On the use of aggregation functions within tests of symmetry

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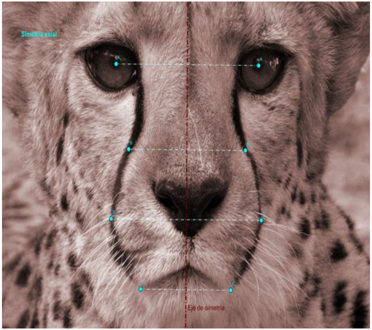
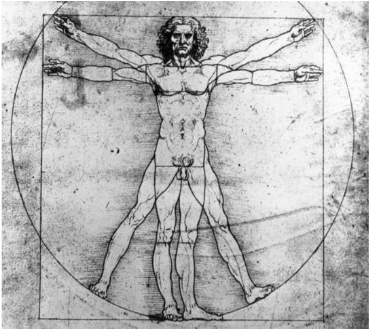
Symmetry in nature



Symmetry in art

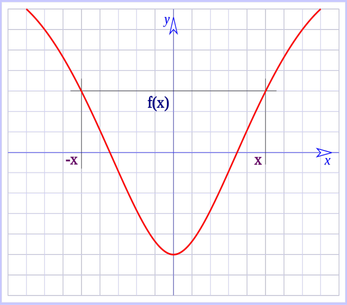


Vitruvian man
(Leonardo Da Vinci, 1490)

The starry night
(Vincent Van Gogh, 1889)

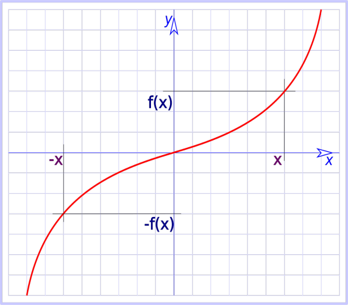
Symmetry in mathematics (analysis)

Even function



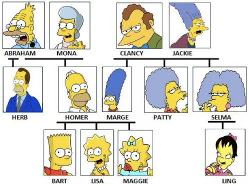
Axial symmetry
about y axis

Odd function



Rotational symmetry about
the origins of the coordinates

Symmetry in mathematics (relational algebra)



Symmetric relationship: $xRy \Rightarrow yRx$

Example: 'x is y's sibling'

Asymmetric relationship: $xRy \Rightarrow y\not{R}x$

Example: 'x is y's child'

Symmetry in mathematics (linear algebra)

A matrix M is symmetric if

$$\boxed{M = M^T}$$

Typical example in probabilities: variances and covariances' matrix

$$\Sigma = \begin{pmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} \\ \sigma_{XY} & \sigma_Y^2 & \sigma_{YZ} \\ \sigma_{XZ} & \sigma_{XY} & \sigma_Z^2 \end{pmatrix}$$

Symmetry in a random variable

- A random variable X is called **symmetric about** $x_0 \in \mathbb{R}$ if

$$P(X \leq x_0 - x) = P(X \geq x_0 + x)\,, \ \text{for any } x \in \mathbb{R}$$

- If there exists no point $x \in \mathbb{R}$ such that $P(X = x) > 0$, then X is symmetric about $x_0 \in \mathbb{R}$ if and only if

$$F(x_0 - x) = 1 - F(x_0 + x)\,, \ \text{for any } x \in \mathbb{R}$$

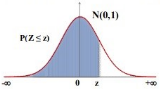
- If X is a continuous random variable with a density function f, then X is symmetric about $x_0 \in \mathbb{R}$ if and only if

$$f(x_0 - x) = f(x_0 + x)\,, \ \text{for any } x \in \mathbb{R}$$

A random variable X is called **symmetric** if there exists $x_0 \in \mathbb{R}$ about which X is symmetric

Tables for symmetric variables: half the effort!

