Optimization meets machine learning for neuroimaging

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PARIETAL

informatics mathematics

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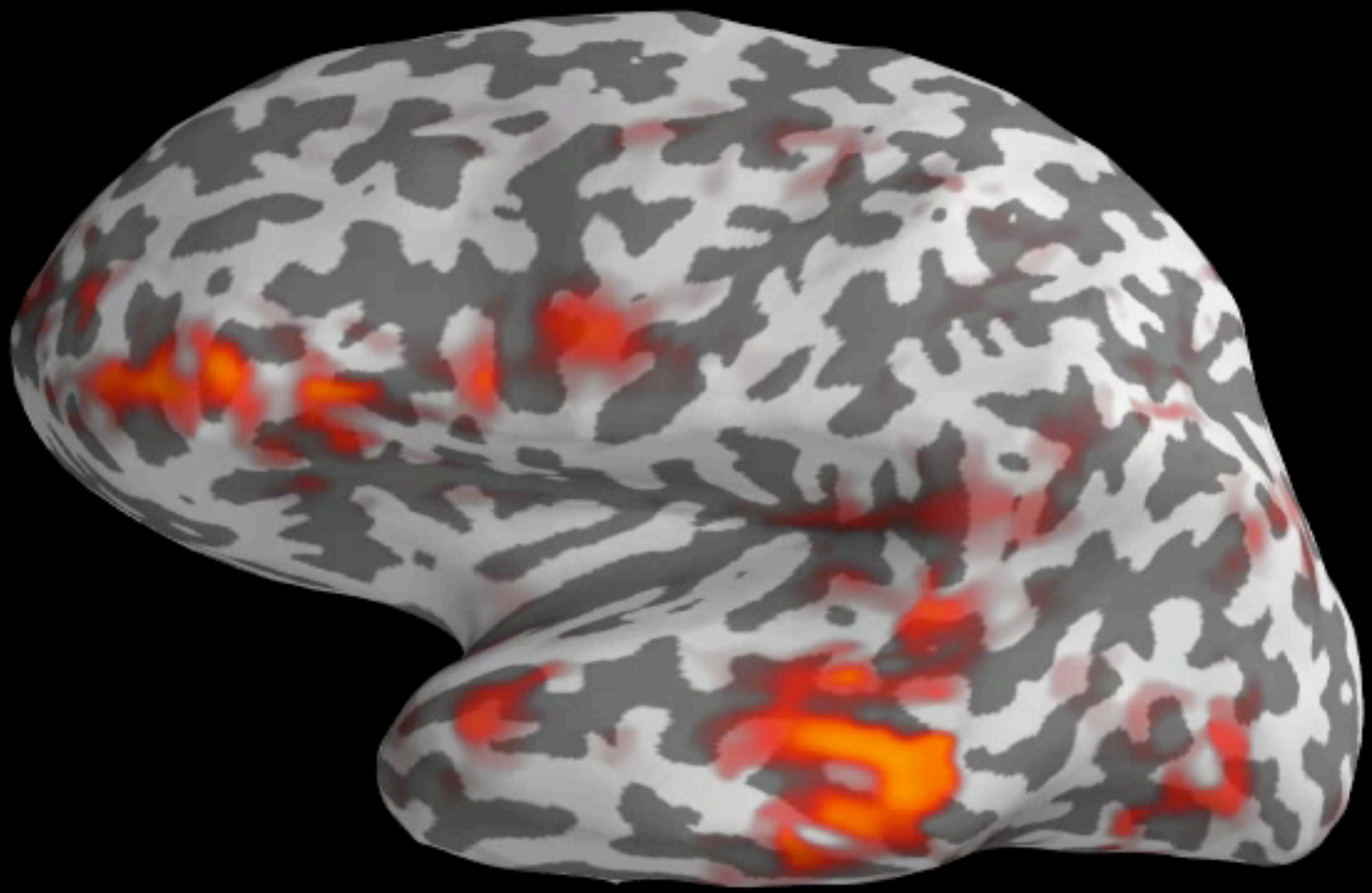
Imaging and machine learning workshop - IHP 2019

Source imaging with magnetoencephalography (MEG)



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http://youtu.be/Uxr5Pz7JPrs

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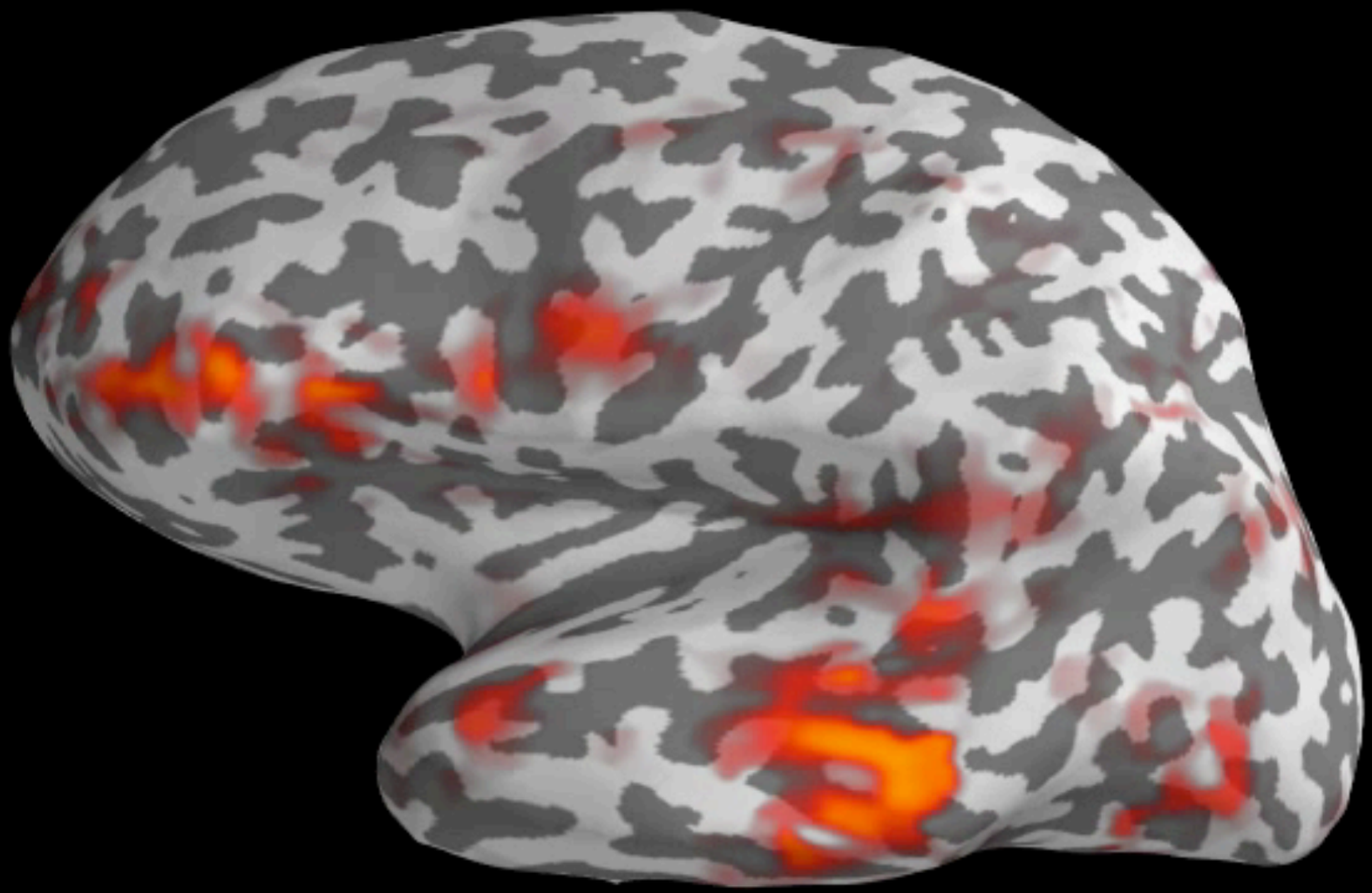

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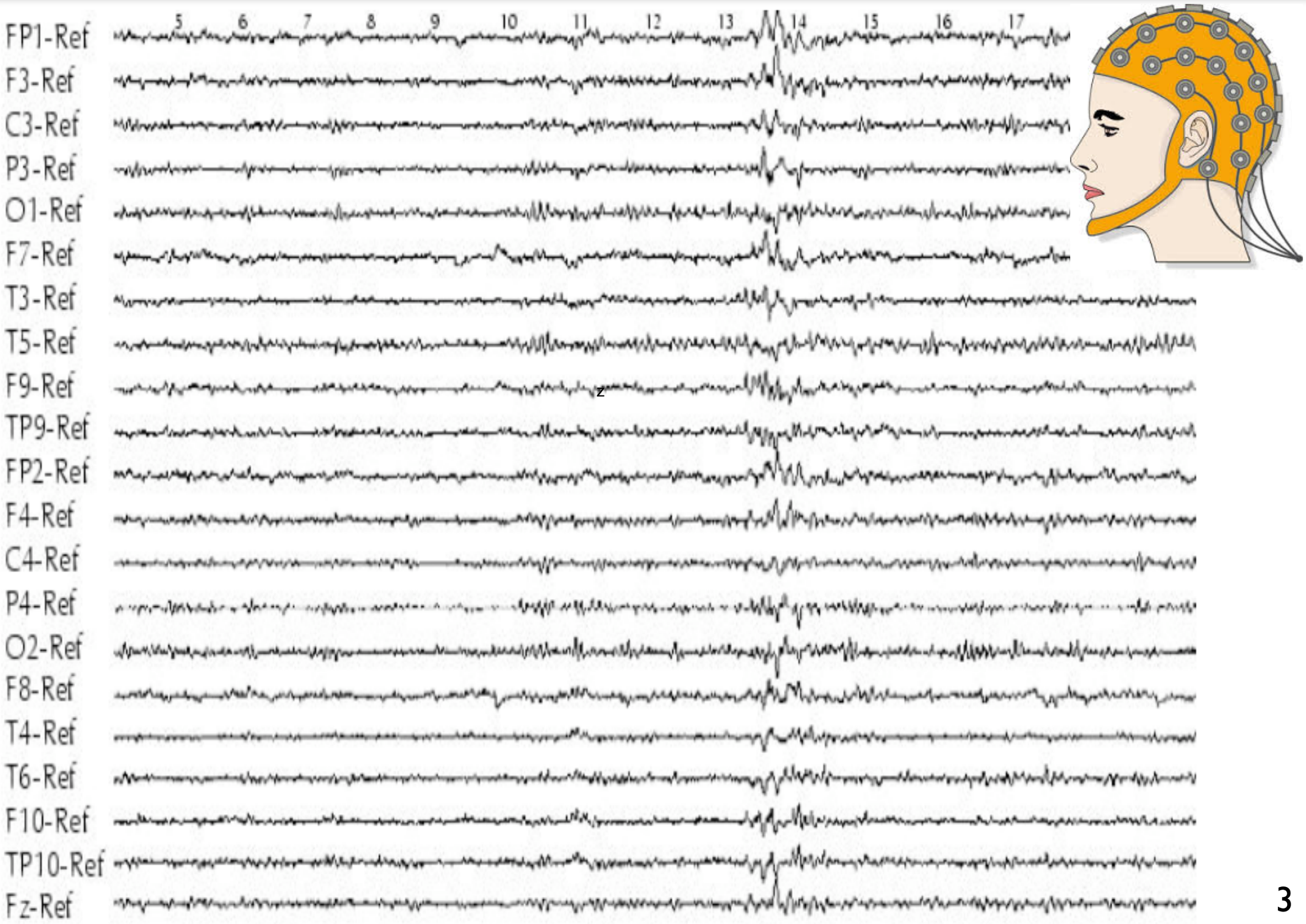
Electroencephalography (EEG))



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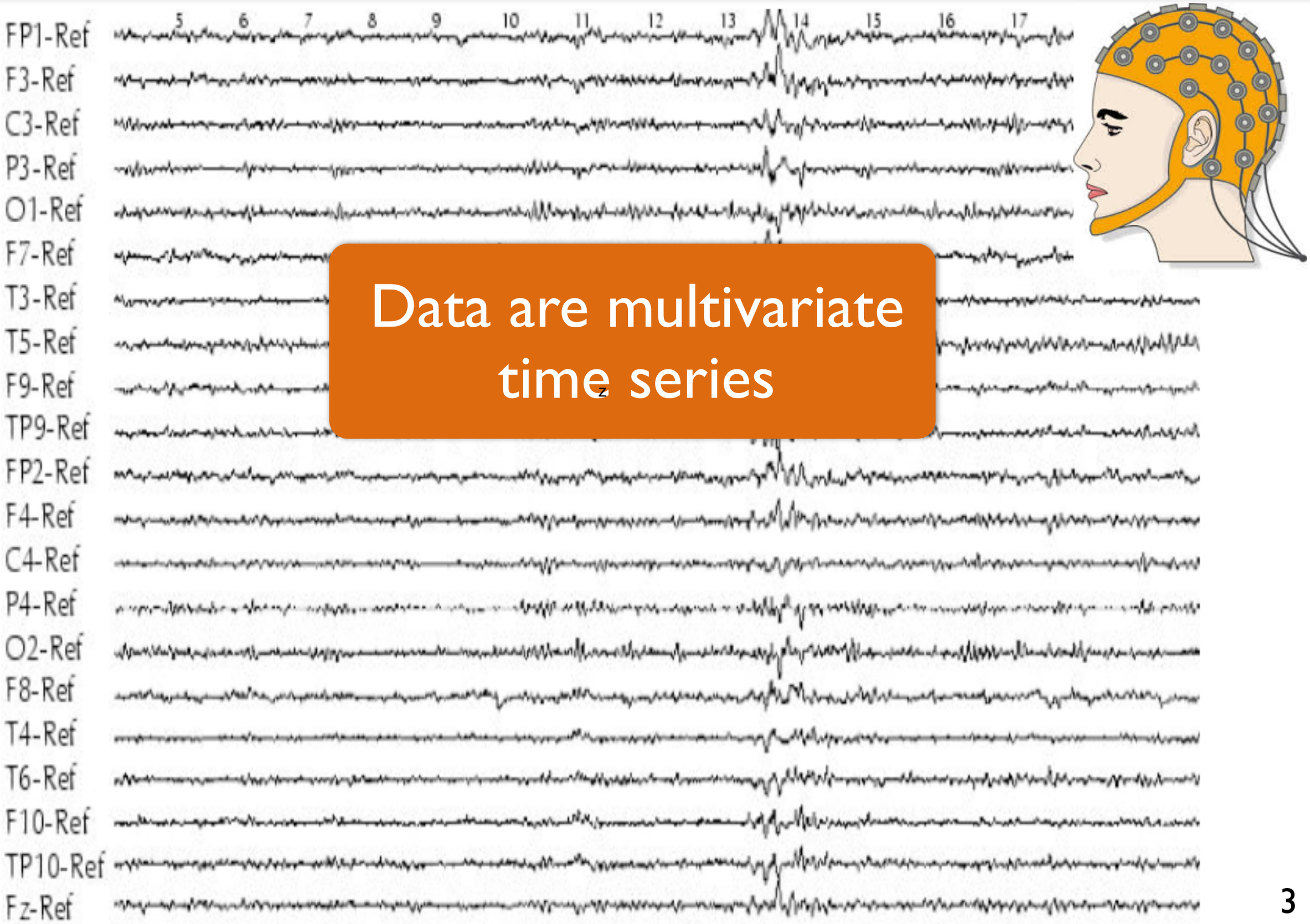
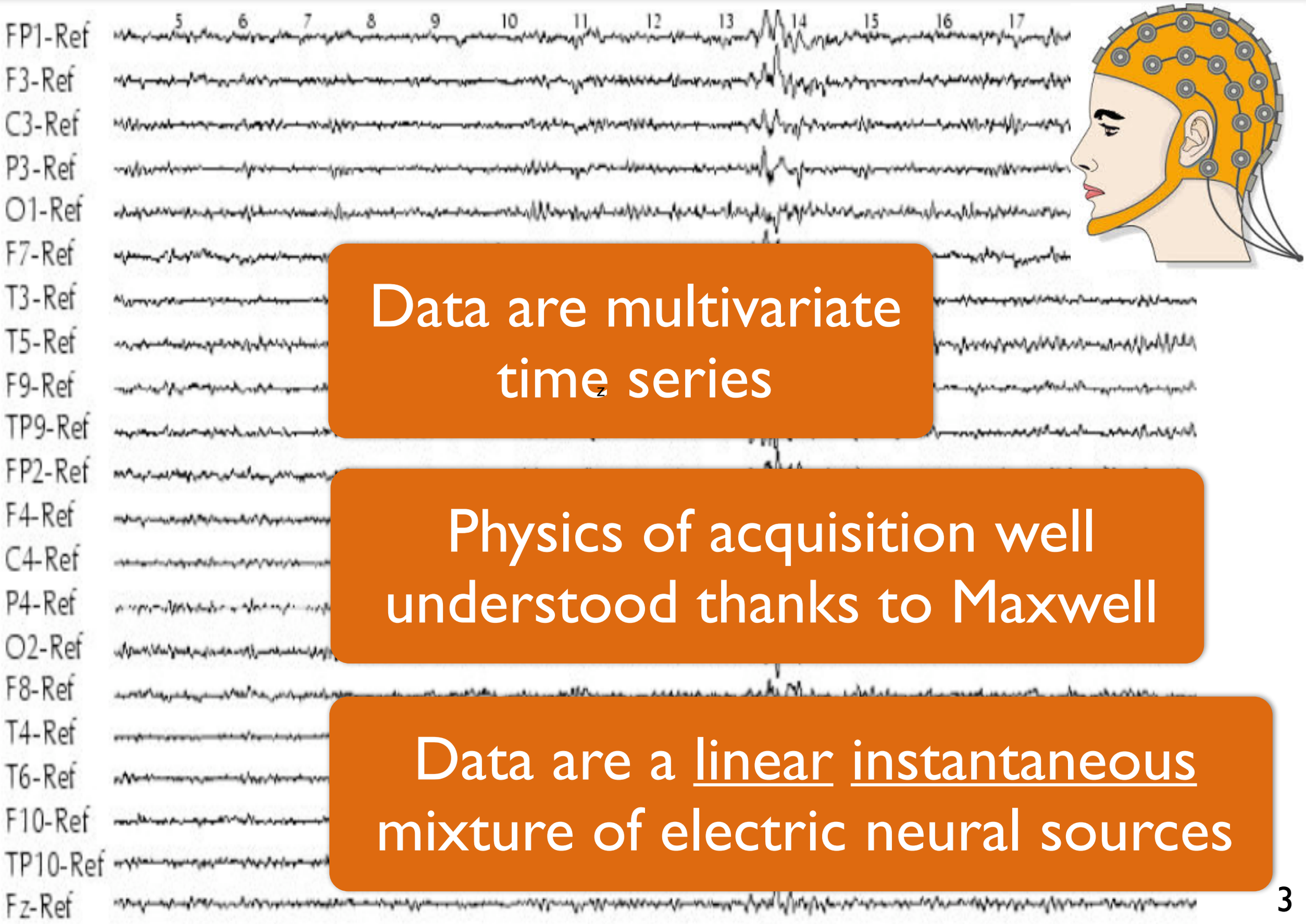


Data are multivariate time series

Electroencephalography (EEG))

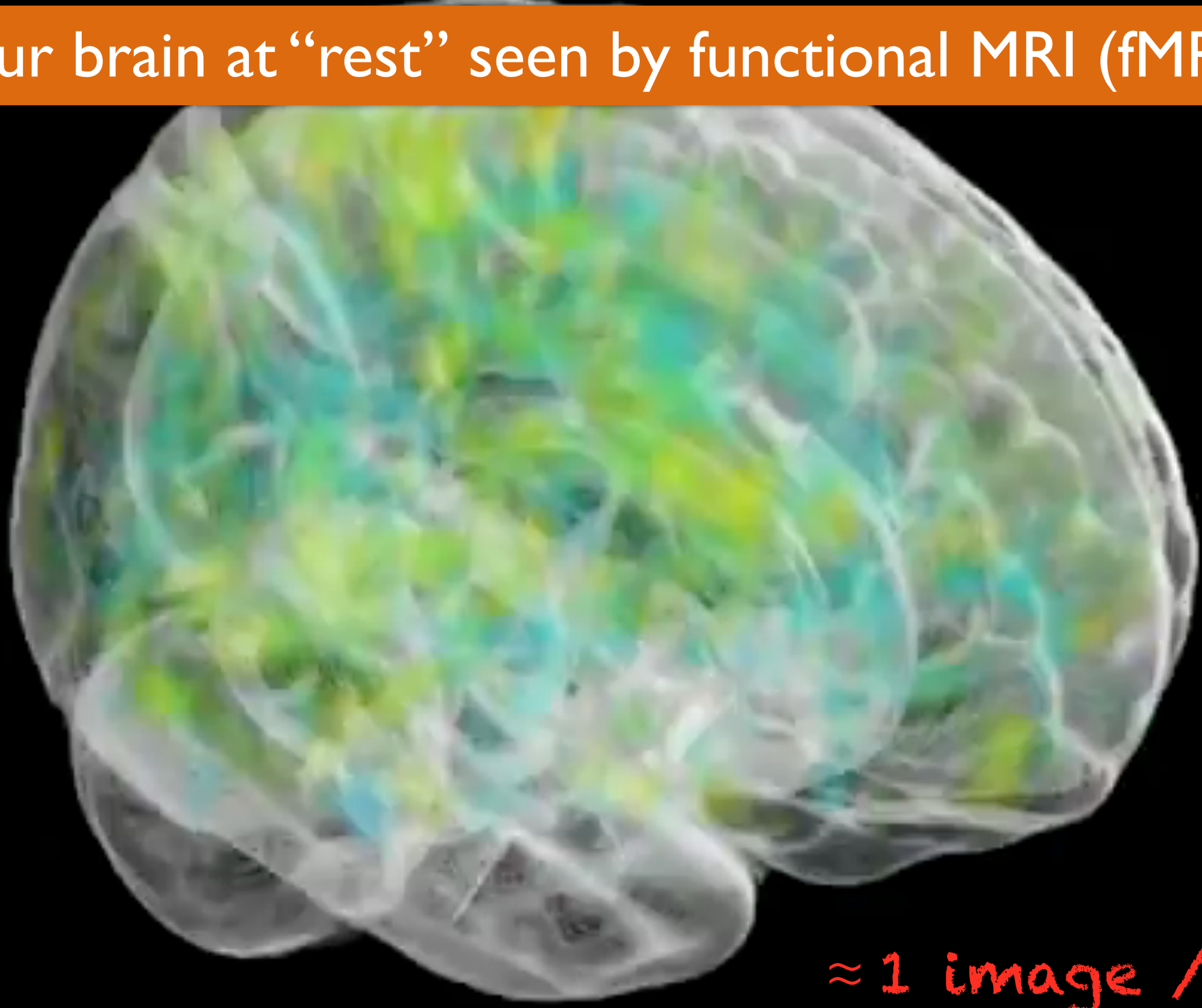


Data are multivariate time series

Physics of acquisition well understood thanks to Maxwell

Data are a linear instantaneous mixture of electric neural sources

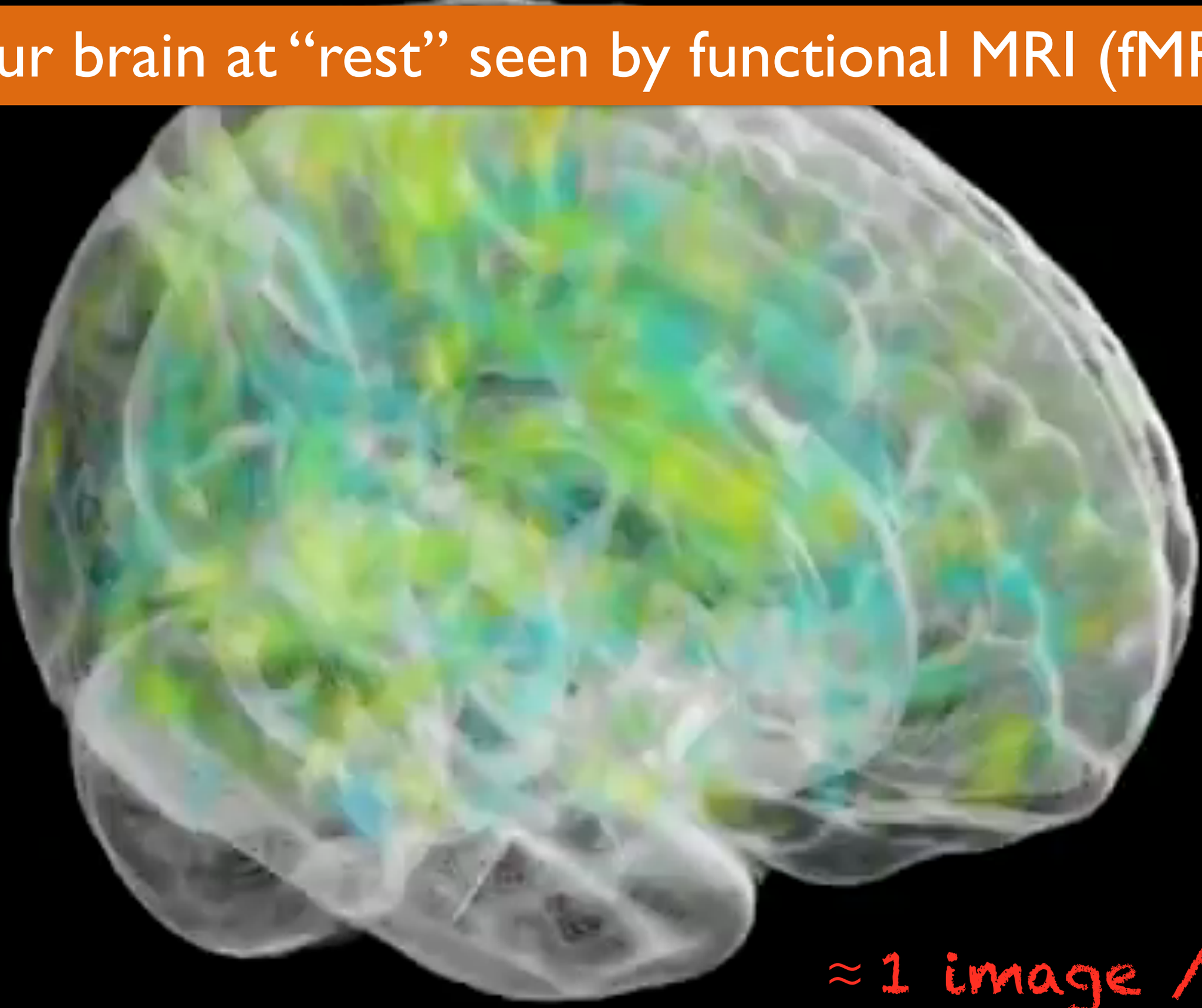
Your brain at "rest" seen by functional MRI (fMRI)



≈ 1 image / 2s

http://www.youtube.com/watch?v=uhCF-zlk0jY courtesy of Gael Varoquaux

Your brain at "rest" seen by functional MRI (fMRI)



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Question 1: What can unsupervised machine learning tell us about such data (the brain)?

