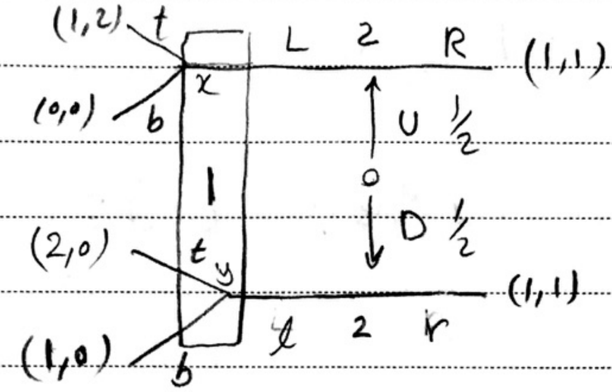
PS5 مهدی امیر - محمدعلی بختیاری کانژاد - مرتضی نورالهی تمرین سری ۵

Subject:
Year. Month. Date.. ( )

Add 2



a)

assame $P_1, P_2 > 0$

$\sigma_2(L) = P_1 \Rightarrow \sigma_2(R) = 1 - P_1$

$\sigma_2(l) = P_2 \Rightarrow \sigma_2(r) = 1 - P_2$

$\Rightarrow P_{\sigma_2}(1) = \frac{P_1 + P_2}{2} \Rightarrow \mu_1(x) = \frac{\frac{1}{2} \times P_1}{\frac{P_1 + P_2}{2}} = \frac{P_1}{P_1 + P_2}$

$\Rightarrow \mu_1(y) = \frac{P_2}{P_1 + P_2}$

Assum $U_1(t) \geq U_1(b) \Rightarrow \frac{P_1}{P_1+P_2} + \frac{2P_2}{P_1+P_2} \geq \frac{P_2}{P_1+P_2}$

~~if $P_1, P_2 > 0$~~

$\Rightarrow P_1 + P_2 > 0 \Rightarrow$ trivial $\Rightarrow t$, x_D : 2 choos r with $P = 1$

x_U : 2 cheese L with $P = 1$

$P_1 = 0, P_2 > 0 \Rightarrow \mu_1(x) = 0 \Rightarrow t$

$\mu_2(y) = 1$

$\Rightarrow$

$P_2 = 0, P_1 > 0 \Rightarrow \mu_1(y) = 0 \Rightarrow t$

$\mu_1(x) = 1$

$P_1 = P_2 = 0 \Rightarrow t, b$

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$2 - \frac{1}{3} = \frac{5}{3}$

Subject:
Year. Month. Date. ( ) $\frac{4}{3} + \frac{1}{3} = \frac{5}{3}$

$\Rightarrow$ all PBE : 1- $(U, t, L, \mu_1(x|1) = 1)$

مرزبنی بودن است $\rightarrow$ 2- $(D, t, r, \mu_1(y|1) \in [0,1])$ =

b) (D, b, r) } is Nash eq but Not PBE
(U, b, l) }

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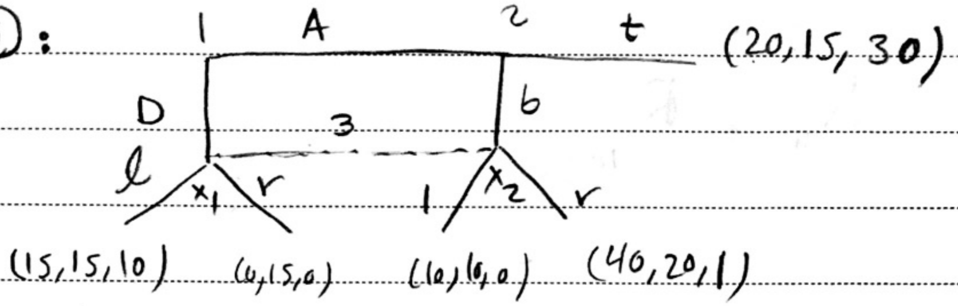
Add 3:



a)

3 \ 2	t	b
l	15,15,10	15,15,10
r	0,15,0	0,15,0

1 chooses D

3 \ 2	t	b
l	20,15,30	10,10,0
r	20,15,30	40,20,1

1 chooses A

or if we define L r for player 3

b) For player 1: $D \succ A$ when $\sigma_3 = l$, $\sigma_2 = b$

$A \succ B$ when $\sigma_3 = l$, $\sigma_2 = t$

so Player 1 has no weakly dominated strategy

For player 2: $\sigma_1 = A$, $\sigma_3 = l$ $t \succ b$

$\sigma_1 = A$, $\sigma_3 = r$ $b \succ t$

So player 2 has no weakly dominated strategy.

