Topics in Macro: Problem set 1

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1 Cake-eating problem

You are endowed with a cake of size $W > 0$. Eating cake provides utility $u(c_t, z_t)$, where c_t is the amount of eaten cake and z_t is a taste shock which affects how delicious the cake would be if it is eaten that period. Taste shocks take on N different values and follow a Markov chain. At the beginning of each period, the taste shifter realizes, and then you decide how much cake to eat and how much to save for next period. From the leftovers, a fraction $1 - \rho \in (0, 1)$ of the cake gets eaten by your roommate and is not available for consumption any longer. The future is discounted at rate $\beta \in (0, 1)$, a measure of voracity, determining how patient you are in your cake preferences. You live forever.

- Write the sequential problem.
- Write the recursive problem. State all the elements, such as the value function(s), the states, the controls, the laws of motion, and other assumptions you make, for instance, the flavor of the cake.
- Characterize the optimal cake-eating rule and value function. How do they depend on β, ρ, and C?

1.1 More structure on preferences and taste shocks

Preferences We assume a CRRA period utility function with multiplicative taste shocks as follows:

$$u(c_t, z_t) = \begin{cases} z_t \left[\frac{c_t^{1-\sigma}-1}{1-\sigma}\right] & \text{if } \sigma \neq 1 \\ z_t \log c_t & \text{if } \sigma = 1 \end{cases}$$

Taste shocks Assume that taste shocks take on five discrete values uniformly spaced in the interval $[1-\gamma/2, 1+\gamma/2]$ and with a transition probability matrix Q with elements q_{ij}. The transition probabilities are zero except for the following entries: $q_{1,2} = q_{4,5} = 2(1 - \theta)$, $q_{2,3} = q_{3,4} = 3(1 - \theta)$, with the upper triangle of the transition matrix symmetrical to the lower triangle and the diagonal elements equal to one minus the sums of the non-diagonal elements.

- Interpret the parameters γ and θ *(Hint: Compute analytically the following moments* $\mathbb{E}[z_t]$*,* $\mathbb{V}[z_t]$*, and autocorrelation* $Cov[z_t, z_{t-1}]$*).*

1.2 Computation

1. Write donw your own code to analytically solve this problem. Table 1 sets the benchmark values for the parameters.

Table I: **Benchmark Parameters**

Parameter	W	σ	β	γ	θ	ρ
Value	100	1	0.99	0.5	0.85	0.9

2. Plot the value function and the policy.

3. Do a comparative statics analysis for the parameters $\{\sigma, \beta, \gamma, \theta\}$. Write an outer loop that varies one parameter at a time (in a neighborhood around the benchmark values in Table 1) and show how the policy and value function change. Interpret your results.

2 Present Bias Preferences

Now we want to consider how present bias affects the optimal decision of cake eating. Consider the preference change over time such that at each point in time, s, preference is instead:

$$u(c_s, z_s) + \delta \mathbb{E} \sum_{t=1}^{\infty} \beta^t u(c_{s+t}, z_{s+t}) \tag{1}$$

Notice that δ is the parameter of present bias. If $\delta = 1$ there are no present bias and if $\delta < 1$ the agent discounts tomorrow relative to today, more than it discounts $n + 1$ days from now over n days from now.

1. Is this preference stationary?

2. Can you write down a Bellman equation for this problem?

3 Stopping Time Problem

Now assume that cakes are packed such that once the pack is opened it must totally be consumed. There is no storage after opening of the pack. So the question is at what period should you eat the whole cake? Assume that the cake is melting at rate $\rho < 1$. Write down the sequence problem as well as the Bellman equation. Calibrate the model for the same parameter values to do similar comparative statics.