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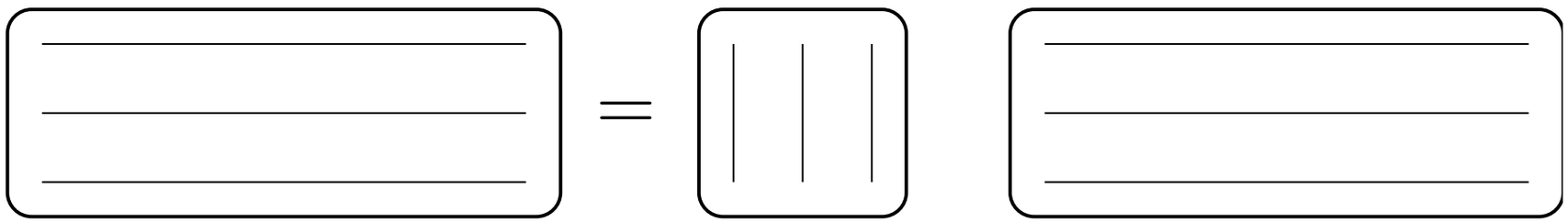
Question 2: Can we use the known physics to find adequate models?

Question 3: Can we come up with efficient learning/optimization algorithms?

Linear ICA model

- **Assumption:** Observed signals are a linear mix of independent identically distributed signals.



$$X = A \quad S$$

- N: Number of signals
- T: Number of samples
- X: Observed signals. Size N x T
- S: Independent **sources** signals. Size N x T

Both A and S are unknown

[Jutten & Herault 91]

ICA vs Dictionary Learning

- **ICA:**

$$X = AS$$

Uses independence assumption

- **Dictionary learning:**

$$\min_{A,S} \|X - AS\|_F^2 + \lambda \|S\|_p \qquad p \leq 1$$

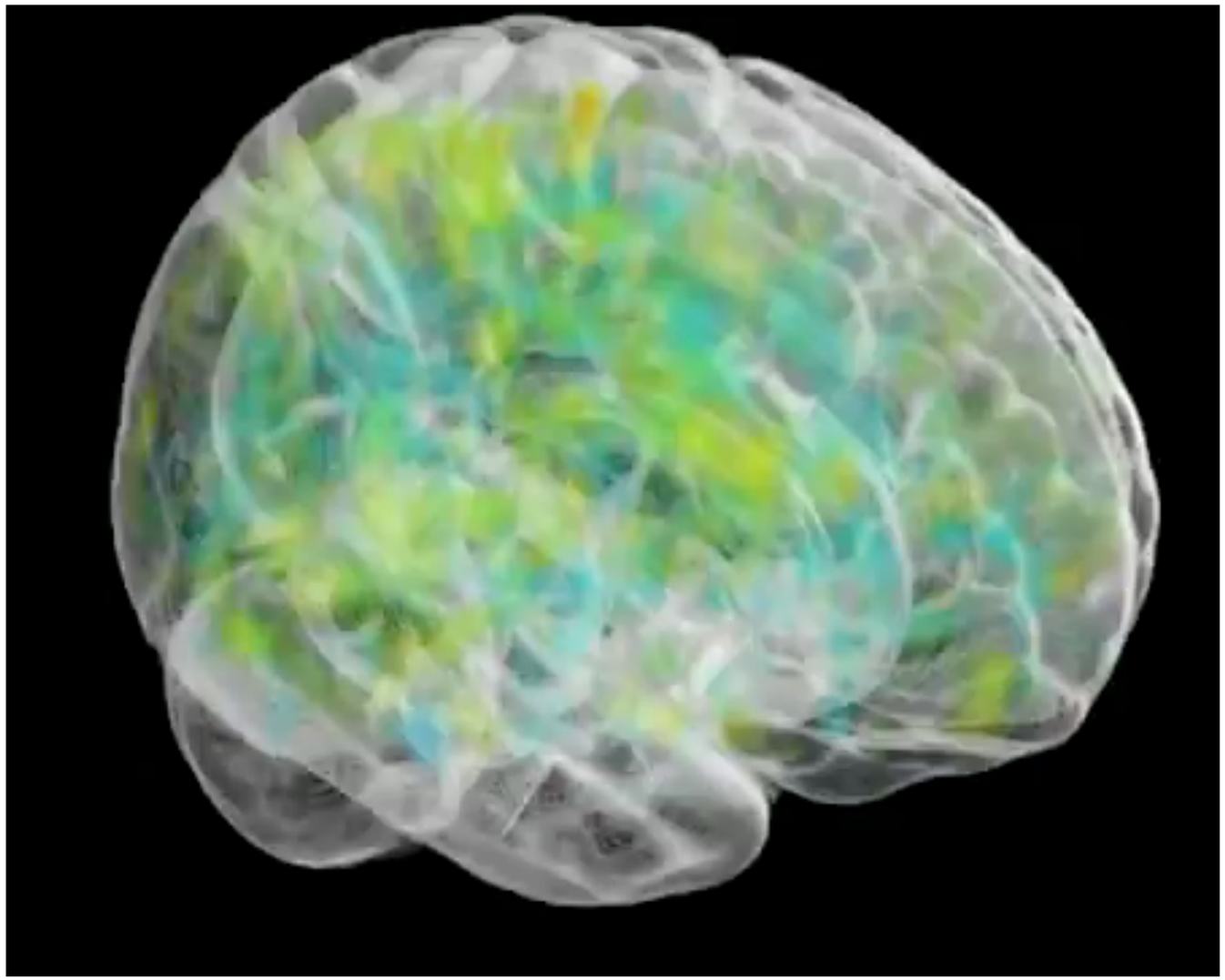
$$\text{s.t. } \|a_j\| \leq 1$$

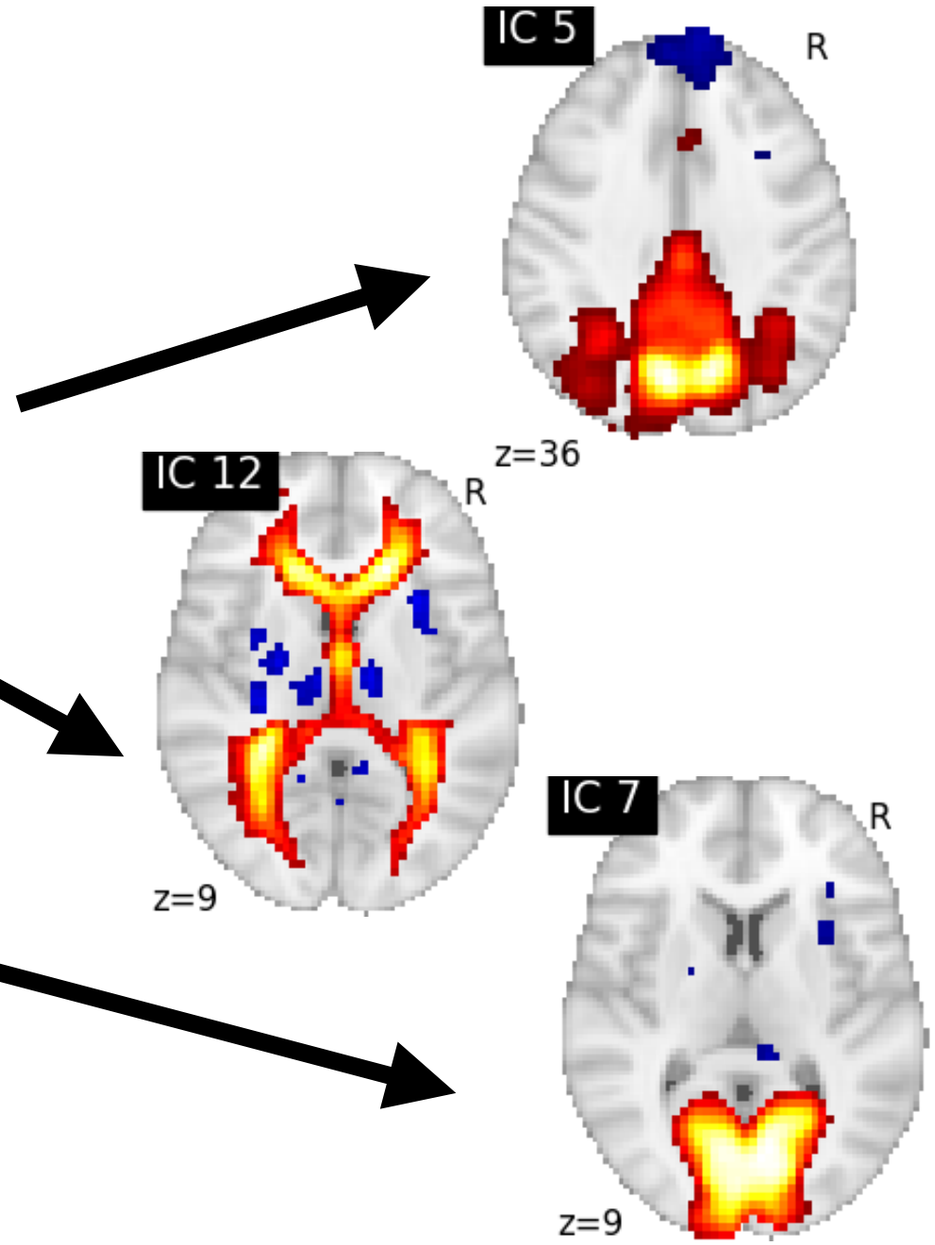
Uses sparsity assumption

[Olshausen et al., Elad et al., Mairal et al. etc…]

ICA on fMRI data

- Used in neuroimaging to find "brain networks" (not neural networks…) in functional MRI data





Source: http://nilearn.github.io/auto_examples/03_connectivity/plot_canica_resting_state.html

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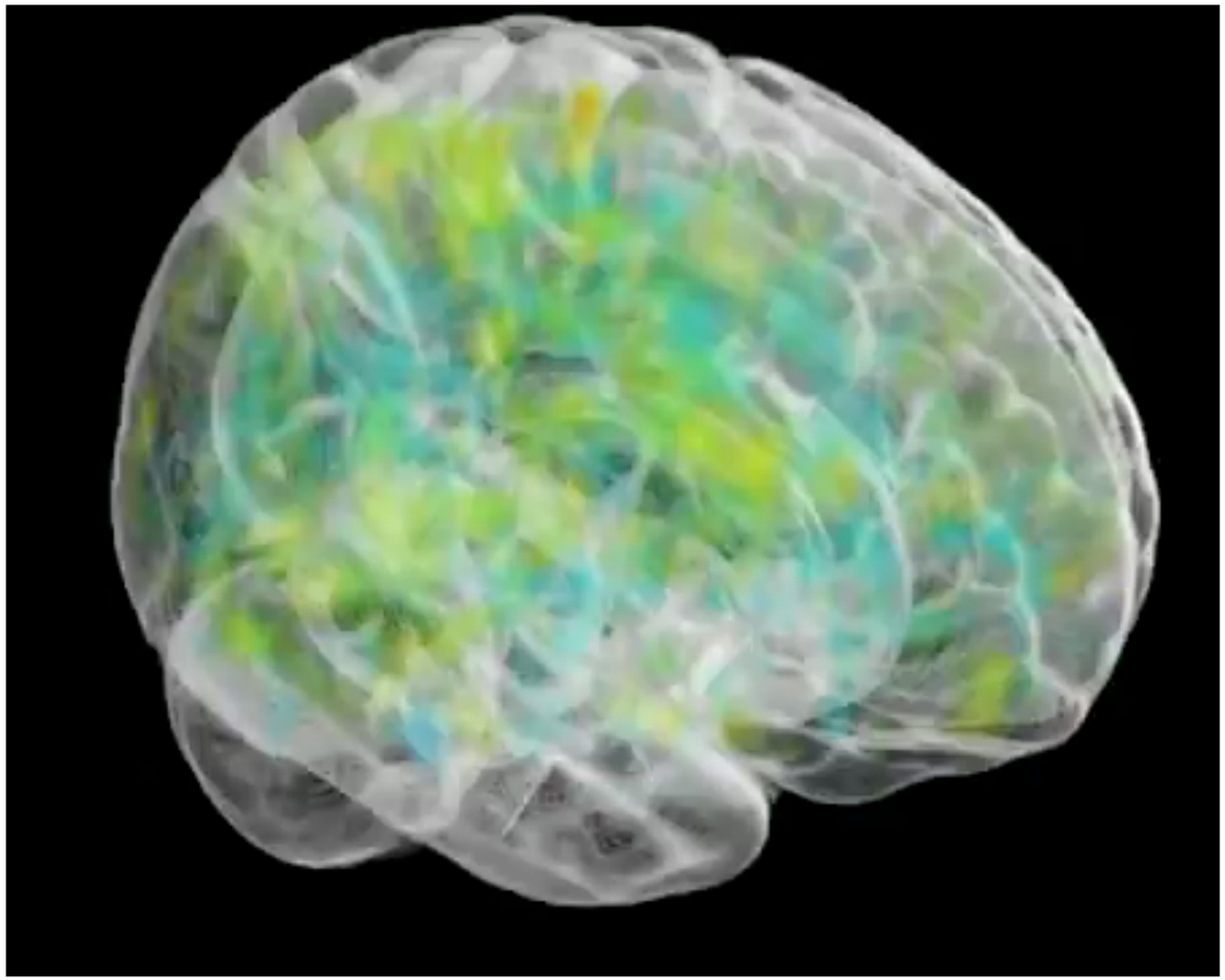

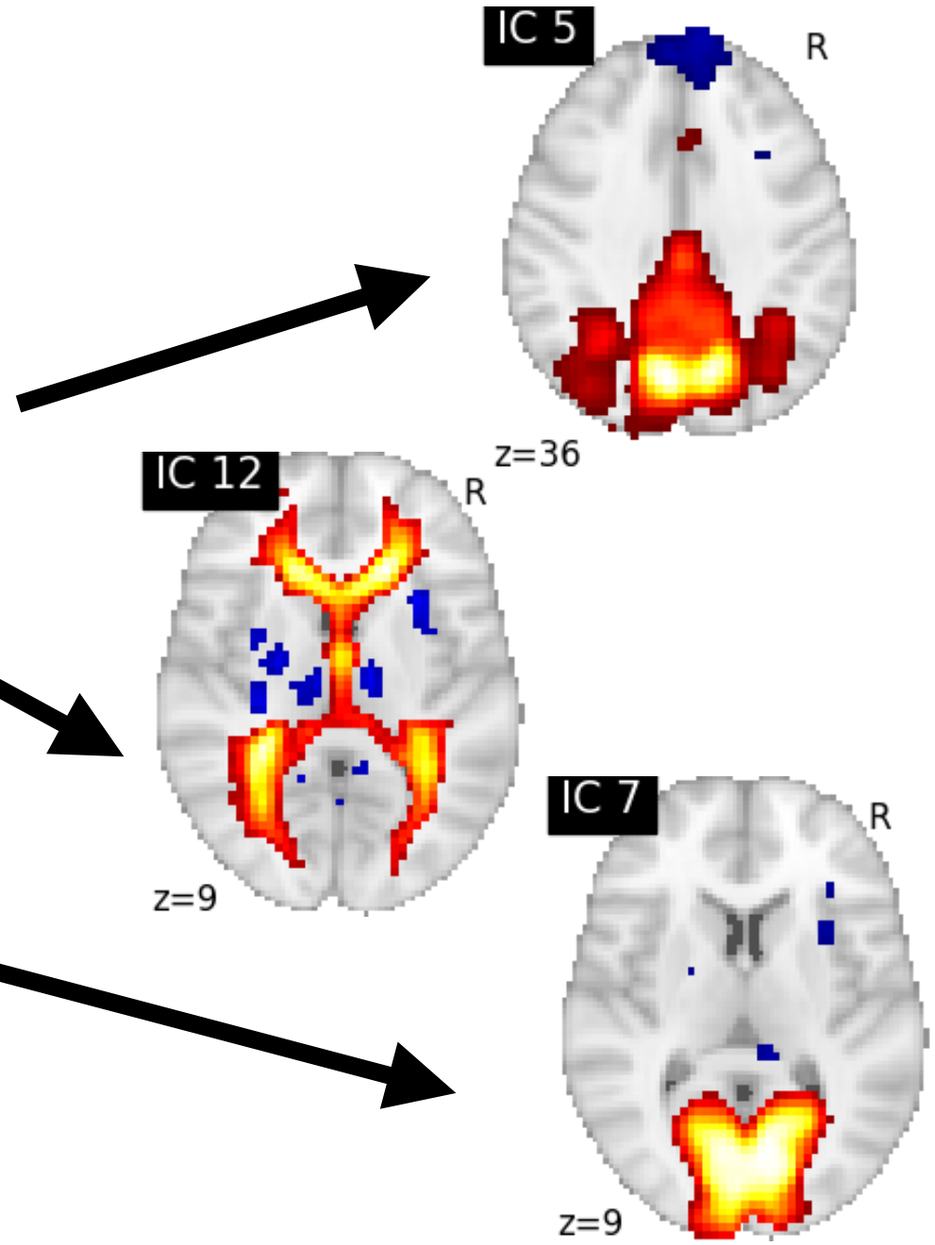

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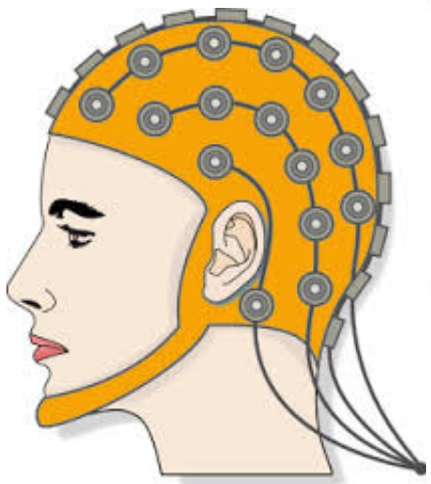
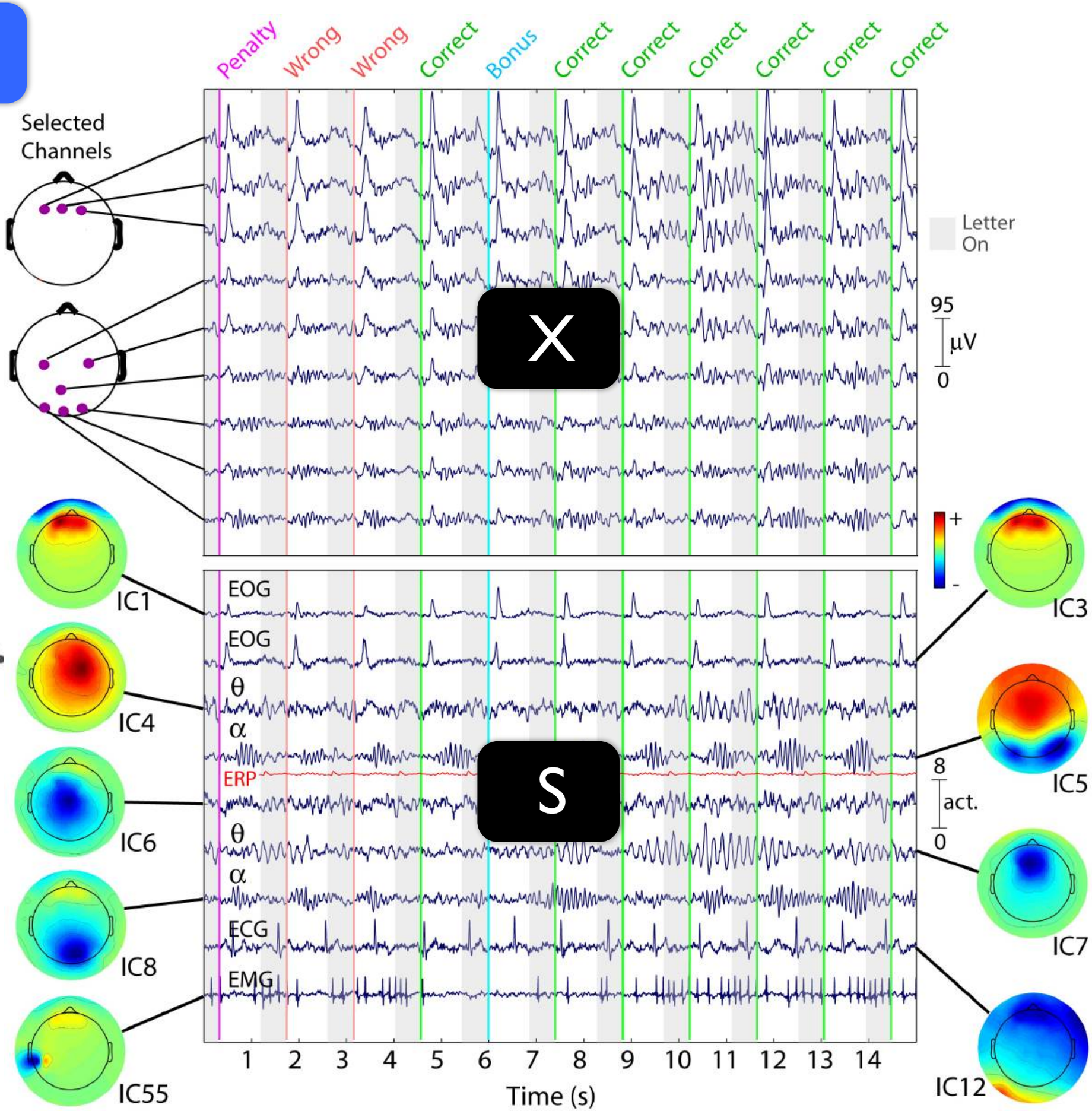
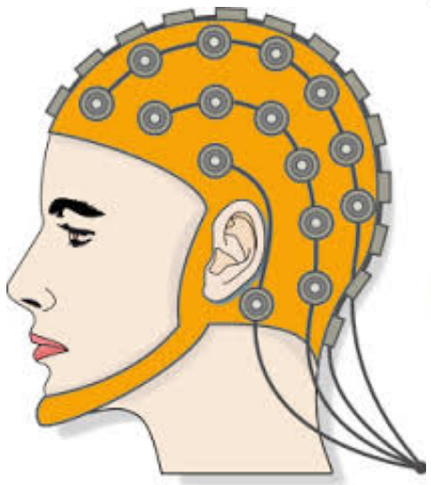
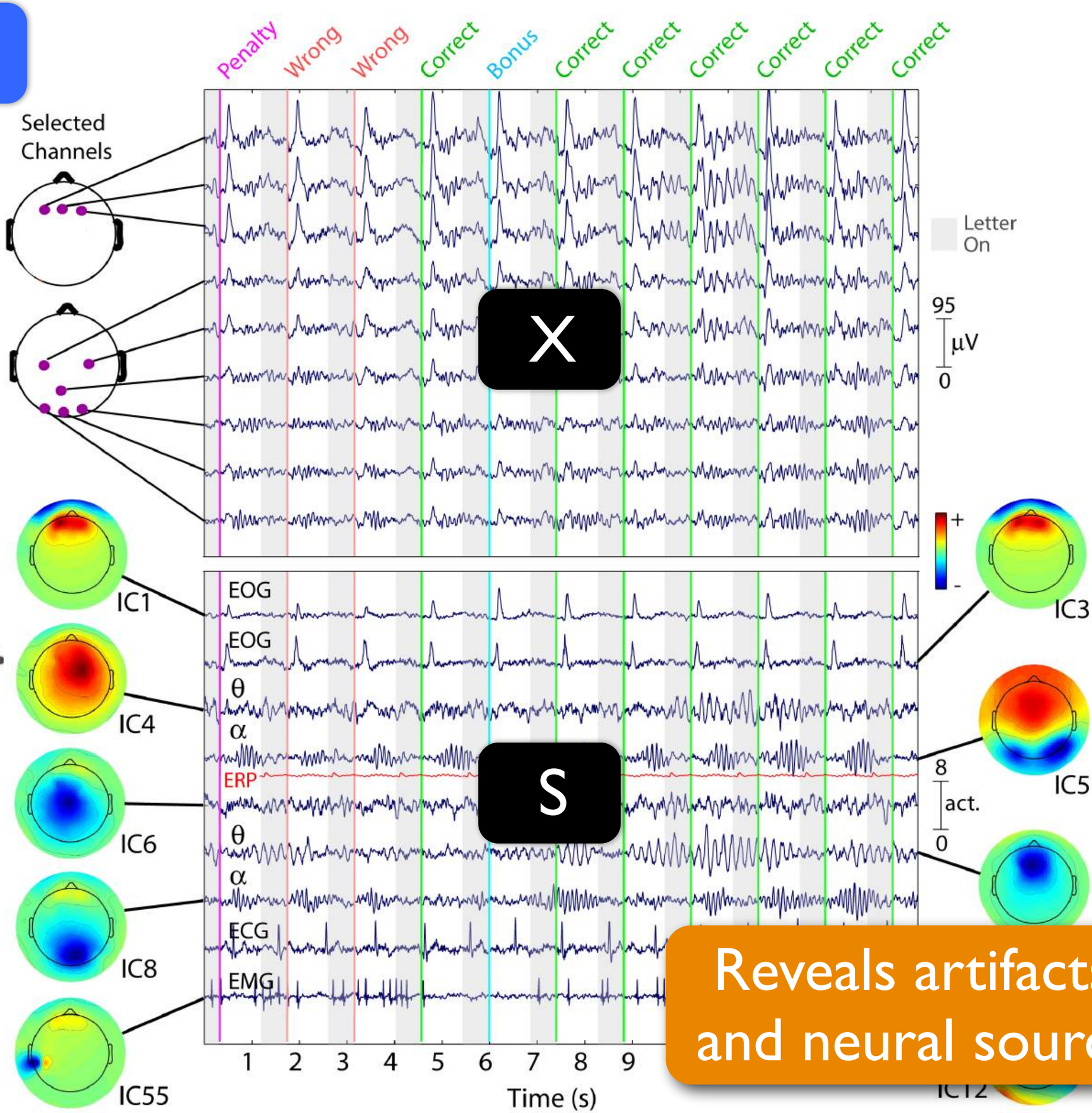
On EEG




Penalty
Wrong
Wrong
Correct
Bonus
Correct
Correct
Correct
Correct
Correct
Correct
Selected Channels
X
Letter On
95
μV
0
EOG
EOG
θ
α
ERP
S
θ
α
ECG
EMG
8
act.
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Time (s)
IC1
IC3
IC4
IC5
IC6
IC7
IC8
IC12
IC55

