Subject: PS4_ Micro II

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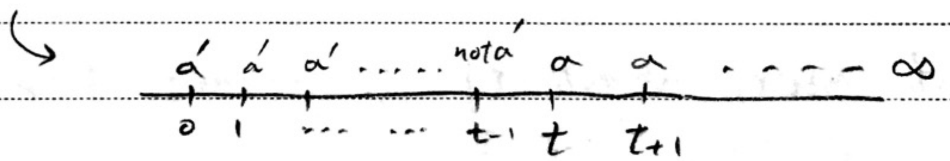
Add ①:

$a \in A$ is NE.

$\exists a' \in A : \forall i, u_i(a') > u_i(a)$

We define strategy profile of σ in this way:

$\sigma :=$ "In period 0 each player plays a' and then for subsequent preiods each player continues to play a' as long as the realized action were a' in all previous periods. At any period that at least 1 player don't plays a' each player will play a for the rest of the game.



For σ is to be subgame perfect equilibrium it suffices that we show no player has a profitable one-shot

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deviation.

That is: $(1-\delta) \max_{a \in A} g_i(a) + \underbrace{\delta u_i(a)}_{\text{sum of utilities after deviating}} \leqslant V_i$ where

g_i denotes stage payoff for player i

if this inequality holds after histories with no unilateral deviations, i doesn't benefit from deviation.

But we know $V_i \geqslant u_i(a)$ so if we choose δ lagrge enough namely $\underline{\delta}$ then $V_i \geqslant (1-\delta) \max_{a \in A} g_i(a) + \delta u_i(a)$ holds.

So after a history in which player i was the first to deviate the continuation strategy profile is "play a" but we know a is a nash equilibrium so no player would have one-shot deviation thereafter.

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Add (2): period 1: a_i

Because the game is totally symmetric

we only consider one player.

$a_i \to c_i \to b_i \to b_i \cdots \infty$

by Principle of optimality or one-shot deviation principle:

stage game:

4,4	3, 2	1,1
2,3	2, 2	1,1
1,1	1,1	-1,-1

$a_i \to c_i \to b_i^{\infty}$

$t=1$ $t=2$ $t=3$

for each player 1 at $t=3$ payoffs are as below

$u_1(h^{\infty}) = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} u_i(a^t)$

if $\forall t,\ a_i = 2 \Rightarrow U_1(h^{\infty}) = 2$

	$\delta(3\delta-1)+3(1-\delta)$	
	2	

if player 1 wants to one-shot deviate obviously choose a_1 strategy because $3>2$ and $3>1$

but if player 1 one-shot deviates payoff are in the next period both players

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will Play c_i then they return to b_i so we have:

$$(a_1,a_2),\ (c_1,c_2),\ \overset{\text{one-shot deviate}}{(a_1,b_2)},\ (c_1,c_2),\ (b_1,b_2)\ldots$$

They pay off would be for player 1:

$$(1-\delta)\,3+(1-\delta)(-1)\times\delta+(1-\delta)\underbrace{\sum_{t=3}^{\infty}\delta^{t-1}\times 2}_{\frac{\delta^2}{1-\delta}\times 2}$$

$$=3(1-\delta)+2\delta^2+\delta(\delta-1)=3(1-\delta)+3(1-\delta)$$

now for player 1 not to one-shot deviate we must have:

$$2\geq\delta(3\delta-1)+3(1-\delta)$$

$$\rightarrow\quad 3\delta^2-4\delta+1\leq 0\quad\Rightarrow\quad(1-\delta)(1-3\delta)\leq 0$$

$$\Rightarrow \overset{\text{trivial}}{1\geq\delta}\geq\frac{1}{3}\quad \text{(I)}$$

But at $t=2$ player 1 must cannot have one-shot deviate

So we have:

		$1-\delta+\delta(3\delta-1)$
		$1-\delta+\delta(3\delta-1)$
		$-(1-\delta)+2\delta$

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For player 1 not to deviate we must have:

$$2\delta - (1-\delta) \geqslant 1-\delta + \delta(3\delta-1)$$

$$\Rightarrow \quad 2-5\delta+3\delta^2 \leqslant 0 \quad \Rightarrow \quad (1-\delta)(2-3\delta) \leqslant 0$$

$$\Rightarrow \quad \delta \geqslant 2/3 \quad \text{II}$$

by I and II ; I $\cap$ II ; $\boxed{\delta \geqslant \frac{2}{3}}$

so if $\delta \geqslant \frac{2}{3}$ then σ is subgame perfect equilibrium.

