

Variational methods for photometric 3D-reconstruction

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Outline

- 1 Shape-from-Shading**
- 2 Variational Solving of Shape-from-Shading**
- 3 Photometric Depth Super-Resolution for RGBD Sensors**
- 4 Combining Variational Methods with Deep Learning**
- 5 Uncalibrated Photometric Stereo**

Outline

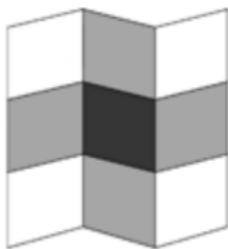
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Shape-from-shading: A Classic Ill-posed Problem

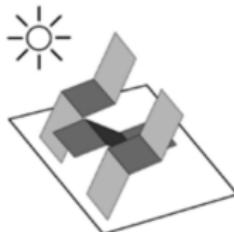
Given an image $\mathbf{I} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^m$, *Shape-from-Shading* (SfS) consists in inverting the forward photometric model (*image irradiance equation*)

$$\mathbf{I} = \mathcal{R}(z, \rho, \ell) \quad (1)$$

with \mathcal{R} a *radiance* function depending on the unknown depth $z : \Omega \rightarrow \mathbb{R}$, surface reflectance $\rho : \Omega \rightarrow \mathbb{R}^m$, and incident lighting $\ell : \Omega \rightarrow \mathbb{S}^2$.



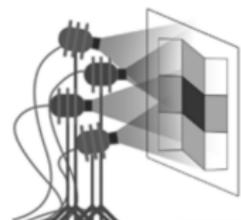
RGB image:
 \mathbf{I}



Sculptor's
explanation:
 z



Painter's
explanation:
 ρ



Gaffer's
explanation:
 ℓ

Illustration of SfS's Ill-posedness

Even with known surface reflectance ρ and incident lighting ℓ , shape estimation by SfS is an ill-posed inverse problem (Horn, 1970).

Example: two solutions of $\mathbf{I} = \mathcal{R}(\mathbf{z}, \rho, \ell)$ with $\mathbf{I} :=$ Lena, white reflectance ($\rho \equiv 1$) and frontal lighting ($\ell \equiv [0, 0, -1]^T$):



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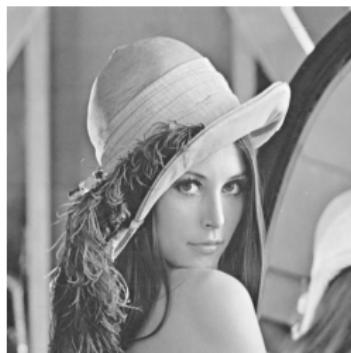


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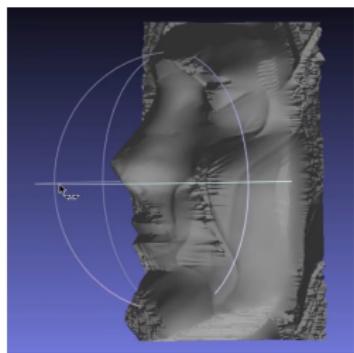


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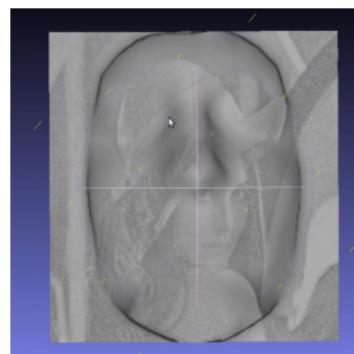
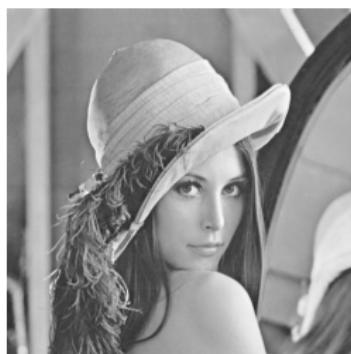


Illustration of SfS's Ill-posedness

Even with known surface reflectance ρ and incident lighting ℓ ,
shape estimation by SfS is an ill-posed inverse problem
(Horn, 1970).



Maximal viscosity solution
[Cristiani and Falcone 2007]

Variational solution
[Quéau et al. 2017]

Parameterization the irradiance equation $\mathbf{I} = \mathcal{R}(z, \rho, \ell)$

Basic Lambertian model:



RGB image
 $\mathbf{I} : \Omega \rightarrow \mathbb{R}^3$

=



Albedo
 $\rho : \Omega \rightarrow \mathbb{R}^3$

\odot



Shading
 $S(z, \ell) : \Omega \rightarrow \mathbb{R}$

where albedo (Lambertian reflectance) \equiv color, and shading \equiv lighting-geometry interaction.