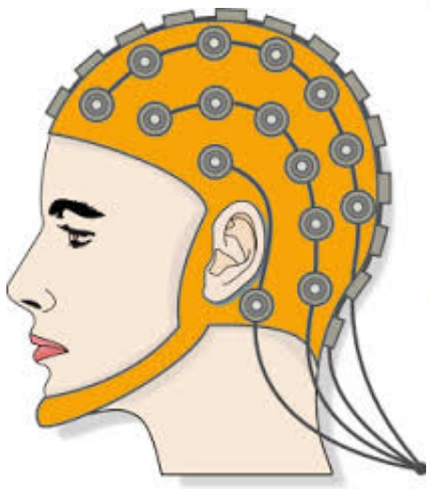
On EEG

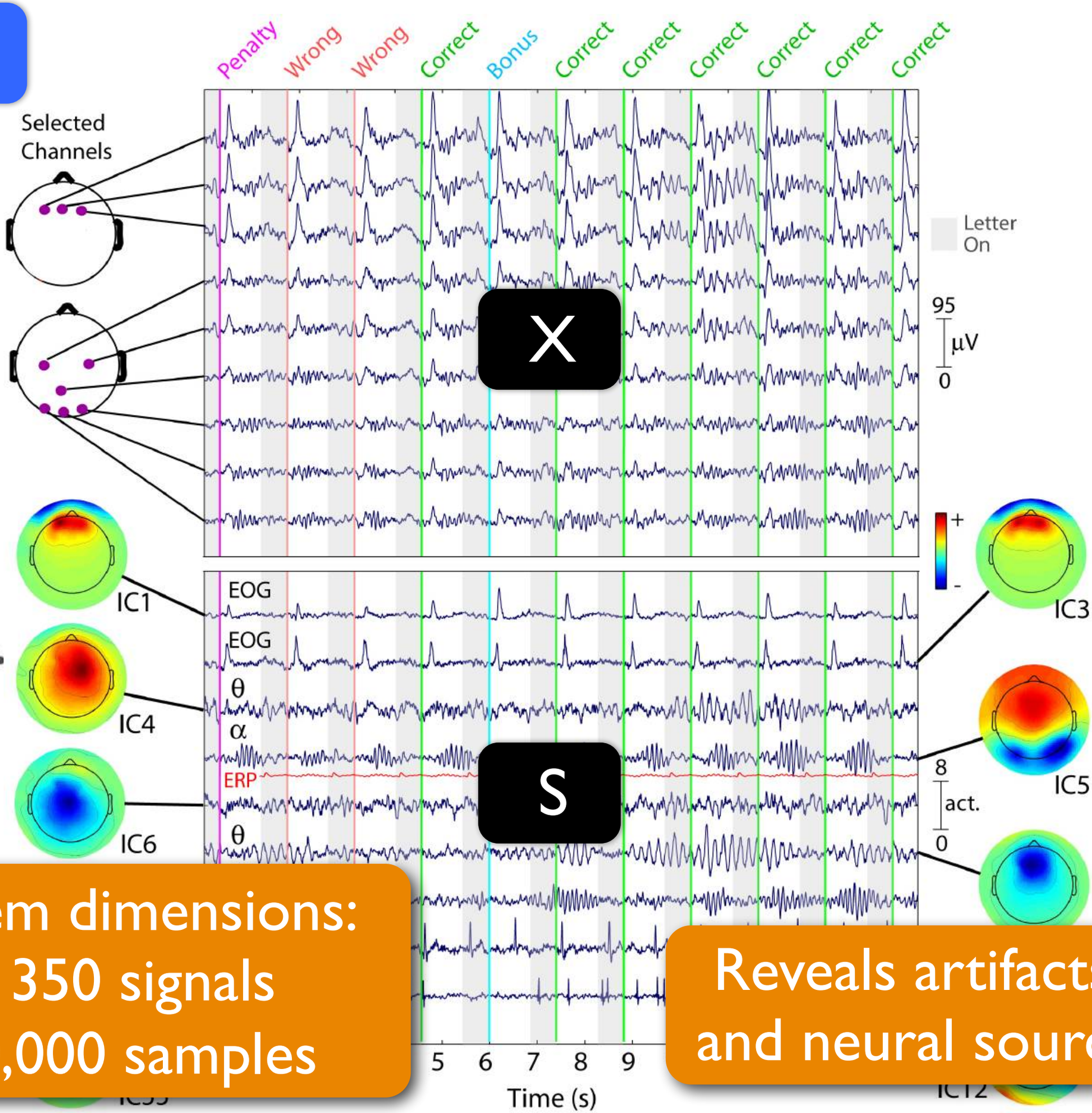




Reveals artifacts and neural sources

On EEG





Problem dimensions:

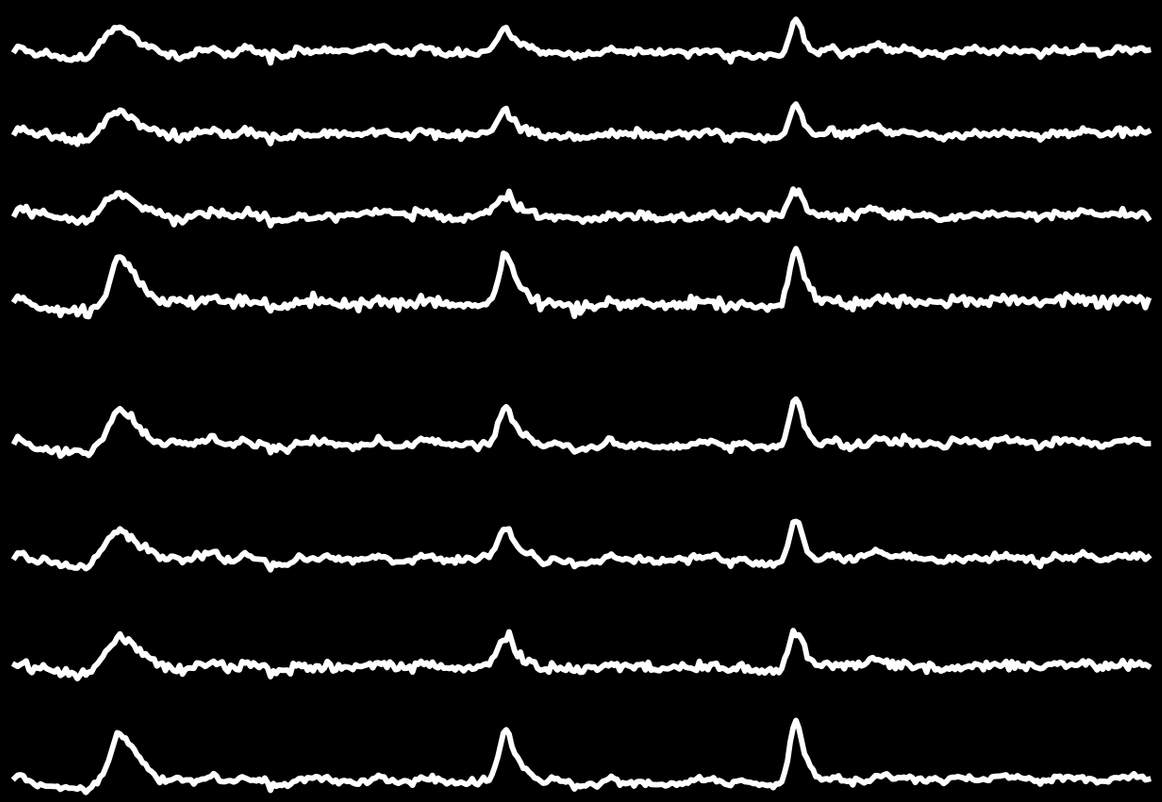
- 30 to 350 signals
- ≈100,000 samples

Reveals artifacts and neural sources

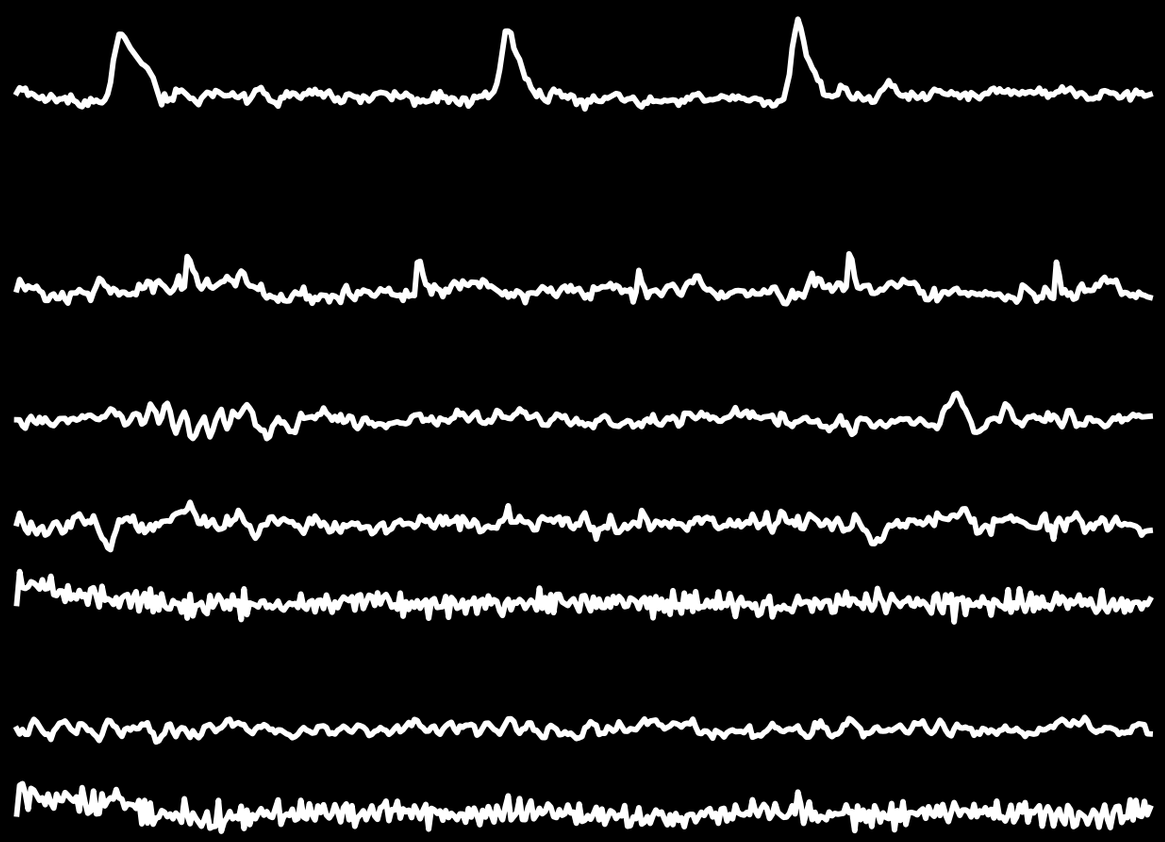
1

How to accelerate Independent Component Analysis (ICA) solvers?

Observations (raw EEG)



ICA recovered sources



[Faster independent component analysis by preconditioning with Hessian approximations, P. Ablin, J.-F. Cardoso & A. Gramfort 2017 IEEE Trans. Signal Processing]

[EM algorithms for ICA, P. Ablin, A. Gramfort, J.F. Cardoso and F. Bach, AISTATS 2019]

Code: https://pierreablin.github.io/picard

Statistical Principles of ICA

$$X = AS$$

Objective:

Find W s.t. $Y = WX$ has maximally independent rows.

Note:

If more than 2 Gaussian sources, ICA problem is not possible.

[Comon 94]

Remark:

Different ways to quantify independence lead to different methods.

Maximum likelihood & ICA

Maximum Likelihood Formulation

- **Model**: $Y = WX$ has independent rows

$$Y = [y_1, \ldots, y_N]^\top \qquad p_i \text{ p.d.f. of } y_i$$

- Independence:

$$p(Y(t)) = p_1(y_1(t))p_2(y_2(t)) \ldots p_N(y_N(t))$$

- Likelihood of a time sample of X:

$$p(X(t)) = |\det(W)|p_1(y_1(t))p_2(y_2(t)) \ldots p_N(y_N(t))$$

Maximum Likelihood Formulation

ICA = minimizing the averaged negative log-likelihood:

$$\mathcal{L}(W) = -\log|\det(W)| - \sum_{i=1}^{N} E_x[\log(p_i([Wx]_i))]$$

[Pham & Garat 1997]

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ICA boils down to minimizing an objective function

- Infomax model *[Bell & Sejnowski 1995]*

$$\psi_i(\cdot) = -\log(p_i(\cdot))' = \tanh(\cdot/2)$$

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Most widely used ICA technique in neuroscience

Geometry of the problem

- non-convex (multiple minima)
- Optimization on the invertible matrices manifold
- Use of **relative changes** in solver iterations:

Absolute

$$W_{n+1} = W_n + dW$$

Relative

$$W_{n+1} = (I + \mathcal{E})W_n$$

n: iteration number

Relative gradient / Hessian

Relative matrix form Taylor expansion:

$$\mathcal{L}((I+\mathcal{E})W) = \mathcal{L}(W) + \langle G|\mathcal{E}\rangle + \frac{1}{2}\langle \mathcal{E}|H|\mathcal{E}\rangle + \mathcal{O}(||\mathcal{E}||^3)$$

- Gradient is a $N \times N$ matrix:

$$G_{ij} = E[\psi_i(y_i)y_j] - \delta_{ij}$$

closed form solutions

- Hessian is a $N \times N \times N \times N$ 4th order tensor.

$$H_{ijkl} = \delta_{il}\delta_{jk} + \delta_{ik}E[\psi_i'(y_i)y_j y_l]$$

[Cardoso & Laheld 1996]

(relative) Newton method

$$H_{ijkl} = \delta_{il}\delta_{jk} + \delta_{ik}E[\psi_i'(y_i)y_j y_l]$$

One iteration: $$W_{n+1} = (I - H^{-1}G)W_n$$

Problem:

Large linear system / not positive (regularization needed)

Newton method is possible but not practical

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Newton method is possible but not practical

but if model holds:

$$\delta_{ik}E[\psi_i'(y_i)y_j y_l] \;=\; \delta_{ik}\delta_{jl}E[\psi_i'(y_i)]E[y_j^2]$$

Hessian approximations

True hessian:

$$H_{ijkl} = \delta_{il}\delta_{jk} + \delta_{ik}E[\psi_i'(y_i)y_j y_l]$$

- If we **suppose that the signals are independent**
- Two Hessian approximations :

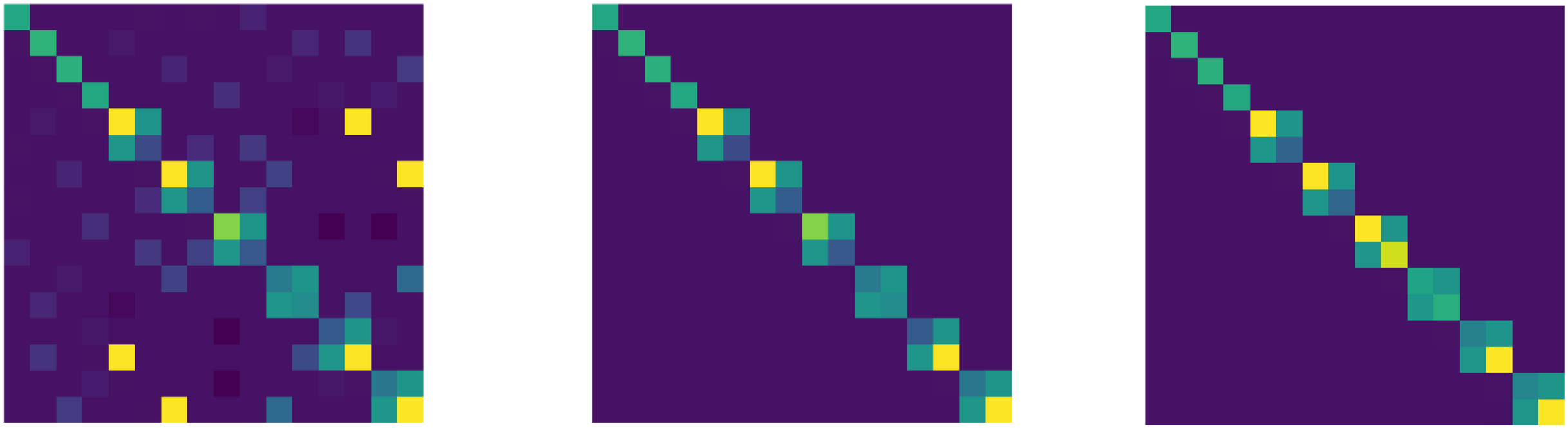
Complexity:

$$H^2_{ijkl} = \delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}E[\psi_i'(y_i)y_j^2] \qquad \text{O(T N}^2\text{)}$$

$$H^1_{ijkl} = \delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}E[\psi_i'(y_i)]E[y_j^2] \qquad \text{O(T N)}$$

Hessian approximations (cont.)

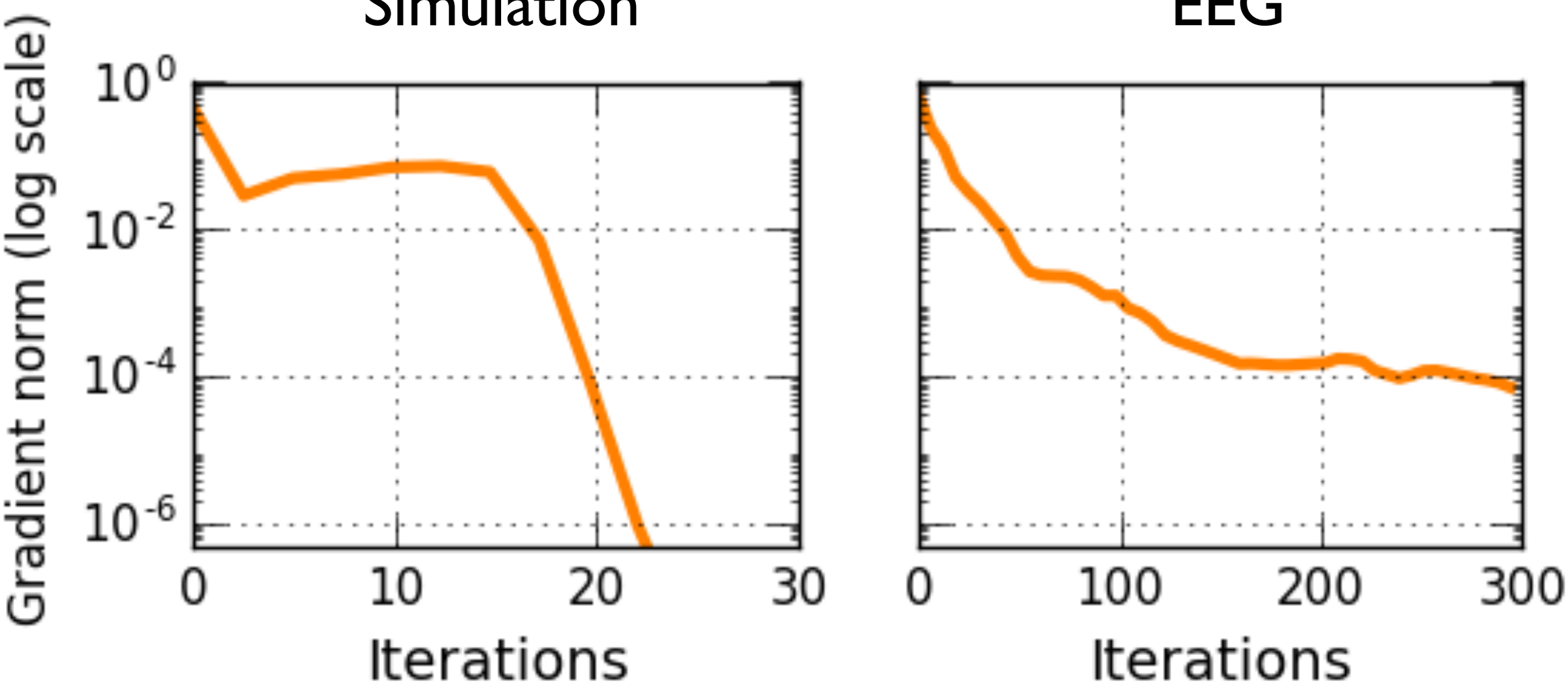
H H^2 H^1



- Hessians approximations are block-diagonal hence much easier to invert
- **Idea:**

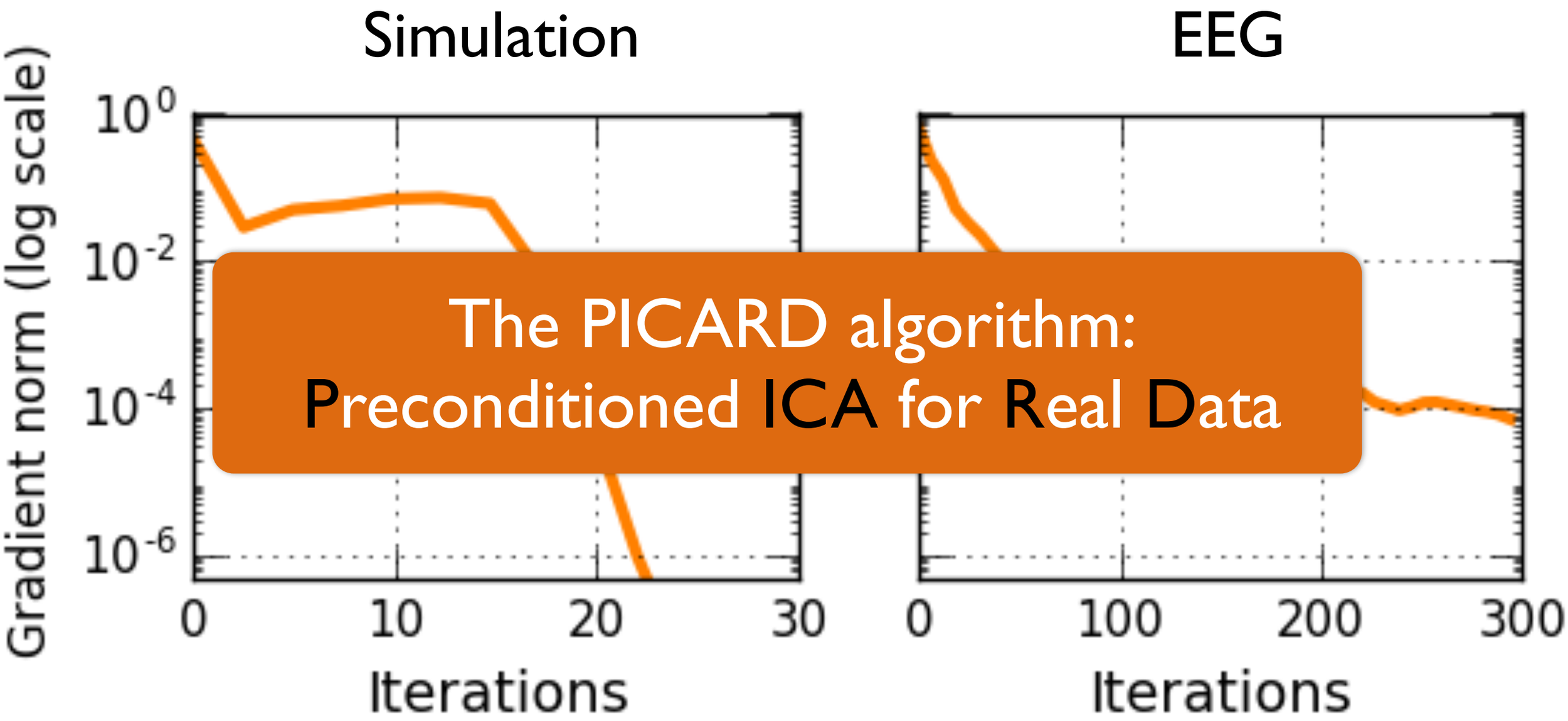
Do Newton with the approximated Hessian !

But…



It does not work with real data !

But…



The PICARD algorithm:
Preconditioned ICA for Real Data

It does not work with real data !

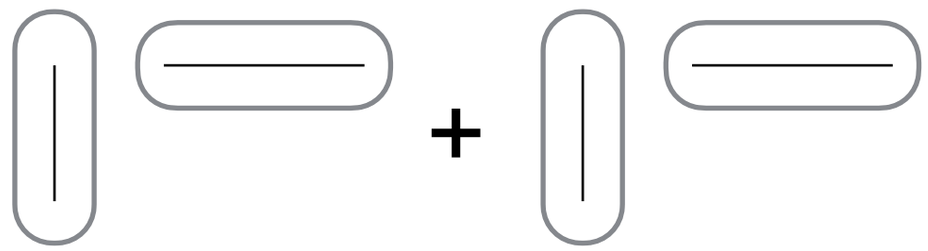
Can these guys help us retrieve missing information about curvature without computing the full Hessian ?

- C. G. Broyden, « The Convergence of a Class of Double-rank Minimization Algorithms », Journal of the Institute of Mathematics and Its Applications, 1970
- R. Fletcher, « A New Approach to Variable Metric Algorithms », Computer Journal, 1970
- D. Goldfarb, « A Family of Variable Metric Updates Derived by Variational Means », Mathematics of Computation, 1970
- D. F. Shanno, « Conditioning of Quasi-Newton Methods for Function Minimization », Mathematics of Computation, 1970

Picard relies on L-BFGS algorithm

- BFGS builds an approximation of the Hessian using only the past function and gradient evaluations

- Does low-rank corrections of an initial guess $H^{(0)}$ of the Hessian

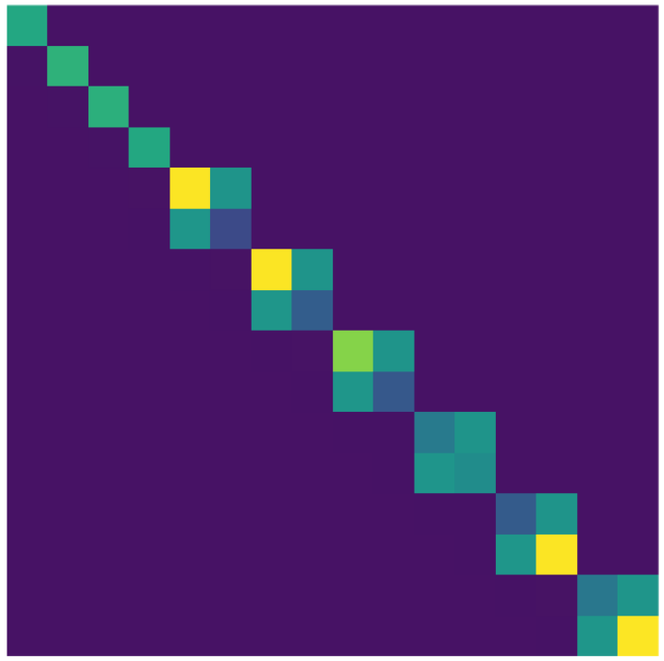
$H^{(q)}$ =  + $H^{(q-1)}$ (rank-2 updates)

- Initial guess $H^{(0)}$ should be easy to invert and is commonly a multiple of identity

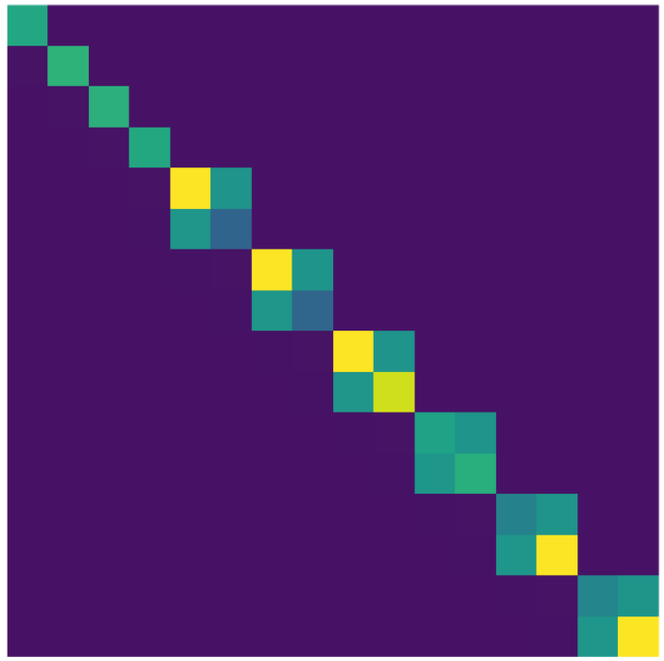
[Liu, D. C., & Nocedal, J. « On the limited memory BFGS method for large scale optimization. » *Mathematical programming, 1989]*

L-BFGS algorithm with Hessian approx.

H^2



H^1



Use these matrices as $H^{(0)}$ in L-BFGS

[P. Ablin, J.-F. Cardoso, A. Gramfort, Faster ICA by preconditioning with Hessian approximations, IEEE Trans. Signal Processing]

What are other state-of-the art solvers?

