Topics in Macro: Problem set 1

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1 Incomplete markets

Consider the following economy. There is a continuum of measure 1 of infinitely lived households and a large number of identical firms. All agents behave competitively. Households are subject to idiosyncratic employment risk but can save in capital. The households' problem is given as:

$$V_{i0} = \max_{c_{it}, k_{it+1}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\gamma}}{1-\gamma}$$

where

$$\begin{aligned} k_{it+1} &= w s_{it} + (1+R) k_{it} - c_{it} \\ R &= r - \delta \end{aligned}$$

$$c_{it} = w s_{it} + (1+R) k_{it} - k_{it+1}$$

where c_{it} denotes consumption, $\gamma > 0$ is the coeficient of relative risk aversion, $\beta \in (0,1)$ is the discount factor, k_{it} denotes capital, δ is the depreciation rate, w is the real wage and s_{it} is an idiosyncratic employment shock. R denotes the net return on capital. We assume there is the following limit on capital holdings:

$$k_{it+1} \geq \phi$$

where $\phi > 0$. Hence, agents are required to always hold a strictly positive amount of capital. We assume that $s_{it} \in \{0,1\}$. Output is produced by firms according to the technology"

$$y_t = K_t^{\alpha} N_t^{1-\alpha}$$

$$\pi = y_t - WN - rK$$

$$(1-\alpha)\frac{Y}{N} = w \Rightarrow (1-\alpha) = \frac{WN}{Y}, \quad \alpha = \frac{rK}{Y},$$

with $\alpha \in (0,1)$: Here, K_t denotes the aggregate capital stock and N_t is aggregate employment. Firms rent capital and labour from the households at price r_t and w_t, respectively.

a) Set up the firms' profit-maximization problem and derive the first-order conditions for the labour and capital choice. Show that firms make no profits.

Consider the following calibration: $\alpha = 0.4, \beta = 0.96, \delta = 0.1, \gamma = 1.5$ and $\phi = 0.01$. Suppose that the probability of being employed in the next period ($s_{it+1} = 1$) is 90 percent regardless of the householdis current employment situation.

b) Write a program that solve the individual's problem using value function iteration for a given level of R. Use value function iteration or policy function iteration with 300 grid points.

c) Construct a grid for R ranging from ϵ to $1/\beta - 1 - \epsilon$ where ϵ is a very small strictly positive number. Use 10 equidistant grid points. For each of the 10 values for R, solve the individual's problem, and compute the stationary distribution of capital holdings. Next, plot aggregate demand curve for capital and the aggregate supply curve for capital in a figure with K on the x-axis and r on the y-axis.

d) Now consider a version of the model with complete financial markets. Plot the new demand and supply curves together with those computed for the incomplete markets model. Discuss the demand and the supply effects of the introduction of complete markets.

e) Write a program that solves explicitly for the value of r in the stationary equilibrium. Compute r for the model with incomplete markets and for the model with complete markets. Discuss the result.

f) Plot the Lorenz curve for the incomplete markets model and compute the Gini coefficient. What fraction of the population is at the constraint in the stationary equilibrium?