FUNCIÓN DE DISTRIBUCIÓN NORMAL N(0,1)



z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,99653	0,99664	0,99674	0,99683	0,99693	0,99702	0,99711	0,99720	0,99728	0,99736
2,8	0,99744	0,99752	0,99760	0,99767	0,99774	0,99781	0,99788	0,99795	0,99801	0,99807
2,9	0,99813	0,99819	0,99825	0,99831	0,99836	0,99841	0,99846	0,99851	0,99856	0,99861
3,0	0,99865	0,99869	0,99874	0,99878	0,99882	0,99886	0,99889	0,99893	0,99896	0,99900
3,1	0,99903	0,99906	0,99910	0,99913	0,99916	0,99918	0,99921	0,99924	0,99926	0,99929
3,2	0,99931	0,99934	0,99936	0,99938	0,99940	0,99942	0,99944	0,99946	0,99948	0,99950
3,3	0,99952	0,99953	0,99955	0,99957	0,99958	0,99960	0,99961	0,99962	0,99964	0,99965
3,4	0,99966	0,99968	0,99969	0,99970	0,99971	0,99972	0,99973	0,99974	0,99975	0,99976
3,5	0,99977	0,99978	0,99978	0,99979	0,99980	0,99981	0,99981	0,99982	0,99983	0,99983
3,6	0,99984	0,99985	0,99985	0,99986	0,99986	0,99987	0,99987	0,99988	0,99988	0,99989
3,7	0,99989	0,99990	0,99990	0,99990	0,99991	0,99991	0,99992	0,99992	0,99992	0,99992
3,8	0,99993	0,99993	0,99993	0,99994	0,99994	0,99994	0,99994	0,99995	0,99995	0,99995
3,9	0,99995	0,99995	0,99996	0,99996	0,99996	0,99996	0,99996	0,99996	0,99997	0,99997
4,0	0,99997	0,99997	0,99997	0,99997	0,99997	0,99997	0,99998	0,99998	0,99998	0,99998

Necessary assumption for some non-parametric tests

Wilcoxon signed-rank test (Wilcoxon, 1945):

Let X be a continuous and **symmetric** random variable about which we test

$$\begin{aligned} H_0 : &\quad \mathrm{Me} = m \\ H_1 : &\quad \mathrm{Me} \neq m \end{aligned}$$

Using the statistic

$$T^+ = \sum_{X_i > m} \mathrm{rank}(|X_i - m|)$$

and the

$$RR = \{T^+ < c_1, T^+ > c_2\}$$

Fun fact: If we knew the population median instead of the symmetry property, we may perform a test of symmetry (Thas, Rayner and Best, 2015)

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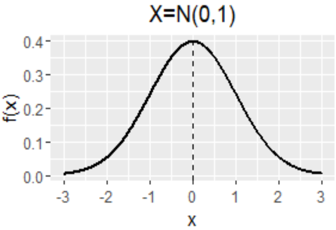
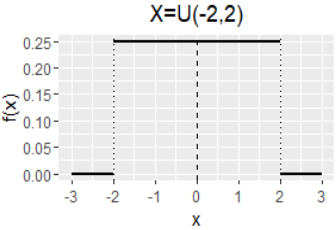
An axiomatization of the notion of symmetry (Oja, 1981)

Axiomatic definition of a **skewness coefficient** γ:

1. $\gamma(X) = 0$ if X is symmetric





2. $\gamma(cX + d) = \gamma(X)$ for any $c, d \in \mathbb{R}$ such that $c > 0$
3. $\gamma(-X) = -\gamma(X)$

Sometimes, it is required

4. $\gamma(X) \leq \gamma(Y)$ if $X \precsim Y$ with $\precsim$ an order convexity
5. $\gamma(X) \in [-1, 1]$

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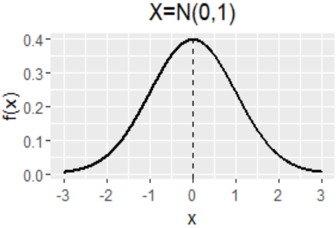