- Stochastic (relative) gradient a.k.a. Infomax

[Bell and Sejnowski 1995]

- Batch (relative) gradient descent
- Truncated Newton with full Hessian (using conjugate gradient solver and "Hessian free" products)

[Tillet et al. 2017]

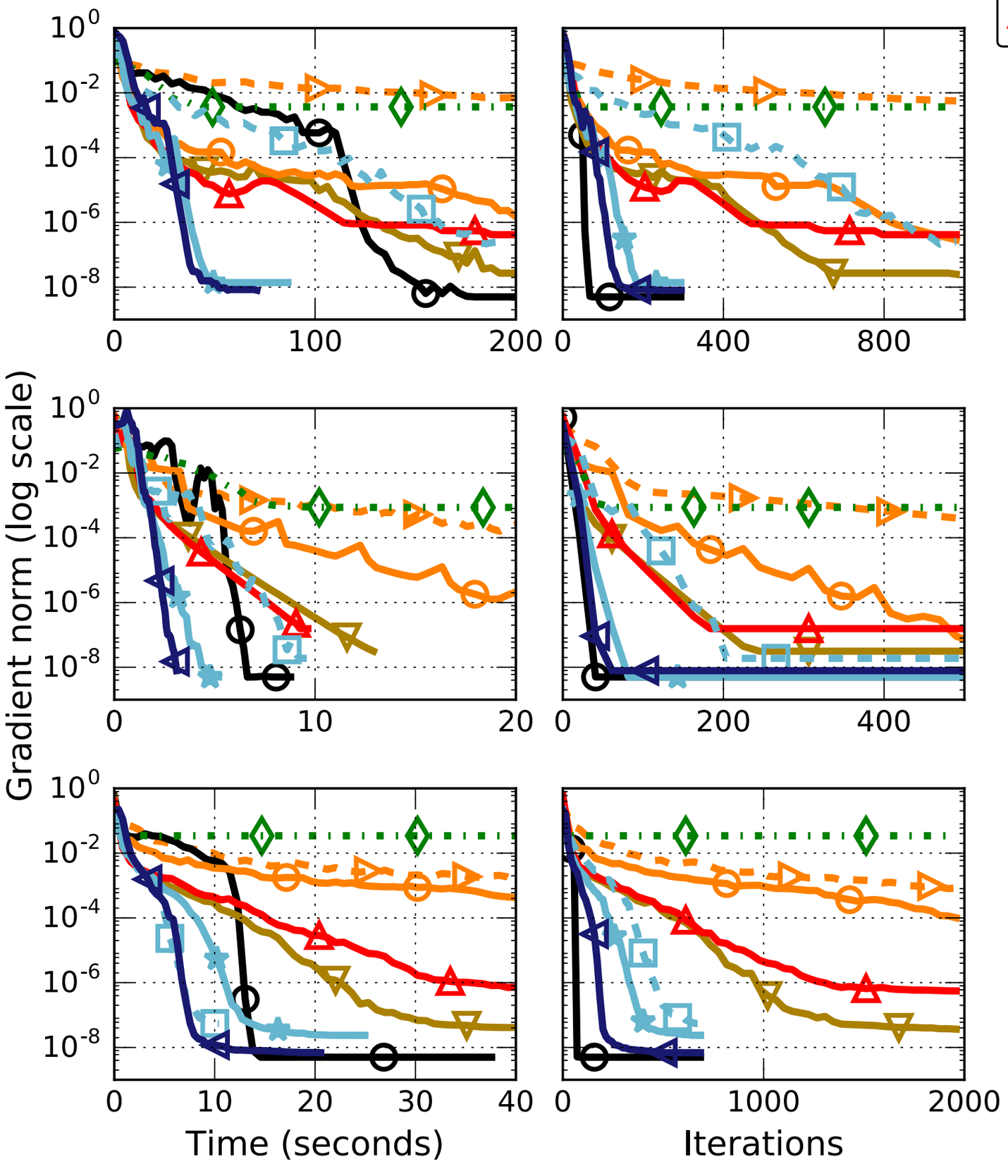
- Quasi-Newton with H1 or H2 Hessians

close to [Palmer et al. 2012]

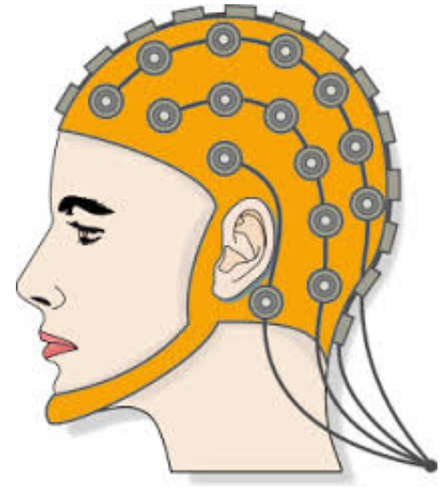
- Trust region ICA with H2 approximation

[Choi et al. 2007]

Real data



EEG



Functional MRI

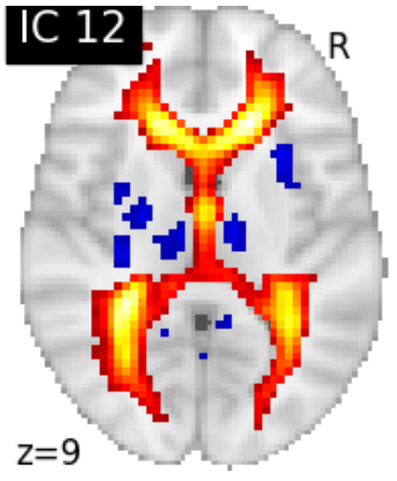
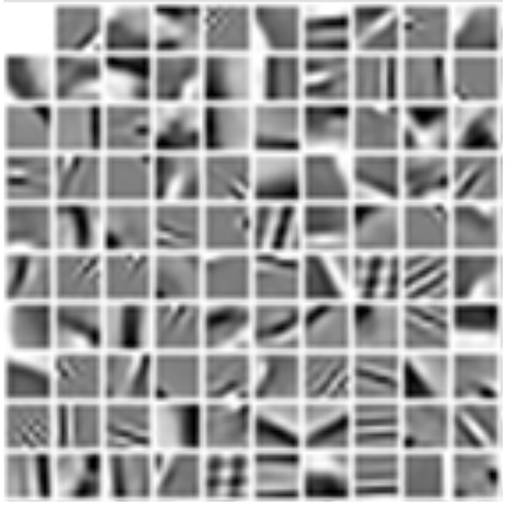


Image patches

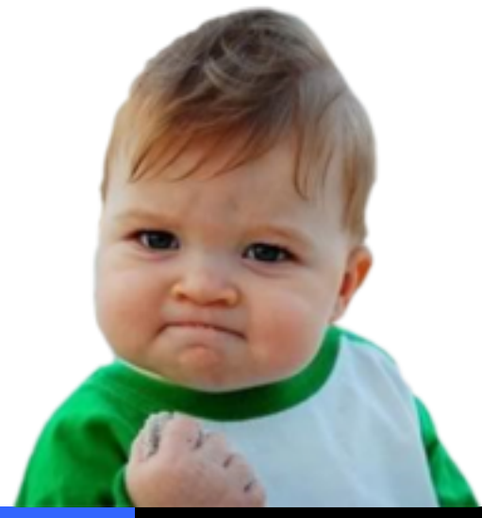


Can we reuse the Hessian approx. idea?

[P. Ablin, J.F. Cardoso and A. Gramfort.
Beyond Pham's algorithm for joint diagonalization.
Proc. ESANN 2019, https://hal.archives-ouvertes.fr/hal-01936887v1]

[P. Ablin, D. Fagot, H. Wendt, A. Gramfort and C. Févotte.
A Quasi-Newton algorithm on the orthogonal manifold for NMF with transform learning.
Proc. ICASSP 2019, https://hal.archives-ouvertes.fr/hal-01912918/document]

[P. Ablin, J.F. Cardoso and A. Gramfort.
Faster ICA under orthogonal constraint.
Proc. ICASSP'18, https://hal.inria.fr/hal-01651842/document]



Problem 1: Picard is a batch method

Problem 2: SGD (here Infomax)

- has no descent guarantee
- requires careful tuning of learning rate

[EM algorithms for ICA, P. Ablin, A. Gramfort, J.F. Cardoso and F. Bach, AISTATS 2019]

Problem 1: Picard is a batch method

Problem 2: SGD (here Infomax)

- has no descent guarantee
- requires careful tuning of learning rate

Question: Can we find an online algorithm not more costly than SGD with descent guarantee and no learning rate?

[EM algorithms for ICA, P. Ablin, A. Gramfort, J.F. Cardoso and F. Bach, AISTATS 2019]

ICA, EM and quadratic majorant

Assumption: $p_i \overset{\text{def}}{=} d$ Same density per source

We denote: $G(y) = -\log(d(y))$

Objective function:

$$\mathcal{L}(W) = -\log|\det(W)| + \frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{T} G([WX]_{ij})$$

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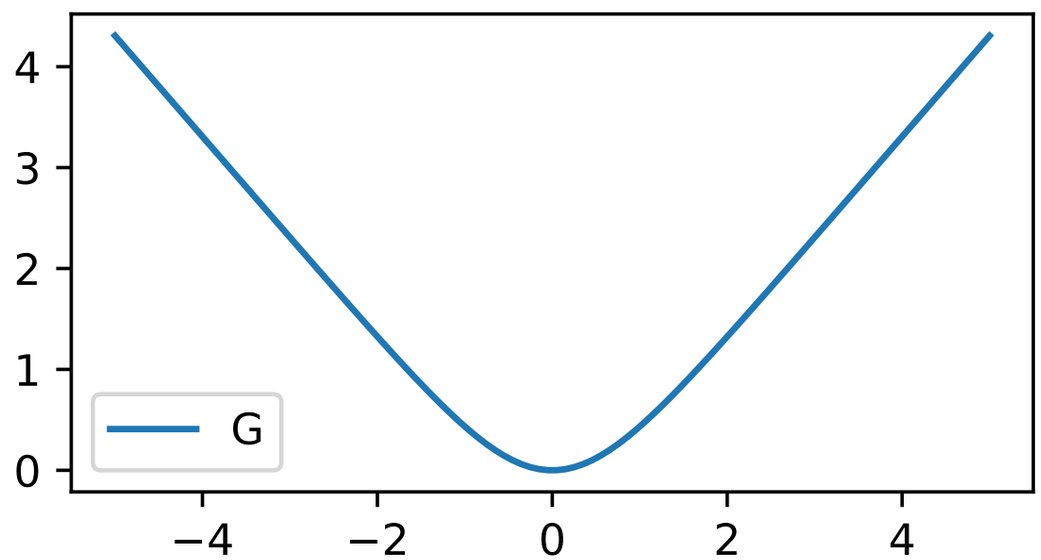
Key assumption: $G(\sqrt{\cdot})$ is concave (super-Gaussian source)

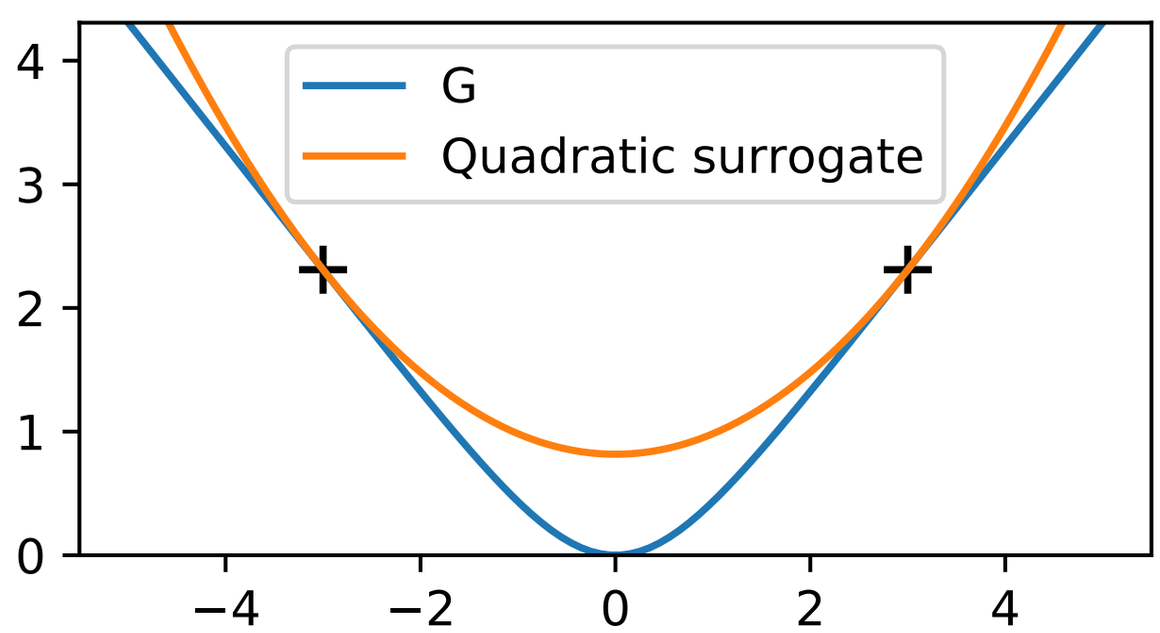


ICA, EM and quadratic majorant

If $G(\sqrt{\cdot})$ is concave (super-Gaussian source)

There exists a function f such that:

$$G(y) = \min_{u \geq 0} \frac{uy^2}{2} + f(u) \quad \text{(cf. Fenchel duality)}$$



Minimum is reached at $u^*(y) = \dfrac{G'(y)}{y}$

ICA, EM and quadratic majorant

The new function reads:

$$\mathcal{L}(W, U) =$$

$$-\log|\det(W)| + \frac{1}{2T}\sum_{i=1}^{N}\sum_{j=1}^{T} U_{ij}\,[WX]_{ij}^2 + \frac{1}{T}\sum_{i=1}^{N}\sum_{j=1}^{T} f(U_{ij})$$

[EM algorithms for ICA, P. Ablin, A. Gramfort, J.F. Cardoso and F. Bach, AISTATS 2019]

ICA, EM and quadratic majorant

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One can minimize by alternate minimization

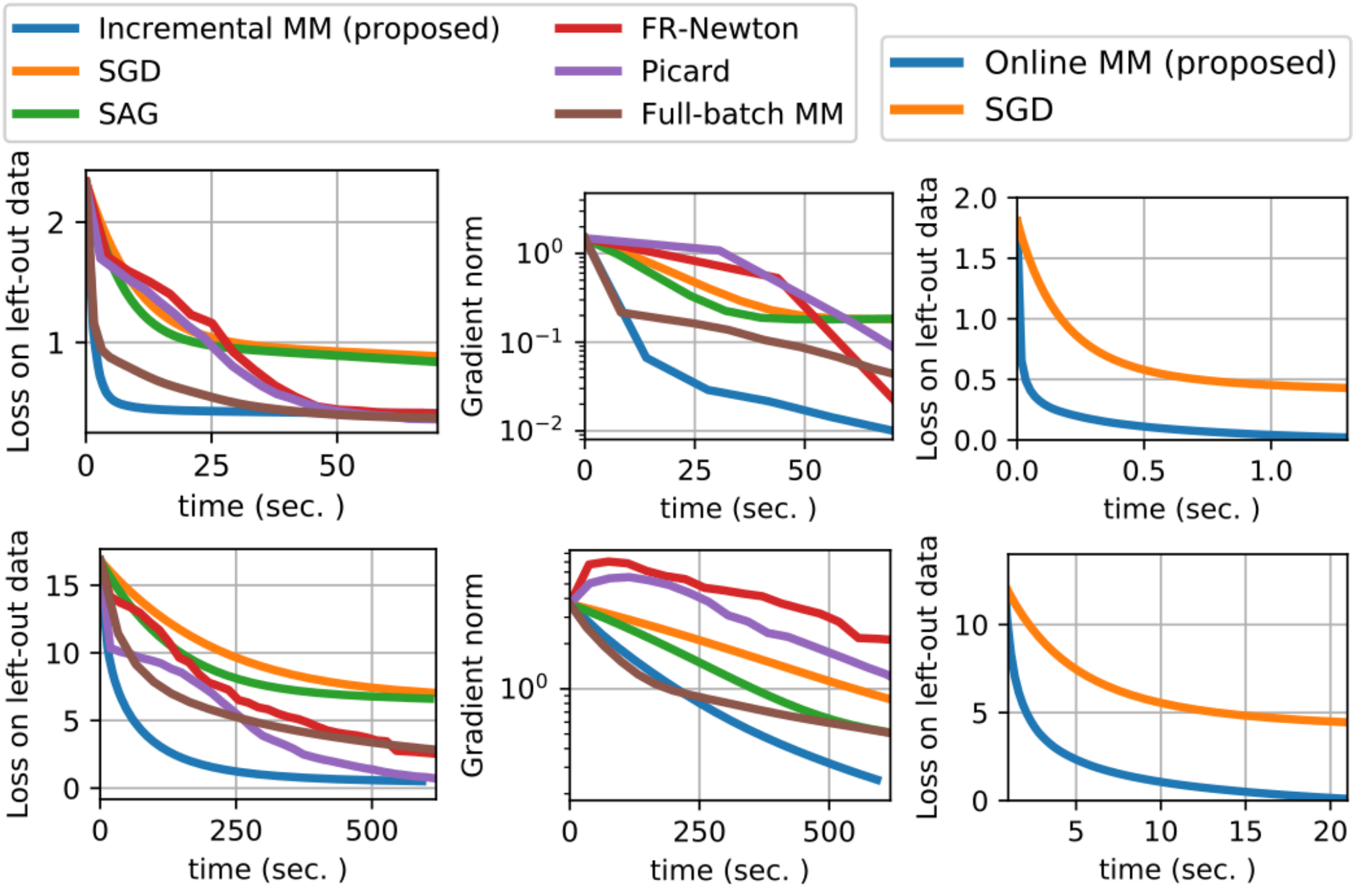
… using ideas from variance reduced stochastic methods (eg. SAG)

… with memory and some accumulation of the right quantities

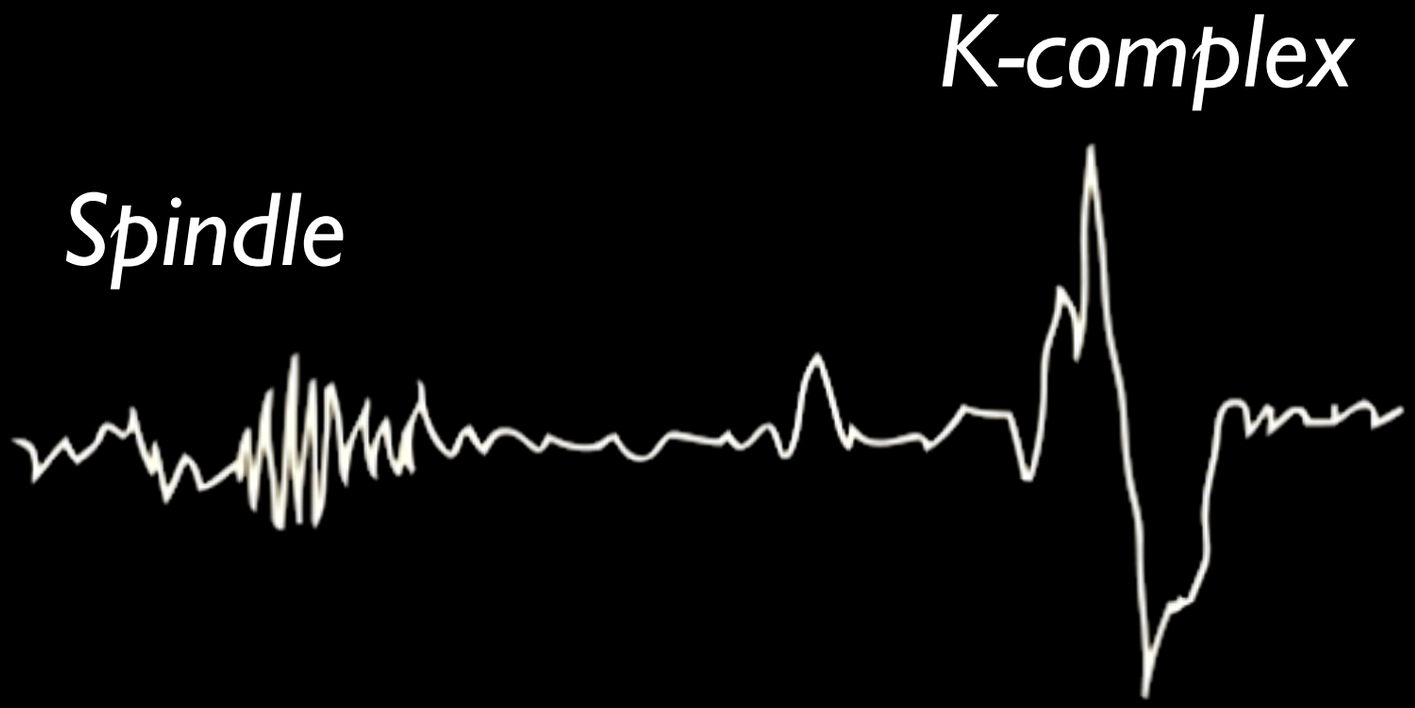
… and carefully coordinate descent steps

[EM algorithms for ICA, P. Ablin, A. Gramfort, J.F. Cardoso and F. Bach, AISTATS 2019]

Experiments on real data



Learning patterns / waveforms from (multivariate) signals



[Learning the Morphology of Brain Signals Using Alpha-Stable Convolutional Sparse Coding, (2017), M. Jas, T. Dupré la Tour, U. Simsekli, A. Gramfort, Proc. NeurIPS Conf.]

[Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals, (2018), T. Dupré la Tour, T. Moreau, M. Jas, A. Gramfort, Proc. NeurIPS Conf.]

[Distributed Convolutional Dictionary Learning (DiCoDiLe): Pattern Discovery in Large Images and Signals (2019), T. Moreau and A. Gramfort, ArXiv.]

Code: https://alphacsc.github.io

Learning patterns / waveforms from (multivariate) signals

Objective:
Physics informed model
Fast solvers

[Learning the Morphology of Brain Signals Using Alpha-Stable Convolutional Sparse Coding, (2017), M. Jas, T. Dupré la Tour, U. Simsekli, A. Gramfort, Proc. NeurIPS Conf.]

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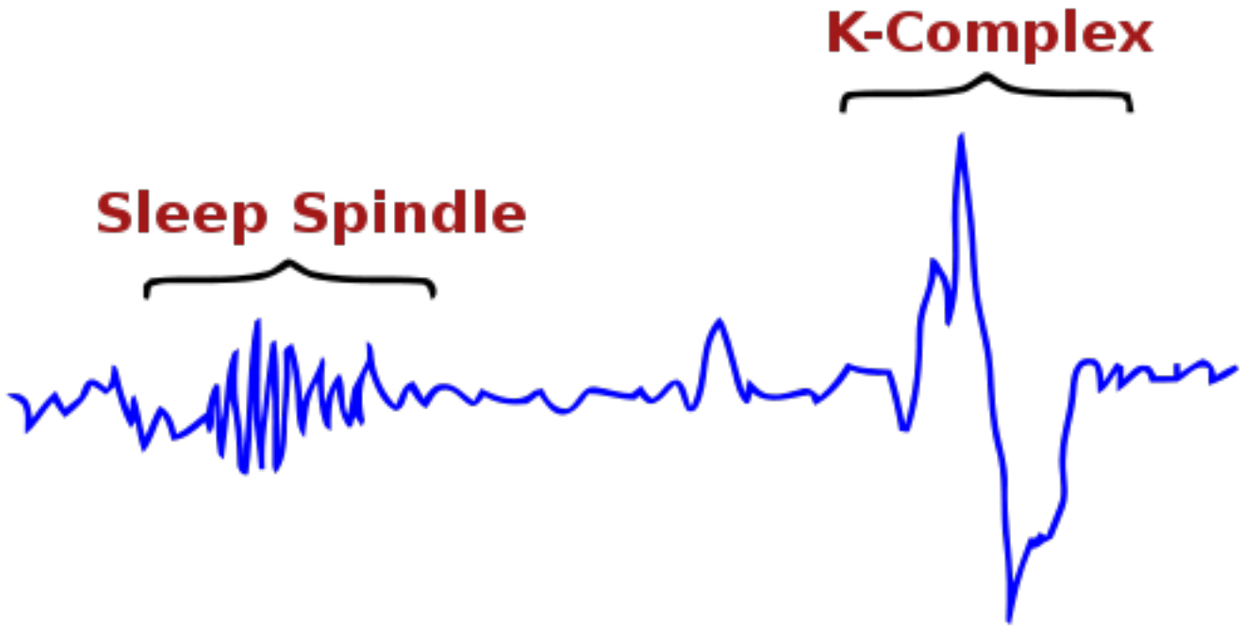
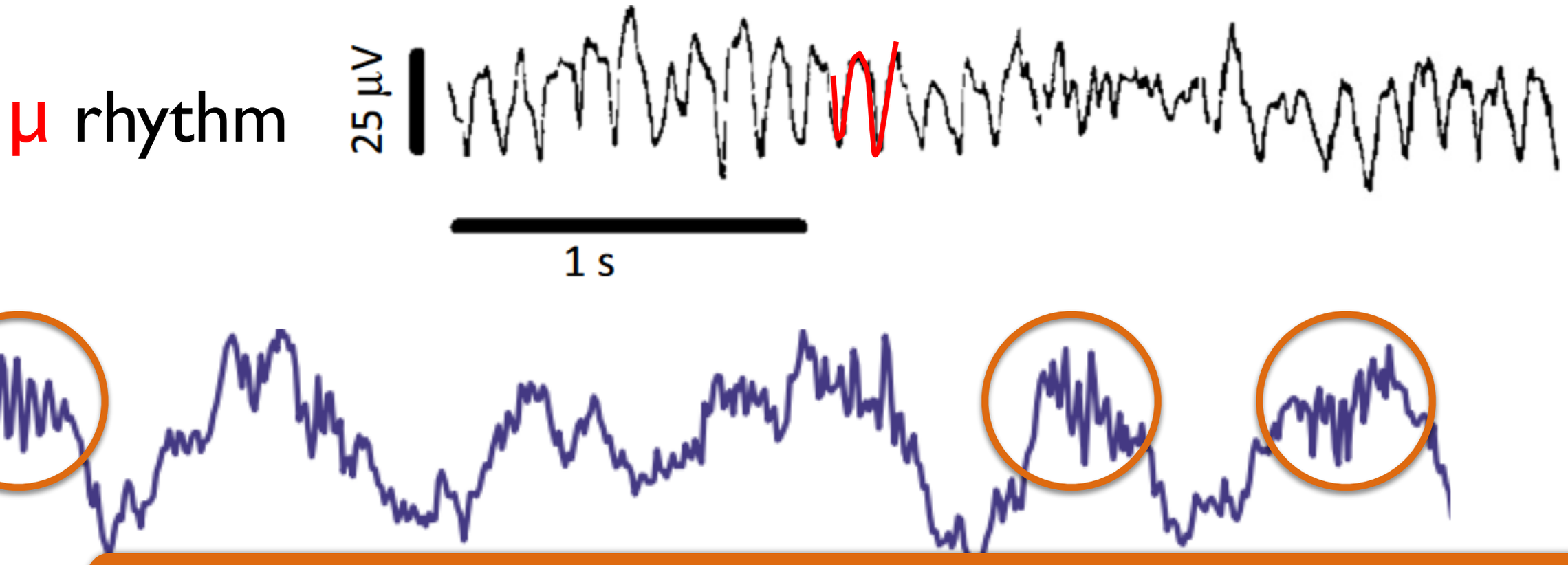


