

# AN EMPIRICAL EQUILIBRIUM SEARCH MODEL OF THE LABOR MARKET using Labor Market Data of Iran Van den Berg and Ridder (1998)

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- ▶ Equilibrium search models:
  - ▶ Albrecht and Axell (1984)
  - ▶ Burdett and Mortensen (1998)
- ▶ Empirical analyses:
  - ▶ Eckstein and Wolpin (1990)
  - ▶ Van den Berg and Ridder (1998)

- ▶ Labor market search model as the outcome of optimal choices by both workers and employers.
- ▶ The optimal strategy of the workers usually has the reservation wage property.
- ▶ A dispersed wage offer distribution as a result of a dispersed distribution of reservation wage.
- ▶ Parameter changes that affect the reservation wages of job searchers also affect the wage offer distribution that they face.

- ▶ In an equilibrium search model the wage offer distribution is endogenous.
- ▶ It results from optimal wage setting by firms that take account of the responses by job seekers and other firms, and hence is affected by a change in unemployment income.
- ▶ In AA model job-to-job transitions or layoffs are not allowed, contrary to BM model.
- ▶ AA model require workers and firms heterogeneous in order to obtain a dispersed wage offer distribution.

# The Literature

- ▶ Job-to-job transitions are common and it is a source of dispersion in reservation wage and hence the wage offer distribution.
- ▶ In BM model even if all workers and firms are identical, on the job search and the risk of becoming unemployed produce a dispersed wage offer distribution.
- ▶ In the latter case there is an explicit solution for the equilibrium wage offer and earnings distributions.
- ▶ Allowing for observed and unobserved population heterogeneity makes the model more realistic and more able to give an acceptable fit to the data.

- ▶ BM consider a labor market that consists of a large number of segments. Every segment is a labor market of its own, and all workers and firms in a particular segment are identical.  
(between-market heterogeneity)
- ▶ Eckstein and Wolpin (1990), in their empirical analysis of the Albrecht-Axell model, consider a single labor market with unobserved differences in the value of leisure between workers and unobserved differences in productivity between firms (they do not have observed differences across workers or firms).  
(within-market heterogeneity)

- ▶ VR estimate the model by maximum likelihood.
- ▶ Using panel data on unemployed and employed individuals in The Netherlands in the 80s.
- ▶ For most individuals in the data, multiple durations (like unemployment durations and job durations) are observed.

- ▶ On average, the arrival rate of job offers is only slightly larger when employed.
- ▶ A small number of observed personal characteristics is sufficient to capture the heterogeneity in arrival and separation rates, but insufficient to capture heterogeneity in the productivity of firms.
- ▶ Contrary to Eckstein and Wolpin we find that a relatively small fraction of wage variation is explained by measurement error and that about a fifth is pure wage variation as generated by the presence of search frictions.



# Assumptions

1. There are continua of workers and firms with measures  $m$  and  $1$ , respectively.
2. Workers receive job offers at rate  $\lambda_0$  if unemployed and  $\lambda_1$  if employed. A job offer is an i.i.d. drawing from a wage offer distribution with c.d.f.  $F(w)$ . An offer has to be accepted or rejected upon arrival. During tenure of a job, the wage is constant. The utility flow of being employed at a wage  $w$  equals  $w$ .
3. Job-worker matches break up at rate  $\delta$ . If this happens, the worker becomes unemployed. The utility flow of being unemployed is  $b$ .
4. Firms have a linear production function and the marginal (= average) revenue product is  $p$ . A firm pays all its workers the same wage  $w$ .
5. Workers maximize their expected wealth and firms maximize their expected steady-state profit flow.
6. The firms cannot set their wage below the mandatory minimum wage  $w_0$ .

# The Model

- ▶ First, we consider the model of a labor market with homogeneous workers and firms, as developed by Burdett and Mortensen (1998).
- ▶ In the limiting case of zero discounting, the reservation wage  $r$  can be shown to be:

$$r = b + (\lambda_0 - \lambda_1) \int_r^\infty \frac{\bar{F}(w)}{\delta + \lambda_1 \bar{F}(w)} dw \quad \text{with } \bar{F} = 1 - F$$

- ▶ Workers are continuously searching for a better paying job. An employed individual accepts a wage offer if and only if it exceeds his current wage.
- ▶ Wage offer distribution :  $F$  is distribution of wages offered to job seekers.
- ▶ Earnings distribution:  $G$  is the distribution of wages received by workers who are currently employed.