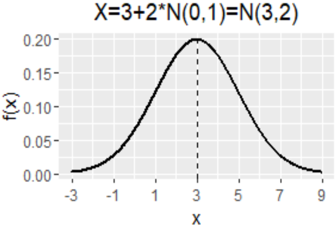

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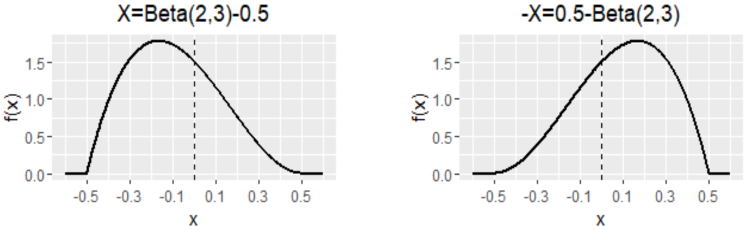

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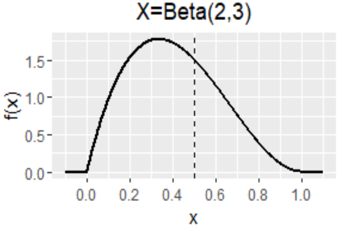

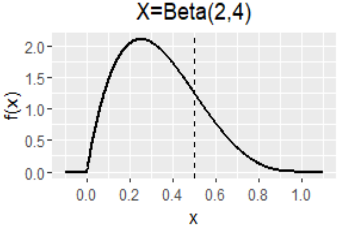

5. $\gamma(X) \in [-1, 1]$

Some classic skewness coefficients

- **Moment skewness coefficient** (Pearson, 1895)

$$\gamma_{\mathrm{M}}(X) = \frac{E\left((X-\mu)^3\right)}{\sigma^3}$$

- **Non-parametric skewness coefficient** (Pearson, 1895)

$$\gamma_{\mathrm{NP}}(X) = \frac{\mu-\mathrm{Me}(X)}{\sigma}$$

- **Groeneveld-Meeden skewness coefficient** (1984, 1995)

$$\gamma_{\mathrm{GM}}(X) = \frac{\mu-\mathrm{Me}(X)}{E(|X-\mathrm{Me}(X)|)}$$

Some robust skewness coefficients

Hinkley skewness coefficients (1975)

For any $p \in]0,1[$

$$\gamma_p(X) = \frac{\left(C_{1-p}(X) - C_{0.5}(X)\right) - \left(C_{0.5}(X) - C_p(X)\right)}{C_{1-p}(X) - C_p(X)}$$

- **Bowley skewness coefficient** (1901)

$$\gamma_{\mathrm{B}}(X) = \frac{Q_3(X)+Q_1(X)-2\mathrm{Me}(X)}{Q_3(X)-Q_1(X)}$$

- **Octile skewness coefficient** (Hinkley, 1975)

$$\gamma_{\mathrm{OCT}}(X) = \frac{\left(C_{0.875}(X)-C_{0.5}(X)\right)-\left(C_{0.5}(X)-C_{0.125}(X)\right)}{C_{0.875}(X)-C_{0.125}(X)}$$

Some robust skewness coefficients

Medcouple skewness coefficient (Brys, Hubert and Struyf, 2003)

$$\gamma_{\mathrm{MC}}(\mathbf{x}) = \underset{x_i \le \mathrm{Me}(\mathbf{x}) \le x_j}{\mathrm{Me}} h(x_i, x_j)$$

where $h(x_i, x_j) = \frac{(x_j - \mathrm{Me}(\mathbf{x})) - (\mathrm{Me}(\mathbf{x}) - x_i)}{x_j - x_i}$ (if $x_i \neq x_j$)

Population expression for medcouple

$$MC = H_F^{-1}(0.5)$$

where

$$H_F(u) = 4 \int_{\mathrm{Me}(X)}^{\infty} F\left(\frac{x(u-1) + 2\mathrm{Me}(X)}{u+1}\right) dF(x)$$

Asymptotically normal skewness coefficients

Under some regularity conditions, the sample versions $\hat{\gamma}$ of all the skewness coefficents above are **asymptotically normal estimators of the corresponding population skewness coefficient** γ_F, i.e.,

$$\sqrt{n}\,\frac{\hat{\gamma}-\gamma_F}{\sqrt{V(\gamma,F)}} \rightsquigarrow N(0,1)$$