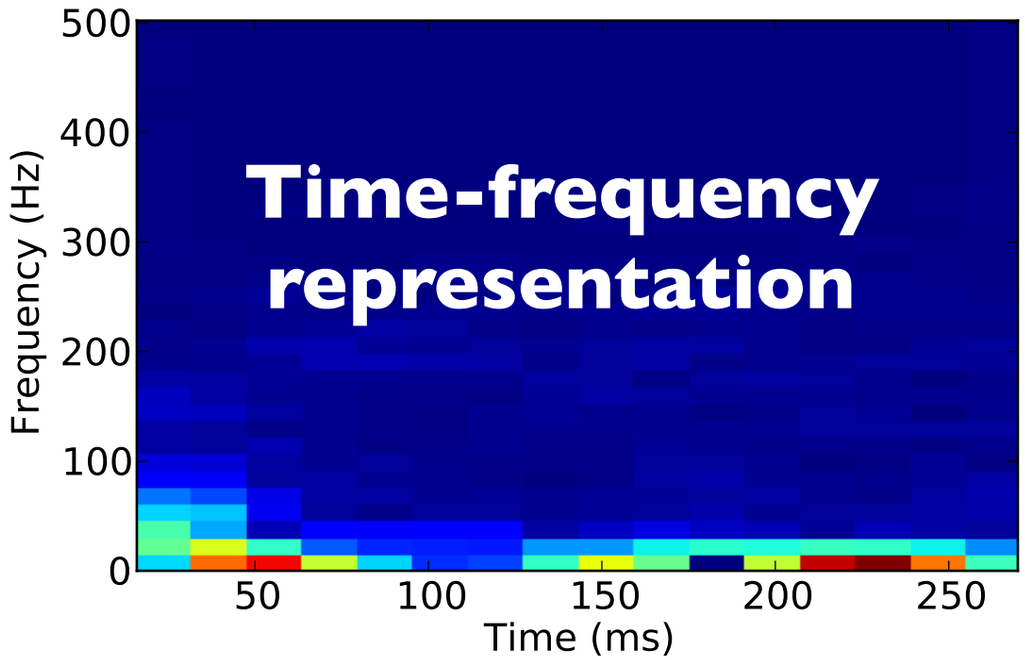
Neural signals exhibit diverse and complex morphologies

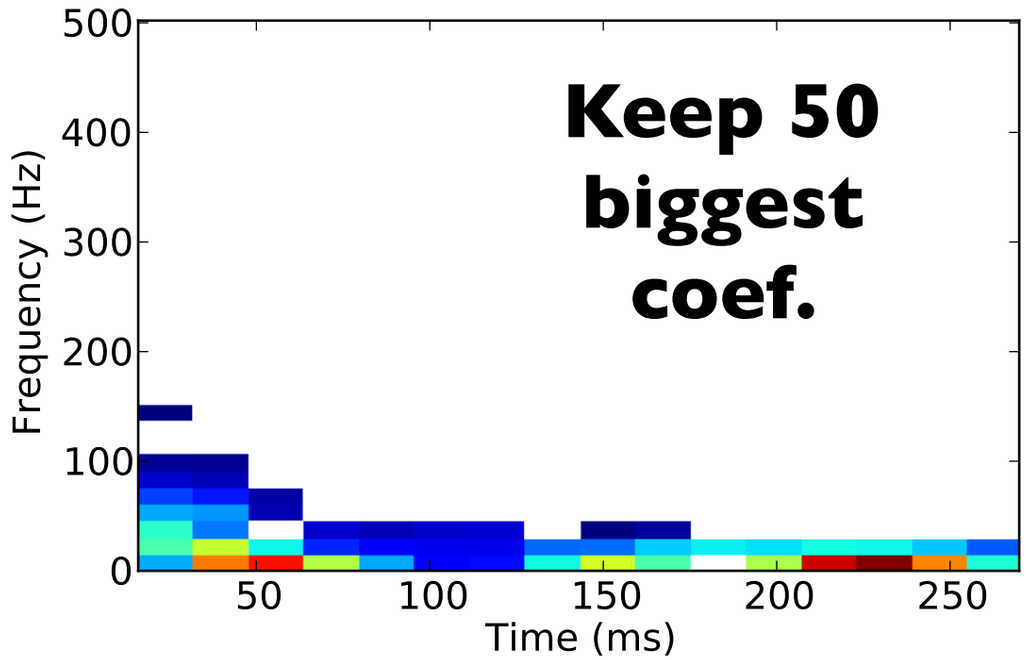


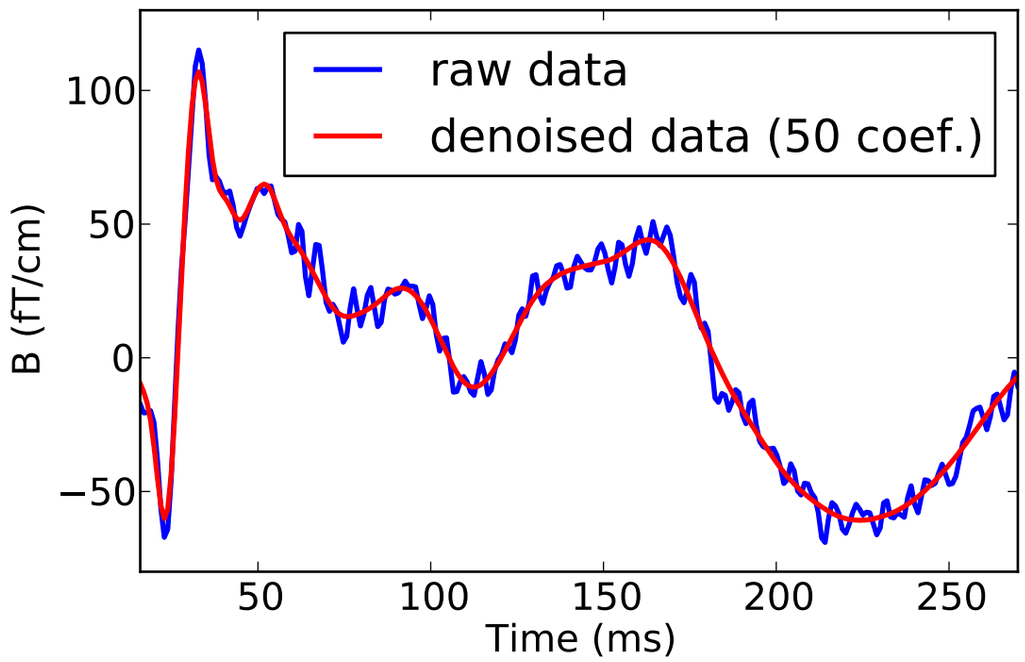
CFC: High frequency bursts coupled with slow waves

[T. Dupré la Tour, L. Tallot, L. Grabot, V. Doyère, V. van Wassenhove, Y. Grenier, A. Gramfort, (2017) PLOS Computational biology]

Why signal representations?







Having adequate signal representation help: denoising, prediction, inverse problems etc.

Signal representations

- Sparse representations: wavelet basis

[Morlet 70', Meyer 80', Mallat 90' etc.]

- Sparse coding / dictionary learning

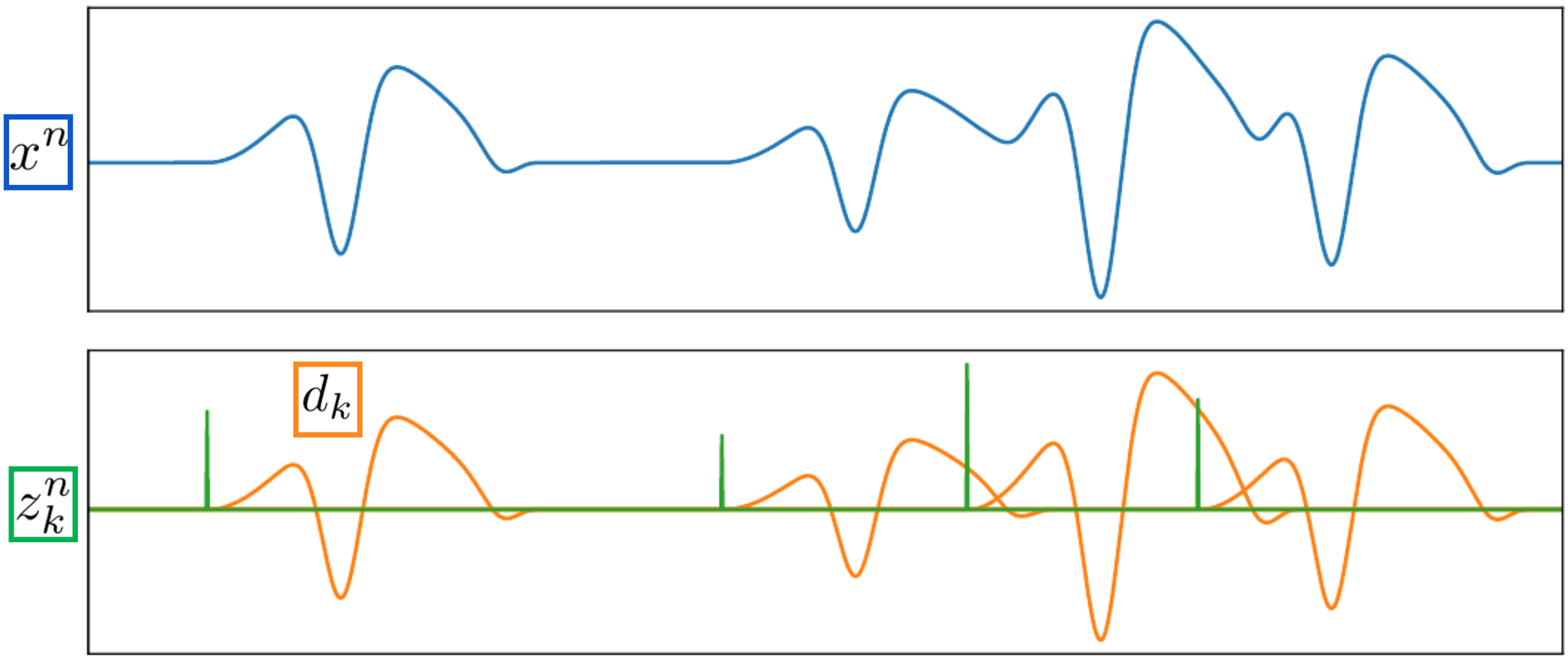
[Olshausen and Field, 1996, Elad and Aharon, 2006]

- Shift-invariant representations

[Lewicki and Sejnowski, 1999, Grosse et al, 2007]

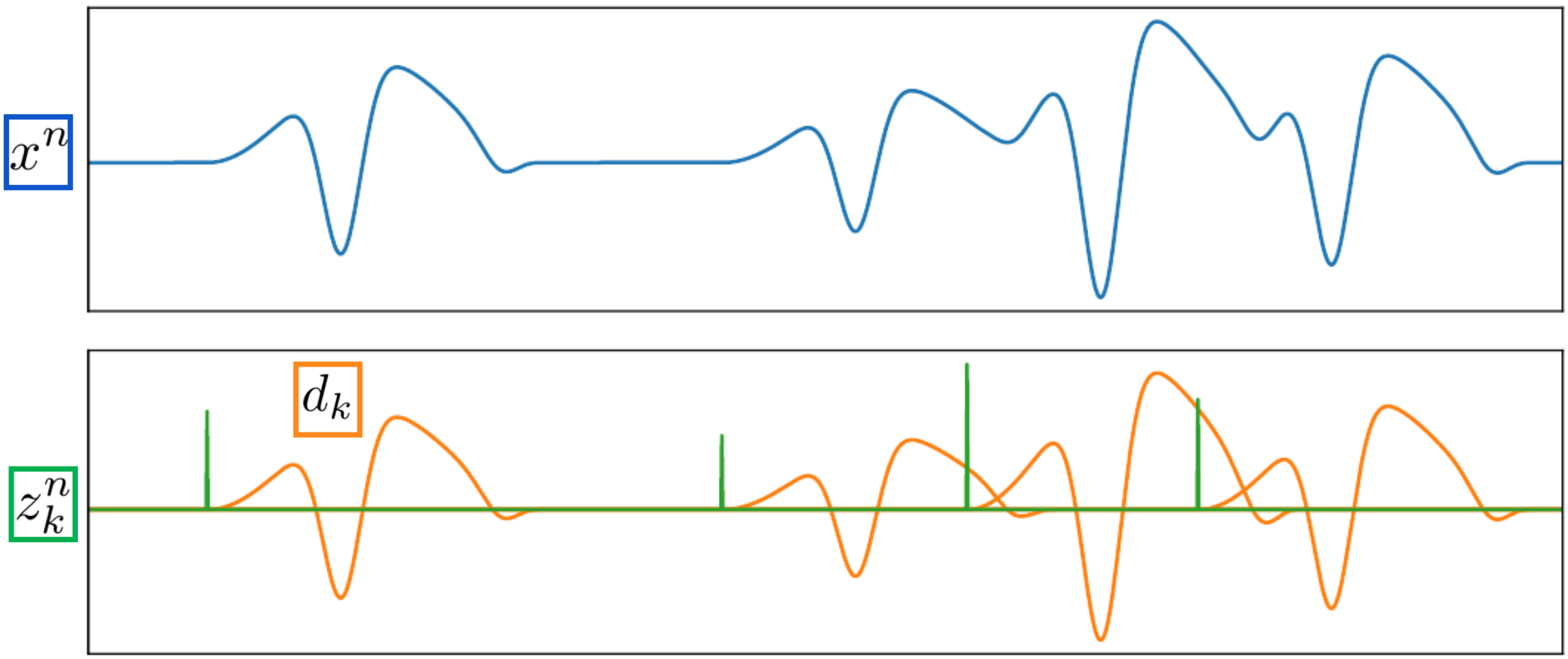
- In neurophysiology:
  - Matching of time-invariant filters (MOTIF) [Jost et al, 2006]
  - Multivariate orthogonal matching pursuit [Barthélemy et al, 2012]
  - Matching pursuit and heuristics [Brokmeier and Principe, 2016]
  - Sliding window machine [Gips et al, 2017]
  - Adaptive waveform learning [Hitziger et al, 2017]

Convolutional sparse coding



[Grosse et al, 2007]

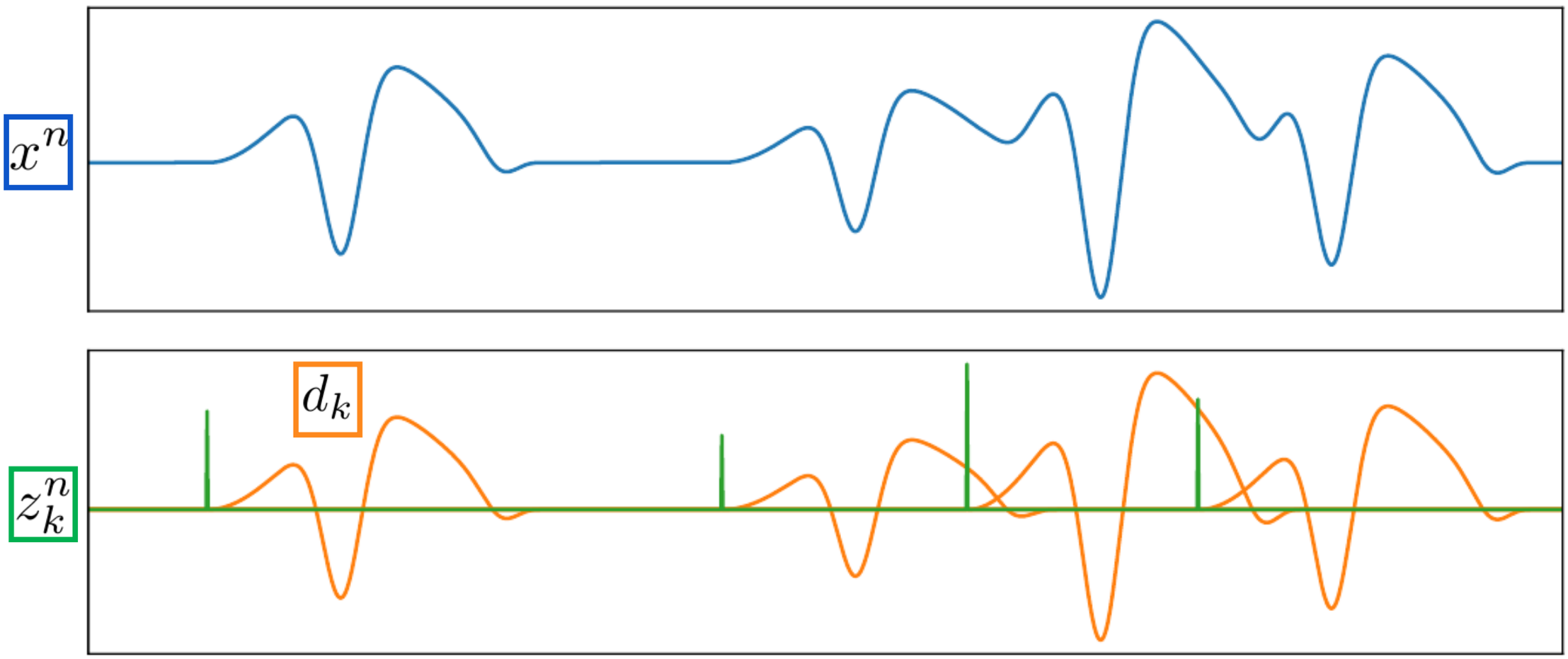
Convolutional sparse coding



$$x^n[t] = \sum_{k=1}^{K} (z_k^n * d_k)[t] + \varepsilon[t]$$

[Grosse et al, 2007]

Convolutional sparse coding



$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| x^n - \sum_{k=1}^{K} z_k^n * d_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$

$$\text{s.t.} \quad \|d_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

[Grosse et al, 2007]

Optimization strategy

$$\min_{d,z} \sum_{n=1}^{N} \frac{1}{2} \left\| x^n - \sum_{k=1}^{K} z_k^n * d_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$
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Block-coordinate descent (update Z, D, Z, D, Z…):

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Block-coordinate descent (update Z, D, Z, D, Z…):

- Z-step
  - GCD [Kavukcuoglu et al, 2010]
  - FISTA [Chalasani et al, 2013]
  - ADMM [Bristow et al, 2013]
  - ADMM + FFT [Wohlberg, 2016]
  - L-BFGS [Jas et al, 2017]
  - LGCD [Dupré la Tour et al, 2018]

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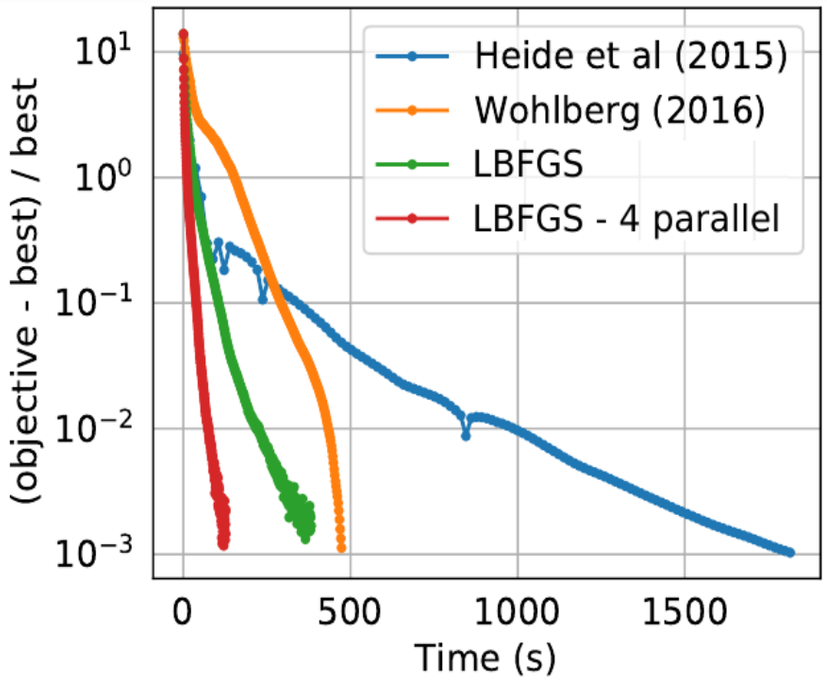
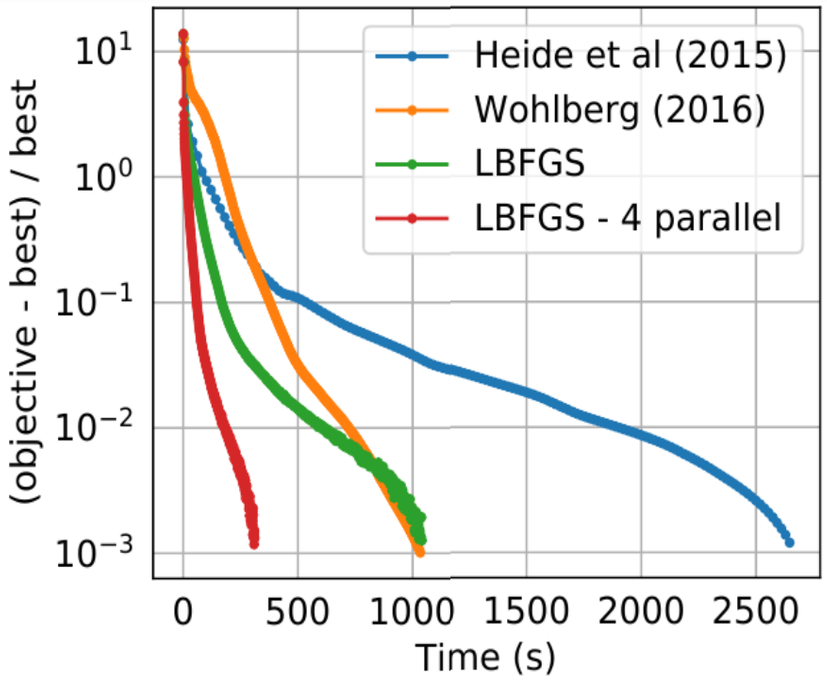
  - L-BFGS [Jas et al, 2017]
  - LGCD [Dupré la Tour et al, 2018]
- D-step
  - FFT [Grosse et al, 2007]
  - ADMM + FFT [Heide et al, 2015]
  - ADMM + FFT [Wohlberg, 2016]
  - L-BFGS (dual) [Jas et al, 2017]
  - PGD [Dupré la Tour et al, 2018]

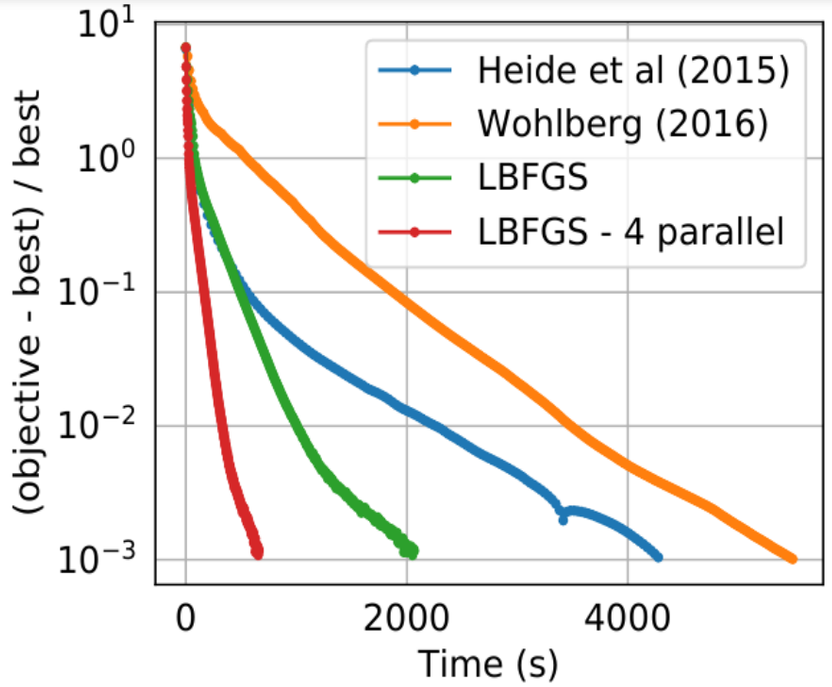
Speed benchmarks



(a) $K = 2$, $L = 32$.



(b) $K = 2$, $L = 128$.

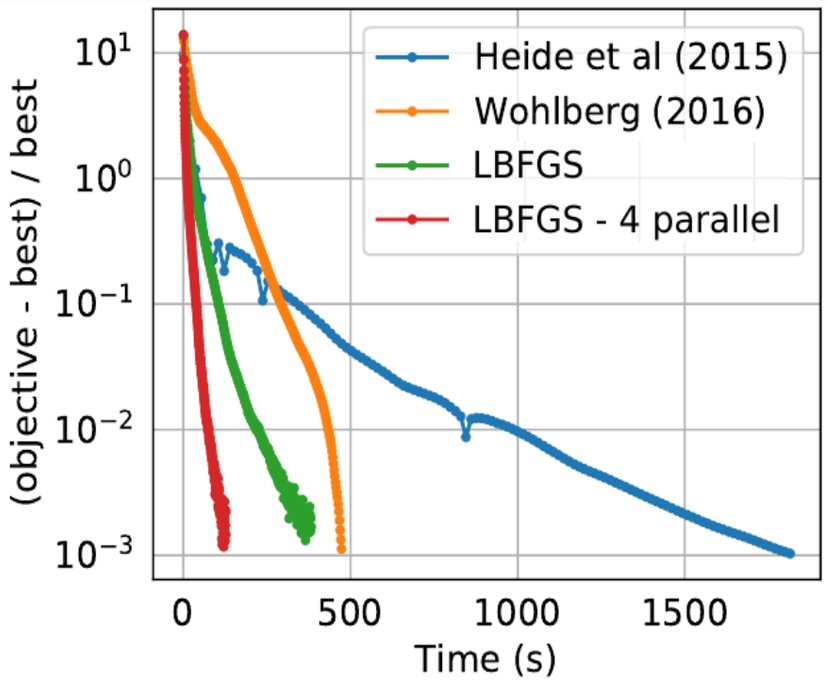


(c) $K = 10$, $L = 32$.

