Coupling Variational Method with CNN for Image Colorization

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Variational methods and optimization in imaging.
To the memory of our dear friend and colleague Mila Nikolova
2019.02.04

Joint work with :
Marie-Odile Berger and Thomas Mouzon.

General problem of colorization

Input.

Output.

The YUV color space

Definition of the gray-scale channel from RGB :

$$Y = 0.299R + 0.587G + 0.114B.$$

Chrominance channel :

- U and V, enable to recover the RGB image ;
- invertible linear map between YUV and RGB.

Challenge.

Recovering an RGB image from the luminance channel alone is an ill-posed problem and requires additional chrominance information.

The manual colorization

Two approaches :

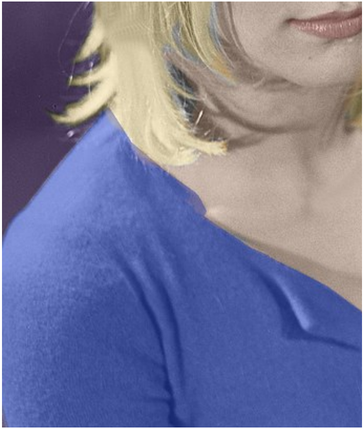
- fully manual (polygonal masks) ;
- automatic diffusion.

Input

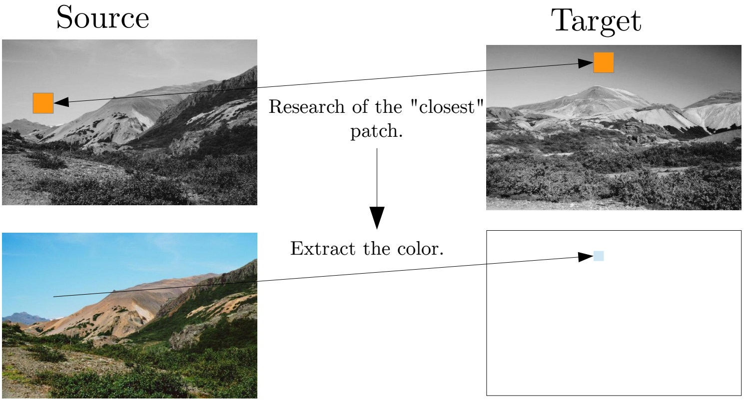
Levin et al. SIGGRAPH 2004.

Colorization with masks

The manual colorization

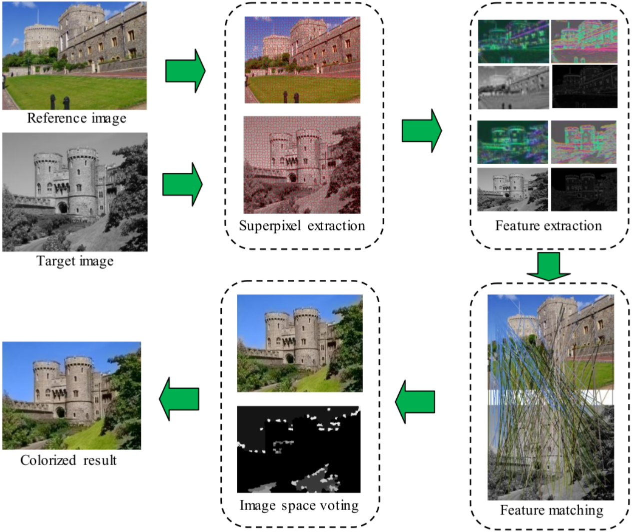


Exemplar-based colorization



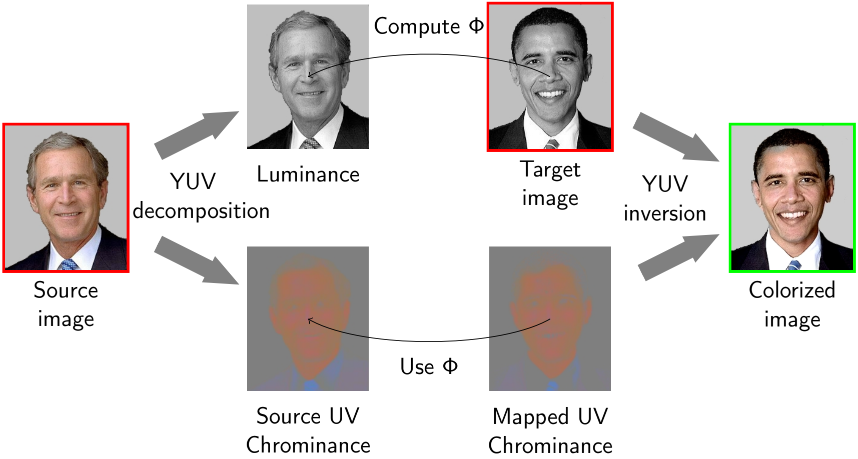
Welsch *et al.* 2002.

