Markov Point Process for Multiple Object Detection

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Outline

Motivation

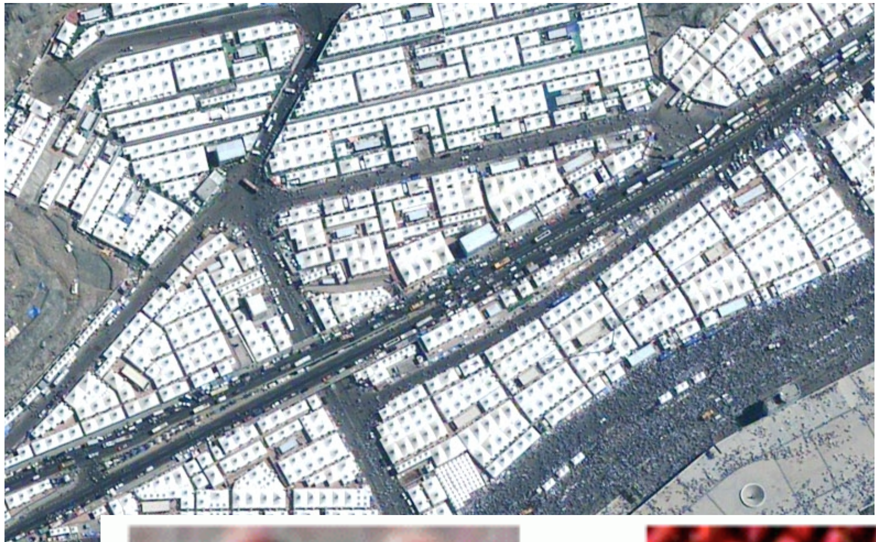
The different ingredients :

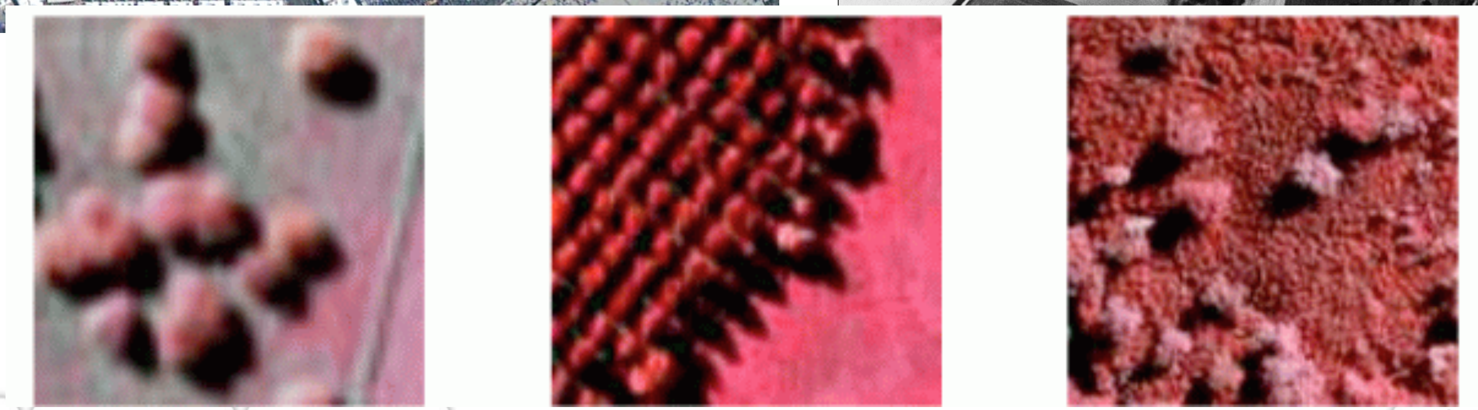
- objects
- reference measure
- prior
- data term
- optimization

Some results on microscopy images

Motivation : from context to geometry

1) High resolution data : the object geometry is an important source of information
2) The pixel scale does not contain the main information





Motivation : from context to geometry

1) Consider prior information (Bayesian approach, Markov Random Fields, interactions)

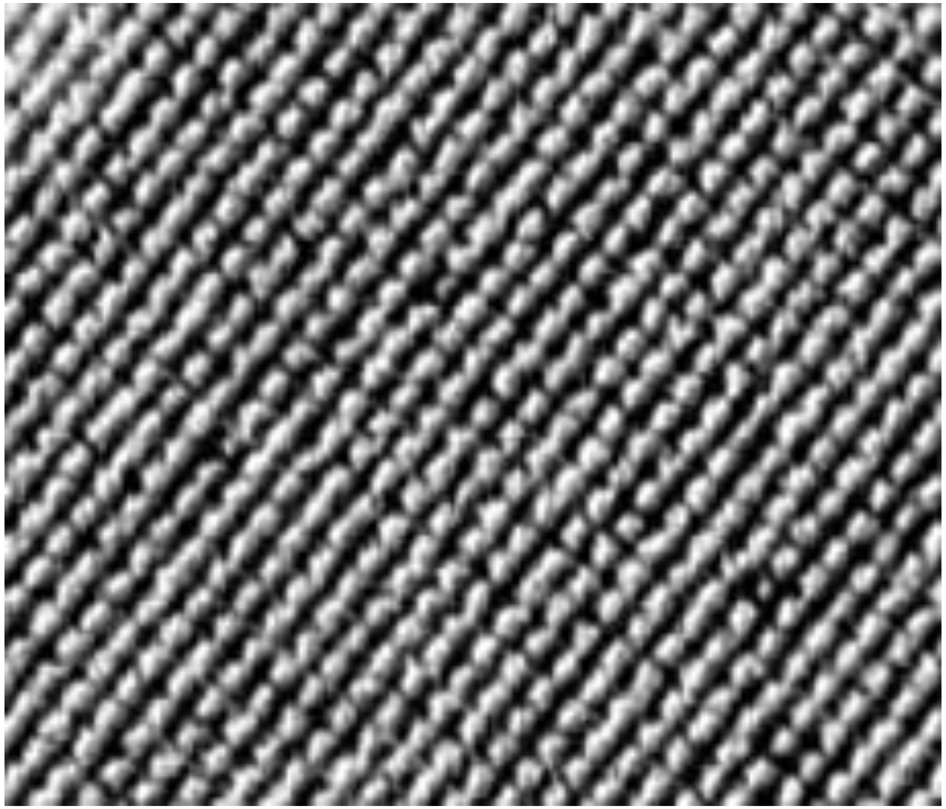
2) Embed geometric information (graph of objects)

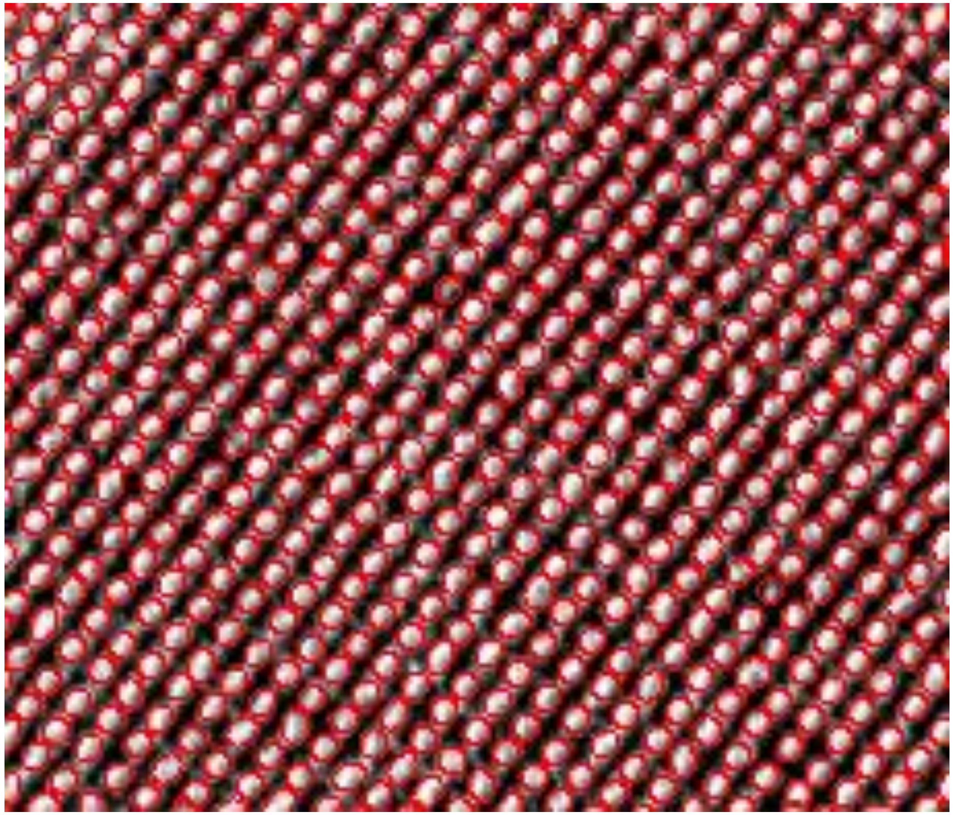
3) Modeling the scene structure (interactions between objects, unknown number of objects)