For player 3: if $\sigma_1 = B$, $\sigma_2 = t$, $l \succ r$

if $\sigma_1 = A$, $\sigma_2 = b$, $r \succ l$

so player 3 has no weakly dominated strategy.

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c) For (A, t, p) be a NE:

[1] A: $E(U_1) = 20p_1 + 20(1-p) = 20$

D, $E = 15$

[2]: t, $E(U_2) = 15$, $b = 10p + 20(1-p) = 20 - 10p$

$20 - 10p \leq 15 \Rightarrow 5 \leq 10p \Rightarrow p \geq \frac{1}{2}$

3: $E(U_3(L)) = E(U_3(r))$

d) (A, t, p), $p \geq \frac{1}{2}$

$\sigma_1(A) = \varepsilon_1$, $\sigma_1(D) = 1 - \varepsilon_1$

$\sigma_2(t) = \varepsilon_2$, $\sigma_2(b) = 1 - \varepsilon_2$

$\varepsilon_3(0, \frac{1}{2})$ $\sigma_3(l) = p - \varepsilon_3$

$P_\sigma(3) = 1 - \varepsilon_1 + \varepsilon_1(1 - \varepsilon_2) \Rightarrow \mu_3(x_1) = \frac{1 - \varepsilon_1}{1 - \varepsilon_1 + \varepsilon_1(1 - \varepsilon_2)}$

$\{\sigma\}_{k=1}^{\infty}$: $\begin{cases} \varepsilon_1 \to 1 & \to \mu_3(x_1) = 0 \\ \varepsilon_2 \to 1 & \mu_3(x_2) = \end{cases}$ $\mu_3(x_2) = \frac{\varepsilon_1(1 - \varepsilon_2)}{1 - \varepsilon_1 + \varepsilon_1(1 - \varepsilon_2)}$

$(1 - \varepsilon_1) \times p \times 10 + \underbrace{\varepsilon_1(1 - \varepsilon_2) \times p \times 0}_{0} > \varepsilon_1(1 - \varepsilon_2)$

$\Rightarrow ((A, t, p), \mu_3(x_1) = \mu_3(x_2) = 0)$ is a sequential Eq.
$p \geq \frac{1}{2}$

So (A, t, p), $p \geq \frac{1}{2}$ is part of sequential Equilibrium

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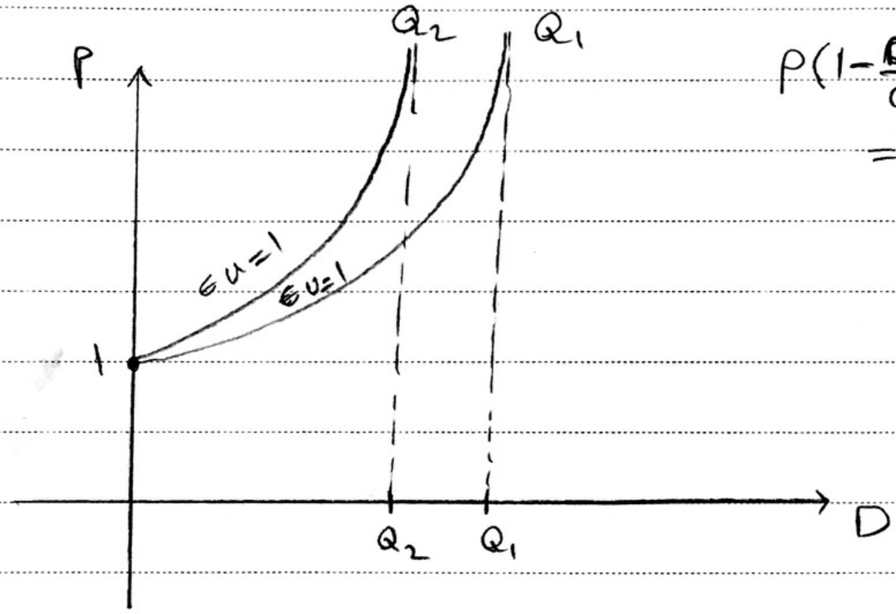
Add 4 : a) Q, P, $U_s = \begin{cases} 0 & P = \frac{D}{Q} \quad \text{breaks before } D \\ P & P = 1 - \frac{D}{Q} \quad \text{braks after } D \end{cases}$

$$\Rightarrow E(U_s) = \frac{D}{Q} \times 0 + \left(1 - \frac{D}{Q}\right) \times P = P\left(1 - \frac{D}{Q}\right)$$

b)