# Earnings Distribution

- ▶ Outflow:

$$\lambda_1 \bar{F}(w) G(w) (m - u) \\ \delta G(w) \cdot (m - u)$$

- ▶ Inflow:

$$\lambda_0 (F(w) - F(r)) u$$

- ▶ The result:

$$G(w) = \frac{F(w)}{\delta + \lambda_1 \bar{F}(w)} \cdot \frac{\lambda_0 u}{(m - u)}.$$

# Model

$$\frac{u}{m} = \frac{\delta}{\delta + \lambda_0}.$$

The measure of individuals earning a wage  $w$  equals  $g(w)(m - u)dw$ , and the measure of firms offering a wage  $w$  equals  $f(w)dw$ .

$$l(w; r, F) = \frac{g(w)dw}{f(w)dw} (m - u) = \frac{m \lambda_0 \delta (\delta + \lambda_1)}{(\delta + \lambda_0) (\delta + \lambda_1 \bar{F}(w))^2} \quad \text{on } [\underline{w}, \bar{w}].$$

# Model (Production)

$$F(w) = \frac{\delta + \lambda_1}{\lambda_1} \cdot \left( 1 - \sqrt{\frac{p - w}{p - \underline{w}}} \right) \quad \text{on } [\underline{w}, \bar{w}]$$

$$f(w) = \frac{\delta + \lambda_1}{2\lambda_1\sqrt{p - \underline{w}}} \cdot \frac{1}{\sqrt{p - w}} \quad \text{on } [\underline{w}, \bar{w}].$$

$$r = \frac{(\delta + \lambda_1)^2 \cdot b + (\lambda_0 - \lambda_1)\lambda_1 \cdot p + \delta_0(\lambda_0 - \lambda_1) \cdot \underline{w}_L}{(\delta + \lambda_0)(\delta + \lambda_1)} \quad \text{if } r < \underline{w}_L,$$

$$r = \frac{(\delta + \lambda_1)^2 \cdot b + (\lambda_0 - \lambda_1)\lambda_1 \cdot p}{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1)\lambda_1} \quad \text{if } r \geq \underline{w}_L,$$

$$\bar{w} = \left( \frac{\delta}{\delta + \lambda_1} \right)^2 \cdot \underline{w} + \left( 1 - \left( \frac{\delta}{\delta + \lambda_1} \right)^2 \right) \cdot p.$$

# Model (Productivity)

$$g(w) = \frac{\delta \sqrt{p - \underline{w}}}{2\lambda_1} \cdot \frac{1}{(p - w)^{3/2}} \quad \text{on } [\underline{w}, \bar{w}].$$

Given an arbitrary transformation of  $w$ ,  $y = \frac{p-w}{p-\underline{w}}$ , the excess wage equation is  $w - \underline{w} = (1 - y)(p - \underline{w})$ . ( $\eta = \frac{\delta}{\delta + \lambda_1}$ )

$\frac{\lambda_1}{\delta}$  : expected number of wage offers during a spell of employment

$$f_y(y) = \frac{1}{2(1 - \eta)} y^{-1/2}, \quad \eta^2 \leq y \leq 1,$$

$$g_y(y) = \frac{\eta}{2(1 - \eta)} y^{-3/2}, \quad \eta^2 \leq y \leq 1,$$

# The Likelihood Function

- The probability of being unemployed at a randomly chosen date :  
 $\frac{\delta}{\delta + \lambda_0}$
- The elapsed unemployment duration  $t_{0b}$  and the residual unemployment duration  $t_{0f}$  are i.i.d. and have an exponential distribution with parameter  $\lambda_0$
- $d_{0b}$  denote a dummy that is one if it is only known that the elapsed duration exceeds a certain value, i.e. is right-censored, and zero otherwise
- $d_{0f}$  is for residual duration

$$\mathcal{L}_0 = \frac{\delta}{\delta + \lambda_0} \cdot \lambda_0^{1 - d_{0b} + 1 - d_{0f}} \cdot \exp(-\lambda_0(t_{0b} + t_{0f})).$$

# The Likelihood Function

- The job duration  $t_1$  has an exponential distribution with parameter  $\delta + \lambda_1 \bar{F}(w)$
- Exit into unemployment occurs with probability  $\frac{\delta}{\delta + \lambda_1 \bar{F}(w)}$  and exit into another job with probability  $\frac{\lambda_1 \bar{F}(w)}{\delta + \lambda_1 \bar{F}(w)}$
- Measurement errors in the wage data is represented by  $\epsilon$ . The observed wage  $\tilde{w}$  equals the true wage  $w$  times an error term  $\epsilon$
- $d_1 = 1$  if  $\tilde{w}$  is missing and zero otherwise
- If  $d_{0f} = 1$  or  $d_1 = 1$ , then we do not follow the individual any further
- $d_2 = 1$  if  $t_1$  is right-censored and zero otherwise
- $d_3 = 1$  if the destination following exit out of the job is unknown and zero otherwise
- $d_4 = 1$  if the destination is another job and zero if the destination is unemployment