4) Need algorithms for simulating, optimizing the models

→ Marked point processes

Motivation : a scene as a collection of objects





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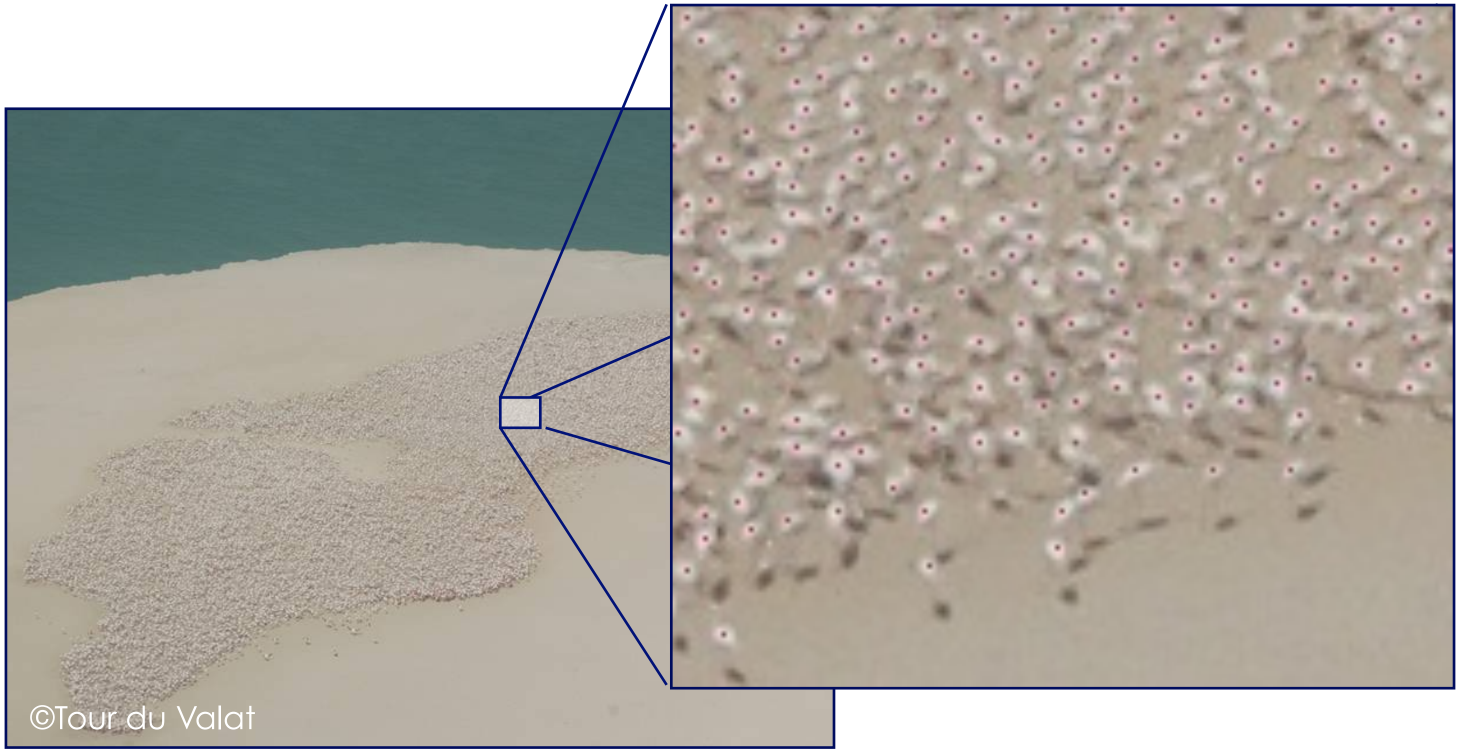

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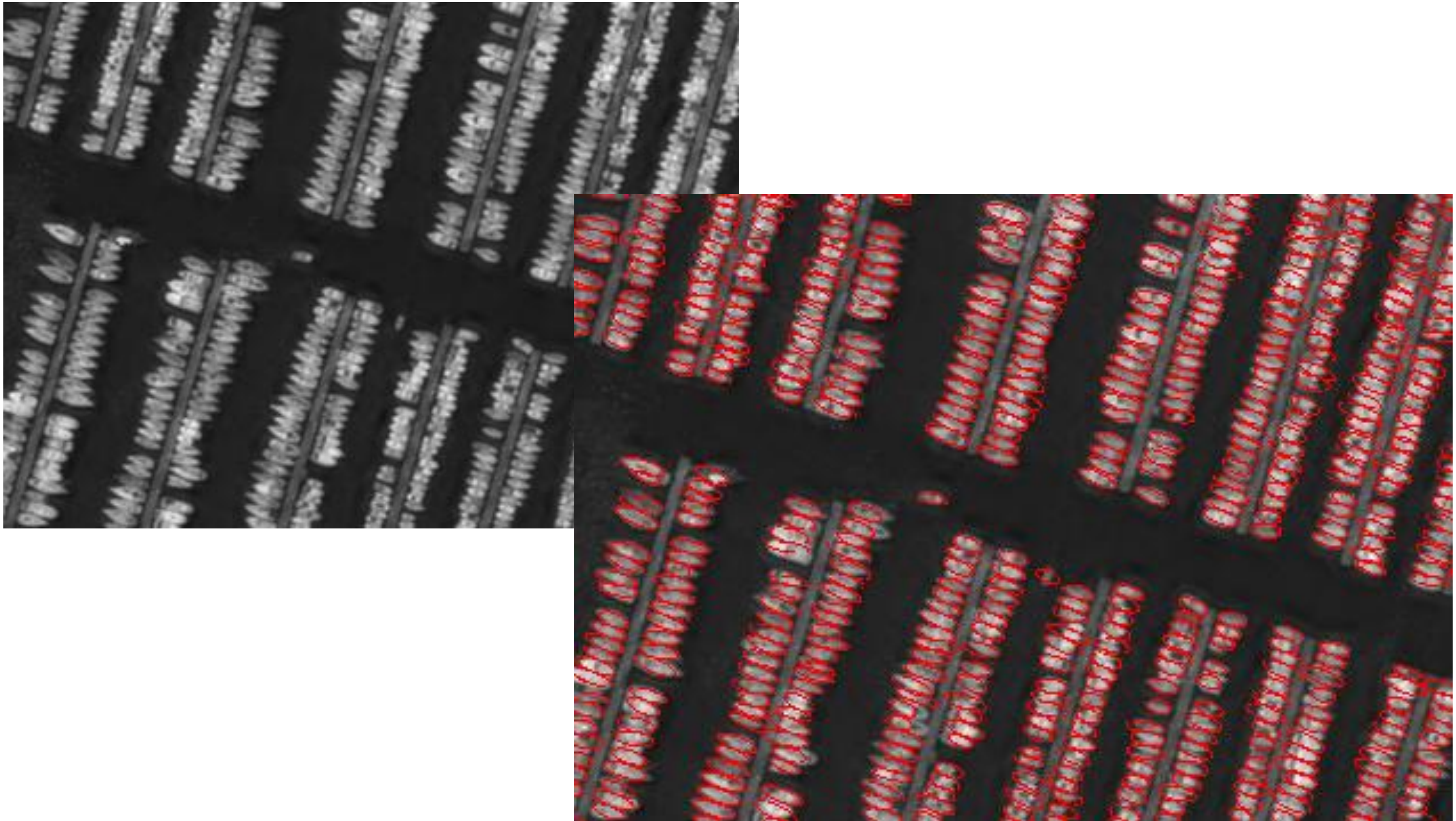


Specific Problems in Microscopy Imaging

Issue 1: How to address the intensity heterogeneity that prevents from considering a global threshold on the intensity in order to separate to objects from background ?

