Microeconomics HW2

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March 1, 2019

Problem 1

A)

We must show that $\prec^*$ is strictly partially order in H_i. In other word we must show that $\prec^*$ is irreflexive, transitive, and asymmetric.

- **Irreflexive**: By definition of extensive form game we know that $\forall i, j, h_j \in H_i$:

$$\forall t, t' \in h_j : t \nprec t', t' \nprec t \Rightarrow h_j \nprec h_j$$

- **Asymmetric**: If $h \prec^* h'$ we have $\exists t \in h, t' \in h'$ such that $t \prec t'$. From definition of extensive form game we know that $\forall t'' \in h', \exists t'''' \in h$ such that $t'''' \prec t''$. Therefore, if we have

$$h \prec^* h' \Rightarrow h' \nprec^* h$$

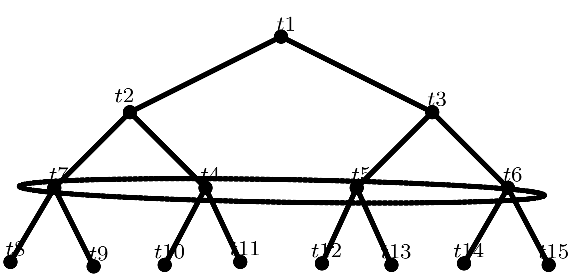
- **Transitivity**: If $h \prec^* h'$ we have $\exists t \in h, s \in h'$ such that $t \prec s$. Likewise, if $h' \prec^* h''$ we have $\exists s' \in h', z \in h''$ such that $s \prec z$. From definition of extensive form game we know that $\forall z' \in h'', \exists s'' \in h'$ such that $s'' \prec z'$. Therefore, for s:

$$\exists z'' \in h'' : s \prec z'' \Rightarrow t \prec z''$$

and as a result:

$$\Longrightarrow h \prec^* h''$$

B)



Assume that $t1, t4, t5, t6, t7$ is for player 1 and $t2, t3, t8, t9, ..., t15$ is for player 2. $t2 \in h$ and $\{t4, t5, t6, t7\} \in h'$ and $t13 \in h''$. Thus, $h \prec h'$ and $h' \prec h''$ but neither $t2 \prec t13$ nor $t13 \prec t2$. Therefore, we have:

$$h \nprec h'' \& h'' \nprec h$$

.

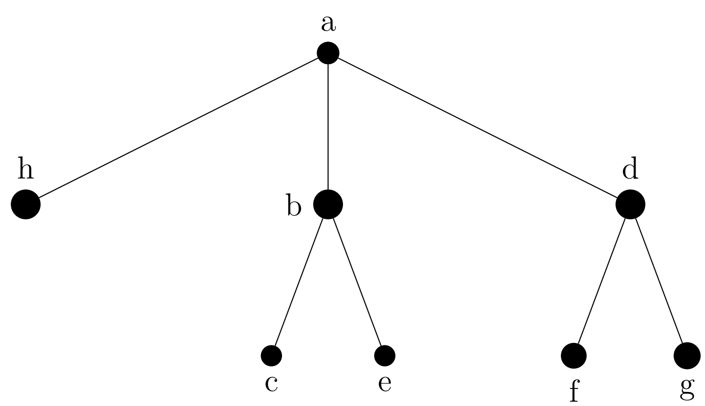
Homework 2

C)

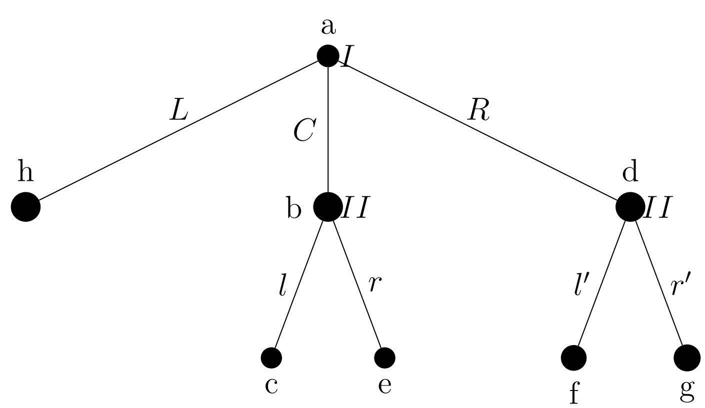
From definition of extensive form game we know that if $\exists t' \in h', t \in h : t \prec t'$ then we have $\forall t" \in h', \exists t* \in h$ such that $t* \prec t"$. Therefore, if $h \prec h'$ we have the desired of the question.