Parameterization the irradiance equation $\mathbf{I} = \mathcal{R}(z, \rho, \ell)$

Shading \equiv lighting-geometry interaction:



RGB image
 $\mathbf{I} : \Omega \rightarrow \mathbb{R}^3$

=



Albedo
 $\rho : \Omega \rightarrow \mathbb{R}^3$

\odot



Lighting
 $\ell \in \mathbb{S}^2$



Normals
 $\mathbf{n}(z) : \Omega \rightarrow \mathbb{S}^2$

where the surface normal \mathbf{n} relates to the depth map z in a **nonlinear** way:

$$\mathbf{n}(z) = \frac{1}{\sqrt{|f\nabla z|^2 + (-z - \langle \mathbf{p}, \nabla z \rangle)^2}} \begin{bmatrix} f\nabla z \\ -z - \langle \mathbf{p}, \nabla z \rangle \end{bmatrix}$$

($f > 0$: length, and $\mathbf{p} : \Omega \rightarrow \mathbb{R}^2$: centered pixel coordinates).

Parameterization the irradiance equation $\mathbf{I} = \mathcal{R}(z, \rho, \ell)$

Extension to first-order spherical harmonics lighting $\ell \in \mathbb{R}^4$:



RGB image
 $\mathbf{I} : \Omega \rightarrow \mathbb{R}^3$



Albedo
 $\rho : \Omega \rightarrow \mathbb{R}^3$



Lighting
 $\ell \in \mathbb{R}^4$



Geometry

$$\begin{bmatrix} \mathbf{n} \\ 1 \end{bmatrix} (\mathbf{z}) : \Omega \rightarrow \mathbb{R}^4$$

$$I = \mathcal{R}(z, \rho, \ell) := \rho \langle \ell, \begin{bmatrix} \mathbf{n}(z) \\ 1 \end{bmatrix} \rangle$$

where the surface normal \mathbf{n} relates to the depth map z in a **nonlinear** way:

$$\mathbf{n}(z) = \frac{1}{\sqrt{|f\nabla z|^2 + (-z - \langle \mathbf{p}, \nabla z \rangle)^2}} \begin{bmatrix} f\nabla z \\ -z - \langle \mathbf{p}, \nabla z \rangle \end{bmatrix}$$

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Variational Solving of SfS Equation $I = \mathcal{R}(z, \rho, \ell)$

Assume (for now) that ρ and ℓ are known. Problem reduces to a nonlinear PDE $I := \mathcal{R}(\nabla z)$.

Horn and Brooks 1986: Regularization

Set $(p, q) := \nabla z$ over $\Omega \subset \mathbb{R}^2$

- 1) Estimate gradient components satisfying integrability:

$$\min_{p,q} \iint_{\Omega} (I - \mathcal{R}(p, q))^2 + \lambda (\partial_y p - \partial_x q)^2 \, dx dy$$

- 2) Integrate: $\min_z \iint_{\Omega} \|(p, q) - \nabla z\|^2 \, dx dy$

Quéau et al. 2017 (EMMCVPR): Hard constraint

(p, q) is conservative *by construction* \rightarrow Integrated estimation of gradient and depth:

$$\min_{p,q,z} \iint (I - \mathcal{R}(p, q))^2 \, dx dy$$

$$\text{s.t. } (p, q) = \nabla z$$

Regularized SfS Model

- Minimal surface regularization over Ω
- (Incomplete) depth prior over $\Omega' \subset \Omega$

$$\begin{aligned} \min_{p,q,z} & \iint_{\Omega} \lambda (I - \mathcal{R}(p, q))^2 + \nu \sqrt{1 + p^2 + q^2} \, dx \, dy \\ & + \iint_{\Omega'} \mu (z - z^0)^2 \, dx \, dy \\ \text{s.t. } & (p, q) = \nabla z \end{aligned}$$

By tuning λ , μ and ν , we may achieve SfS, depth denoising and inpainting, or shading-based depth refinement.

Solving the Regularized SfS Model using ADMM

$(p^{(k+1)}, q^{(k+1)})$ solution of the **local**, nonlinear least-squares problem: use **parallel** BFGS iterations

$$\begin{aligned} \min_{(p,q)} & \lambda \|I - \mathcal{R}(p, q)\|_{\ell^2(\Omega)}^2 + \nu \left\| \sqrt{1 + p^2 + q^2} \right\|_{\ell^1(\Omega)} \\ & + \frac{1}{2\beta} \left\| (p, q) - \nabla z^{(k)} + \theta^{(k)} \right\|_{\ell^2(\Omega)}^2 \end{aligned}$$