Skewness-based goodness-of-fit tests

Let $\mathcal{F}$ be a location-scale distribution family. We can set out

$$\boxed{\begin{array}{ll} H_0: & X \rightsquigarrow \mathcal{F} \\ H_1: & X \not\rightsquigarrow \mathcal{F} \end{array}}$$

For any $F, F' \in \mathcal{F}$ it holds that $\gamma_F = \gamma_{F'}$ and $V(\gamma, F) = V(\gamma, F')$

Under asymptotic normality, the rejection region will be defined as

$$\boxed{RR = \left\{ \mathbf{x} \in \mathbb{R}^n \;\middle|\; \sqrt{n} \frac{|\gamma(\mathbf{x}) - \gamma_F|}{\sqrt{V(\gamma, F)}} > z_{1-\alpha/2} \right\}}$$

where $z_{1-\alpha/2}$ is the quantile of order $1 - \alpha/2$ of a normal distribution

Tests of symmetry (i)

It is set out

$$\begin{array}{lll} H_0: & \gamma(X) = 0 & (\approx X \text{ is symmetric}) \\ H_1: & \gamma(X) \neq 0 & (\approx X \text{ is not symmetric}) \end{array}$$

Under asymptotic normality, we could define the rejection region

$$RR = \left\{ \mathbf{x} \in \mathbb{R}^n \,\middle|\, \sqrt{n}\,\frac{|\gamma(\mathbf{x}) - 0|}{\sqrt{V(\gamma, F)}} > z_{1-\alpha/2} \right\}$$

where $z_{1-\alpha/2}$ is the quantile of order $1 - \alpha/2$ of a normal distribution

Problem: $V(\gamma, F)$ is unknown!

Tests of symmetry (ii)

It is set out

$$\begin{array}{lll} H_0: & \gamma(X) = 0 & (\approx X \text{ is symmetric}) \\ H_1: & \gamma(X) \neq 0 & (\approx X \text{ is not symmetric}) \end{array}$$

Problem: $V(\gamma, F)$ is unknown!

- **Solution 1:** $V(\gamma, F)$ is estimated considering a reference distribution (e.g., ϕ)
- **Solution 2:** $V(\gamma, F)$ is estimated with some techniques for estimation of density function
- **Solution 3:** $V(\gamma, F)$ is estimated with its bootstrap estimation $V^*(\gamma, \mathbf{x})$

Tests of symmetry (iii)

It is set out

$$\begin{array}{lll} H_0: & \gamma(X) = 0 & (\approx X \text{ is symmetric}) \\ H_1: & \gamma(X) \neq 0 & (\approx X \text{ is not symmetric}) \end{array}$$

Problem: $V(\gamma, F)$ is unknown!

Adopted solution: $V(\gamma, F)$ is estimated with its bootstrap estimation $V^*(\gamma, \mathbf{x})$

Then, under asymptotic normality, we can consider

$$RR = \left\{ \mathbf{x} \in \mathbb{R}^n \,\middle|\, \sqrt{n}\, \frac{|\gamma(\mathbf{x}) - 0|}{\sqrt{V^*(\gamma, \mathbf{x})}} > z_{1-\alpha/2} \right\}$$

where $z_{1-\alpha/2}$ is the quantile of order $1 - \alpha/2$ of a normal distribution

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Experimental framework (i)

Objective

Compare the power of the here-presented tests with the *m*-out-of-*n* bootstrap counterparts version implemented in the R package *lawstat* of two tests:

- Cabilio and Masaro test (1996)
- Miao, Gel and Gastwirth test (2006)

- Significance level $\alpha = 0.05$
- The power of the tests is estimated by Monte Carlo simulation (10^4 replications) and the number of bootstrap replications is set to $B = 10^3$
- 4 sample sizes are studied: $n \in \{25, 50, 100, 200\}$
- All tests are applied to the same samples with the aim of reducing the influence of the sampling

