Problem Set 2

Due before recitation session on 1397/12/12.

1. (a) Prove that each player i's information sets H_i are strictly partially ordered by the precedence ranking $\prec^*$,where we define $h' \prec^* h$ if there exists $t' \in h'$ and $t \in h$ such that $t' \prec t$.

   (b) Give an example showing that the set of all information sets is not similarly strictly partially ordered.

   (c) Prove that if $h' \prec^* h$ for $h', h \in H_i$, then for all $t \in h$, there exists $t' \in h'$ such that $t' \prec t$.

2. Suppose $T = \{a, b, c, d, e, f, g, h\}$ and $\prec$ is given by (i) $a \prec b, c, d, e, f, g, h$, (ii) $b \prec c, e$, and (iii) $d \prec f, g$.

   (a) Draw the implied tree. Be sure to label all nodes.

   (b) Suppose this is a two player game, with player 2 owning b and d. Specify an action labeling for the game, and a pair of strategies for each of players 1 and 2 with the property that the four resulting strategy profiles have precisely c, f, g as outcomes.

   (c) Suppose now that player 2 cannot distinguish between the two nodes b and d. Describe player 2's information set(s). Is it possible to specify an action labeling for the game, and a pair of strategies for each of players 1 and 2 with the property that the four resulting strategy profiles have precisely c, f, g as outcomes? Why or why not?

   (d) What is the implied tree if (ii) is given by $b \prec c, d, e$?

3. Suppose $\{(S_i, U_i)_{i=1}^n\}$ is a normal form game, and $\hat{s}_1 \in S_1$ is a weakly dominated strategy for player 1. Let $S_1' = S_1 \backslash \{\hat{s}_1\}$, and $S_i' = S_i$ for $i \neq 1$. Suppose s is a Nash equilibrium of $\{(S_i', U_i)_{i=1}^n\}$. Prove that s is a Nash equilibrium of $\{(S_i, U_i)_{i=1}^n\}$.

4. Consider the Cournot duopoly example discussed in class. What is the Nash Equilibrium of the n-firm Cournot oligopoly? What happens as n approaches infinity?

5. Consider now the Cournot duopoly where inverse demand is $P(Q) = a - Q$ but firms have asymmetric marginal costs: c_i for firm i, $i = 1, 2$.

   (a) What is the Nash equilibrium when $0 < c_i < a/2$ for $i = 1, 2$?
   What happens to firm 2's equilibrium output when firm 1's costs, c_1, increase? Can you give an intuitive explanation?

   (b) What is the Nash equilibrium when $c_1 < c_2 < a$ but $2c_2 > a + c_1$?

6. The number 1000000 is written on a board. A and B take turns, each turn consisting of replacing the number n on the board with $n-1$ or $\lfloor \frac{n+1}{2} \rfloor$. The player who writes the number 1 wins. Using backward induction, who has a winning strategy?