Issue 2: How to deal with nuisance objects that do not belong to the targeted class of objects but cannot be considered as background neither ?

Issue 3: How to deal with a high density of objects that generates clusters of possibly overlapping objects ?

Issue 4: How to handle the shape variability between objects ?

Issue 5: How to detect objects that consist of a few pixels ?

Issue 6: How to deal with both 2D and 3D datasets ?

The configuration space

« Simple » parametric shapes : $S = \{ s = (x,m), x \in K, m \in M \}$

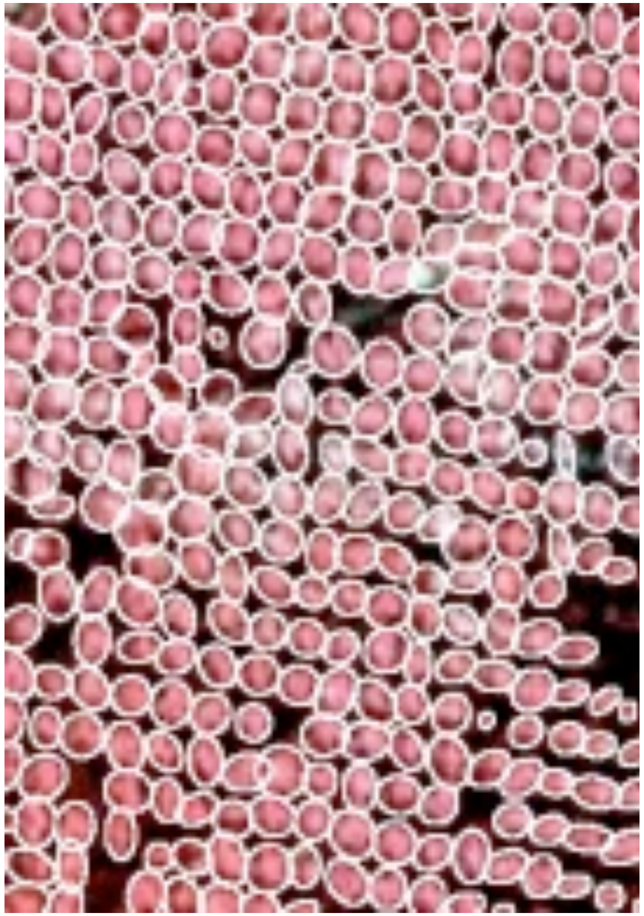
(x,r) (x,a,b,θ) (x,L,l,θ) (x,L,θ)

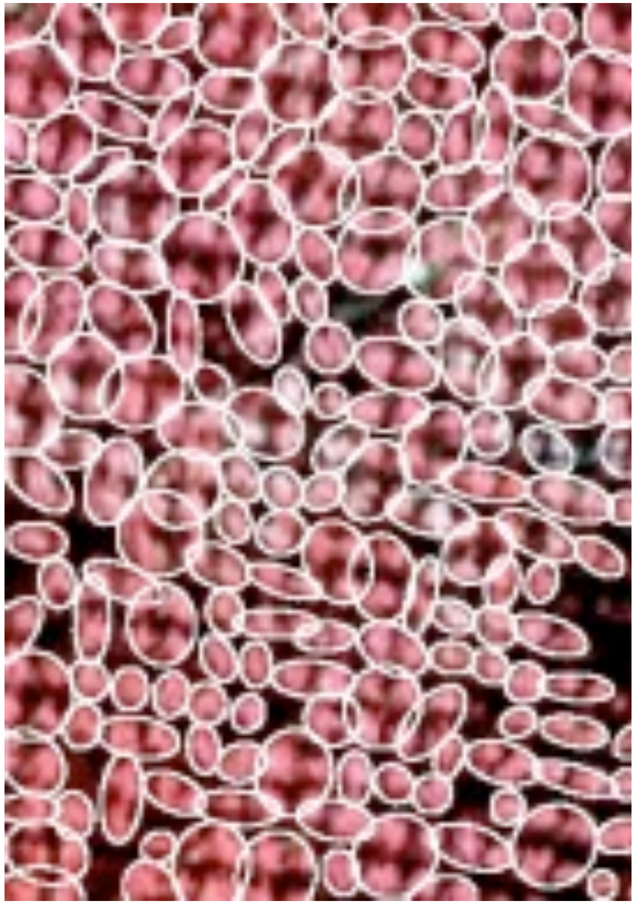
$\Omega_n = \{ \{ s_1, \ldots, s_n \}, s_i \in S \}$

$\Omega_0 = \varnothing$

$$\Omega = \bigcup_n \Omega_n$$

Influence of the marks



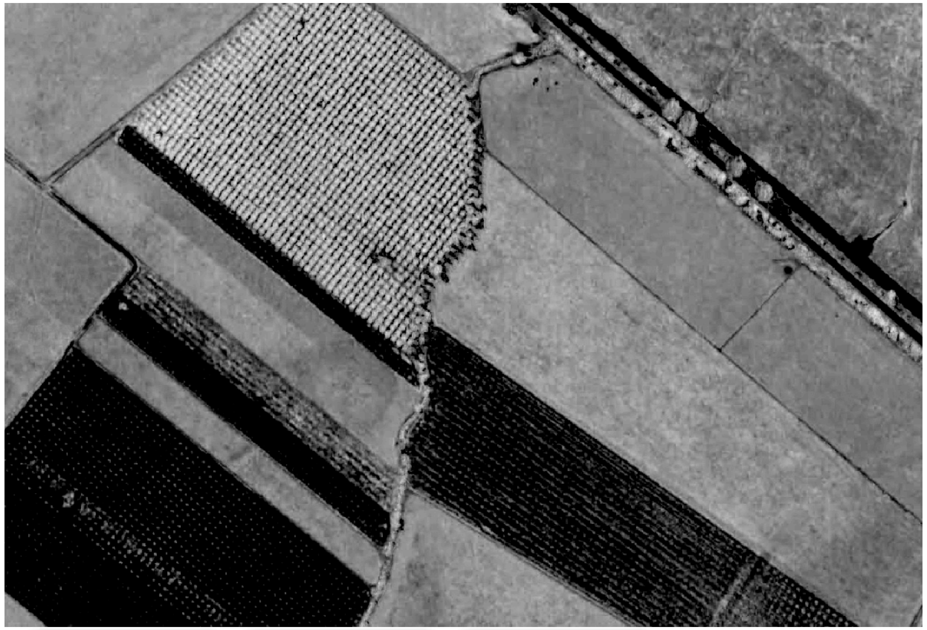


The reference measure

Usually the Poisson measure : $$\pi_\nu(B) = e^{-\nu(\chi)}\left(1_{[\emptyset \in B]} + \sum_{n=0}^{\infty} \frac{\pi_{\nu_n}(B)}{n!}\right)$$

$$\pi_{\nu_n}(B) = \int_\nu \cdots \int_\nu 1_{[\{x_1,\cdots,x_n\}\in B]} \nu(dx_1)\cdots\nu(dx_n)$$