$z^{(k+1)}$ solution of the global, **linear** least-squares problem: use preconditioned **conjugate gradient** iterations

$$\min_z \mu \|z - z^0\|_{\ell^2(\Omega')}^2 + \frac{1}{2\beta} \left\| (p^{(k+1)}, q^{(k+1)}) - \nabla z + \theta^{(k)} \right\|_{\ell^2(\Omega)}^2$$

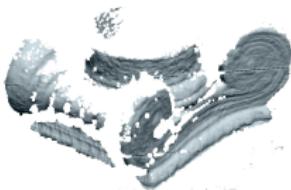
Auxiliary variable update

$$\theta^{(k+1)} = \theta^{(k)} + (p^{(k+1)}, q^{(k+1)}) - \nabla z^{(k+1)}$$

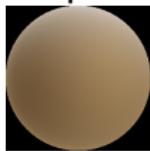
Application 1: Depth Refinement for MVS Techniques



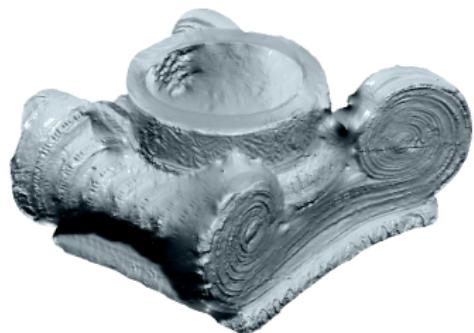
Input images I^1 and I^2



Input depth
map z^2



Estimated
lighting



Shading-based refinement

Application 2: SfS under Natural Illumination



Input RGB
image



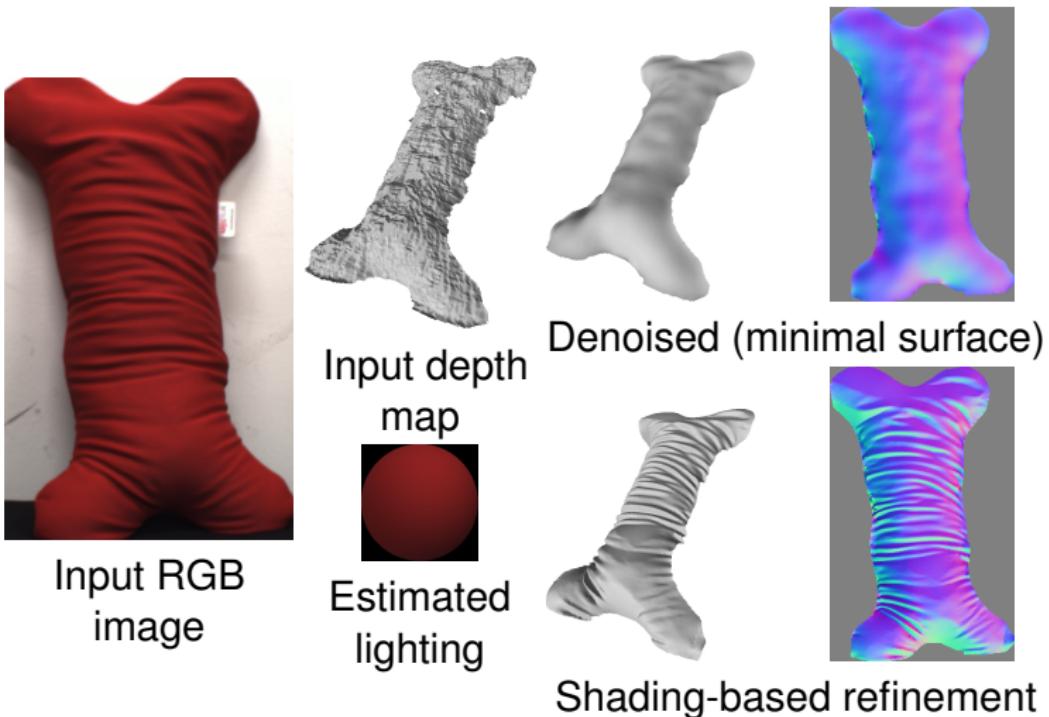
Calibrated
lighting



SfS 3D-reconstruction



Application 3: Depth Refinement for RGB-D Sensors



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Problem with RGB-D Sensors



Depth image

Shape

RGB image

Depth image has

- noise and quantization,
- missing areas,
- **coarse resolution.**

RGB image has

- less noise and quantization,
- no missing area,
- **high resolution.**

Goal:

Combine data to get **high-resolution shape**

Problem with RGB-D Sensors



Depth
image



Shape



RGB image



High-resolution
shape

Depth image has

- noise and quantization,
- missing areas,
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RGB image has

- less noise and quantization,
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Goal:

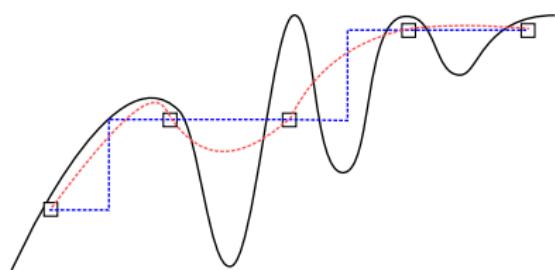
Combine data to get **high-resolution shape**

III-posedness in Depth Super-Resolution

Given a low-resolution depth map $z_0 : \Omega_{LR} \rightarrow \mathbb{R}$, *Depth Super-Resolution* (SR) consists in inverting the forward downsampling model

$$z_0 = Dz$$

with $z : \Omega_{HR} \rightarrow \mathbb{R}$ the (unknown) high-resolution depth, and D a (rank-deficient) downsampling operator



Genuine surface (black line)
can be approximated in ∞
many ways (dashed lines)
given sparse observations
(rectangles)

⇒ Use SfS to find a shape interpolation which is consistent with the high-resolution RGB image

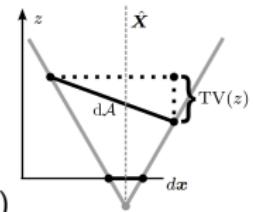
- Peng et al. 2017 (ICCV)
- Haefner, Quéau, et al. 2018 (CVPR)
- Haefner, Peng, et al. 2019 (PAMI)

Variational Formulation

$$\min_{\begin{array}{l} \mathbf{z}: \Omega_{HR} \rightarrow \mathbb{R} \\ \boldsymbol{\rho}: \Omega_{HR} \rightarrow \mathbb{R}^3 \\ \boldsymbol{\ell} \in \mathbb{R}^4 \end{array}} \left\| \mathbf{I} - \boldsymbol{\rho} < \boldsymbol{\ell}, \begin{bmatrix} \mathbf{n}(\mathbf{z}) \\ 1 \end{bmatrix} \right\|_{\ell_2(\Omega_{HR})}^2 + \mu \|\mathbf{z}_0 - D\mathbf{z}\|_{\ell_2(\Omega_{LR})}^2 + \nu P_1(\mathbf{z}) + \lambda P_2(\boldsymbol{\rho})$$

P_1 is a minimal surface regularization term:

$$\begin{aligned} P_1(\mathbf{z}) &= \|dA(\mathbf{z})\|_{\ell_1(\Omega_{HR})} \\ &= \left\| \frac{\mathbf{z}}{f} \sqrt{|f \nabla \mathbf{z}|^2 + (-\mathbf{z} - \langle \mathbf{p}, \nabla \mathbf{z} \rangle)^2} \right\|_{\ell_1(\Omega_{HR})} \end{aligned}$$



P_2 is a Potts regularization term (**nondifferentiable** and **nonconvex**),

$$P_2(\boldsymbol{\rho}) = \|\nabla \boldsymbol{\rho}\|_{\ell_0(\Omega_{HR})} = \sum_{\mathbf{p} \in \Omega_{HR}} \begin{cases} 0, & \text{if } |\nabla \boldsymbol{\rho}(\mathbf{p})|_F = 0, \\ 1, & \text{otherwise.} \end{cases}$$

Numerical Resolution - Splitting Strategy

Normal \mathbf{n} and minimal surface term dA depend on \mathbf{z} and $\nabla \mathbf{z}$:

$$\mathbf{n}(\mathbf{z}) := \mathbf{n}(\mathbf{z}, \nabla \mathbf{z}) = \frac{1}{\sqrt{|f \nabla \mathbf{z}|^2 + (-\mathbf{z} - \langle \mathbf{p}, \nabla \mathbf{z} \rangle)^2}} \begin{bmatrix} f \nabla \mathbf{z} \\ -\mathbf{z} - \langle \mathbf{p}, \nabla \mathbf{z} \rangle \end{bmatrix}$$

$$dA(\mathbf{z}) := dA(\mathbf{z}, \nabla \mathbf{z}) = \frac{z}{f} \sqrt{|f \nabla \mathbf{z}|^2 + (-\mathbf{z} - \langle \mathbf{p}, \nabla \mathbf{z} \rangle)^2}$$