Problem 2

A)



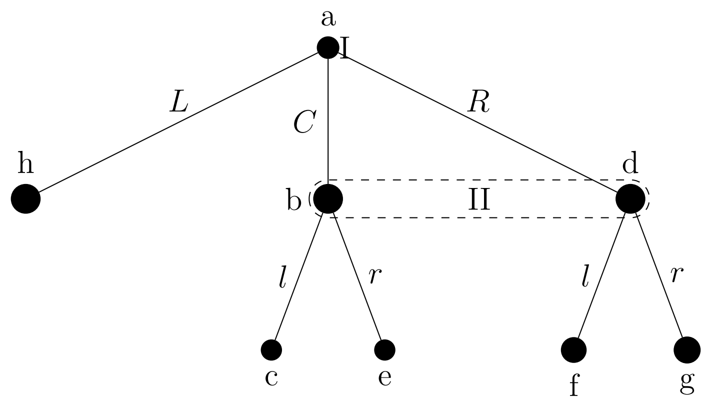
B)



The strategies are: $s_1 \in \{C, R\}$ and $s_2 \in \{ll', lr'\}$.

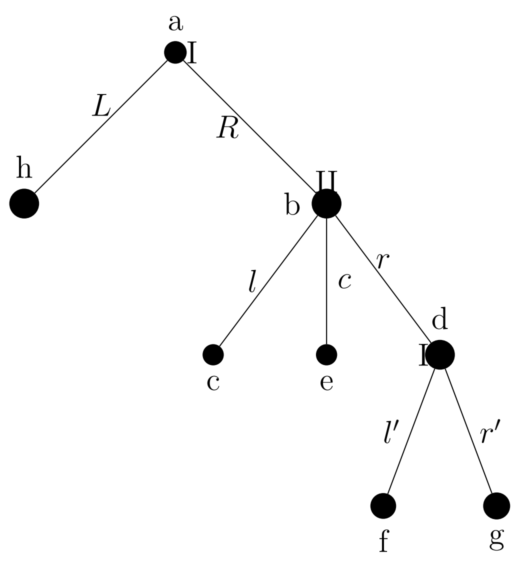
Homework 2

C)



As it is shown in the picture, it is possible to specify action labeling. However, since the player 2 cannot distinguish between node b and d, it is not possible to find strategy profiles that end to c, f, g but not e. We should notice that $S_1 = \{C, R\}$ and $S_2 = \{l, r\}$. For reaching to node g, player 2 must choose r and For reaching to node c, player 1 must choose C. Therefore, it is possible to reach node e too.

D)



Problem 3

The s is Nash equilibrium for game $\{(S'_n, U_n)_{i=1}^n\}$. Therefore, we have:

$$U_i(s_i, s_{-i}) \geq U_i(s', s_{-i}) \qquad \forall i, s' \in S'_i, s_i \in S'_{-i}.$$

Since $S'_i = S_i$ for $i = 2, 3, ..., n$ then, we have:

$$U_i(s_i, s_{-i}) \geq U_i(s', s_{-i}) \qquad \forall s' \in S_i, s_{-i} \in S'_{-i} \quad i = 2, 3, ..., n.$$

Therefore, in the game $\{(S_n, U_n)_{i=1}^n\}$ the players $i = 2, 3, ..., n$ will not change their action to any other action of players $i = 2, 3, ..., n$.

Also, we have that $\hat{s}$ is weakly dominated strategy for player 1, i.e. :

$$U_1(s', s_{-1}) \geq U_1(\hat{s}, s_{-1}) \qquad \forall s' \in S_1, \forall s_{-1} \in S_{-1},$$

and

$$\exists i : \quad U_1(s', s_{-1}) > U_1(\hat{s}, s_{-1}) \quad s' \in S_1, s_{-i} \in S_{-1}.$$

Now, we must show that in the game $\{(S_n, U_n)_{i=1}^n\}$ player1 will not change his action when s_{-i} is played and player i $i = 2, 3, ..., n$ will not change his action neither.

Since $\hat{s}$ is weakly dominated strategy for player 1, for some s'_{-1} it is definitely not chosen for some s'_{-1} he is indifferent to choose is it or choose another one. Therefore, player 1 will not change his action in response to s_{-1}. Moreover, player i $i = 2, 3, ..., n$ will not change his action too in response to s_1. Because we already know that s_i is best response to s_{-i}.