$\{x_1,\cdots,x_n\}$: **unordered** set of simple points



Intensity measure: uniform or not

$$\nu(A)\int_A \lambda(x)dx$$

NDVI MAP

The density

The model is defined by a density (usually un-normalised) w.r.t the reference measure:

$$f:\Omega \rightarrow [0,\infty[, \int_{\Omega} f(x)\,d\pi_{\nu}(x) < \infty$$

Mimicking the Bayesian approach:

$$f(x) = g(x)h_I(x)$$

Prior

Data (I) term

An optimization algorithm

MAP criterion

$$\hat{X} = \operatorname{argmax} f(X|Y)$$

$$f(X|Y) \propto f(Y|X)f(X)$$

$$\hat{X} = lim_{T\to 0} f^{\frac{1}{T}}(X|Y)$$

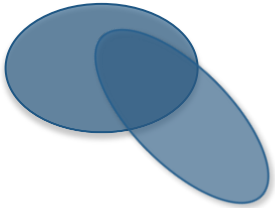
We sample :

$$f^{\frac{1}{T}}(X|Y)d\pi(x)$$

reference measure vs prior

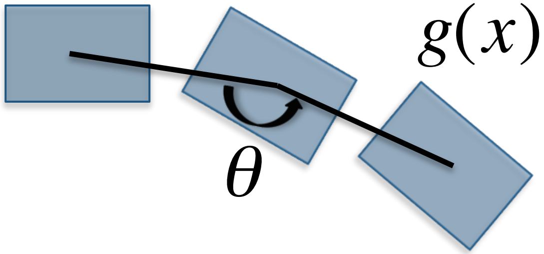
The prior

Overlap penalization (pairwise interaction):



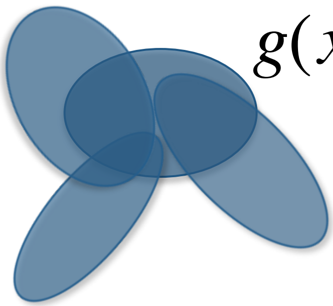
$$g(x)=\prod_{i\sim j}\varphi\left(x_i,x_j\right)\qquad \varphi\left(x_i,x_j\right)=\Phi\left(\frac{\left|s_i\cap s_j\right|}{\min\left(\left|s_i\right|,\left|s_j\right|\right)}\right)$$

Alignment :



$$g(x)=\prod_{i\sim j\sim k}\varphi\left(x_i,x_j,x_k\right)\quad \varphi\left(x_i,x_j,x_k\right)=\Phi\left(\left|\pi-\theta_{i,j,k}\right|\right)$$

Overlap penalization :



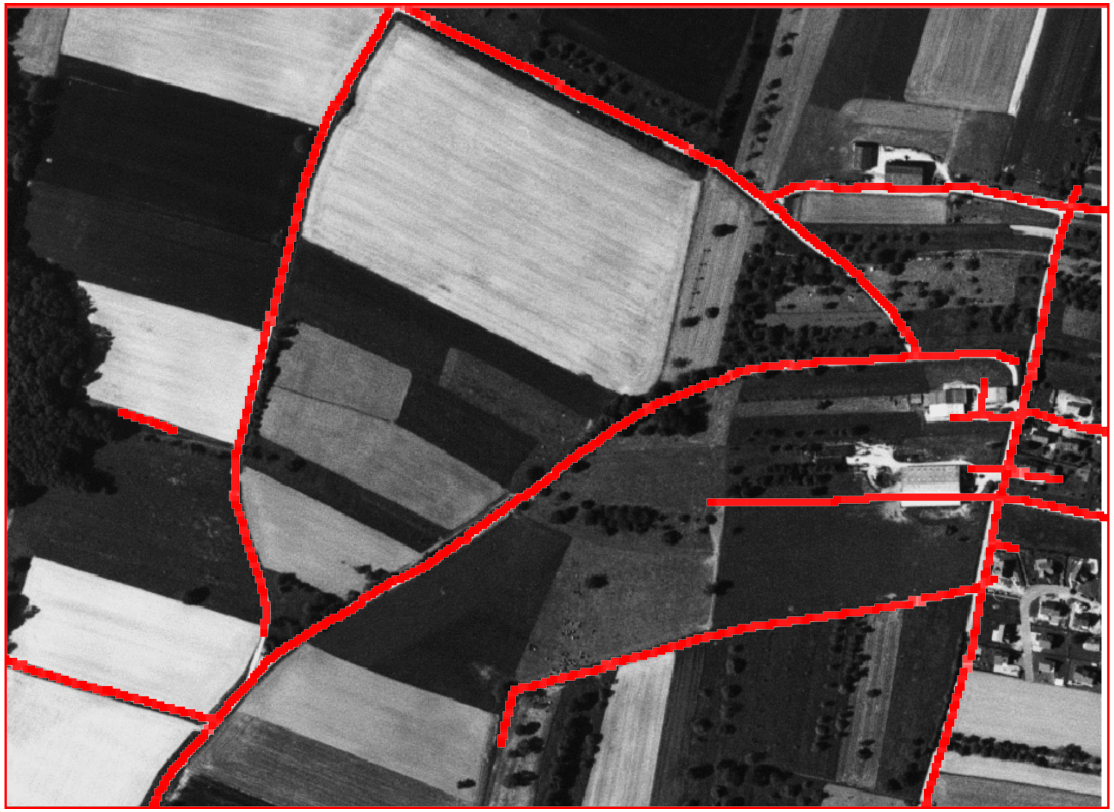
$$g(x)=\prod_{i}\varphi\left(x_i,x_j,j\sim i\right)$$

$$\varphi\left(x_i,x_j,j\sim i\right)=\Phi\left(\max_{j\sim i}\left(\frac{\left|s_i\cap s_j\right|}{\min\left(\left|s_i\right|,\left|s_j\right|\right)}\right)\right)$$

And many more …

Influence of the prior

Influence of the prior

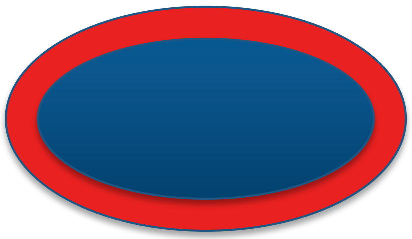


The data term

Bayesian approach: likelihood

$$h_Y(x) = \prod_{i \text{ inside objects}} 1_{\text{object}}(y_i) \prod_{i \text{ outside objects}} 1_{\text{background}}(y_i)$$

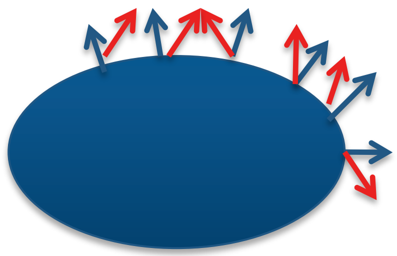
Detector like term : Distance between interior and exterior



$$h_Y(x) = \prod_{\text{objects}} dissimilarity(\text{red pixels, blue pixels})$$