Introduce splitting $\boldsymbol{\theta} := (\mathbf{z}, \nabla \mathbf{z})$ to make optimization tractable:

$$\begin{aligned} & \min_{\substack{\mathbf{z}: \Omega_{HR} \rightarrow \mathbb{R} \\ \boldsymbol{\rho}: \Omega_{HR} \rightarrow \mathbb{R}^3 \\ \boldsymbol{\ell} \in \mathbb{R}^4 \\ \boldsymbol{\theta}: \Omega_{HR} \rightarrow \mathbb{R}^3}} \left\| \mathbf{I} - \boldsymbol{\rho} < \boldsymbol{\ell}, \begin{bmatrix} \mathbf{n}(\boldsymbol{\theta}) \\ 1 \end{bmatrix} \right\|_{\ell_2(\Omega_{HR})}^2 + \mu \| \mathbf{z}_0 - D\mathbf{z} \|_{\ell_2(\Omega_{LR})}^2 \\ & \quad + \nu \| dA(\boldsymbol{\theta}) \|_{\ell_1(\Omega_{HR})} + \lambda \| \nabla \boldsymbol{\rho} \|_{\ell_0(\Omega_{HR})} \end{aligned}$$

$$\text{s.t. } \boldsymbol{\theta} = (\mathbf{z}, \nabla \mathbf{z})$$

Numerical Resolution - Multi-block ADMM

Given $(\rho^{(k)}, \ell^{(k)}, \theta^{(k)}, z^{(k)})$ at iteration k , we update:

$$\rho^{(k+1)} = \operatorname{argmin}_{\rho} \left\| \mathbf{I} - \rho < \ell^{(k)}, \begin{bmatrix} \mathbf{n}(\theta^{(k)}) \\ 1 \end{bmatrix} \right\|_{\ell^2(\Omega_{HR})}^2 + \lambda \|\nabla \rho\|_{\ell^0(\Omega_{HR})}$$

$$\ell^{(k+1)} = \operatorname{argmin}_{\ell} \left\| \mathbf{I} - \rho^{(k+1)} < \ell, \begin{bmatrix} \mathbf{n}(\theta^{(k)}) \\ 1 \end{bmatrix} \right\|_{\ell^2(\Omega_{HR})}^2$$

$$\theta^{(k+1)} = \operatorname{argmin}_{\theta} \left\| \mathbf{I} - \rho^{(k+1)} < \ell^{(k+1)}, \begin{bmatrix} \mathbf{n}(\theta) \\ 1 \end{bmatrix} \right\|_{\ell^2(\Omega_{HR})}^2$$

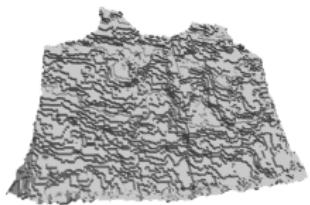
$$+ \nu \|dA_\theta\|_{\ell^1(\Omega_{HR})} + \frac{\kappa}{2} \left\| \theta - (z, \nabla z)^{(k)} + u^{(k)} \right\|_{\ell^2(\Omega_{HR})}^2$$

$$z^{(k+1)} = \operatorname{argmin}_z \mu \|z_0 - Dz\|_{\ell^2(\Omega_{LR})}^2 + \frac{\kappa}{2} \left\| \theta^{(k+1)} - (z, \nabla z) + u^{(k)} \right\|_{\ell^2(\Omega_F)}^2$$

$$u^{(k+1)} = u^{(k)} + \theta^{(k+1)} - (z, \nabla z)^{(k+1)}$$



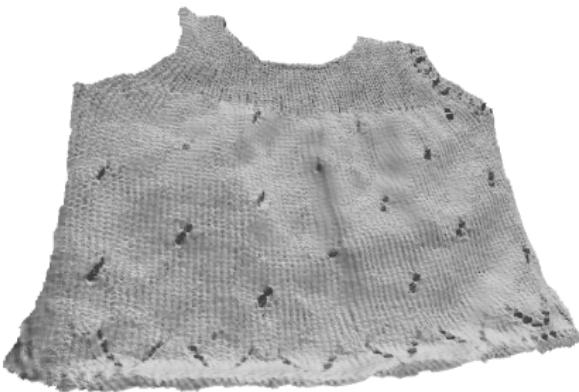
Qualitative Evaluation



Input depth



Input RGB



Depth estimate

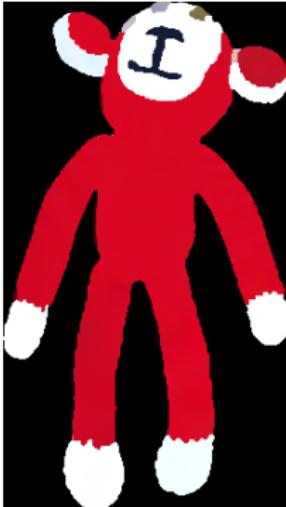
Qualitative Evaluation



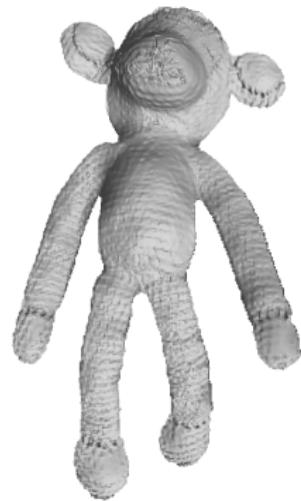
Input RGB



Input
depth



Albedo
estimate



Depth estimate

Qualitative Evaluation



Input RGB



Input
depth



Albedo
estimate



Depth estimate

Qualitative Evaluation



Input RGB



Albedo estimate



Depth image



Depth estimate

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Motivation: Failure Case of the Previous Approach