[Jas et al. 2017]

[Dupré la Tour et al. 2018]

Speed benchmarks



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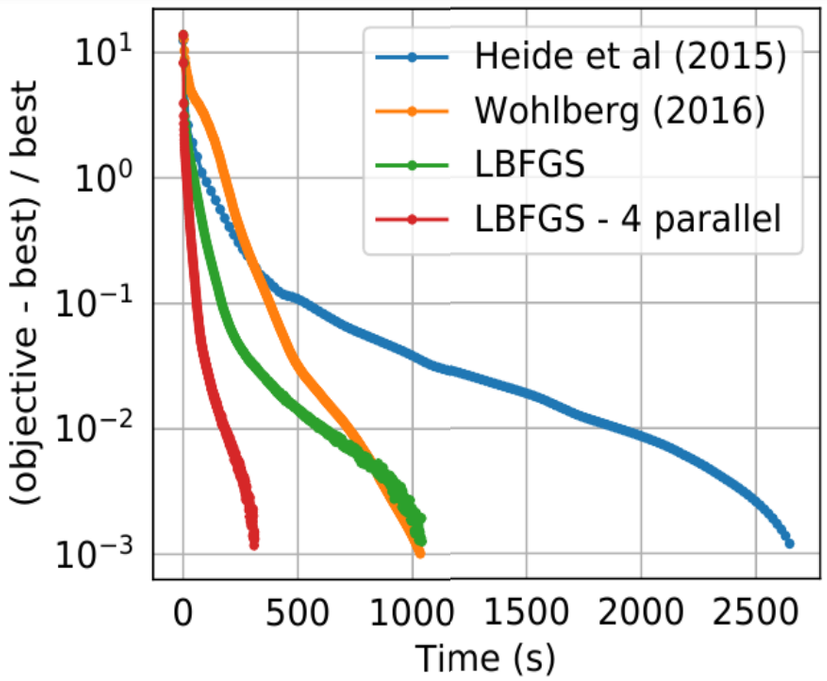

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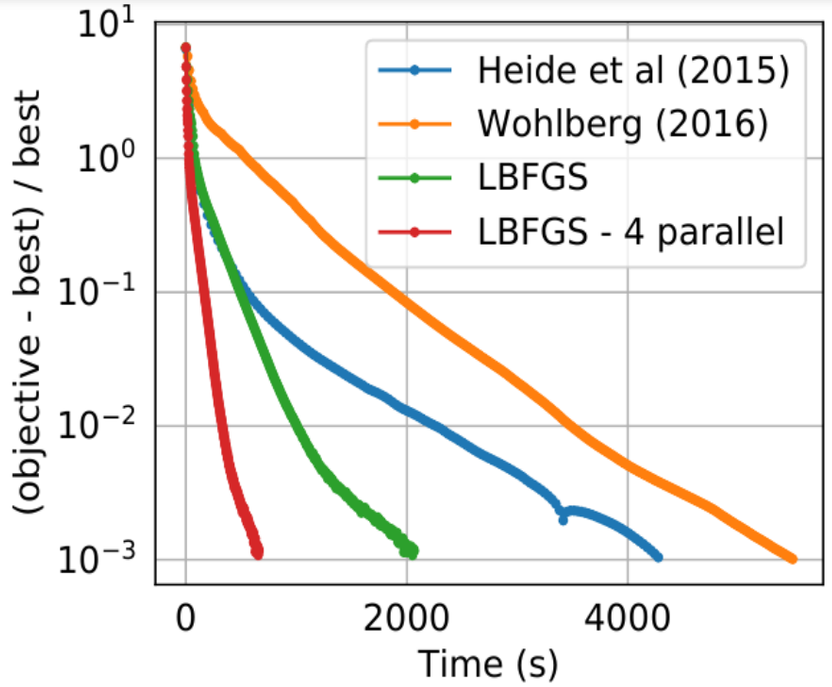

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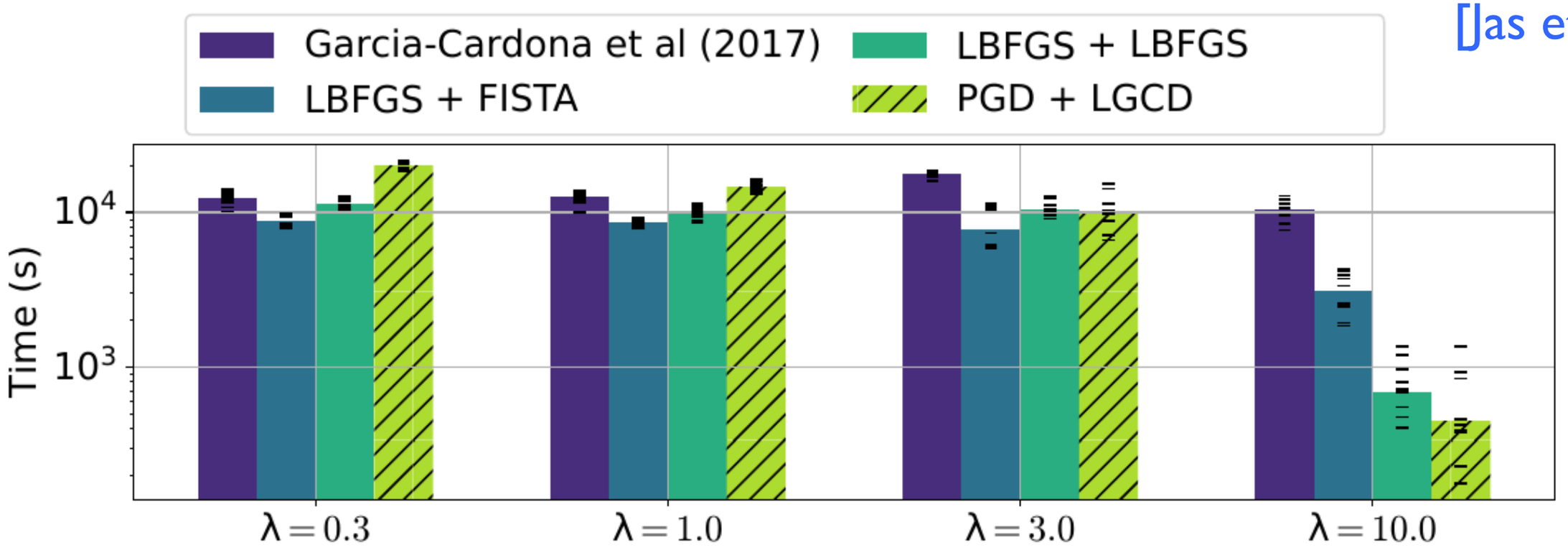
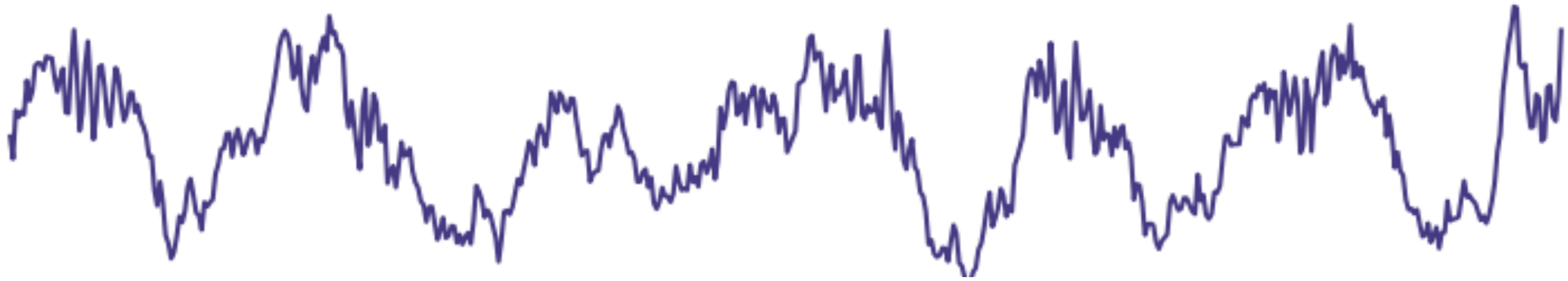
[Jas et al. 2017]

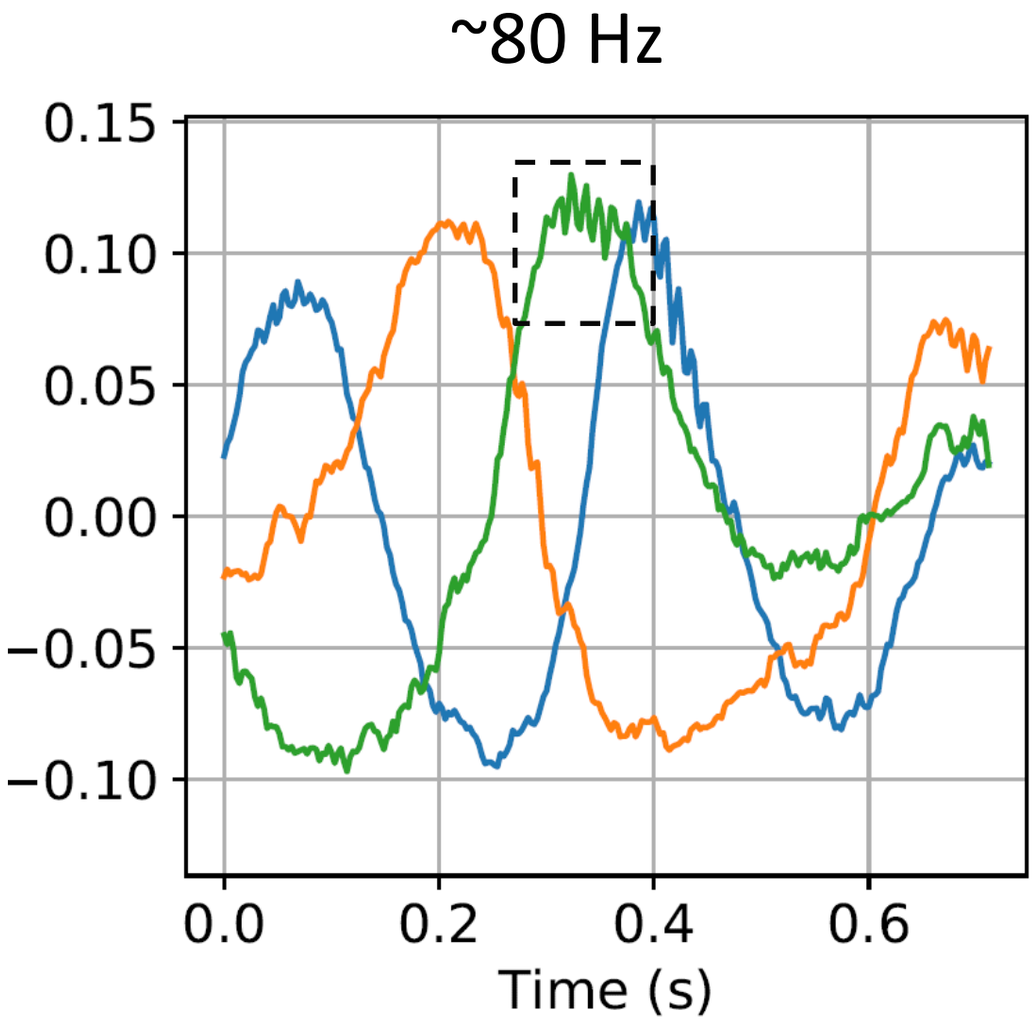


[Dupré la Tour et al. 2018]

Learned atoms

Data:





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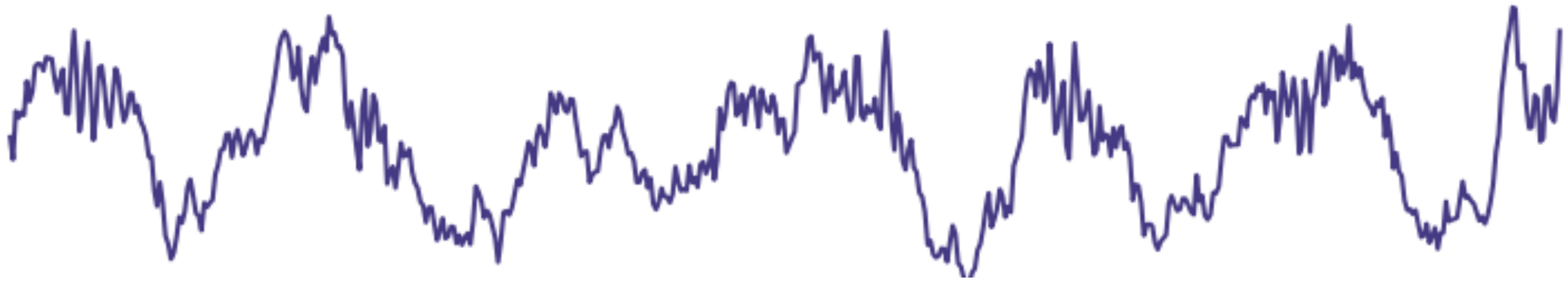
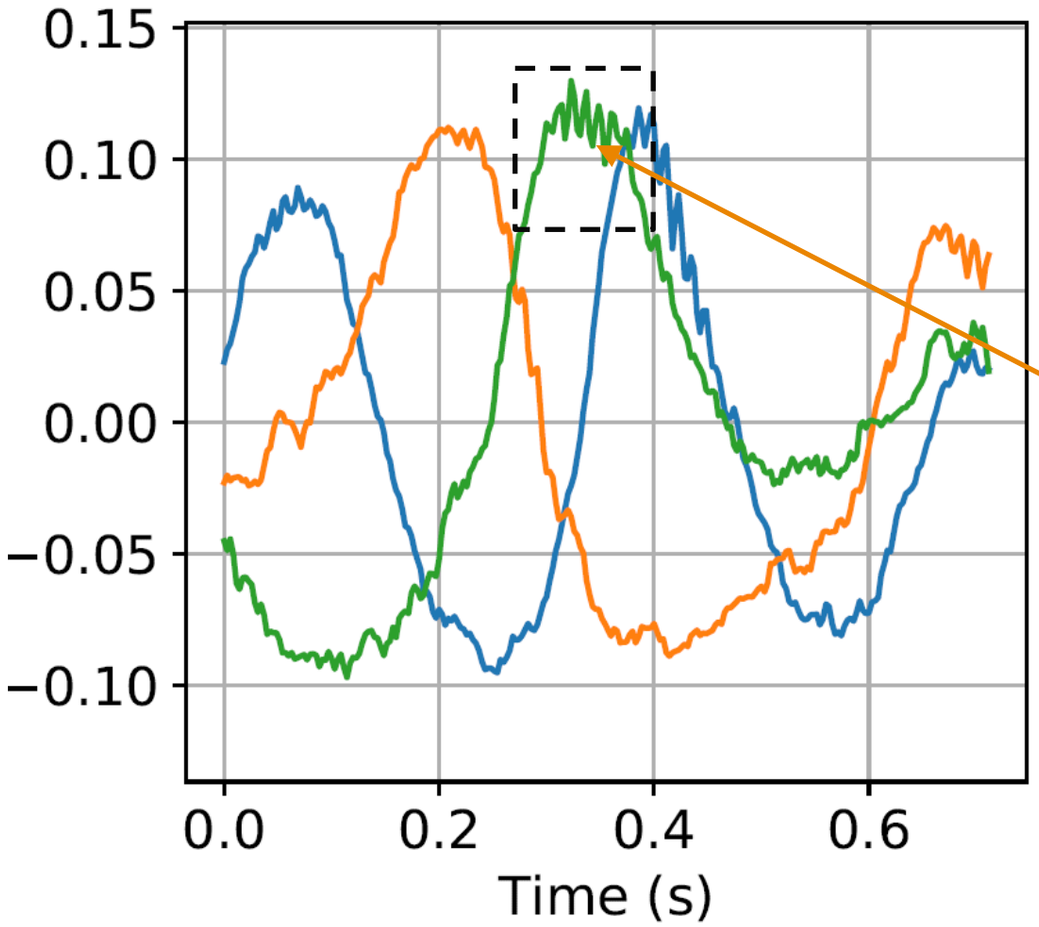


~80 Hz



CSC reveals "nested oscillations"

[Learning the Morphology of Brain Signals Using Alpha-Stable Convolutional Sparse Coding, (2017), M. Jas, T. Dupré la Tour, U. Simsekli, A. Gramfort, Proc. NeurIPS Conf.]

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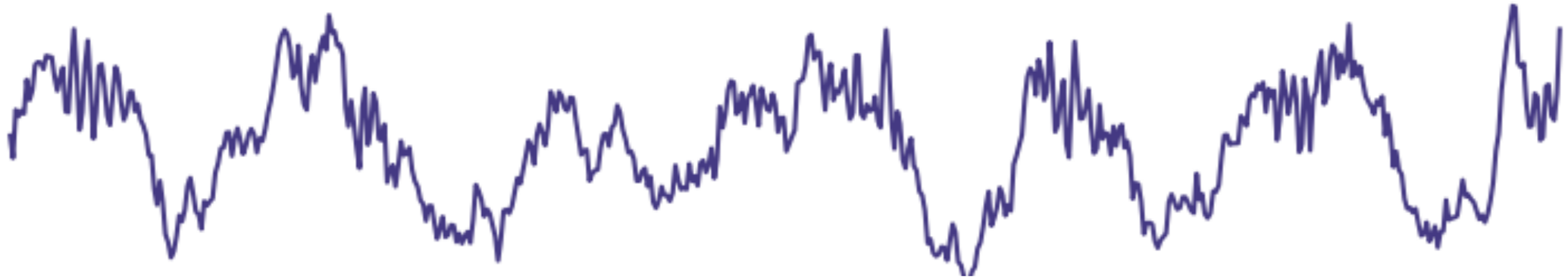
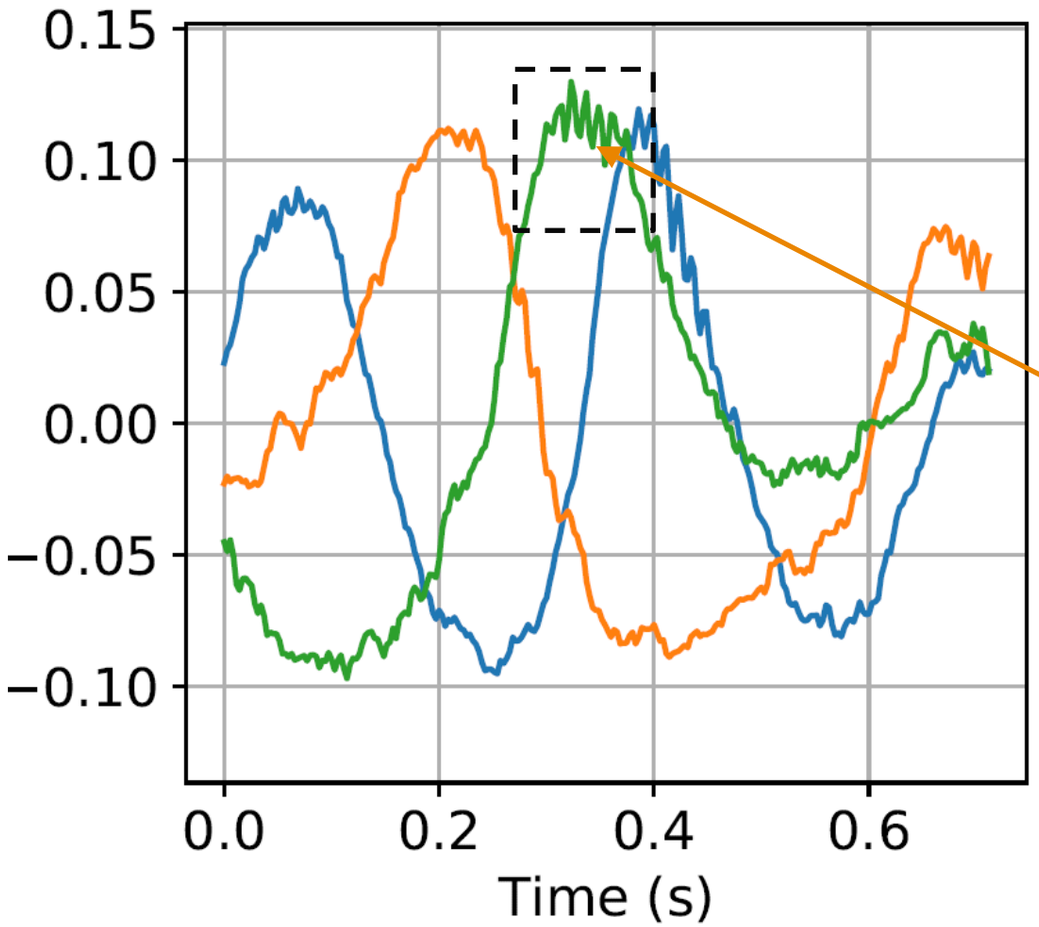


~80 Hz



How about if I have many channels?

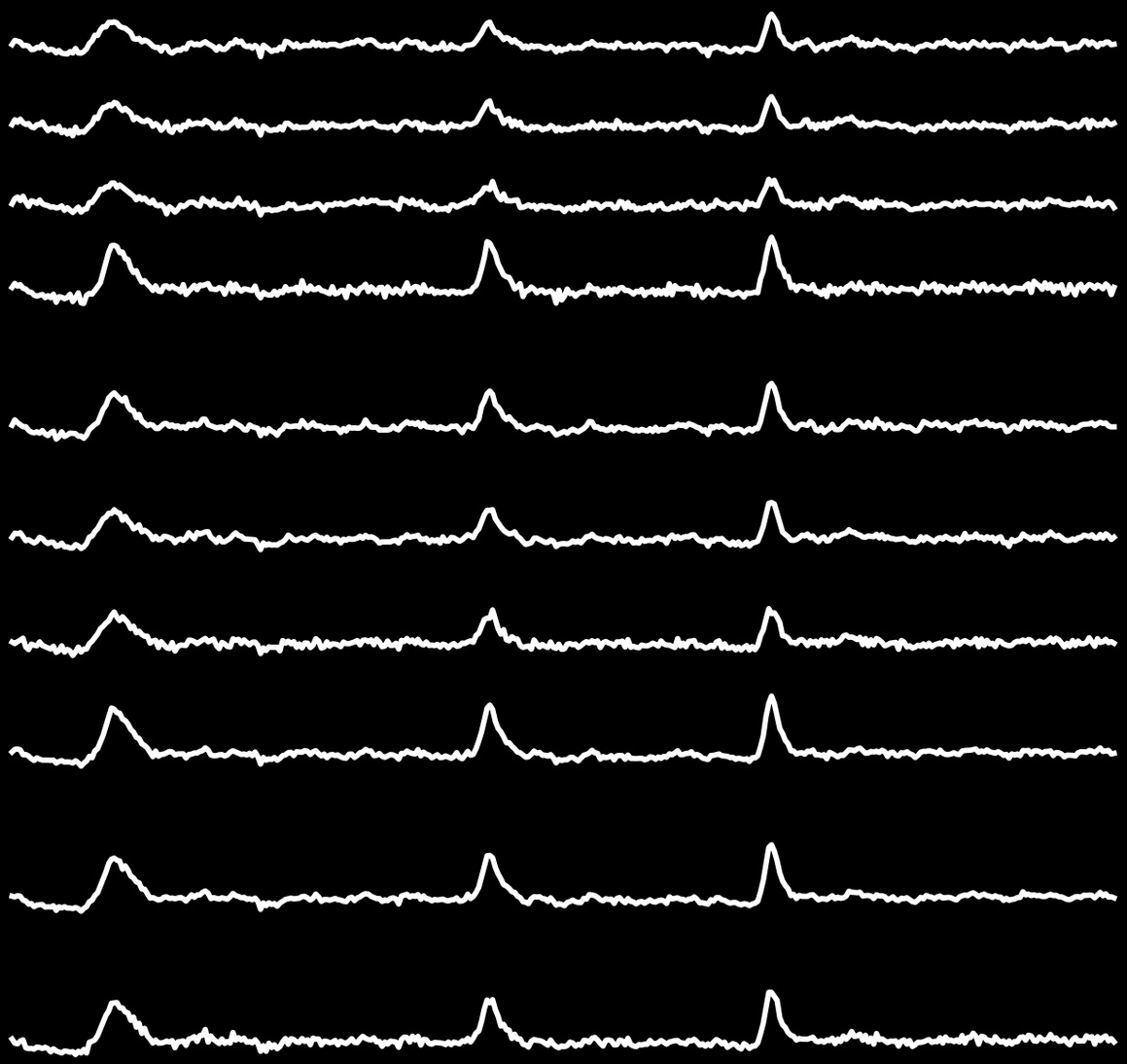
CSC reveals "nested oscillations"

[Learning the Morphology of Brain Signals Using Alpha-Stable Convolutional Sparse Coding, (2017), M. Jas, T. Dupré la Tour, U. Simsekli, A. Gramfort, Proc. NeurIPS Conf.]

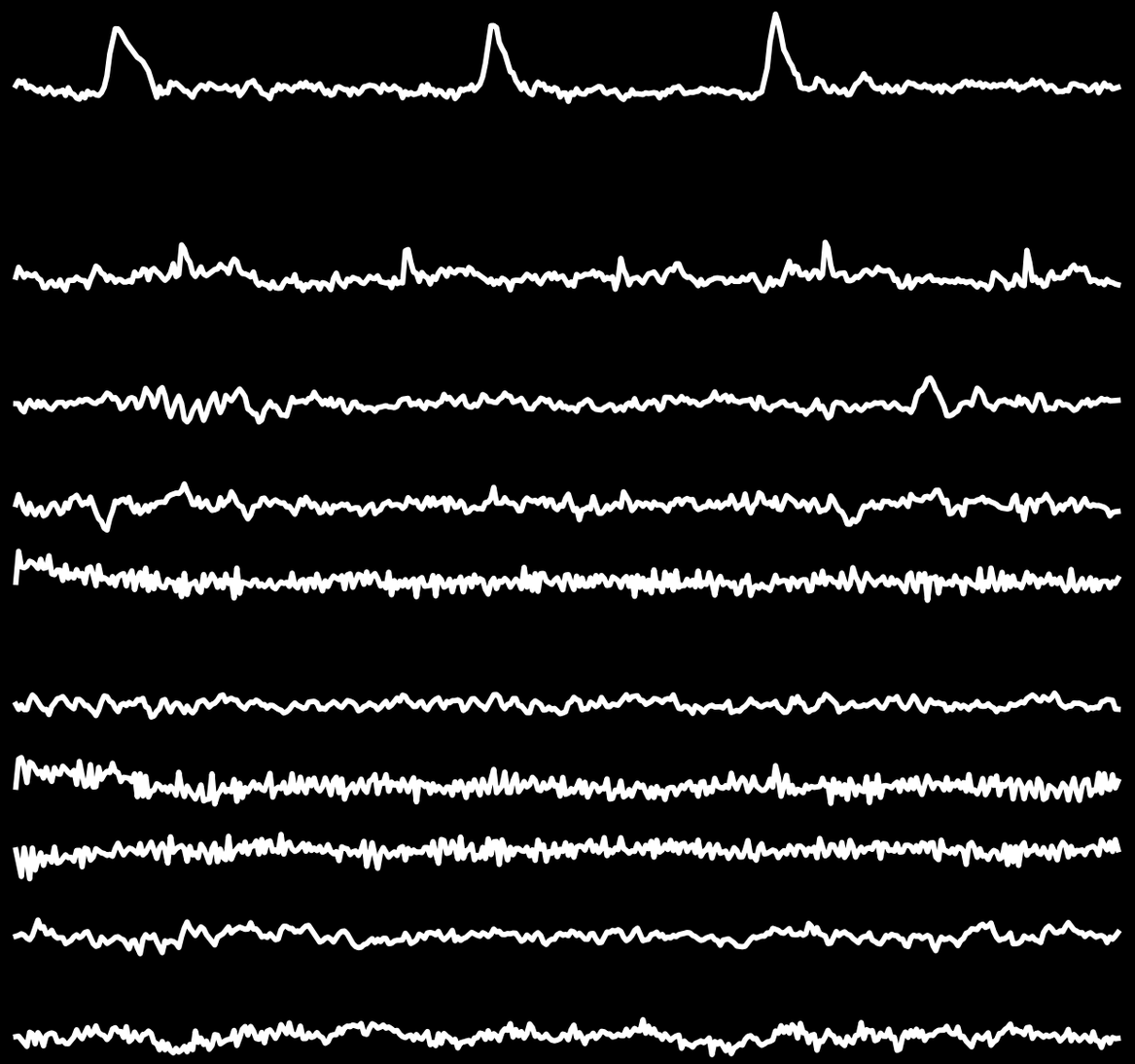
From ICA to CSC

Independent Component Analysis (ICA)

Observations (raw EEG)



