 2022.

- Distortion models for estimating human error probabilities. Alonso, Montes, Miranda. SS 2023.
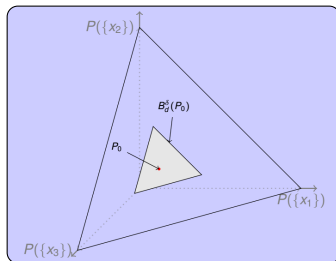
Distortion models

Ingredients

P_0: probability measure

δ: distortion parameter

d: distorting function

Example

The model

$$B_d^\delta(P_0) = \{P \mid d(P, P_0) \leq \delta\}$$

d continuous and convex

$$\underline{P}_d(f) = \inf\{P(f) \mid P \in B_d^\delta(P_0)\}$$

$$\mathcal{M}(\underline{P}_d) = \{P \mid P(f) \geq \underline{P}_d(f) \ \forall f\}$$

Particular models

Pari Mutuel Model

$$\underline{P}_{PMM}(A) = \max\{0, (1+\delta)P_0(A) - \delta\}$$

Pelessoni et al., 2010
Montes et al., 2019
Walley, 1991

Linear vacuous model

$$\underline{P}_{LV}(A) = (1-\delta)P_0(A) \qquad A \neq \mathcal{X}$$

Huber, 1981
Walley, 1991

Constant odds ratio

$$\underline{P}_{COR}(A) = \frac{(1-\delta)P_0(A)}{1-\delta P_0(A)}$$

Benavoli and Zaffalon, 2013
Walley, 1991

Total Variation model

$$\underline{P}_{TV}(A) = \max\{0, P_0(A) - \delta\} \qquad A \neq \mathcal{X}$$

Montes et al., 2020
Herron et al., 1997

Vertical barrier models

$$\underline{P}_{VB}(A) = \max\{bP_0(A) + a, 0\} \qquad \begin{array}{l} a \leq 0, b > 0 \\ a + b \in (0,1) \end{array}$$

Pelessoni et al., 2021
Corsato et al., 2019

Increasing transformation

$$\underline{P}(A) = g(P_0(A)) \qquad \begin{array}{l} g : [0,1] \to [0,1] \text{ increasing} \\ g(0) = 0, g(1) = 1 \end{array}$$

Bronevich, 2005

Overview

Euclidean model

Euclidean model

$$d_E(P, Q) = \sqrt{\sum_{i=1}^{n}(p_i - q_i)^2}$$

$P_0 \searrow$

$\qquad B_E^{\delta}(P_0)$

$\delta \nearrow$

$P(\{x_2\})$

$P(\{x_1\})$

$P(\{x_3\})$

$B_E^{\delta}(P_0)$

P_0
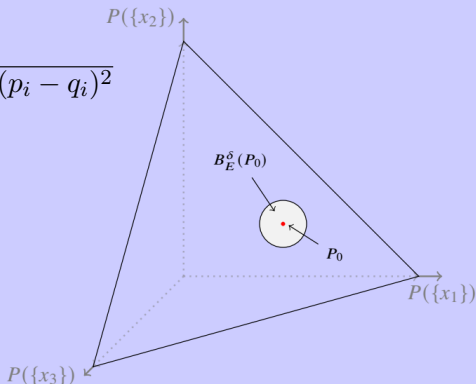
Euclidean model

$$f \equiv (a_1, \ldots, a_n) \qquad \underline{P}_E(f) = E_{P_0}(f) - \delta\sqrt{n}S_f$$

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