When estimated albedo is highly undersegmented, albedo information propagates to geometry:



Input RGB



Input
depth



Albedo
estimate



Depth estimate

Beyond Regularization: Reflectance Learning

Replacing Potts regularization of reflectance by a deep learning framework circumvents the difficulties of choosing an appropriate regularizer, and simplifies numerics.



Input RGB

Input
depth

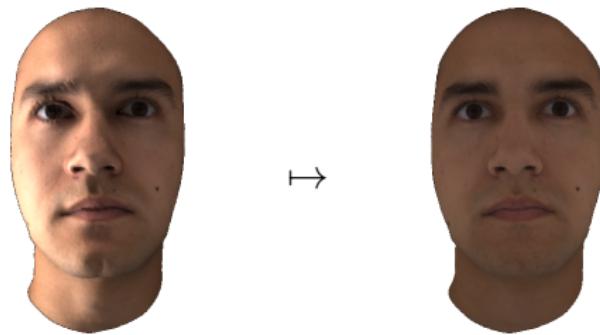
CNN-albedo
estimate

Depth estimate

[Haefner et al., "Photometric Depth Super-Resolution",
PAMI 2019]

Reflectance Learning: Idea

Learn a black-box mapping from image to albedo to get rid of man-made Potts prior. Leave the rest (geometry and lighting estimation) to the physics-based variational approach.



RGB
image
(input)

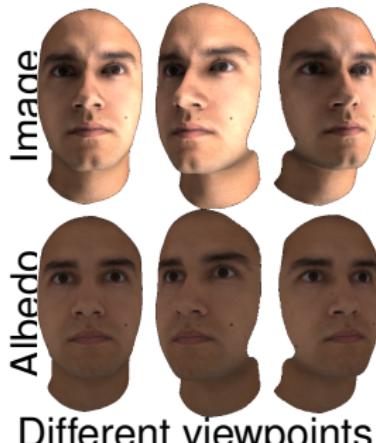
Albedo
(output)

Advantage: Move from “piecewise constant albedo” assumption to “same class of objects” assumption.

Reflectance Learning: Database Creation

Rendering (with Blender) of ≈ 5000 **faces** with known reflectance:

- 21 faces, each with 15 different expressions
- three different viewpoints
- multiple different lighting conditions



Different lighting conditions

Reflectance Learning: Results

Image-albedo mapping learnt using a U-Net CNN on the synthetic database. Testing on real-world images:



Image

Albedo
estimate

Image

Albedo
estimate

Reflectance Learning: Back to Depth Super-resolution

The optimization framework gets simpler, as no optimization over $\rho = \rho_{\text{CNN}}$ is needed, so the Potts term $\lambda P_2(\rho)$ disappears:

$$\min_{\substack{\mathbf{z}: \Omega_{HR} \rightarrow \mathbb{R} \\ \rho: \Omega_{HR} \rightarrow \mathbb{R}^3 \\ \ell \in \mathbb{R}^4}} \left\| \mathbf{l} - \rho_{\text{CNN}} < \ell, \begin{bmatrix} \mathbf{n}(\mathbf{z}) \\ 1 \end{bmatrix} \right\|_{\ell_2(\Omega_{HR})}^2 + \mu \|\mathbf{z}_0 - D\mathbf{z}\|_{\ell_2(\Omega_{LR})}^2 + \nu P_1(\mathbf{z}) + \cancel{\lambda P_2(\rho)}$$

Take Away Message

- Use variational methods if the physics-based model is simple and realistic (here, for micro-geometry estimation)
- If the physics-based model is over-complicated or unrealistic, prefer a black-box (here, for reflectance estimation)