Power under symmetry

Distribution	n	γ_M	γ_{NP}	γ_{GM}	γ_B	γ_{OCT}	γ_{MC}	CM	MGG
Normal	25	0.0652	0.0174	0.0167	0.0036	0.0169	0.0086	0.0298	0.0313
	50	0.0809	0.0269	0.0265	0.0152	0.0298	0.0158	0.0293	0.0315
	100	0.0739	0.0366	0.0355	0.0278	0.0360	0.0247	0.0426	0.0467
	200	0.0654	0.0420	0.0414	0.0363	0.0425	0.0330	0.0419	0.0458
Cauchy	25	0.4092	0.1329	0.2136	0.0105	0.0531	0.0262	0.2673	0.0304
	50	0.4185	0.1527	0.2216	0.0281	0.0633	0.0323	0.3638	0.0421
	100	0.4189	0.1511	0.2090	0.0389	0.0640	0.0356	0.4314	0.0563
	200	0.4216	0.1407	0.1980	0.0360	0.0584	0.0359	0.4231	0.0453
t_3	25	0.2272	0.0381	0.0438	0.0038	0.0184	0.0114	0.0485	0.0304
	50	0.2472	0.0535	0.0528	0.0150	0.0338	0.0157	0.0492	0.0347
	100	0.2253	0.0597	0.0563	0.0274	0.0397	0.0221	0.0679	0.0567
	200	0.1969	0.0568	0.0512	0.0326	0.0441	0.0312	0.0564	0.0502
Logistic	25	0.1108	0.0198	0.0192	0.0023	0.0163	0.0086	0.0299	0.0261
	50	0.1268	0.0363	0.0358	0.0155	0.0299	0.0171	0.0351	0.0329
	100	0.1144	0.0384	0.0377	0.0279	0.0373	0.0228	0.0415	0.0453
	200	0.0902	0.0425	0.0422	0.0338	0.0407	0.0315	0.0437	0.0447
Laplace	25	0.1653	0.0343	0.0372	0.0045	0.0253	0.0127	0.0401	0.0299
	50	0.1586	0.0435	0.0428	0.0196	0.0366	0.0171	0.0394	0.0360
	100	0.1474	0.0492	0.0475	0.0328	0.0438	0.0241	0.0516	0.0488
	200	0.1147	0.0485	0.0479	0.0356	0.0456	0.0301	0.0486	0.0480
Uniform	25	0.0308	0.0248	0.0235	0.0038	0.0269	0.0134	0.0480	0.0553
	50	0.0445	0.0422	0.0403	0.0228	0.0453	0.0287	0.0499	0.0554
	100	0.0492	0.0484	0.0462	0.0301	0.0491	0.0387	0.0590	0.0714
	200	0.0494	0.0521	0.0502	0.0391	0.0535	0.0463	0.0525	0.0604