Exemplar-based colorization



Gupta *et al.* 2012.

Exemplar-based colorization



Persch *et al.* 2017.

Image database-based colorization

Authors	Color space	Structure of the network	Data base	Cost function
G. Larsson et al. 2006	HCL	VGG	ImageNet	Cross Entropy
R. Zhang et al. 2016	Lab	VGG	ImageNet	Cross Entropy
S. Iizuka et al. 2016	Lab	U-Net + Classifier	MIT Places	L2 + Cross Entropy
S. Guadarrama et al.2017	YCbCr	Pixel CNN	ImageNet	L1 + Cross Entropy
Y. Cao et al. 2017	YUV	cGAN	LSUN	Adv
A. Royer et al. 2017	Lab	Pixel CNN	ImageNet CIFAR10	Cross Entropy
A. Deshpande et al.2017	Lab	VAE + MDN	ImageNet	MD
F. Baldassarre et al.2017	Lab	U-Net + Classifier	ImageNet	L2
R. Zhang et al. 2017	Lab	U-Net + LHN + GHN	ImageNet	Huber Loss
P. Isola et al. 2017	Lab	cGAN	ImageNet	L1
Y. Xiao et al. 2018	Lab	U-Net + LHN + GHN	MIT Places	Huber Loss
Z. Su et al. 2018	YUV	VGG	MIT Places	L1 + L2
K. Nazeri et al. 2018	Lab	cGAN	MIT Places CIFAR10	L1

Lack of regularization with CNN

Target (input)

Result of Zhang *et al.*, 2016

Our model

Limitation of Zhang *et al.*, 2016.

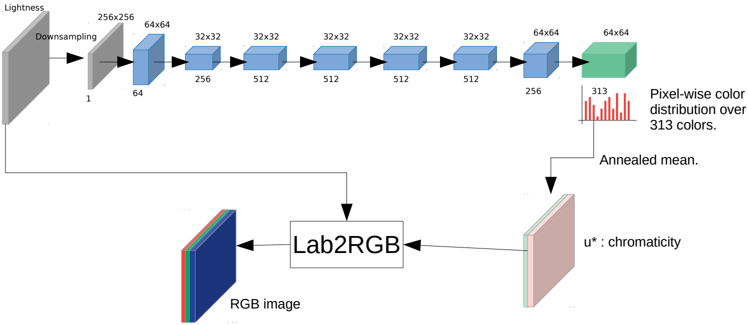
- halo effects ;
- mixing of colors.

Based on a variational model, our method is able to remove such artifacts.

Naive approach

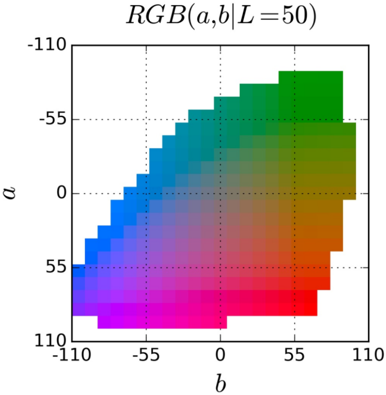
Too simple ! !

CNN of Zhang *et al.*, 2016



This CNN computes color distribution on each pixel.

CNN of Zhang *et al.*, 2016



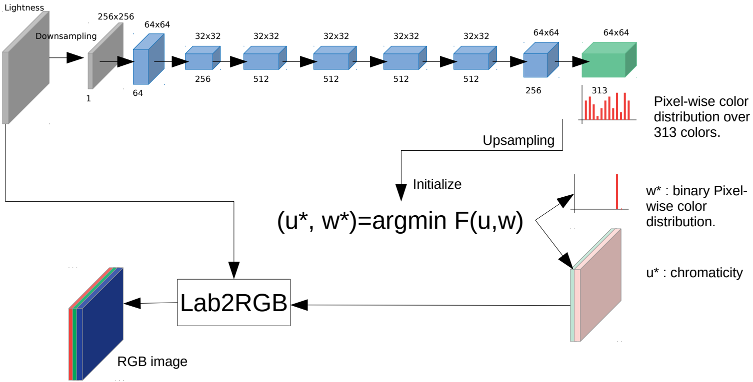
Set of the "labels".