Qualitative Results



Input RGB

Input
depth

CNN-albedo
estimate

Depth estimate

Qualitative Results



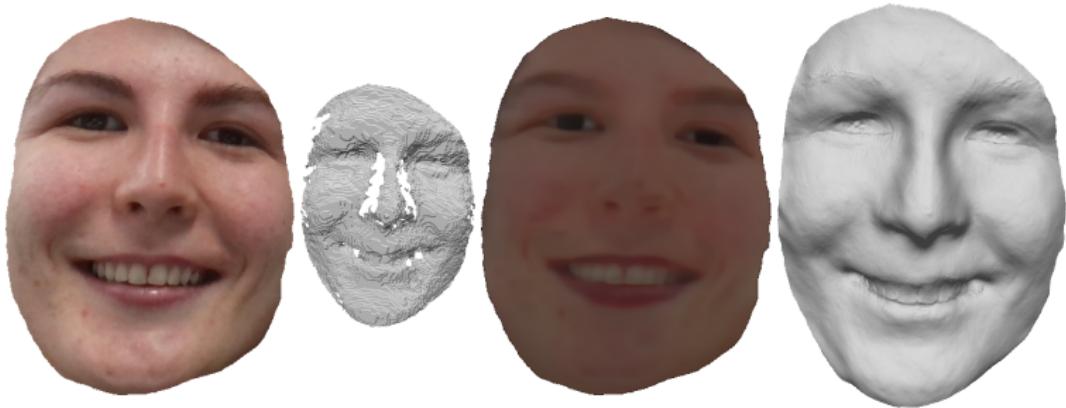
Input RGB

Input
depth

CNN-albedo
estimate

Depth estimate

Qualitative Results



Input RGB

Input
depth

CNN-albedo
estimate

Depth estimate

Failure Case

RGB image is not a face \Rightarrow Reflectance information is propagated to geometry



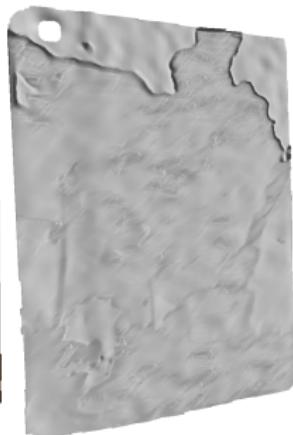
Input RGB



Input
depth



CNN-albedo
estimate



Depth estimate

Possible remedies: larger training set, or multi-shot approach
i.e., photometric stereo.

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Multi-Shot Depth Super-Resolution using Photometric Stereo



n RGB images
under varying
lighting

n depth
images

Albedo
estimate

Depth estimate

[Peng et al., “Depth super-resolution meets uncalibrated photometric stereo”, ICCV 2017]

Idea

Use uncalibrated photometric stereo instead of shape-from-shading, i.e. go from a single image observation

$$\mathbf{I} = \rho < \ell, \begin{bmatrix} \mathbf{n}(z) \\ 1 \end{bmatrix} >$$

to multiple image observations under **varying lighting**

$$\mathbf{I}^i = \rho < \ell^i, \begin{bmatrix} \mathbf{n}(z) \\ 1 \end{bmatrix} >, \quad i \in \{1, \dots, n\}.$$

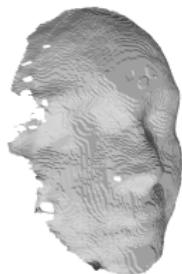
⇒ Results in much more constrained ρ and z , due to their independence on $i \in \{1, \dots, n\}$. No regularization or learning is thus needed:

$$\begin{aligned} \min_{\substack{\mathbf{z}: \Omega_{HR} \rightarrow \mathbb{R} \\ \rho: \Omega_{HR} \rightarrow \mathbb{R}^3 \\ \{\ell^i\} \in \mathbb{R}^4}} & \sum_{i=1}^n \left\| \mathbf{I}^i - \rho < \ell^i, \begin{bmatrix} \mathbf{n}(z) \\ 1 \end{bmatrix} > \right\|_{\ell_2(\Omega_{HR})}^2 + \mu \|z_0 - Dz\|_{\ell_2(\Omega_{LR})}^2 \\ & + \nu P_1(z) + \lambda P_2(\rho) \end{aligned}$$