Power under asymmetry

Distribution	n	γ_M	γ_{NP}	γ_{GM}	γ_B	γ_{OCT}	γ_{MC}	CM	MGG
GLD7	25	0.2585	0.1056	0.1010	0.0106	0.0935	0.0534	0.1346	0.1355
	50	0.5975	0.2485	0.2340	0.0602	0.2315	0.1427	0.2387	0.2514
	100	0.9077	0.4273	0.4129	0.1250	0.4238	0.2902	0.4623	0.4867
	200	0.9976	0.6839	0.6763	0.2292	0.7041	0.5111	0.7320	0.7411
GLD8	25	0.8351	0.4148	0.4113	0.0351	0.2903	0.1662	0.4124	0.3548
	50	0.9927	0.7548	0.7397	0.1612	0.6250	0.3856	0.7491	0.7248
	100	1.0000	0.9506	0.9457	0.3363	0.8996	0.6643	0.9719	0.9715
	200	1.0000	0.9981	0.9980	0.6201	0.9937	0.9135	0.9995	0.9998
GLD9	25	0.3894	0.1187	0.1199	0.0067	0.0795	0.0381	0.1329	0.1176
	50	0.6908	0.3056	0.2976	0.0528	0.2201	0.1160	0.2715	0.2650
	100	0.9272	0.5597	0.5506	0.1113	0.4379	0.2345	0.5691	0.5719
	200	0.9955	0.8420	0.8366	0.2244	0.7259	0.4466	0.8664	0.8656
GLD10	25	0.5967	0.2185	0.2260	0.0133	0.1323	0.0648	0.2235	0.1841
	50	0.8789	0.5007	0.4903	0.0762	0.3293	0.1726	0.4593	0.4336
	100	0.9876	0.7995	0.7901	0.1799	0.6325	0.3691	0.8141	0.8080
	200	0.9998	0.9732	0.9711	0.3489	0.8995	0.6410	0.9819	0.9815
GLD11	25	0.2097	0.0410	0.0449	0.0038	0.0255	0.0150	0.0477	0.0355
	50	0.2550	0.0727	0.0718	0.0196	0.0462	0.0239	0.0657	0.0566
	100	0.2676	0.1064	0.1026	0.0362	0.0703	0.0410	0.1107	0.1024
	200	0.2929	0.1680	0.1622	0.0468	0.1021	0.0585	0.1672	0.1630
GLD12	25	0.3781	0.1136	0.1250	0.0078	0.0601	0.0302	0.1188	0.0849
	50	0.5497	0.2603	0.2562	0.0417	0.1437	0.0741	0.2317	0.1999
	100	0.7081	0.4696	0.4544	0.0805	0.2777	0.1420	0.4676	0.4486
	200	0.8597	0.7633	0.7520	0.1528	0.5164	0.2891	0.7676	0.7600

Robustness in the presence of outliers

Distribution ($n = 200$)	ε	γ_{M}	γ_{NP}	γ_{GM}	γ_{B}	γ_{OCT}	γ_{MC}	CM	MGG
$(1-\varepsilon)\,N(0,1) + \varepsilon\,N(0,3)$	0.00	0.0673	0.0412	0.0408	0.0363	0.0419	0.0324	0.0414	0.0415
	0.01	0.0755	0.0461	0.0456	0.0372	0.0417	0.0332	0.0453	0.0460
	0.05	0.1481	0.0455	0.0430	0.0323	0.0392	0.0285	0.0446	0.0459
	0.10	0.1540	0.0469	0.0445	0.0346	0.0369	0.0303	0.0463	0.0449
	0.20	0.1305	0.0475	0.0470	0.0332	0.0373	0.0304	0.0501	0.0476
$(1-\varepsilon)\,N(0,1) + \varepsilon\,C(0,1)$	0.00	0.0700	0.0369	0.0364	0.0319	0.0429	0.0304	0.0387	0.0407
	0.01	0.1391	0.0491	0.0456	0.0327	0.0438	0.0320	0.0645	0.0426
	0.05	0.3558	0.0824	0.0735	0.0328	0.0421	0.0328	0.1391	0.0444
	0.10	0.4479	0.0994	0.1000	0.0319	0.0417	0.0284	0.2122	0.0439
	0.20	0.4503	0.1346	0.1523	0.0322	0.0405	0.0307	0.3093	0.0597
$(1-\varepsilon)\,N(0,1) + \varepsilon\,N(5,1)$	0.00	0.0684	0.0427	0.0427	0.0364	0.0404	0.0345	0.0434	0.0443
	0.01	0.4099	0.0886	0.0826	0.0308	0.0420	0.0331	0.0829	0.0838
	0.05	0.9954	0.6845	0.6692	0.0430	0.0955	0.0675	0.6744	0.6640
	0.10	1.0000	0.9832	0.9819	0.0772	0.3048	0.2448	0.9842	0.9819
	0.20	1.0000	1.0000	1.0000	0.3170	0.9967	0.9118	1.0000	1.0000

Experimental framework (ii)

Objective

Study the tests of symmetry based on the skewness coefficients belonging to the class introduced by Hinkley (1975)

- Significance level $\alpha = 0.05$
- Sample size $n = 200$
- The power of the tests is estimated by Monte Carlo simulation (10^4 replications) and the number of bootstrap replications is set to $B = 10^3$
- $p \in \{0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.40, 0.45\}$
- All tests are applied to the same samples with the aim of reducing the influence of the sampling