Examples : Bhattacharya distance, Statistical test,…

Geometrical consistency



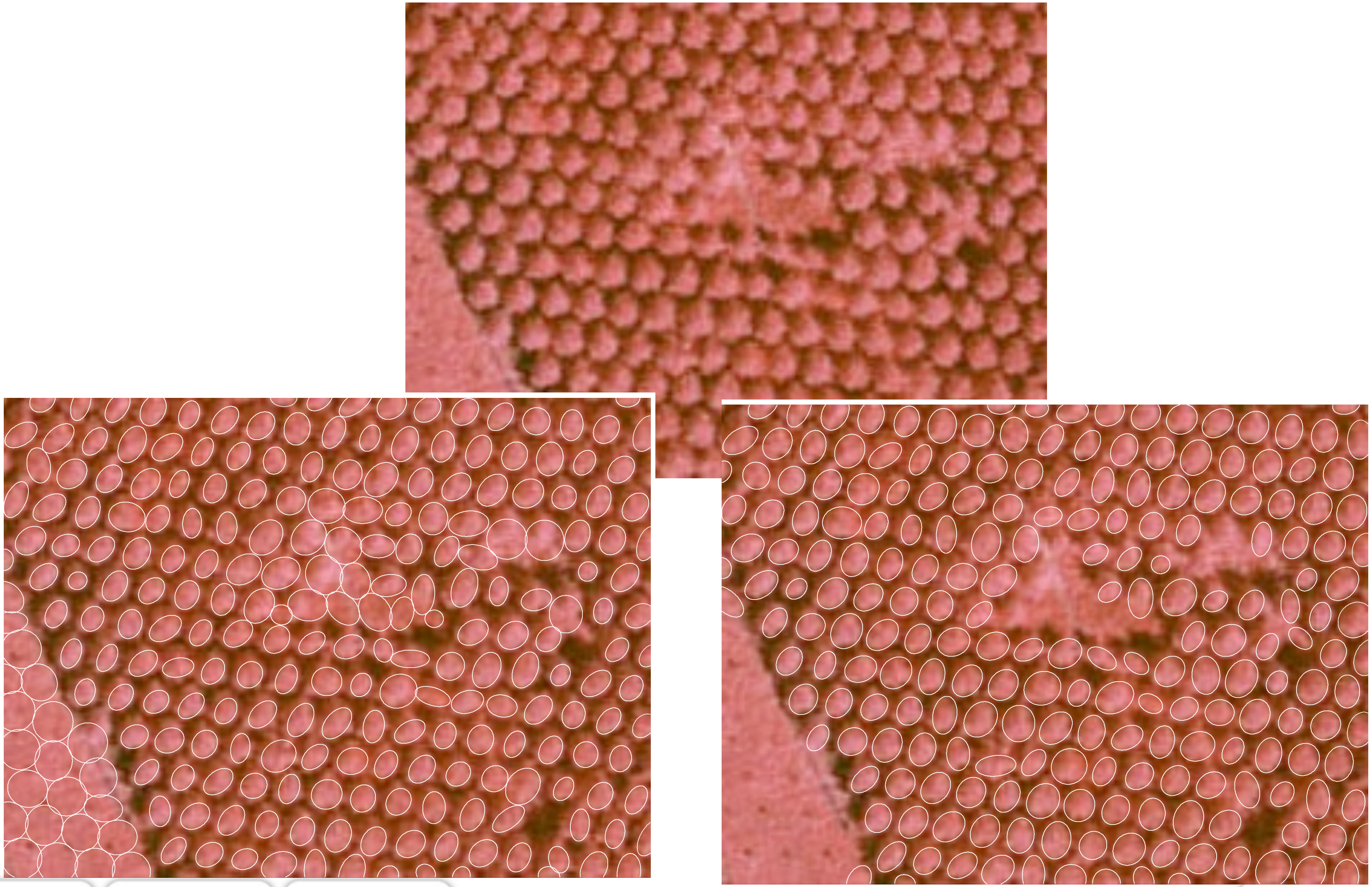
$$h_Y(x) = \prod_{\text{objects}} \exp - U(\omega)$$

$$U(\omega) = \frac{1}{|\partial\omega|} \int_{\partial\omega} \left\langle \frac{\nabla Y(u)}{\sqrt{\|\nabla Y(u)\|^2 + \varepsilon^2}}, n(u) \right\rangle du$$

$$U_d(\omega, Y) = \psi(U(\omega), t)$$

And many more …

Data likelihood versus detector like term



Optimization : RJMCMC

$$(h(x))^{1/T} d\pi(x)$$

- initialize the temperature T and the configuration x (empty set)
- Choose a proposition kernel $Q_m(x,.)$ with probability $p_m(x)$, or let the configuration unchanged probability $1-\Sigma_m p_m(x)$.
- Sample x' according to the chosen kernel
- Compute the acceptation ratio :

$$R_m(x,x') = \frac{D_m(x',x)}{D_m(x,x')} = \frac{(h(x'))^{1/T}\pi(dx')Q_m(x',dx)}{(h(x))^{1/T}\pi(dx)Q_m(x,dx')}$$

- With probability $\alpha = \min(1,R_m)$ set $x_{t+1} = x'$, else reject the proposition : $x_{t+1} = x$.

Some perturbation kernels (proposal)

Adding an object

Removing an object

Modifying an object (translation, rotation, dilation)

Merging/Splitting objects

Optimization : RJMCMC

Pros :

- Generality
- Choice for kernels
- Convergence to the global optimum

Cons :

- Rejection
- Simulated annealing scheme (parameters setting)
- Kernels usually involve one or two objects

Optimization :Multiple births and deaths

Goals :

Avoid rejection

Consider several objects at once

Idea :

Extend Langevin’s dynamics (Stochastic Differential Equation : diffusion process)

Optimization :Multiple births and deaths

1) Precomputing of the data term / birth map

2) Repeat :

2.1) Birth:

For each pixel, add an object with probability :

$$\delta B\left(E_d(u)\right)$$

2.2) Sort the objects with respect to their data term value

2.3) Death:

For each object u taken in the list order, remove it with probability :

$$\frac{\delta \exp\left[\beta\left(E(x)-E(x/u)\right)\right]}{1+\delta \exp\left[\beta\left(E(x)-E(x/u)\right)\right]}$$

Optimization :Multiple births and deaths

<u>Theorem:</u> When $N \to \infty, \beta \to \infty, \delta \to 0$, convergence toward the configuration minimizing the energy

Pros :

No rejection in the birth step
Birth does not depends on the temperature
Convergence to the global optimum

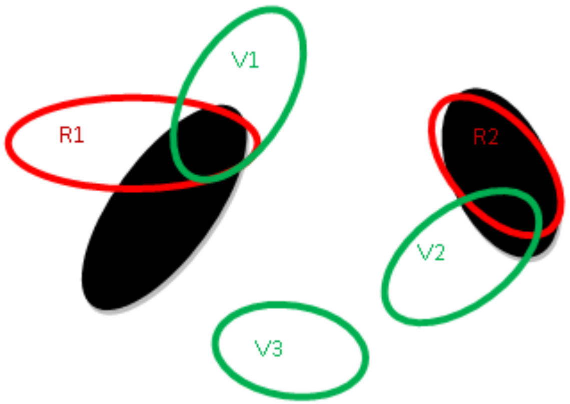
Cons :

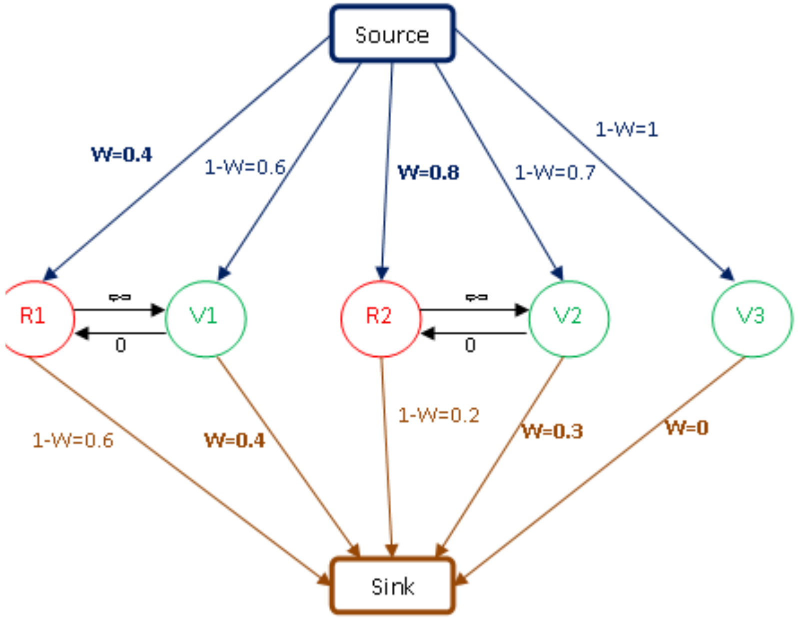
Only births and deaths kernels

Optimization :Multiple births and cut

Idea : Combine multiples births and deaths with graphcut techniques

- **Generate a first configuration x_0 of non overlapping objects and iterate the following steps:**
- ***Birth:***
  - ***generate a new configuration of non overlapping objects x'***
- ***Death:***
  - ***x_{n+1} is defined by $Cut(x_n U x')$***





Optimization :Multiple births and cut

Pros :

No rejection in the birth step
No cooling schedule

Cons :