Qualitative Results



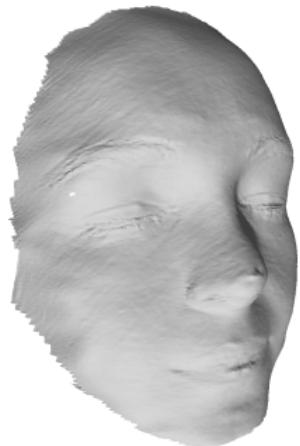
Input RGBs



Input
Depth



Albedo
Estimate



Depth Estimate

Qualitative Results



Input RGBs



Input Depth



Albedo Estimate



Depth Estimate

Qualitative Results



Input RGBs



Input
Depth



Albedo
Estimate

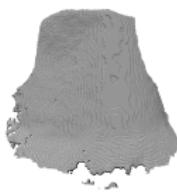


Depth Estimate

Qualitative Results



Input RGBs



Input
Depth



Albedo
Estimate

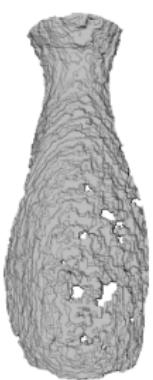


Depth Estimate

Qualitative Results



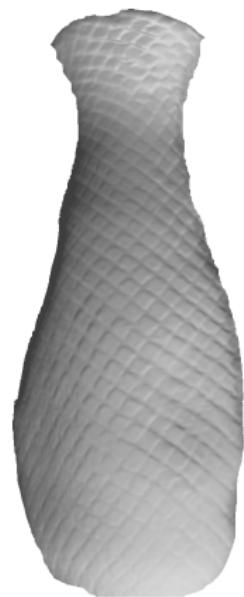
Input RGBs



Input
Depth



Albedo
Estimate



Depth Estimate

What if There is no Depth Prior ?

In theory, the depth prior is not even needed:

Theorem – Brahimi et al. 2019 (Hal 02297643)

There is a **unique** (C^2 -depth, reflectance, lighting) solution of:

$$\mathbf{l}^i = \rho < \ell^i, \begin{bmatrix} \mathbf{n}(\mathbf{z}) \\ 1 \end{bmatrix} >, \quad i \in \{1, \dots, n\},$$

And the solution can be found in closed-form using a spectral approach...

But, spectral approach very sensitive to noise: regularization + non-convex optimization remains the best option.

State-of-the-art heuristic: ballooning initialization, then multi-block ADMM: Haefner, Ye, et al. 2019 (ICCV 2019).

Conclusion and Perspectives

Contributions:

- 1 A flexible splitting-based numerical framework for photometric 3D-reconstruction
- 2 Application to depth super-resolution for RGBD sensors

Ongoing work:

- Simultaneous 3D-reconstruction and (Chan-Vese-like) 2D-segmentation: Haefner, Qu au, and Cremers **2019** (3DV)
- Extension to multi-view stereo: M lou et al. **2019** (SSVM)

- ... Photometric 3D-reconstruction for Arts

Ongoing Work on Cultural Heritage

Bayeux tapestry (represents the conquest of England by William):



High-resolution multi-illumination capture:



Ongoing Work on Cultural Heritage

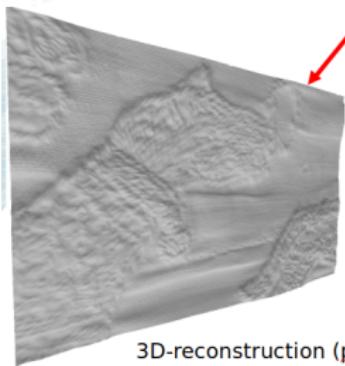
High-resolution 3D-scanning, for 3D-copies which could
be touched by visually-defficients:



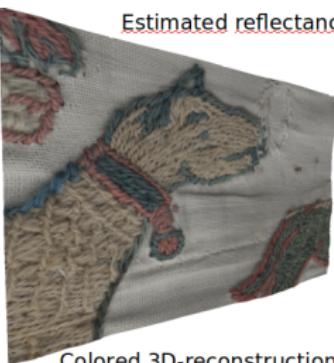
One of the input images



Estimated reflectance (illumination-free)



3D-reconstruction (printable)



Colored 3D-reconstruction

UNILU



Ongoing Work on Optical Illusions

- With one image: ill-posed problem
- With many images: well-posed problem
- ⇒ With two images: can we find a shape explaining any two images under two different lighting?



Monet



Van Gogh

Monet or Van Gogh ?

Thank you for your attention !

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- Haefner, B., S. Peng, et al. (2019). "Photometric Depth Super-Resolution". In: *IEEE Transactions on Pattern Analysis and Machine Intelligence (PAMI)*.
- Haefner, B., Y. Quéau, and D. Cremers (2019). "Photometric Segmentation: Simultaneous Photometric Stereo and Masking". In: *The IEEE International Conference on 3D Vision (3DV)*.
- Haefner, B., Y. Quéau, et al. (2018). "Fight ill-posedness with ill-posedness: Single-shot variational depth super-resolution from shading". In: *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*.
- Haefner, B., Z. Ye, et al. (2019). "On the Well-Posedness of Uncalibrated Photometric Stereo under General Lighting". In: *The IEEE International Conference on Computer Vision (ICCV)*.
- Mélou, J. et al. (2019). "A splitting-based algorithm for multi-view stereopsis of textureless objects". In: *International Conference on Scale Space and Variational Methods in Computer Vision (SSVM)*.
- Peng, S. et al. (2017). "Depth Super-Resolution Meets Uncalibrated Photometric Stereo". In: *The IEEE International Conference on Computer Vision (ICCV)*.
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