Power of the tests

Distribution	ε	$\gamma_{0.05}$	$\gamma_{0.10}$	$\gamma_{0.15}$	$\gamma_{0.20}$	$\gamma_{0.25}$	$\gamma_{0.30}$	$\gamma_{0.35}$	$\gamma_{0.40}$	$\gamma_{0.45}$
Normal		0.0419	0.0435	0.0379	0.0371	0.0338	0.0294	0.0208	0.0073	0.0000
Cauchy		0.0682	0.0587	0.0549	0.0501	0.0408	0.0318	0.0203	0.0086	0.0000
Laplace		0.0471	0.0472	0.0427	0.0416	0.0374	0.0334	0.0212	0.0067	0.0000
Uniform		0.0584	0.0529	0.0500	0.0425	0.0390	0.0326	0.0223	0.0064	0.0001
GLD7		0.9581	0.8092	0.6025	0.4000	0.2303	0.1211	0.0565	0.0164	0.0002
GLD8		1.0000	0.9994	0.9804	0.8657	0.6180	0.3350	0.1326	0.0304	0.0002
GLD9		0.9437	0.8190	0.6177	0.3986	0.2258	0.1144	0.0494	0.0122	0.0000
GLD10		0.9933	0.9530	0.8231	0.5929	0.3488	0.1770	0.0738	0.0156	0.0001
GLD11		0.1506	0.1195	0.0928	0.0681	0.0492	0.0356	0.0241	0.0075	0.0001
GLD12		0.7222	0.6029	0.4268	0.2720	0.1587	0.0876	0.0373	0.0096	0.0000
GLD13		1.0000	1.0000	0.9978	0.9626	0.7959	0.4835	0.2024	0.0418	0.0000
GLD14		1.0000	1.0000	0.9990	0.9743	0.8322	0.5259	0.2207	0.0456	0.0005
$(1-\varepsilon)\,N(0,1)+\varepsilon\,N(5,1)$	0.01	0.0489	0.0454	0.0426	0.0367	0.0308	0.0285	0.0194	0.0077	0.0000
	0.05	0.4221	0.1070	0.0793	0.0556	0.0430	0.0323	0.0224	0.0080	0.0000
	0.10	0.9873	0.5615	0.2132	0.1351	0.0772	0.0435	0.0254	0.0070	0.0000
	0.20	1.0000	0.9998	0.9746	0.7000	0.3170	0.1610	0.0586	0.0120	0.0002

Conclusions

- The tests based on **classic skewness coefficients** exhibit the highest power at asymmetric distributions, but they fail to maintain the significance level at some symmetric distributions
- The tests based on **robust skewness coefficients** are less powerful at asymmetric distributions but they are more robust in the presence of outliers
- Lower values of p in the **skewness coefficients belonging to Hinkley class** result in tests that are more powerful but less robust in the presence of outliers
- Comparison with two classic tests of symmetry:
  - Cabilio and Masaro test (1996) shows worse results than most of the here-presented tests
  - Miao, Gel and Gastwirth test (2006) exhibit an intermediate behaviour between the tests of symmetry based on skewness coefficientes and the robust and classic ones

Future work

- Study a more general family based on OWA estimators:

$$\gamma_{\mathbf{v},\mathbf{w}}(\mathbf{x}) = \frac{\left(\mathbf{v}^T\mathbf{x} - \mathbf{w}^T\mathbf{x}\right) - \left(\mathbf{w}^T\mathbf{x} - \mathbf{v}'^T\mathbf{x}\right)}{\mathbf{v}^T\mathbf{x} - \mathbf{v}'^T\mathbf{x}}$$

  where $\mathbf{v} = 1 - \mathbf{v}'$ and $\mathbf{w}$ are weight vectors, with $\mathbf{w}$ symmetric and $\mathbf{v}$ asymmetric
- Explore the use of different skewness coefficients constructed from different aggregation functions
- Adapt the notion of the skewness coefficient to the context of imprecision and study skewness tests that are robust to the presence of **imprecision in sampling**

On the use of aggregation functions within tests of symmetry

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