

## تمرین های سری ۴

زمان تحویل: چهارشنبه ۱۳۹۸/۱۲/۲۸ ساعت ۸ صبح

### Chapter 8: MWG Exercises 8.D.7a, 8.E.1

#### **Additional Exercise 1:** (Nash reversion pure strategy folk theorem)

Consider an infinitely repeated game  $G(\infty, \delta)$ . Suppose that pure strategy profile  $a \in A$  is a Nash equilibrium of  $G$ . Suppose also that  $a' \in A$  is a pure strategy profile such that  $u_i(a') > u_i(a)$  for every player  $i$ . Show that there exists  $\underline{\delta} \in (0, 1)$  such that if  $\delta > \underline{\delta}$ , then there is a subgame perfect equilibrium  $\sigma$  of  $G(\infty, \delta)$  for which  $u_i(\sigma) = u_i(a')$  for all  $i$ .

**Additional Exercise 2:** Consider the infinitely repeated game  $G(\infty, \delta)$  based on the stage game below. Use the principle of optimality to find the set of discount factors for which the following strategy profile is a subgame perfect equilibrium: 1) in period 1 Player  $i$  plays  $a_i$ ; 2) in every period after period 1, Player  $i$  plays  $b_i$  if  $(b_1, b_2)$  or  $(c_1, c_2)$  was played in the previous period; and 3) in every period after period 1, Player  $i$  plays  $c_i$  if  $(b_1, b_2)$  or  $(c_1, c_2)$  was not played in the previous period.

		Player 2		
		$a_2$	$b_2$	$c_2$
Player 1	$a_1$	4, 4	3, 2	1, 1
	$b_1$	2, 3	2, 2	1, 1
	$c_1$	1, 1	1, 1	-1, -1

**Additional Exercise 3:** (Repeated Prisoners' Dilemma) Consider the following stage game  $G$ :

		Player 2	
		$C$	$D$
Player 1	$C$	-1, -1	-4, 0
	$D$	0, -4	-3, -3

- Show that the finitely repeated game  $G(T, \delta)$  has a unique subgame perfect equilibrium for any  $T$  and any discount factor  $\delta$ . What are the payoffs?
- Show that for high enough  $\delta$  there is a SPE  $\sigma$  of the infinitely repeated game  $G(\infty, \delta)$  for which  $u_1(\sigma) = u_2(\sigma) = -1$ . HINT: Use Additional Exercise 1.