As a result, the strategy s is Nash equilibrium of the game $\{(S_n, U_n)_{i=1}^n\}$ too.

Problem 4

Every firm chooses the quantity that makes his profit maximum, considering production of other firms. Assuming interior solution:

$$\begin{aligned}
&\max_{q_i} \quad \pi_i = q_i(a - q_{-i}) - cq_i \\
FOC: \quad &\frac{\partial \pi_i}{\partial q_i} = 0 \quad \Longrightarrow \quad a - 2q_i - q_i - c = 0 \\
SOC: \quad &\frac{\partial \pi_i^2}{\partial^2 q_i} = -2 < 0 \\
\Longrightarrow \quad &q_i^* = \frac{a - q_{-i} - c}{2}.
\end{aligned}$$

Considering the symmetry of the firms, we have:

$$q_i^* = \frac{a-c}{n+1}.$$

Thus, the production of the industry is:

$$Q = \frac{n(a-c)}{n+1}.$$

As n increases, the production of industry approaches to the production of competitive market, which is $a-c$.

Now, we try to find corner solutions. If $c > 0$ and $P = 0$, then each $q > 0$ is dominated by $q = 0$ because producing leads to negative profit, and we have not Nash equilibrium. However, if $c = 0$ and $\forall i, q_{-i} > -$ then each firm is indifferent between different q. Thus, any $(q_1, ..., q_n)$ which satisfies $\forall i, q_{-i} > -$ is Nash equilibrium. If $\exists i, q_{-i} < a$ the firm i change its q_i to amount obtained in the previous part.So, it is not Nash equilibrium.

Problem 5

A)

Each firm chooses the quantity that makes his profit maximum, considering production of other firms. Assuming interior solution:

$$\begin{aligned}
&\max_{q_i} \quad \pi_i = q_i(a - q_{-i}) - c_i q_i \\
&FOC: \quad \frac{\partial \pi_i}{\partial q_i} = 0 \quad \Longrightarrow \quad a - 2q_i - q_i - c_i = 0 \\
&SOC: \quad \frac{\partial \pi_i^2}{\partial^2 q_i} = -2 < 0 \\
&\Longrightarrow \quad q_1^* = \frac{a - q_2 - c_1}{2} \quad \& \quad q_2^* = \frac{a - q_1 - c_2}{1}
\end{aligned}$$

By solving these equations, we have:

$$q_1^* = \frac{a + c_2 - 2c_1}{3} \quad \& \quad q_2^* = \frac{a + c_1 - 2c_2}{3}$$

For corner solutions we should notice that since $0 < c_i < \frac{a}{2}$ then we have $q_i < \frac{a + \frac{a}{2} - 2\times 0}{3} = \frac{a}{2}$. So, $\sum q_i < a$ and we do not have corner solutions.

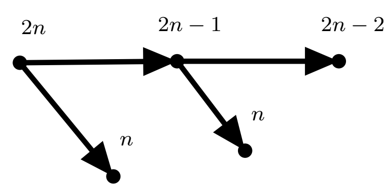
When c_1 increases, q_2 increases too. It is completely intuitive to think that when firm1 has more marginal cost, i.e. older production facilities, the production of firm2 increases.

B)

Since we have $2c_2 > a + c_1$ then best response functions gained in the previous section, is negative for firm2! In fact, firm2 chooses $q_2^* = 0$ which is better for him, and firm1 chooses $q_1 = \frac{a-c_1}{2}$ to maximize his profit.

Problem 6

First, we know that if player i gets 2 he writes 1 and he wins, so 2 is winning number. If player i gets 3 he has to write 2 and the he lose, so 3 is losing number. Secondly, we draw the tree of the game:



The player who plays first and the first number is even can choose between n and $2n-1$. If n is loosing number he chooses n and his opponent loses and he wins. However, if n is winning number, he chooses $2n-1$ and like choosing n he could lose or win. The second player at $2n-1$ has to choose $2n-2$ because n is winning number, if it was not chosen by player1.

Now using backward induction, we have:

- 2 is winning number.
- 3 is losing number.
- 4 is winning number.
- 5 is winning number.
- 6 is winning number.
- 7 is losing number.
- 8 is winning number.
- 9 is losing number.
- 10 is winning number.

Since 10000 is even number and first player has the upper hand, **the first player wins the game.**