# The Likelihood Function

$$\begin{aligned}\mathcal{L}_1 = & f(w) \cdot \exp(-(\delta + \lambda_1 \bar{F}(w)) \cdot t_1) \cdot (\delta + \lambda_1 \bar{F}(w))^{d_3(1-d_2)} \\ & \cdot (\lambda_1 \bar{F}(w))^{d_4(1-d_2)(1-d_3)} \cdot \delta^{(1-d_4)(1-d_3)(1-d_2)}\end{aligned}$$

The total individual likelihood contribution for a respondent who is unemployed at the date of the first interview equals

$$\mathcal{L}_0 \cdot \mathcal{L}_1^{(1-d_{0f})(1-d_1)}.$$

# The Likelihood Function

- The probability of being employed at a randomly chosen date equals  $\frac{\lambda_0}{\delta + \lambda_0}$
- $d_5 = 1$  if  $\tilde{w}$  is unobserved, and zero otherwise
- The elapsed job duration  $t_{1b}$  and the residual job duration  $t_{1f}$  are i.i.d. and have an exponential distribution with parameter  $\delta + \lambda_1 \bar{F}(w_1)$
- $d_{6b}$  denote a dummy that is one if it is only known that the elapsed duration exceeds a certain value, i.e. is right-censored, and zero otherwise.  $d_{6f}$  is for residual duration
- $d_7 = 1$  if the destination following exit out of the job is unknown and zero otherwise
- $d_8 = 1$  if the destination is another job and zero if it is unemployment
- $d_0 = 1$  if  $t_0$ , unemployment duration, is right-censored, and zero otherwise
- Dummy variables  $d_{10}$ ,  $d_{11}$ ,  $d_{12}$ , and  $d_{13}$  indicate whether  $w_2$  is unobserved, whether  $t_2$  is right-censored, whether the destination state is unobserved, and whether the destination state is another job, respectively.

# The Likelihood Function

$$\begin{aligned}
 \mathcal{L} = & \frac{\lambda_0}{\delta + \lambda_0} \cdot g(w_1) \cdot (\delta + \lambda_1 \bar{F}(w_1))^{1-d_{6b}} \cdot \exp(-(\delta + \lambda_1 \bar{F}(w_1)) \cdot (t_{1b} + t_{1f})) \\
 & \cdot (\delta + \lambda_1 \bar{F}(w_1))^{d_7(1-d_{6f})} \cdot [\delta \cdot \lambda_0^{(1-d_9)} \cdot \exp(-\lambda_0 t_0)]^{(1-d_8)(1-d_7)(1-d_{6f})} \\
 & \cdot \left[ \lambda_1 \bar{F}(w_1) \cdot \left[ \frac{f(w_2)}{\bar{F}(w_1)} \cdot (\delta + \lambda_1 \bar{F}(w_2))^{d_{12}(1-d_{11})} \right. \right. \\
 & \quad \cdot \exp(-(\delta + \lambda_1 \bar{F}(w_2)) \cdot t_2) \cdot \delta^{(1-d_{13})(1-d_{12})(1-d_{11})} \\
 & \quad \left. \left. \cdot (\lambda_1 \bar{F}(w_2))^{d_{13}(1-d_{12})(1-d_{11})} \right]^{(1-d_{10})} \right]^{d_8(1-d_7)(1-d_{6f})}
 \end{aligned}$$

# Heterogeneity

- Separate labor markets as different segments of the labor market, for different types of individuals and firms
- The deep structural parameters in the model ( $p$ ,  $\lambda_0$ ,  $\lambda_1$ , and  $\delta$ ) do not vary over the different labor markets significantly
- $x$  is the vector of age, education, and occupation dummies (and a constant). We assume that  $p$ ,  $\lambda_0$ ,  $\lambda_1$ , and  $\delta$  are log-linear functions of  $x$

