Note 1

Proposition: If a strategy is deleted during the iterative deletion of strictly dominated strategies, then that strategy wouldn't exist in any Nash equilibrium of the game.

Proof. Without loss of generality assume that strategy s_1 of player 1 is deleted during the iterative deletion of strictly dominated strategies. Now assume there exist $s_{-1} = \{s_2, s_3, ...s_N\}$ such that (s_1, s_{-1}) forms a Nash equilibrium of the game. If all of strategies in s_{-1} were present in stage of s_1 deletion, then by definition:

$$\exists s_1' : U_1(s_1', s_{-1}) > U_1(s_1, s_{-1})$$

So player 1 has a profitable deviation to s_1'. Thus (s_1, s_{-1}) is not a Nash equilibrium.
Now assume that s_{-1} is not present in the stage of s_1 deletion. It means that at least one of the strategies in s_{-1} is deleted in previous iterations. Now assume the first iteration in which one of strategies in s_{-1}, call it s_i, was strictly dominated. Again by definition

$$\exists s_i' : U_i(s_1, s_2, ..s_i', .., s_N) > U_i(s_i, s_{-i}).$$

Thus agent i has a profitable deviation s_i' and (s_1, s_{-1}) could not be a Nash equilibrium. $\square$