ICA recovered sources



From ICA to CSC

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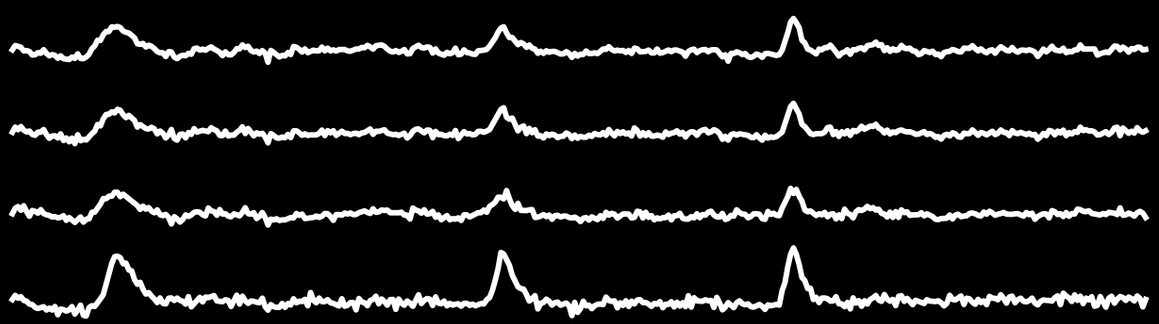
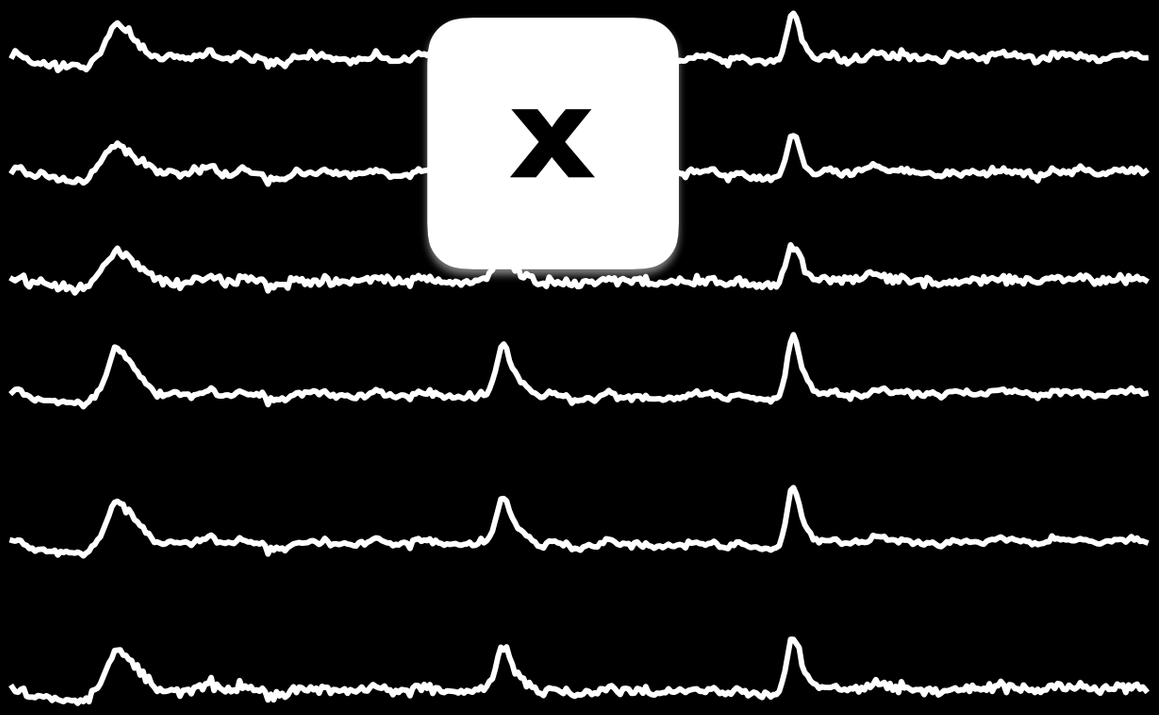
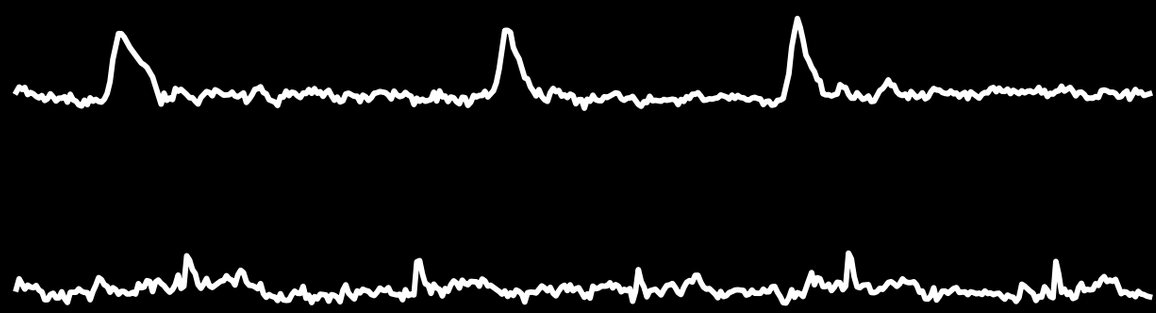
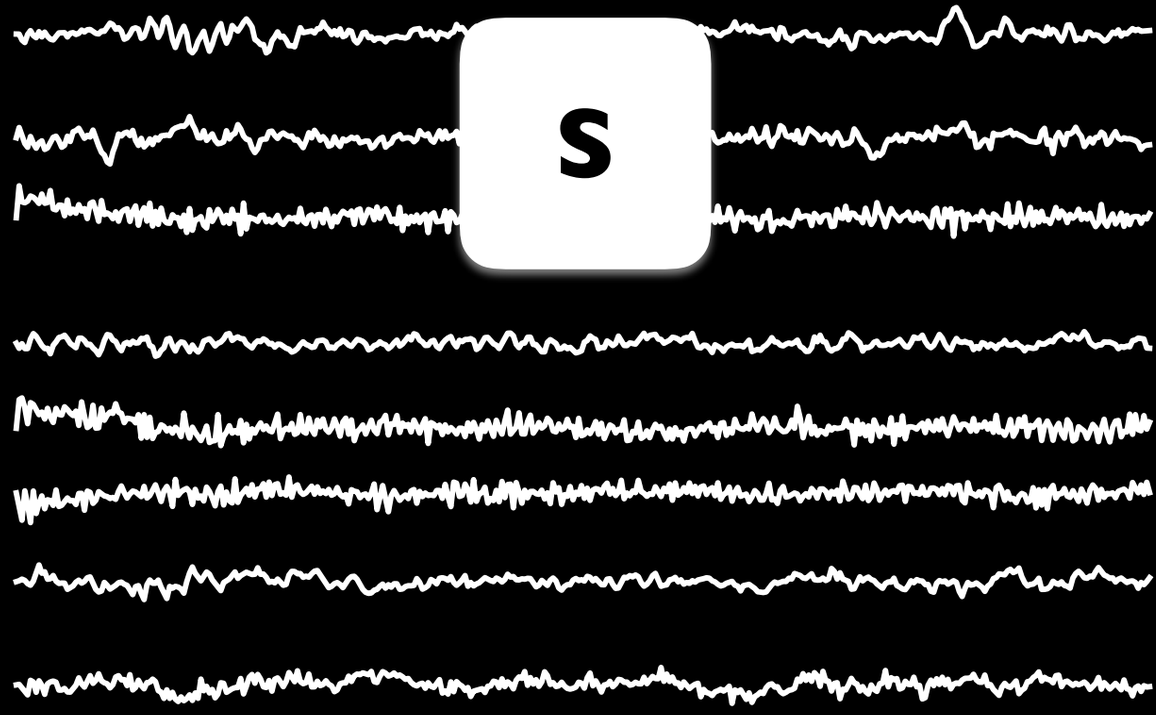
Observations (raw EEG)



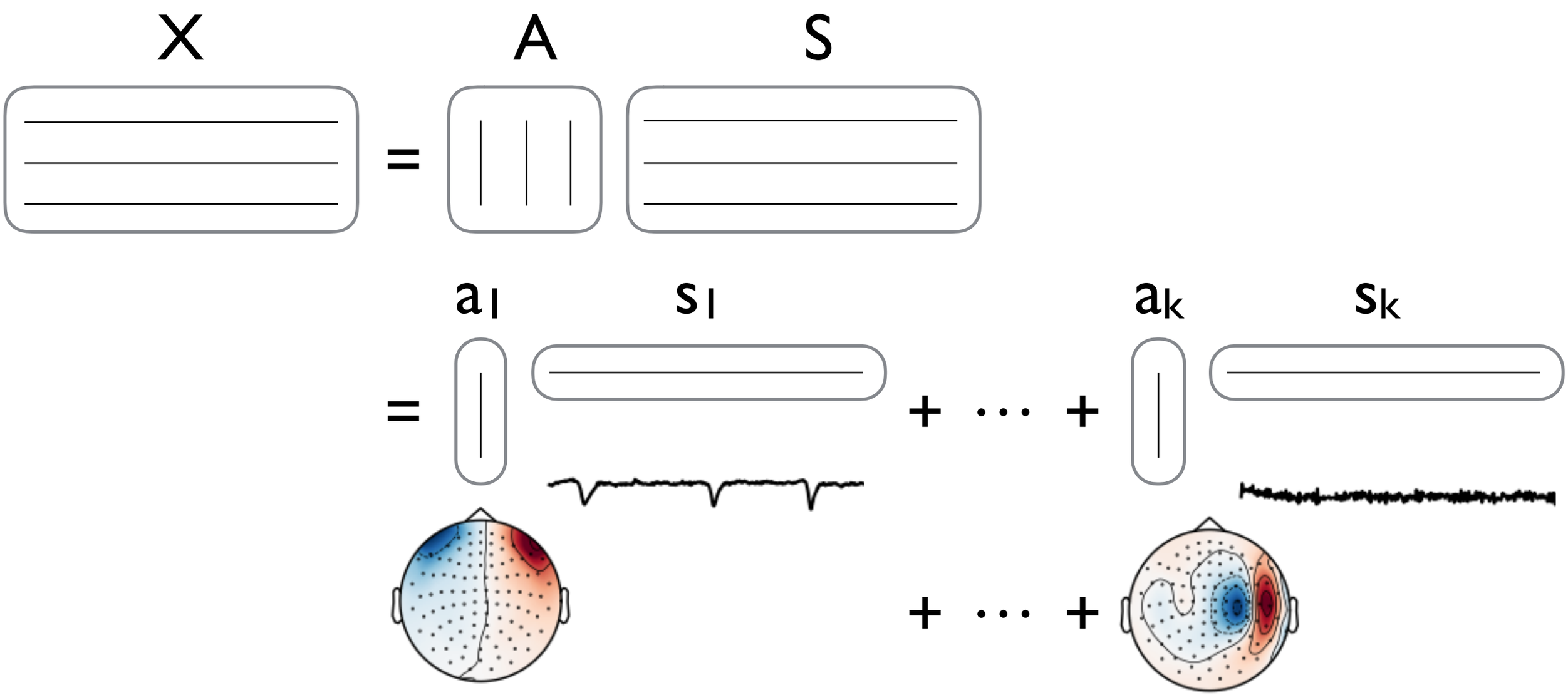


ICA recovered sources





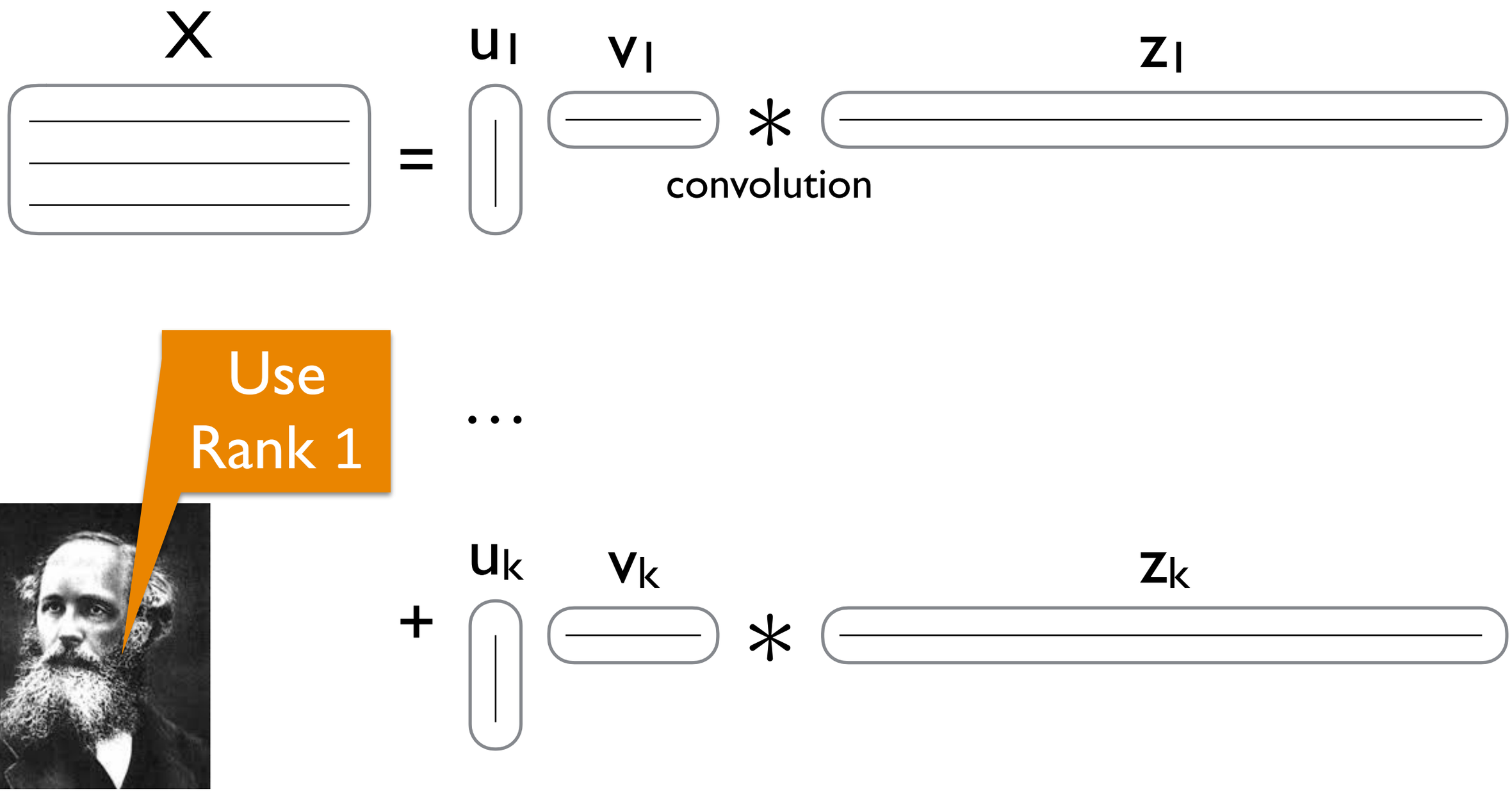
From ICA…



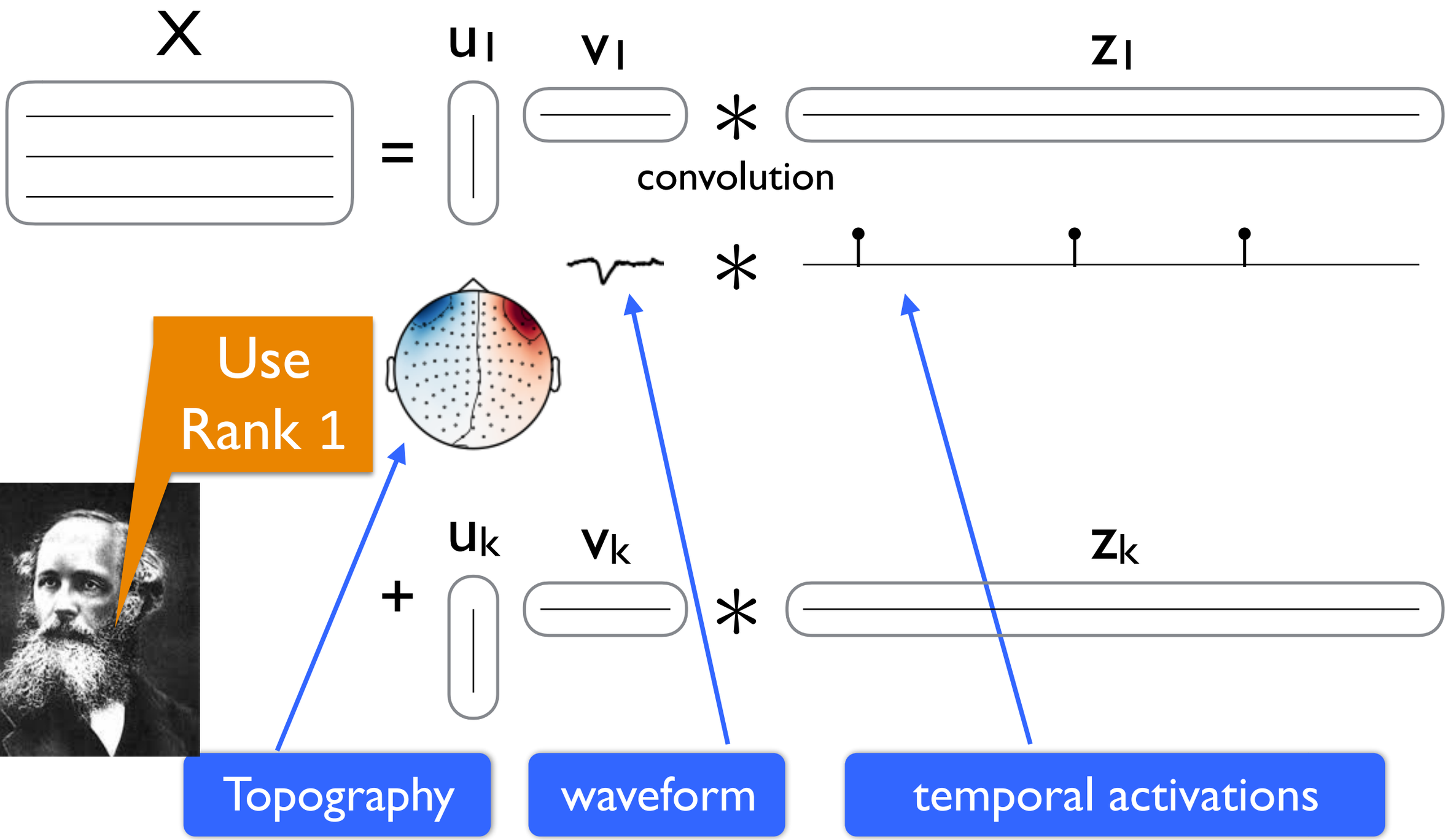
https://www.martinos.org/mne/stable/auto_tutorials/plot_artifacts_correction_ica.html

https://pierreablin.github.io/picard/auto_examples/plot_ica_eeg.html

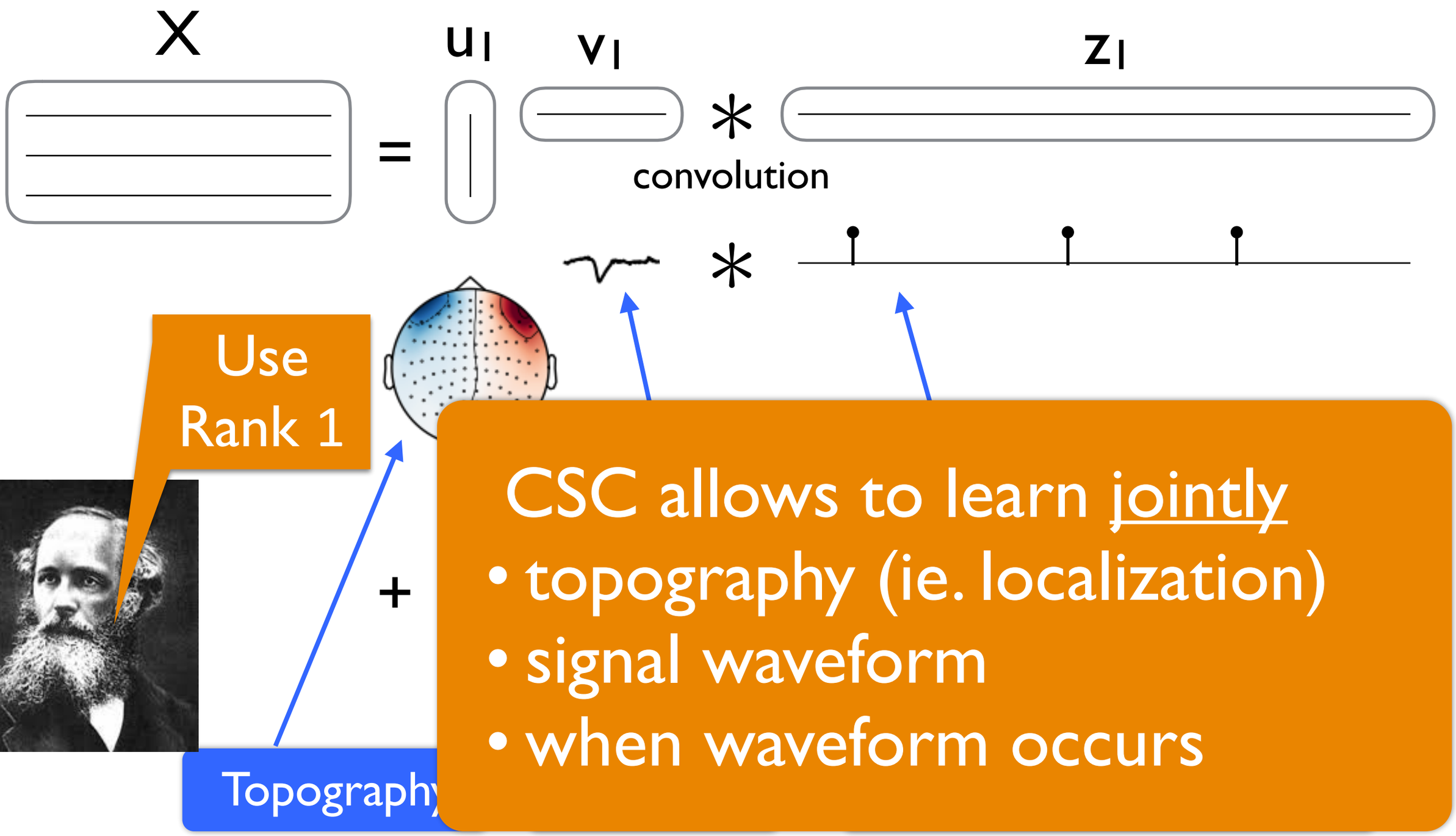
. . . to CSC



… to CSC


X
u1
v1
z1
=
*
convolution
*
Use
Rank 1
uk
vk
zk
+
*
Topography
waveform
temporal activations

… to CSC


X
u_l
v_l
z_l
=
*
convolution
*
Use Rank 1
+
Topography
CSC allows to learn jointly
• topography (ie. localization)
• signal waveform
• when waveform occurs

Multivariate CSC

$$\min_{D,z} \sum_{n=1}^{N} \frac{1}{2} \left\| X^n - \sum_{k=1}^{K} z_k^n * D_k \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1,$$
$$\text{s.t.} \quad \|D_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

[Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals, (2018), T. Dupré la Tour, T. Moreau, M. Jas, A. Gramfort, Proc. NeurIPS Conf.]

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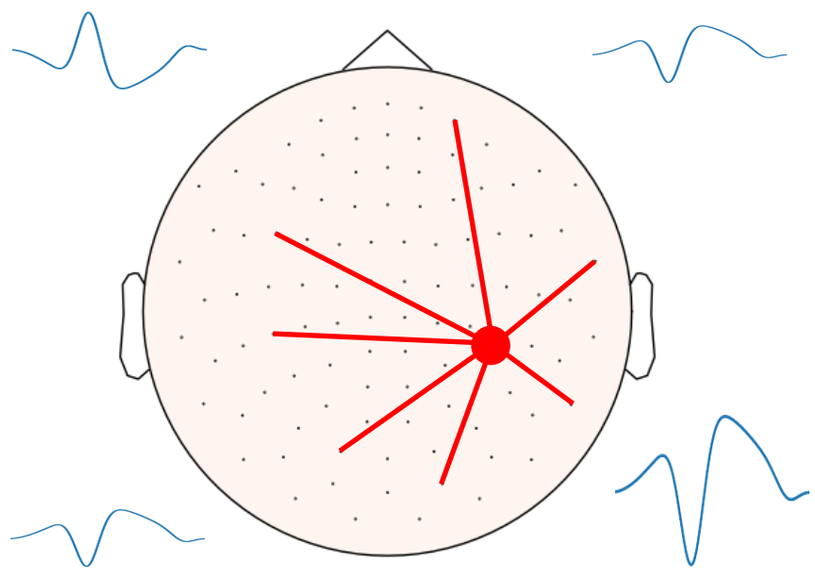

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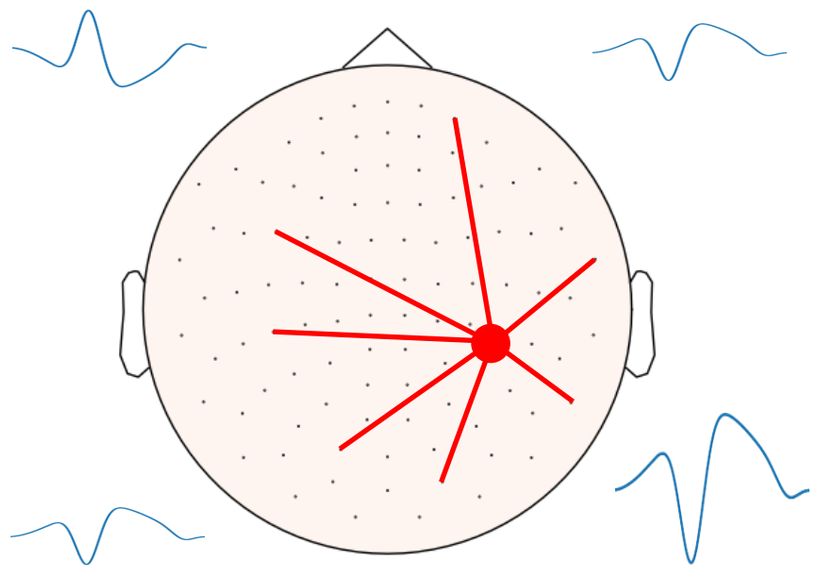
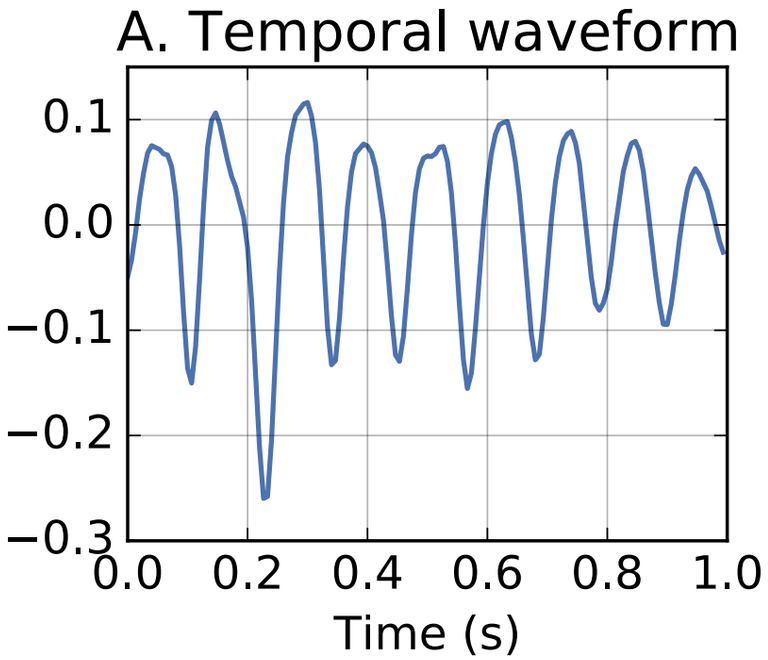


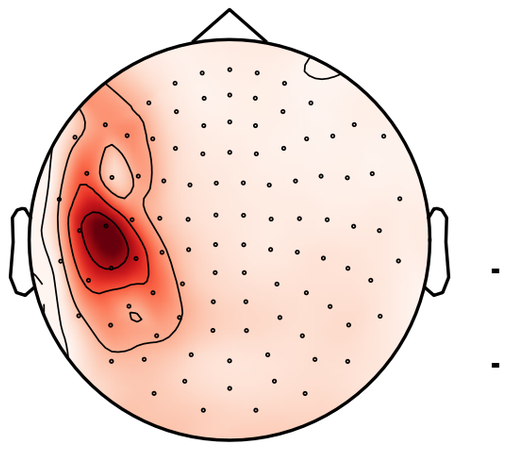
Rank 1 constraint:

$$\min_{u,v,z} \sum_{n=1}^{N} \frac{1}{2} \left\| X^n - \sum_{k=1}^{K} z_k^n * (u_k v_k^\top) \right\|_2^2 + \lambda \sum_{k=1}^{K} \|z_k^n\|_1 ,$$
$$\text{s.t.} \quad \|u_k\|_2^2 \leq 1 \ , \ \|v_k\|_2^2 \leq 1 \text{ and } z_k^n \geq 0.$$

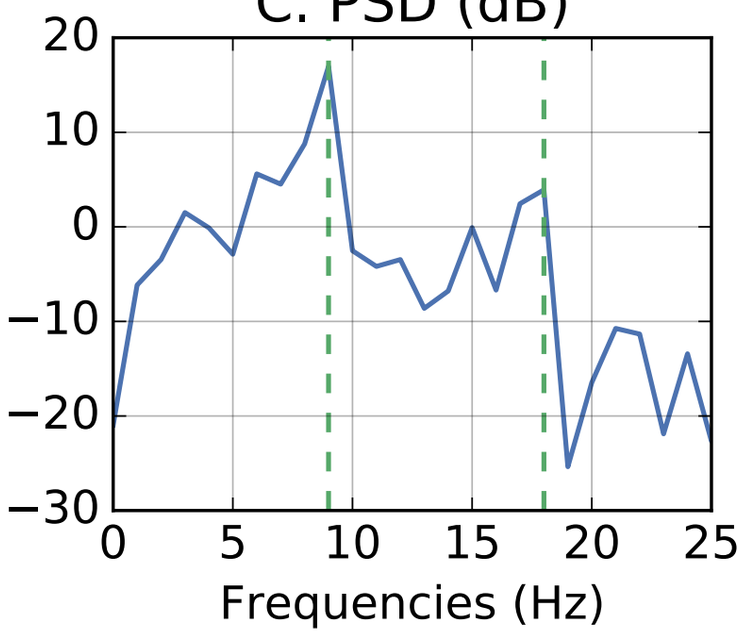
[Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals, (2018), T. Dupré la Tour, T. Moreau, M. Jas, A. Gramfort, Proc. NeurIPS Conf.]