CNN of Zhang *et al.*, 2016

Definition of annealed-mean

$$w_i^* = \frac{\exp(\log(w_i)/T)}{\sum_j \exp(\log(w_j)/T)}$$

CNN of Zhang *et al.*, 2016



A CNN computes color distribution on each pixel that feeds a variational method.

Coupled total variation

Inspired of Pierre et al. 2015 SIAM journal of Imaging Sciences.

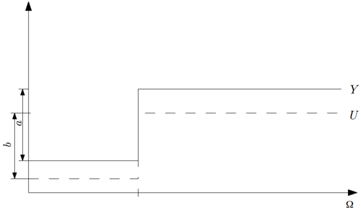
Color regularization.

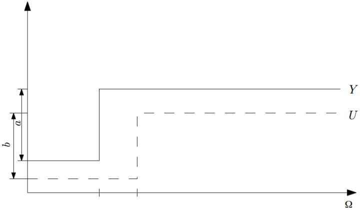
$$\hat{u} = (\hat{U}, \hat{V}) = \mathrm{argmin}_{(U,V)} \mathrm{TV}_{Y_{\mathrm{data}}}(U, V) + \alpha \int_\Omega |U(x) - U_{\mathrm{data}}(x)|^2 + |V(x) - V_{\mathrm{data}}(x)|^2 dx,$$

with

$$\mathrm{TV}_{Y_{\mathrm{data}}}(U, V) := \int_\Omega \sqrt{\gamma |\nabla Y_{\mathrm{data}}|^2 + |\nabla U|^2 + |\nabla V|^2}\, dx.$$

1D interpretation





$$\mathrm{TV}_{Y_{\mathrm{data}}} = \sqrt{\gamma a^2 + b^2} \quad \leq \quad \mathrm{TV}_{Y_{\mathrm{data}}} = \sqrt{\gamma a^2} + \sqrt{b^2}$$

Chrominance inpainting

$$\hat{u} = (\hat{U}, \hat{V}) = \operatorname{argmin}_{(U,V)} \mathrm{TV}_{Y_{\mathbf{data}}}(U, V) + \alpha \int_\Omega M \left(|U(x) - U_{\mathrm{data}}(x)|^2 + |V(x) - V_{\mathrm{data}}(x)|^2 \right) dx,$$

with

$$\mathrm{TV}_{Y_{\mathbf{data}}}(U, V) := \int_\Omega \sqrt{\gamma |\nabla Y_{\mathrm{data}}|^2 + |\nabla U|^2 + |\nabla V|^2}\, dx.$$

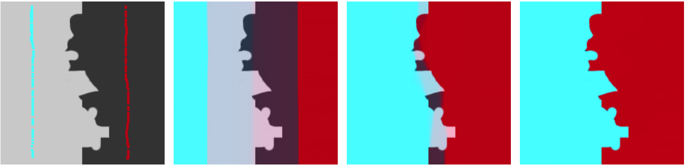
M a mask, and $(U_{\mathrm{data}}, V_{\mathrm{data}})$ some color scribbles given by the user.

Scribbles

No coupling.

With coupling.

Intuition about coupling



Scribbles. $\gamma = 0.$ $\gamma = 1.$ $\gamma = 10.$

Parameter influence.

γ small : chrominance contours have low perimeters.

Variational model for color regularization

Let us minimize the following functional with respect to (u, w) :

$$F(u,w) := TV_{\mathfrak{C}}(u) + \frac{\lambda}{2}\int_{\Omega}\sum_{i=1}^{C} w_i \|u(x) - c_i(x)\|_2^2 \, dx \\ + \chi_{\mathcal{R}}(u(x)) + \chi_{\Delta}(w(x)) \,. \quad (1)$$

The central part of this model is based on the term

$$\int_{\Omega}\sum_{i=1}^{C} w_i(x)\|u(x) - c_i(x)\|_2^2 \, dx. \quad (2)$$

Variational model for color regularization

Assume that u^* is a uniform real-valued random variable over the set $[0, 255]^2$. Let us denote $\mathcal{E}$ the canonical basis of $\mathbb{R}^C$.
The set of minimizers of

$$\int_\Omega \sum_{i=1}^{C} w_i \|u^* - c_i\|_2^2 + \chi_\Delta(w) \tag{3}$$

is reduced to a point $w^*(u^*)$ almost everywhere (*a.e.*).
Moreover, the one of :

$$\int_\Omega \sum_{i=1}^{C} w_i \|u^* - c_i\|_2^2 + \chi_\mathcal{E}(w) \tag{4}$$

is reduced to a point $w^{**}(u^*)$ *a.e.*.
When these two minimizers are unique then $w^{**}(u^*) = w^*(u^*)$.