Only births and deaths
No proof of convergence

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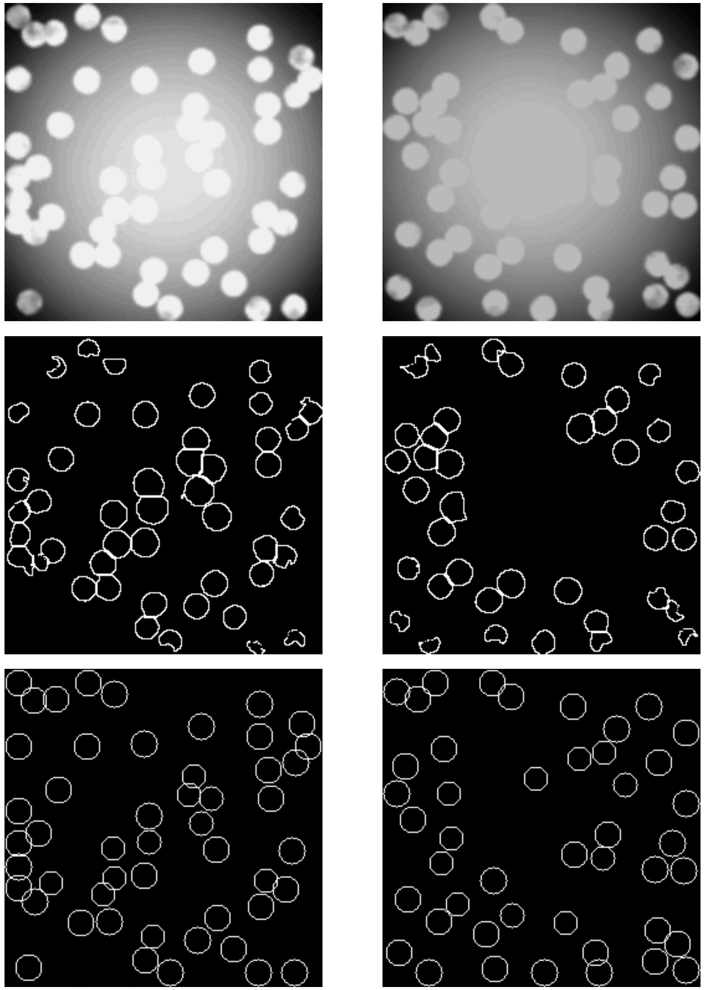

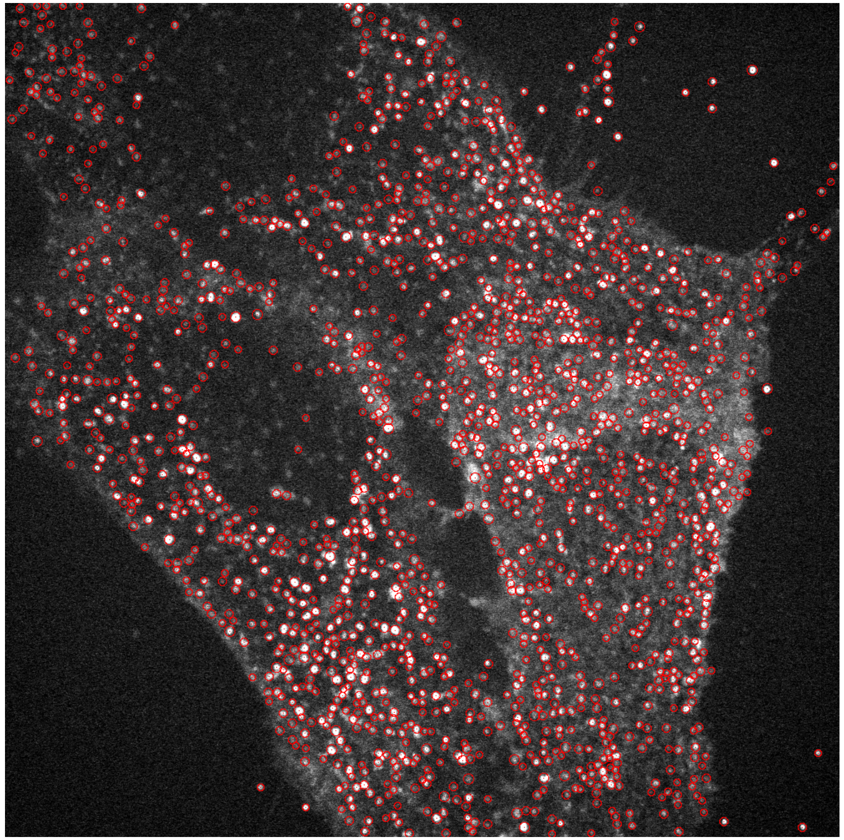

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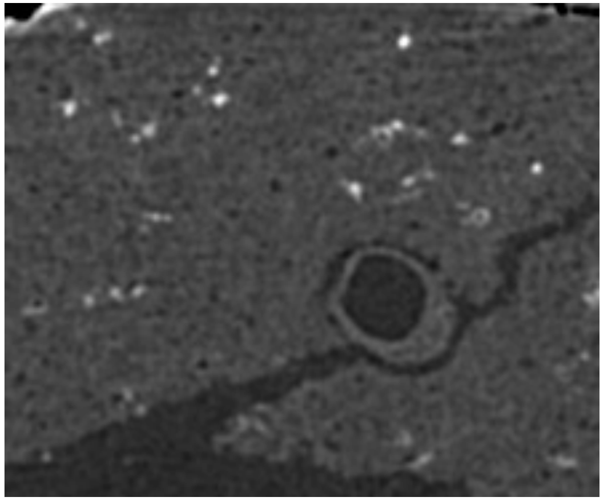

