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دانشگاه خاتم
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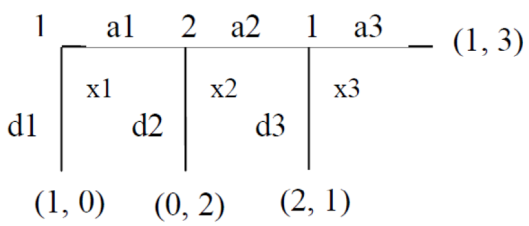
تمرین‌های سری ۳

زمان تحویل: چهارشنبه ۱۳۹۸/۱۲/۲۱ ساعت ۸ صبح

<u>Chapter 9:</u> MWG Exercises 9.B.1, 2, 3, 10.

<u>Additional Exercise 1:</u> Consider the following extensive form game:



Denote a mixed strategy profile $\sigma = (\sigma_1, \sigma_2)$ by $\alpha = \sigma_1(a_1|x_1)$, $\beta = \sigma_2(a_2|x_2)$, and $\gamma = \sigma_1(a_3|x_3)$.

a) Find all the subgame perfect Nash equilibria.
b) Find all the Nash equilibria.

<u>Additional Exercise 2:</u> The following normal form game is played twice:

		Player 2	
		a	b
Player 1	A	10, 10	5, 20
	B	20, 5	7, 7

a) Draw the extensive form game. How many pure strategies does each player have?
b) Find all the subgame perfect Nash equilibria.

<u>Additional Exercise 3:</u> A Stackelberg duopoly has two firms, 1 and 2, with firm 1 choosing output first and firm 2 choosing output second, after observing the choice of firm 1. Suppose that the inverse demand function is $P(Q) = 6 - Q$, where $Q = q_1 + q_2$ is aggregate output. Each firm has constant marginal cost of \$4 per unit, and a capacity constraint of 3 units.

a) Define formally the strategy sets of each firm. (Hint: Firm 2's strategy is a function.)
b) Find a Nash equilibrium in which the Cournot outputs are produced.
c) Find a Nash equilibrium in which firm 2 produces the monopoly output and firm 1 produces nothing.
d) Find the subgame perfect Nash equilibria.

Additional Exercise 4: Suppose n players use an ultimatum procedure to share an apple pie. First, player 1 proposes a division. Then the others simultaneously respond "yes" or "no." If they all say "yes," the proposed division is implemented. Otherwise the pie is fed to Fido the dog. Each player prefers more pie to less, and is indifferent about how much pie any other player or dog consumes.

a) Define formally the strategy sets of each player.
b) Find the subgame perfect Nash equilibria when $n = 2$ and when $n = 3$.

Additional Exercise 5: Consider the following Pirate Game: There are R pirates who must decide how to divide 100 gold pieces among themselves. The gold pieces are indivisible, so a division is feasible only if each pirate gets a whole number of gold pieces. The mechanism they use is as follows: Pirate 1 proposes a division. Then Pirate 2 can accept or reject it. If he accepts, the proposed division is implemented and the game is over. If he rejects, Pirate 1 is thrown to the sharks, and then Pirate 2 proposes a division to Pirate 3, and so on. If Pirate R rejects Pirate $R-1$'s proposal, then Pirate R gets all 100 gold pieces. Pirates prefer more gold to less and are indifferent about how much gold any other pirate gets; being fed to the sharks is their least favorite thing. Watching another pirate being fed to the sharks gives a pirate positive utility, but a pirate always prefers an extra gold piece to watching sharks eat. Describe the subgame perfect Nash equilibria of this game. How does your answer depend on the value of R?

Additional Exercise 6: N people may contribute to a public project in periods $t = 1, \ldots, T \leq \infty$. In the first period in which the total of their present and past contributions is $K > 0$ or more dollars, the project is "complete" and each person receives a dollar benefit of V. They have the same discount factor $\delta \in (0,1)$. If player i's sequence of contributions is $(z_i(1), z_i(2), \ldots, z_i(T))$, and the project is completed in period $T^* \leq T$, his utility is $U_i = \delta^{T^*-1}V - \sum_{t=1}^{T} \delta^{t-1} z_i(t)$. If the project is never completed, $U_i = -\sum_{t=1}^{T} \delta^{t-1} z_i(t)$. Assume that contributions must be nonnegative, and that $K/N < V < K$.

a) Suppose that $N = 2$. Describe the pure-strategy Nash equilibria of the static game. (ie. $T = 1$).

For the rest of the questions, suppose that $N = T$, and that player t can contribute only in period t. At the beginning of period t, let $X(t) = K - \sum_{s=1}^{t-1} z_s(s)$ denote the amount that must be contributed to complete the project. (Let $X(1) = K$.)

b) There is an amount R for which it is a conditionally dominant strategy for player t to complete the project in period t if $X(t) < R$. Find R.
c) Suppose that $N = T = 2$, and that $\delta = 1$. Find the unique subgame perfect equilibrium.
d) Again supposing that $N = T = 2$ and that $\delta = 1$, find a Nash equilibrium in which the project is not completed.