Minimization algorithm

Primal-dual algorithm inspired by Chambolle and Pock 2011.

1: $u^0 \leftarrow \sum_{i=1}^{C} w_0^{n+1} c_i$
2: **for** $n > 0$ **do**
3: $\quad p^{n+1} \leftarrow P_{\mathcal{B}}\left(p^n + \sigma \nabla \overline{u}^n\right)$
4: $\quad w^{n+1} \leftarrow P_{\Delta}\left(w^n - \rho\lambda(\|u^{n+1} - c_i\|_2^2)_i\right)$
5: $\quad u^{n+1} \leftarrow P_{\mathcal{R}}\left(\dfrac{u^n + \tau\left(\mathrm{div}(p^{n+1}) + \lambda \sum_{i=1}^{C} w_i^{n+1} c_i\right)}{1 + \tau\lambda}\right)$
6: $\quad \overline{u}^{n+1} \leftarrow 2u^{n+1} - u^n$

Parameters ρ, τ and σ are the time steps.

No proof of convergence

Regularize the regularizer

Smoothing of the regularizer for a convergent numerical scheme (Tan, Pierre and Nikolova, preprint 2018).

Introducing some regularity for the total variation :

$$\mathrm{TV}_{Y_{\mathrm{data}}}(U, V) := \int_\Omega \sqrt{\max\{1, \gamma|\nabla Y_{\mathrm{data}}|^2\} + |\nabla U|^2 + |\nabla V|^2}\, dx.$$

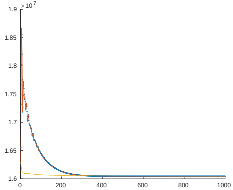
Minimization algorithm

Inertial Bregman-based proximal gradient descent for image colorization

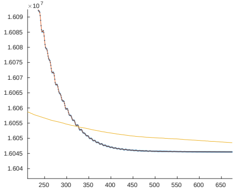
1: $u^0 \leftarrow \sum_{i=1}^{C} w_0^{n+1} c_i$
2: $u^1 \leftarrow u^0$
3: **for** $n > 0$ **do**
4: $\quad \overline{u}^n \leftarrow 2u^n - u^{n-1}$
5: $\quad \hat{u}^n \leftarrow 2u^n - u^{n-1}$
6: $\quad u^{n+1} \leftarrow P_{\mathcal{R}} \left(\dfrac{\hat{u}^n - \tau \nabla \operatorname{TV}_{\mathfrak{C}}(\overline{u}^n) + \tau\lambda \sum_{i=1}^{C} w_i^{n+1} c_i}{1 + \tau\lambda} \right)$
7: $\quad w^{n+1} \leftarrow \dfrac{w_i^n \exp\left(-\sigma\lambda \sum_{j=1}^{C} \|u^{n+1} - c_j\|_2^2\right)}{\sum_{i=1}^{C} w_i^n \exp\left(-\sigma\lambda \sum_{j=1}^{C} \|u^{n+1} - c_j\|_2^2\right)}$

- Convergence guaranteed.
- No need of projection onto simplex.

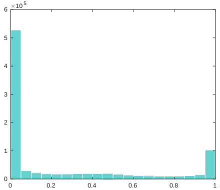
Energy comparison



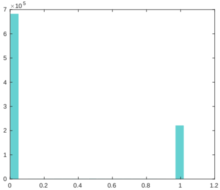
Energy
(yellow : ASAP, Primal-dual : red)



Zoom



Weights, n=500 (ASAP)

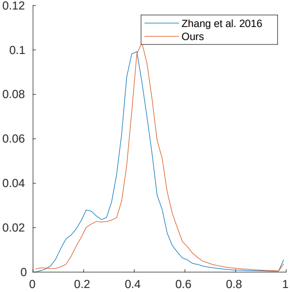


Weights, n=500 (Primal-dual)

Original image

Zhang *et al.* 2016

Our result



Results of Zhang *et al.* 2016 *vs* our (histogram of the saturation).