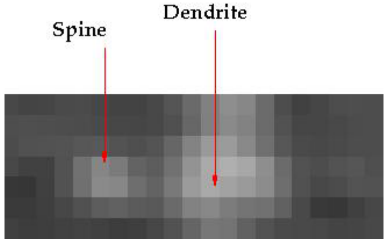

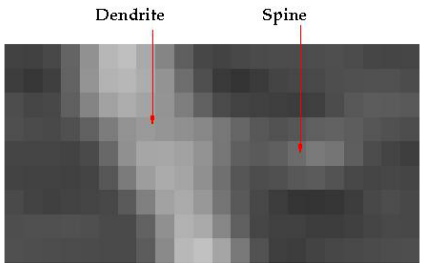

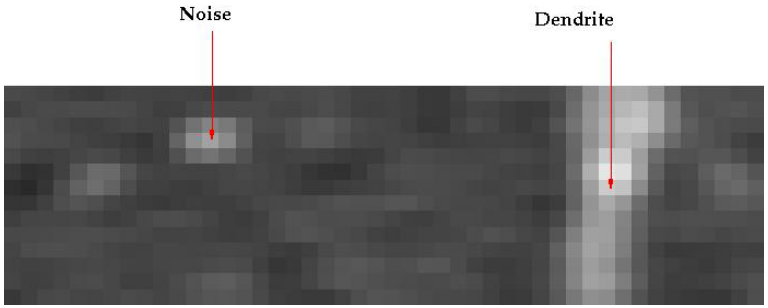

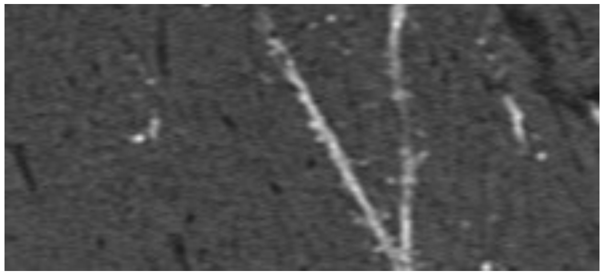

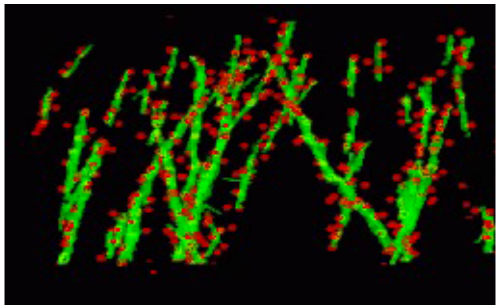

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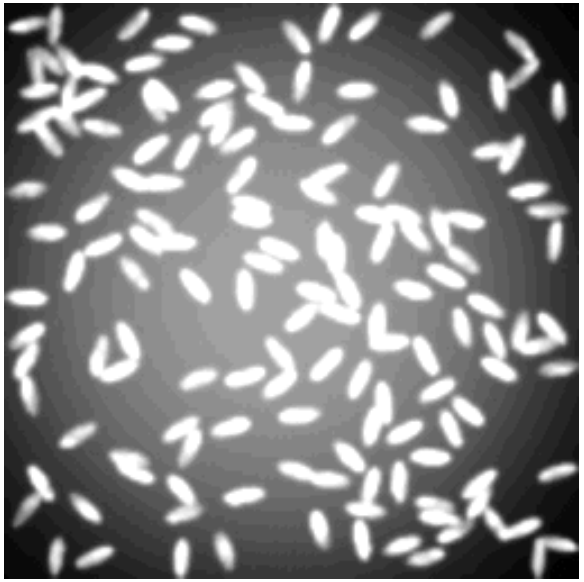
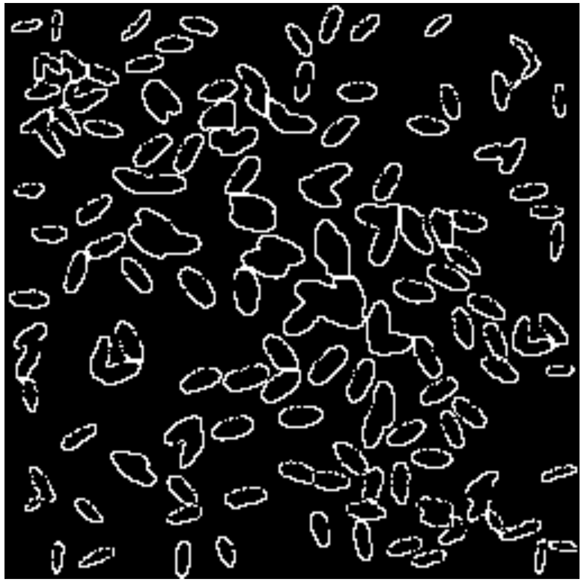
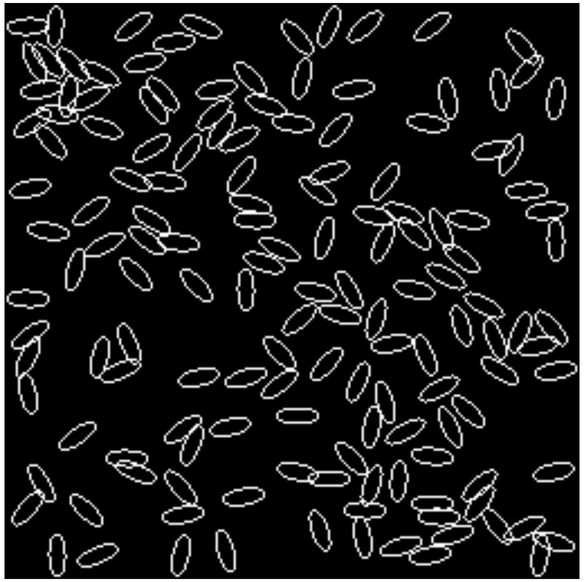


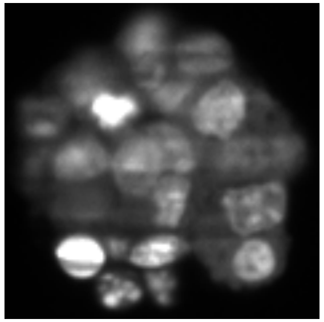


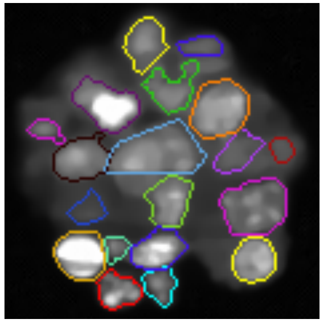
Particle analyzer (Fiji)



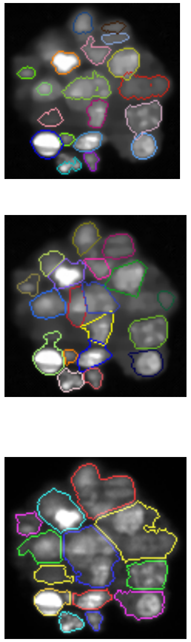
MPP with ellipses and small overlap allowed

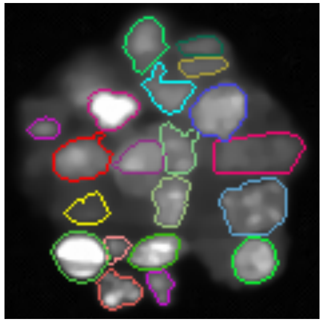
Issue 4: How to handle the shape variability between objects ?





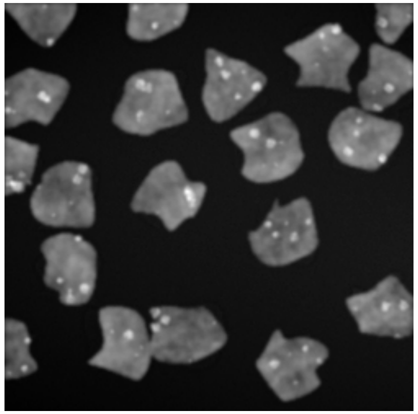
Level sets as objects

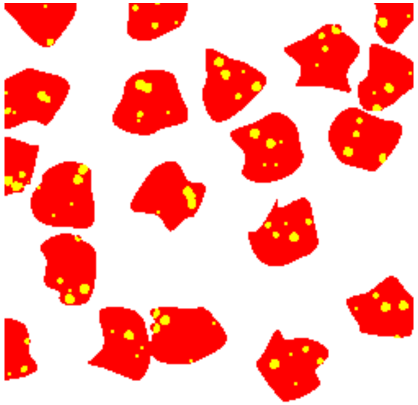


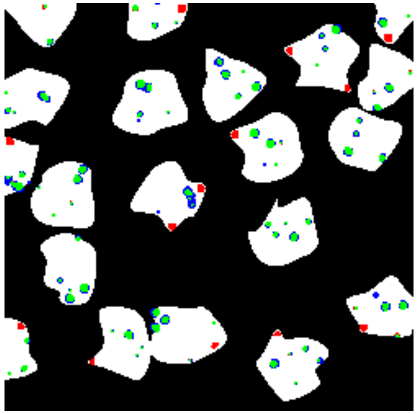


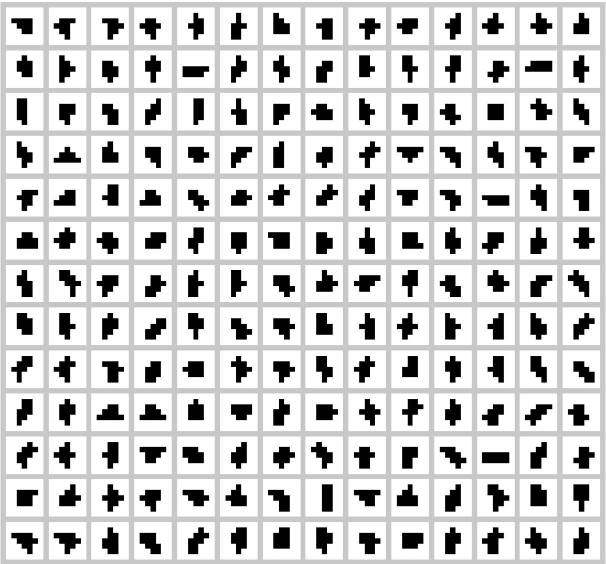
Objects selection among previous segmentation