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Add ③:

		player 2	
		C	D
player 1	C	$-1, -1$	$-4, 0$
	D	$0, -4$	$-3, -3$

a) assume $G(T, \infty)$ be a finite repeated game.

by backward induction at stage $t = T$ regardless of history of the game D strictly dominate C for each player so at $t = T$ They play (D, D)

so play at $T-1$ cannot influence period T.

so again by backward induction in $T-1$ both choose D over C and hence repatin this through period 0. $\Rightarrow$ Thus we proved that always choosing D for each player is the unique subgame perfect equilibrium.

b) for $G(\infty, \delta)$ by Add ① we must have:

$$-1 \geq (1-\delta)(0) + \delta(-3) \Rightarrow -3\delta \leq -1 \Rightarrow \delta \geq \frac{1}{3}$$

So if $\delta \geq \frac{1}{3}$ then there σ that is spe in which

$$\boxed{u_1(\sigma) = u_2(\sigma) = -1}$$

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8.D.7 a.

suppose $w_{-i} = u_i(\sigma_i^*, \sigma_{-i}^*)$. The following is true by definition:

$$\underline{v}_i = \min_{\sigma_{-i}} \{ \max_{\sigma_i} u_i(\sigma_i, \sigma_{-i}) \} \geq \min_{\sigma_{-i}} \{ u_i(\sigma_i^*, \sigma_{-i}) \} = u_i(\sigma_i^*, \sigma_{-i}^*) = w_{-i}$$

8.E.1.

There are four pure strategies contingent on the type of player:

AA: Attack if either weak or strong type.

AN: Attack if strong and Not Attack if weak.

NA: Not Attack if strong and Attack if weak.

NN: Never attack

The expected payoff of each strategies can be easily computed and are are given in below figure:

$\frac{M}{4} - \frac{S+W}{2}, \frac{M}{4} - \frac{S+W}{2}$	$\frac{M}{2} - \frac{S+W}{4}, \frac{M}{4} - \frac{S}{2}$	$\frac{3M}{4} - \frac{S+W}{4}, -\frac{W}{2}$	$M, 0$
$\frac{M}{4} - \frac{S}{2}, \frac{M}{2} - \frac{S+W}{4}$	$\frac{M-S}{4}, \frac{M-S}{4}$	$\frac{M}{2} - \frac{S}{4}, \frac{M-W}{4}$	$M/2, 0$
$-\frac{W}{2}, \frac{3M}{4} - \frac{S+W}{4}$	$\frac{M-W}{4}, \frac{M}{2} - \frac{S}{4}$	$\frac{M-W}{4}, \frac{M-W}{4}$	$M/2, 0$
$0, M$	$0, \frac{M}{2}$	$0, \frac{M}{2}$	$0, 0$

Any NE of this normal form game is a Beysian NE of the original game.

case 1: $M > W > S$, and $W > M/2 > S$

From the above payoffs we can see that (AA, AN) and (AN, AA) are both pure strategy Beysian Nash equilibria.

case 2: $M > W > S$, and $M/2 < S$

From the above payoffs we can see that (AA, NN) and (NN, AA) are both

Pure strategy beysian nash equilibria

Case 3 : $W > M > S$ and $M/2 < S$

From the above payoffs we can see that (AN, AN), (AA, NN) and (NN, AA) are pure strategy Baysian nash equilibria.

Case 4. $W > M > S$, $\frac{M}{2} > S$

From the above payoffs we can see that (AA, AN), (AN, AA) and (AN, AN) are Pure strategy Baysian nash equlibria.