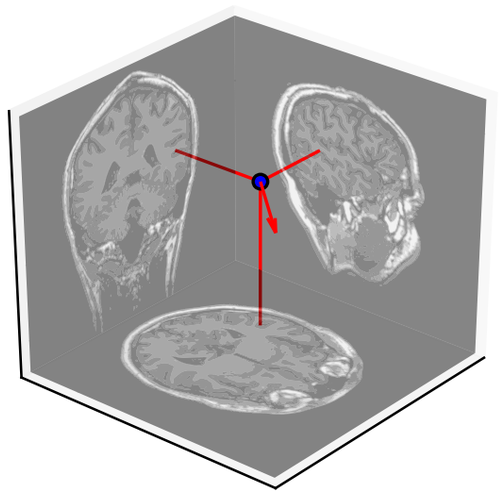
CSC on MEG





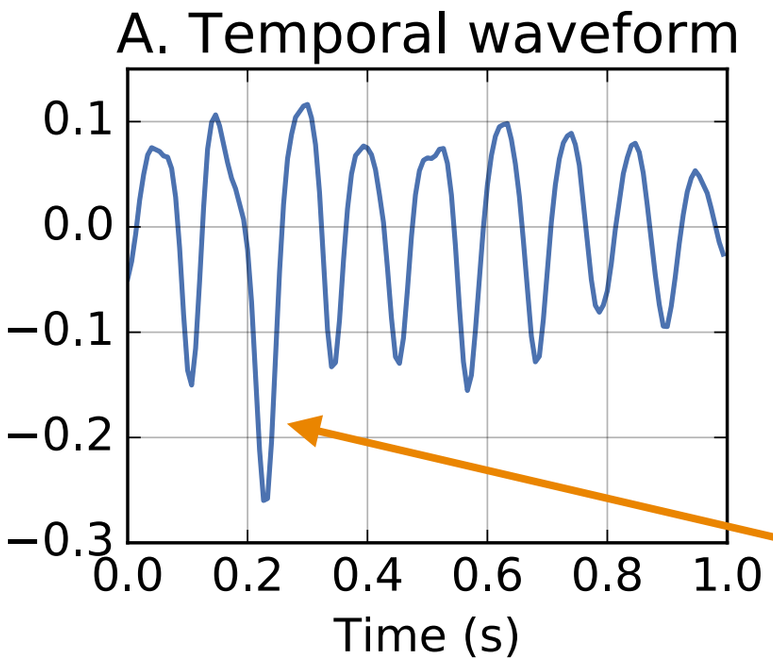
- MEG vectorview
- Median nerve stim.

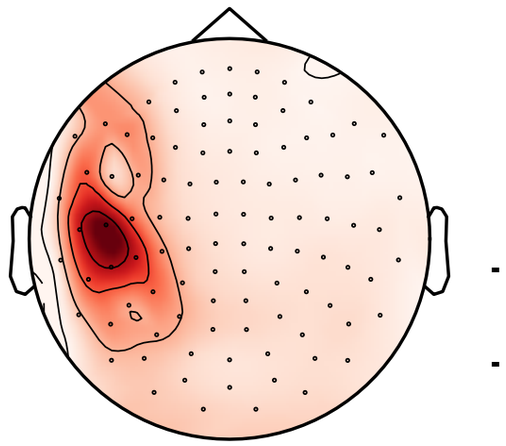




[Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals, (2018), T. Dupré la Tour, T. Moreau, M. Jas, A. Gramfort, Proc. NeurIPS Conf.]

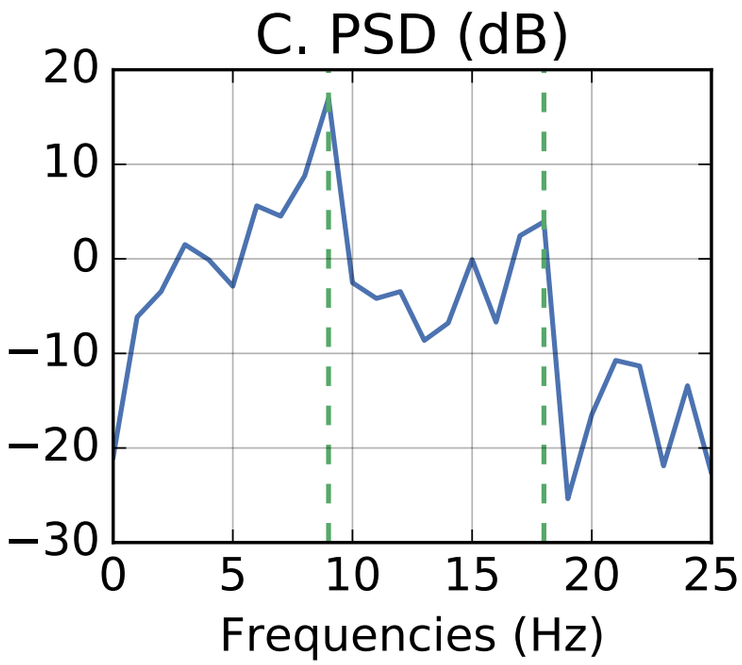
CSC on MEG

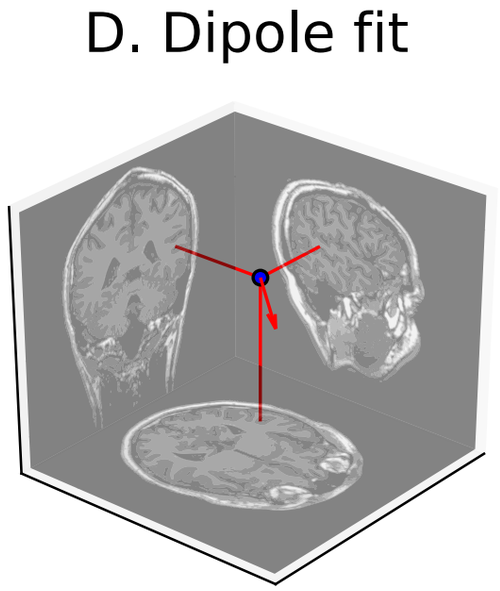




- MEG vectorview
- Median nerve stim.

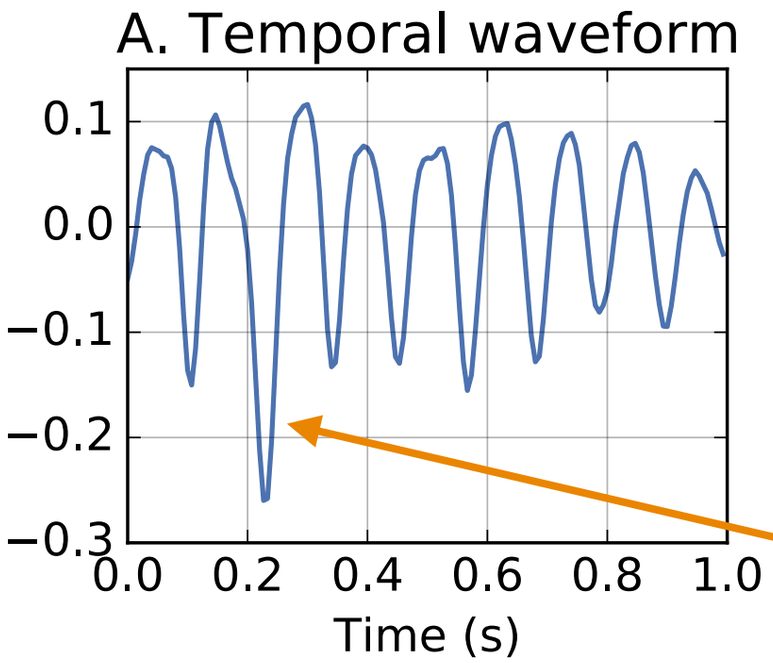
CSC reveals mu-shaped waveforms

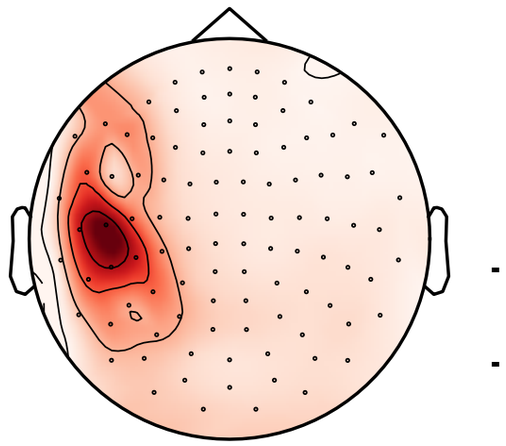




[Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals, (2018), T. Dupré la Tour, T. Moreau, M. Jas, A. Gramfort, Proc. NeurIPS Conf.]

CSC on MEG





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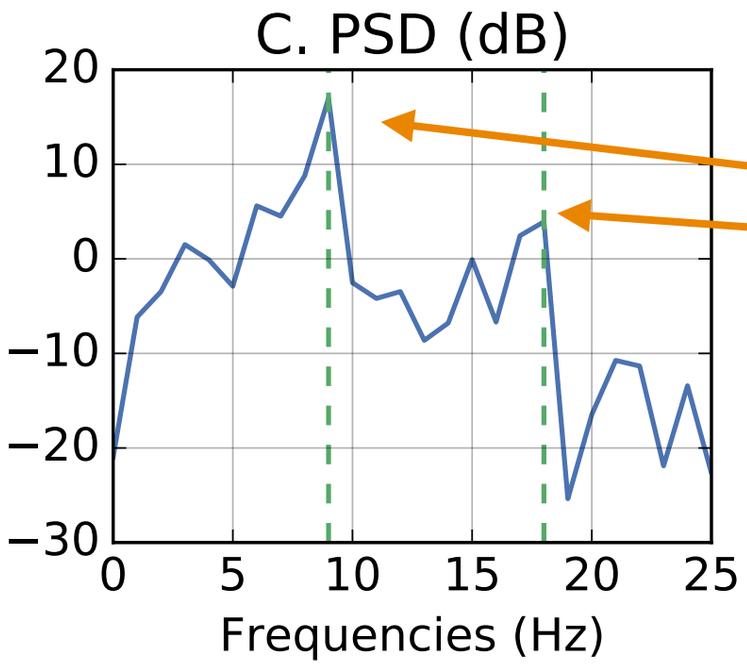
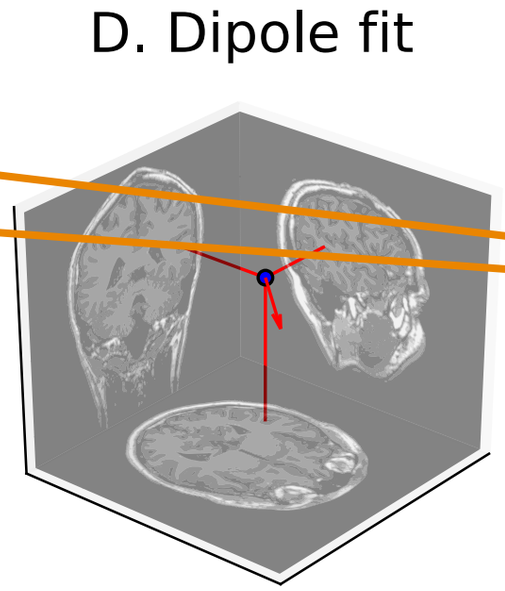
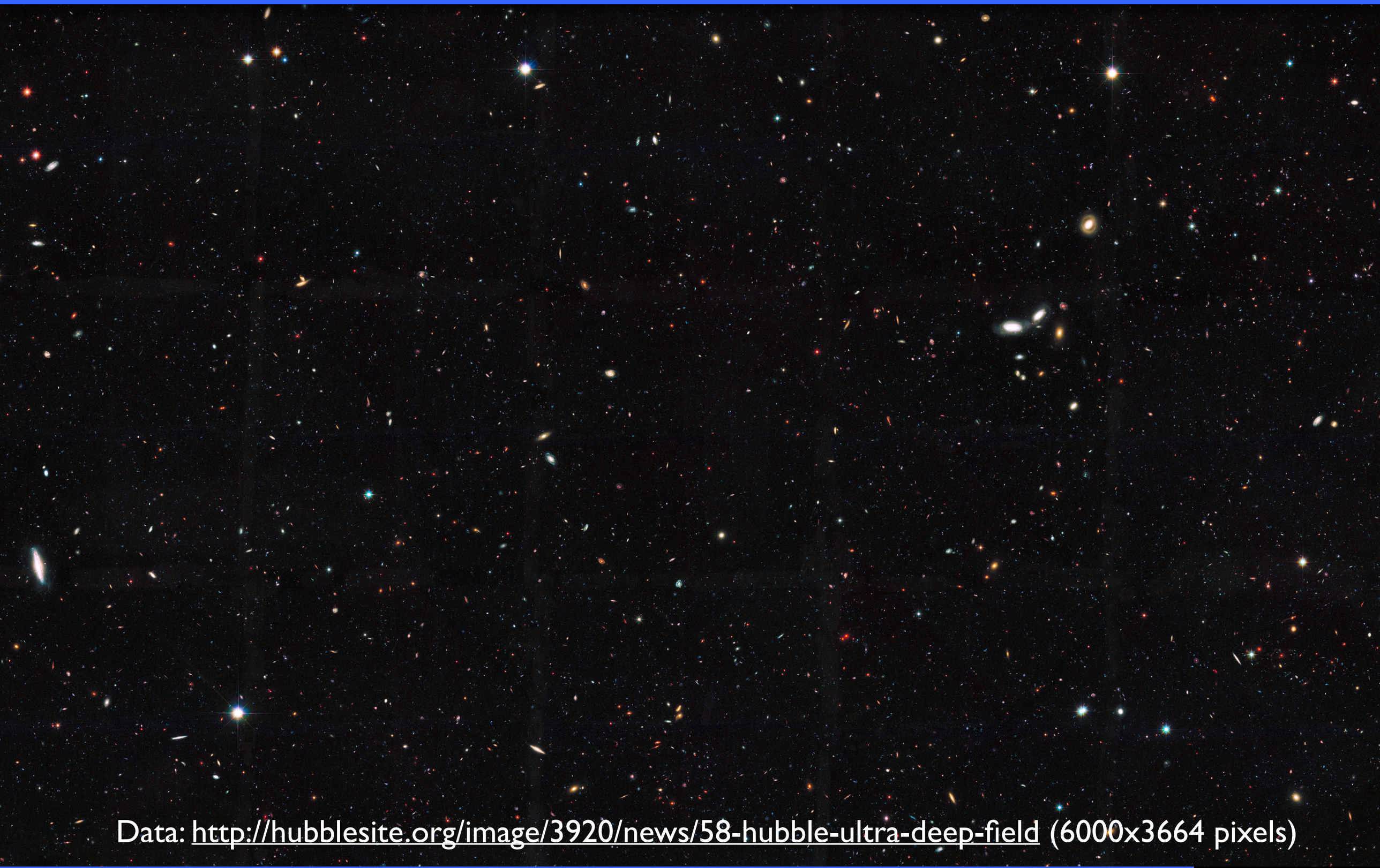




See the frequency harmonics

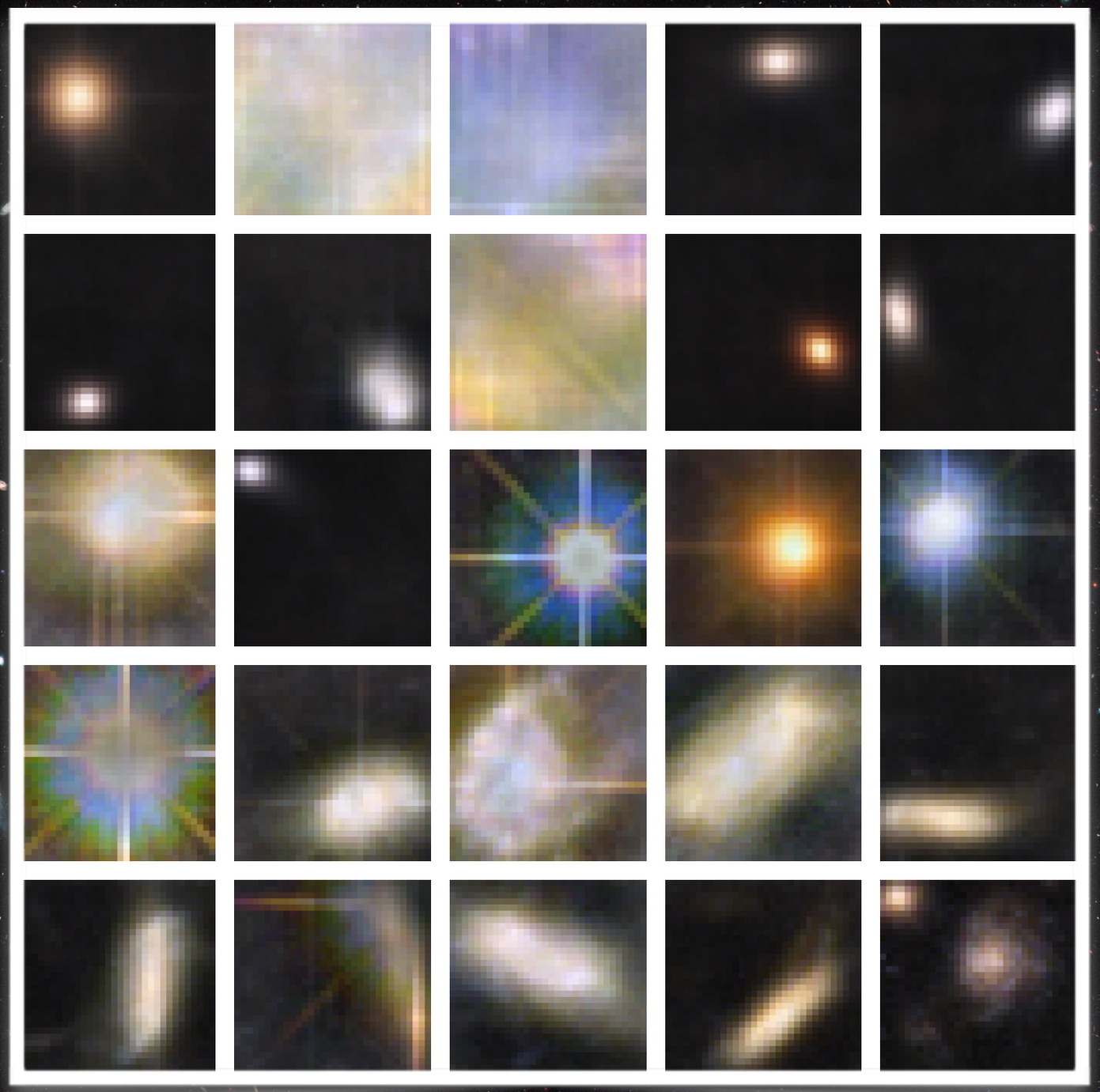
[Multivariate Convolutional Sparse Coding for Electromagnetic Brain Signals, (2018), T. Dupré la Tour, T. Moreau, M. Jas, A. Gramfort, Proc. NeurIPS Conf.]

Example using astrophysics



Data: http://hubblesite.org/image/3920/news/58-hubble-ultra-deep-field (6000x3664 pixels)

Example using astrophysics



[Distributed Convolutional Dictionary Learning (DiCoDiLe): Pattern Discovery in Large Images and Signals (2019), T. Moreau and A. Gramfort, ArXiv https://arxiv.org/abs/1901.09235]

Data: http://hubblesite.org/image/3920/news/58-hubble-ultra-deep-field (6000x3664 pixels)

Let's take a step back…

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Conclusion

- Contrib 1: Use Hessian approximations to inform a quasi-Newton solver (L-BFGS)
- Contrib 2: Convolutional sparse coding model for multivariate series with fast solvers

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- Statistical machine learning for neuroscience data
- High dimensional inference with complex noise and non-stationarities
- Fast algorithms for large scale non-smooth problems
- Software tools

"All models are wrong but some come with good open source implementation and good documentation so use those."

Picard

This is a library to run the Preconditioned ICA for Real Data (PICARD) algorithm [1] and its orthogonal version (PICARD-O) [2]. These algorithms show fast convergence even on real data for which sources independence do not perfectly hold.

Installation

We recommend the Anaconda Python distribution. Otherwise, to install `picard`, you first need to install its dependencies:

```
$ pip install numpy matplotlib numexpr scipy
```

Then install Picard:

```
$ pip install python-picard
```

If you do not have admin privileges on the computer, use the `--user` flag with *pip*. To upgrade, use the `--upgrade` flag provided by *pip*.

To check if everything worked fine, you can do:

```
$ python -c 'import picard'
```

and it should not give any error message.

https://pierreablin.github.io/picard/

Comparison of Picard-O and FastICA on faces data

This example compares FastICA and Picard-O:

Pierre Ablin, Jean-François Cardoso, Alexandre Gramfort "Faster ICA under orthogonal constraint" ICASSP, 2018 https://arxiv.org/abs/1711.10873

On the figure, the number above each bar corresponds to the final gradient norm.

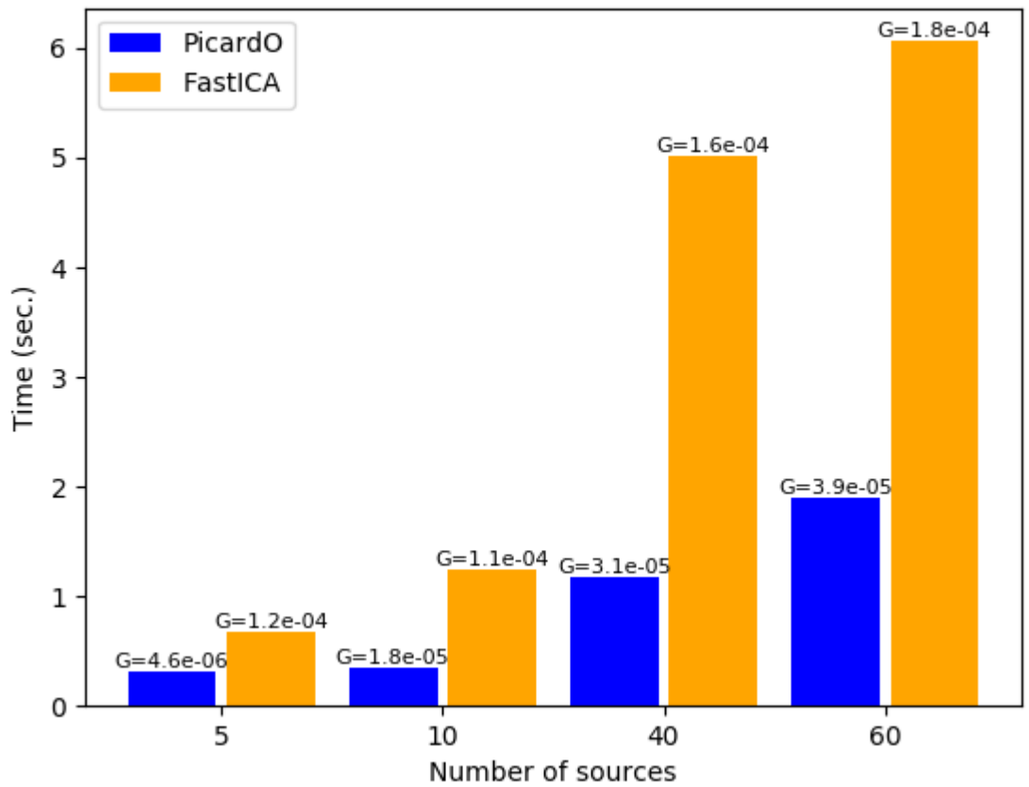
```
# Author: Pierre Ablin <pierre.ablin@inria.fr>
#         Alexandre Gramfort <alexandre.gramfort@inri
# License: BSD 3 clause


import numpy as np
from time import time
import matplotlib.pyplot as plt
from sklearn.datasets import fetch_olivetti_faces
from sklearn.decomposition import fastica

from picard import picard


print(__doc__)

image_shape = (64, 64)
rng = np.random.RandomState(0)
```



https://pierreablin.github.io/picard/

alphaCSC: Convolution sparse coding for time-series

This is a library to perform shift-invariant sparse dictionary learning, also known as convolutional sparse coding (CSC), on time-series data. It includes a number of different models:

1. univariate CSC
2. multivariate CSC
3. multivariate CSC with a rank-1 constraint [1]
4. univariate CSC with an alpha-stable distribution [2]

A mathematical descriptions of these models is available in the documentation.

Installation

To install this package, the easiest way is using `pip`. It will install this package and its dependencies. The `setup.py` depends on `numpy` and `cython` for the installation so it is advised to install them beforehand. To install this package, please run

```
pip install numpy cython
pip install git+https://github.com/alphacsc/alphacsc.git#egg=alphacsc
```

If you do not have admin privileges on the computer, use the `--user` flag with `pip`. To upgrade, use the `--upgrade` flag provided by `pip`.

To check if everything worked fine, you can run:

```
python -c 'import alphacsc'
```

and it should not give any error messages.

Quickstart

https://alphacsc.github.io

Here is an example to present briefly the API:

```
import numpy as np
```

Thanks !

Especially to:
Mainak Jas (PhD 2014-2017)
Tom Dupré La Tour (PhD 2015-2018)
Pierre Ablin (PhD 2016-…)
Thomas Moreau (Post-doc)
Francis Bach (DR, Inria)
Umut Şimşekli (MdC, Télécom ParisTech)
Jean-François Cardoso (DR, CNRS)

Contact

http://alexandre.gramfort.net

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