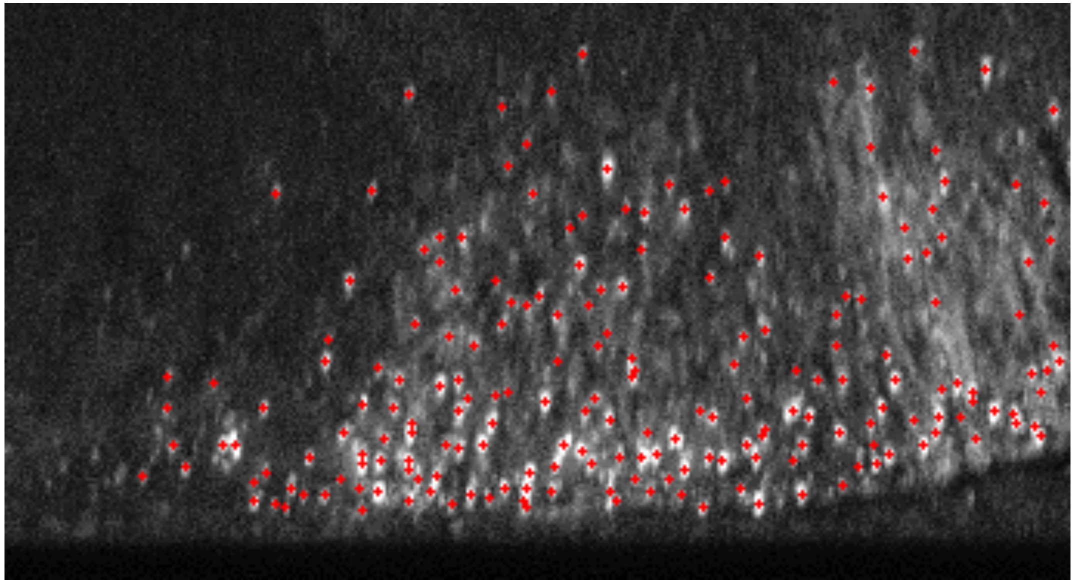
Issue 5: How to detect objects that consist of a few pixels ?











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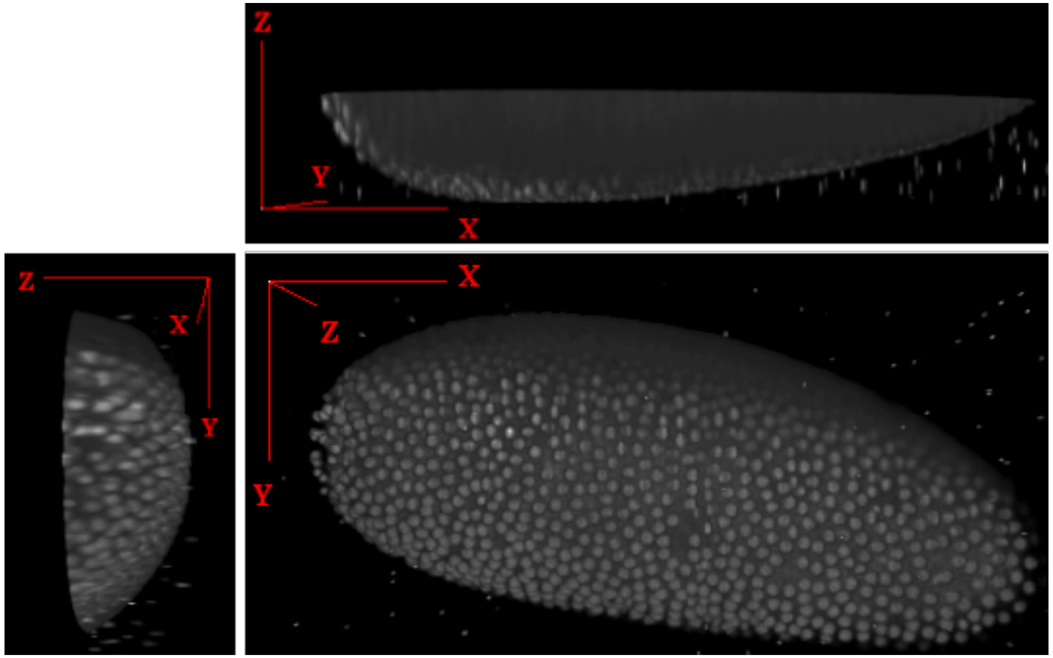
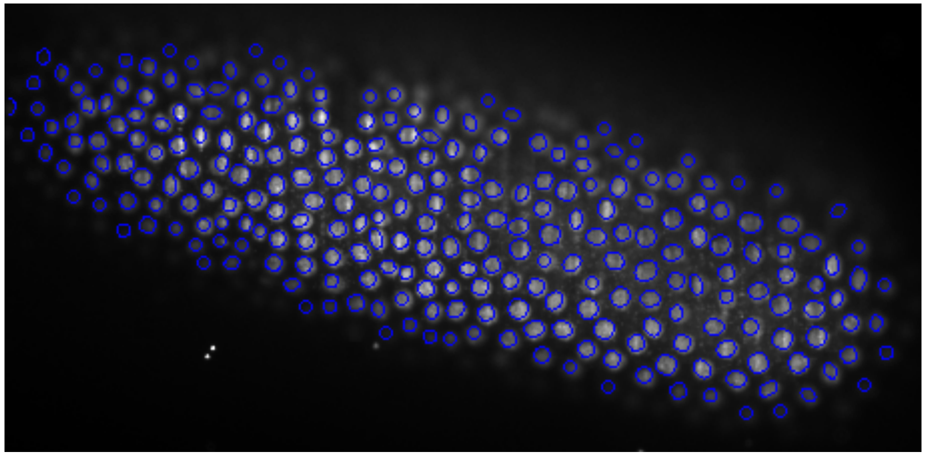
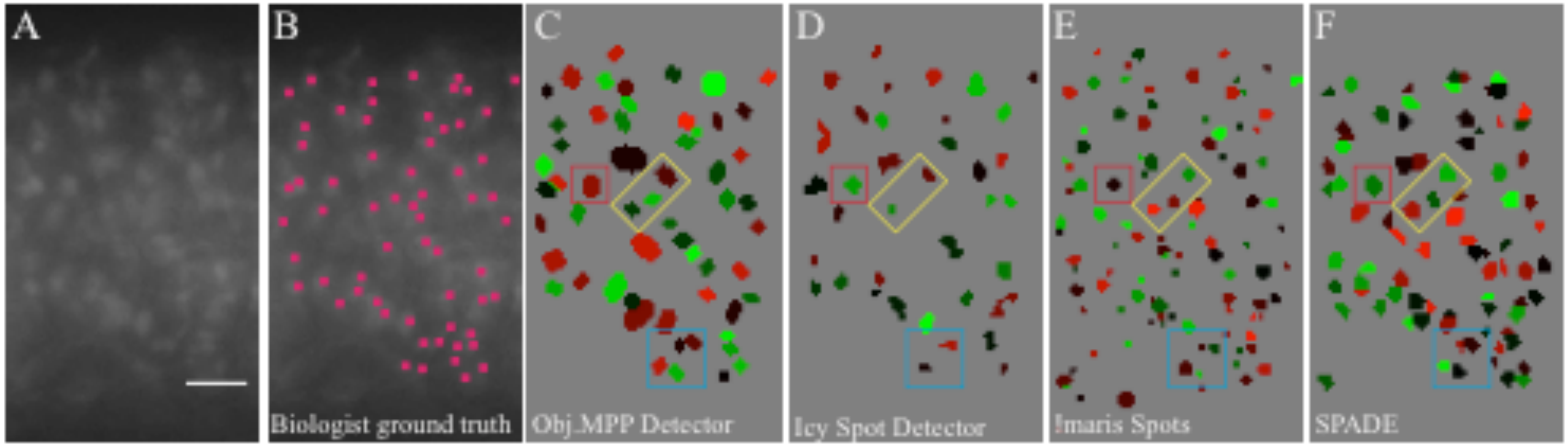




Comparison



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Algorithm	F1 score : mean	F1 score : std dev	F1 score : min	F1 score : max
Obj.MPP Detector	0,816	0,026	0,788	0,857
Icy Spot Detector	0,696	0,071	0,602	0,796
Imaris Spots	0,684	0,111	0,492	0,755
SPADE	0,676	0,0427	0,627	0,725

Conclusion

Pros :

General framework / Numerous application
Embed strong geometric constraint
Adapted to microscopy images for cellular and intra-cellular studies

Future work :

parallelism
parameter estimation
open source software (SPADE, ObjMPP)