Average of saturation : our=**0.4228** ; Zhang *et al.*=**0.3802**.

Halo removal

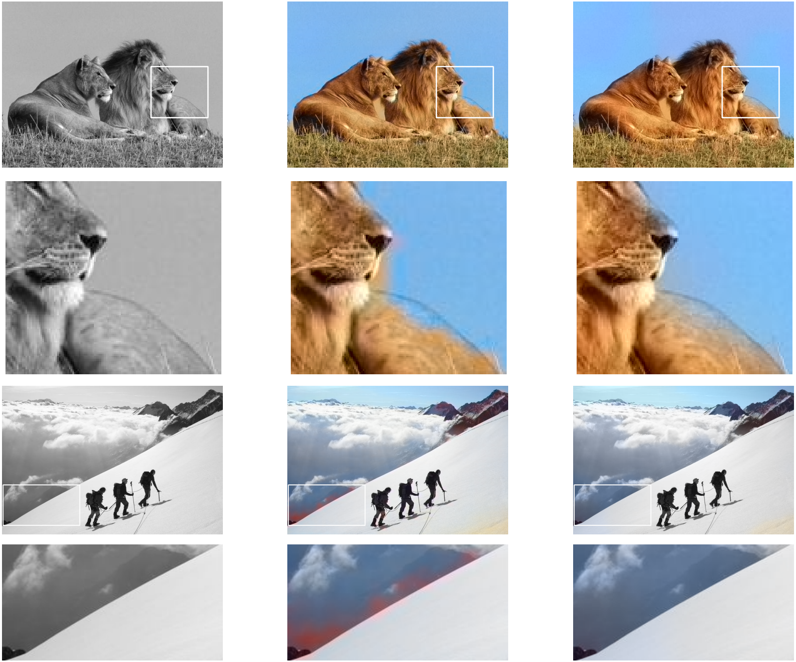
Target (input)

Result of Zhang *et al.* 2016

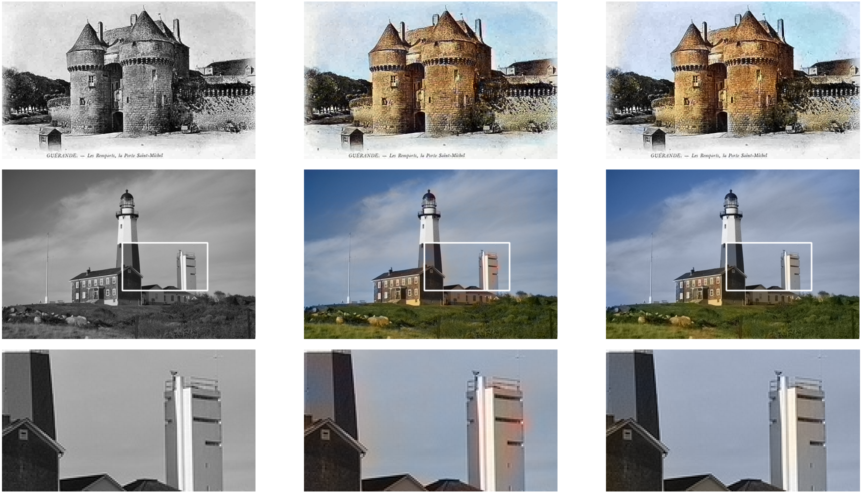
Our model

Toy example

- proof of concept ;
- ability to remove the halo effects of Zhang *et al.* 2016.



Target (input) | Result of Zhang *et al.* 2016 | Our model



Target (input) | Result of Zhang *et al.* 2016 | Our model

Limitation.

The results depend on the database.

Zhang et al. ECCV 2016, **ImageNet** (1.3 millions images)

Larsson et al. ECCV 2016, **ImageNet**

Iizuka et al. SIGGRAPH, 2016, **Places** (2.5 millions images)

Conclusion and future works :

Conclusion :

- system able to colorize images without user intervention ;
- coupling of CNN and variational model.

Further improvement :

- convergence for standard total variation with primal-dual approach and biconvex functions ;
- debiasing of the results.

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To the memory of our dear friend and colleague Mila Nikolova
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Many thanks for your attention.