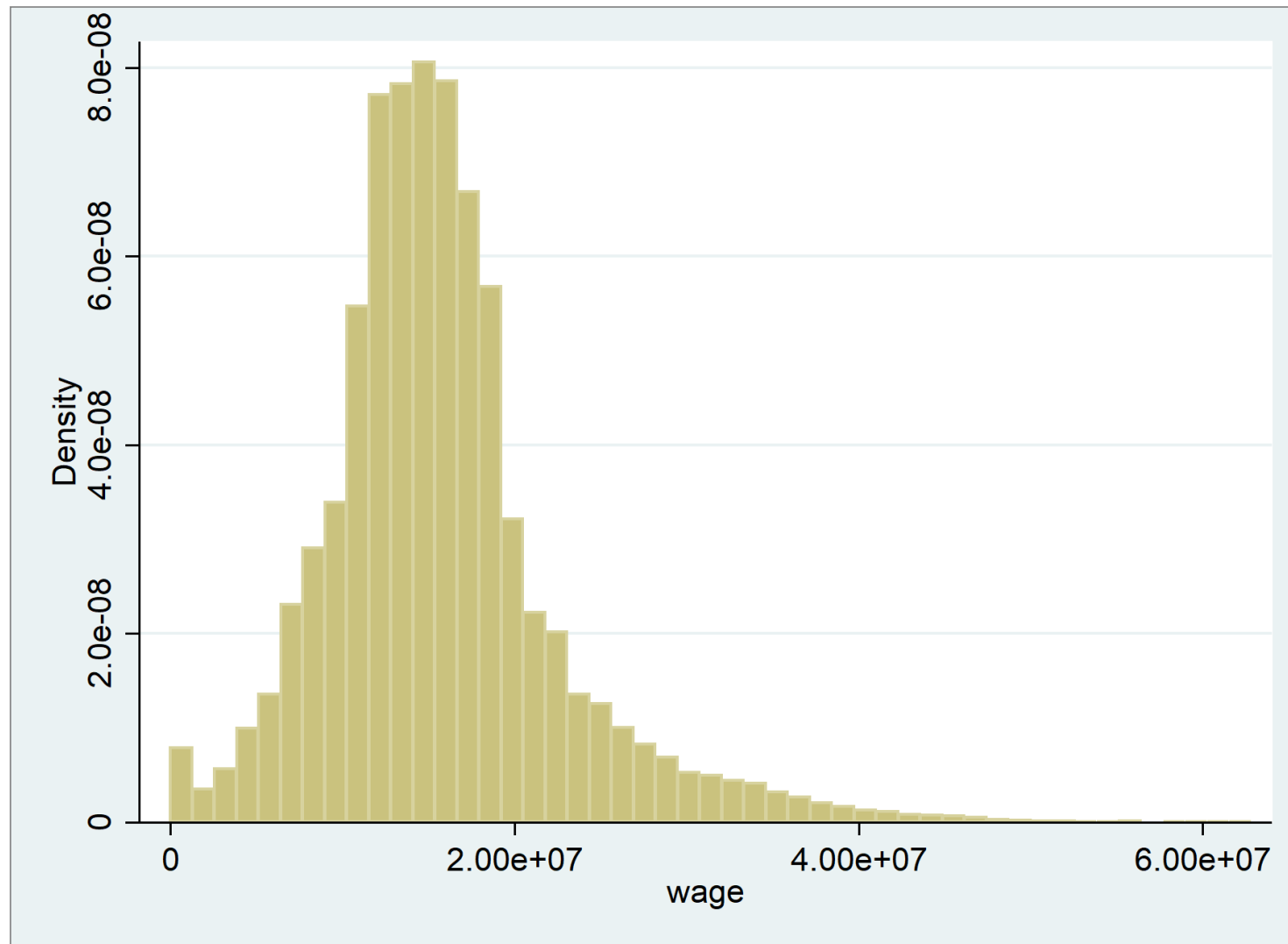
$$\begin{aligned} p &= \exp(\beta_1' x), & \lambda_0 &= \exp(\beta_2' x), \\ \lambda_1 &= \exp(\beta_3' x), & \delta &= \exp(\beta_4' x). \end{aligned}$$

In order to capture unobserved heterogeneity,  $p$  is represented by  $p = v \cdot \exp(\beta_1' x)$  that  $v$  has a discrete distribution with a finite number of unknown points of support.

# Data

- Labor Force Survey(LFS) data of Iran's labor market
- Household Budget Survey(HBS) data of Iran's population

# Data



# Data

Unemployed _t1	Period		Total
	1	2	
0	59,675	27,845	87,520
1	10,780	2,140	12,920
Total	70,455	29,985	100,440

Unemployed _t1	Unemployed_t2		Total
	0	1	
0	25,587	2,258	27,845
1	2,140	0	2,140
Total	27,727	2,258	29,985

Thank you for your attention.  
Any questions?