$$P\left(1 - \frac{D}{Q_1}\right) = 1$$

$$\Rightarrow P_1 = \frac{1}{1 - D/Q_1}$$

$$P_2 = \frac{1}{1 - D/Q_2}$$

(ej): $Q_1 > Q_2$

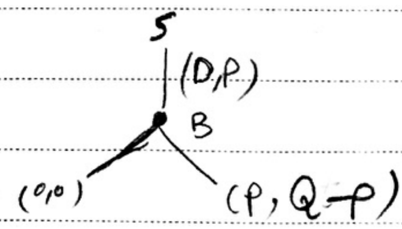
$$\frac{dP_1}{dQ} = \frac{+\frac{1}{Q_1}}{\left(1 - \frac{D}{Q_1}\right)^2}$$

As it is obvious from graph single crossing condition holds.

$$1 - D/Q_1 > 1 - D/Q_2$$

$$\frac{1}{1 - D/Q_1} < \frac{1}{1 - D/Q_2}$$

c) pooling : $\underset{\downarrow}{\overset{Q_1}{}}P_1 = \underset{\downarrow}{\overset{Q_2}{}}P_2 = P$, $(0, P)$



$$E(U_B) = E(2t - P) = 2E(t) - P = Q - P$$

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$\boxed{\text{Add6}}$: a)

Tina

		H	A
Gina	H	3, 0	3, 2
	A	2, 3	2,2

H chooses Gina over Tina

1

T

		H	A
G	H	0, 3	3, 2
	A	2, 3	2, 2

H chooses T over G

2

NE.

$\boxed{(1, H, A), (2, A, H)}$, these are all Nash equilibria of the game.

b) $(1, H, A), (2, A, H)$ is SPE

c) $\mu_2(x_4 \mid h_{T_1}) = 1$, $P_0(h_{T_2}) = \frac{1}{2} + \frac{1}{2} \times p$ — at x_2 Gina chooses hunk with p

$\mu_2(x_6) = \frac{p}{1+p}$, $\mu_2(x_5) = \frac{1}{1+p}$

if $H \succ A$ for 2 at h_{T_2} : $\cancel{\frac{1}{1+p} \times P_2 \times 0} + \frac{p}{1+p} \times P_2 \times 3 > 2(1-P_2)$

$\Rightarrow 3 \frac{p}{1+p} \times P_2 > 2 - 2P_2 \Rightarrow$ $3 \times \frac{p}{1+p} > 2 \to 3p > 2 + 2p$

$\to p > 2$ $\times$

if 2 plays pure it chooses A

at h_{T_1} 2 plays Hunk for sure.

$E(u_2) = 2(1-p) + p$

$\frac{1}{1+p} \times 2 +$

$2 = 3 \times \frac{p}{1+p}$ $\times$

unique PBE is $= ((\underset{\text{Gina}}{H}, A), \mu(x_5) = \mu(x_6) = \frac{1}{2})$

d) If x_1 is reached Gina plays H, $P_0(h_{T_2}) = \frac{1}{6} + \frac{5}{6} \times P_1$,

$2 > \frac{5}{6_2} \times P_1 \times 3 \Rightarrow P_1 < 4/5$

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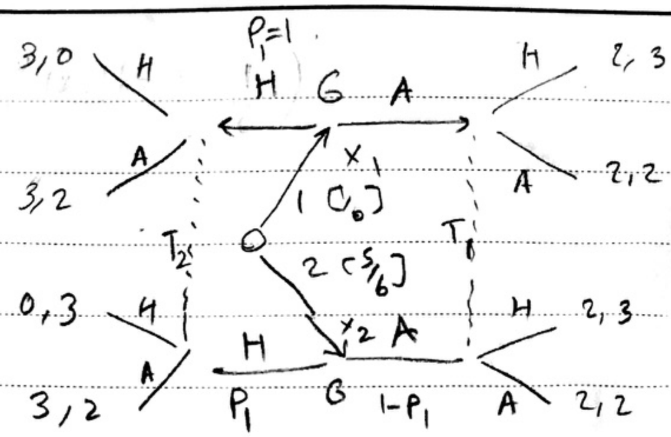
d)



if G take a mixed action T would find out she is at X_2 so she would does H, at x_1 G must play H becasus it is dominant. But if G play H at X_2 then sequential rationality requires T plays H so $E(G)=0$ hence she wouldn't play H at x_2 only remains A at X_2.

$$\Rightarrow \begin{cases} PBE_1 = ((1, H, A), \mu_2(x_3/T_2) = 1) \\ PBE_2 = ((2, A, H), \mu_2(x_4/T_1) = 1) \end{cases}$$

$\frac{1}{6} \times 2 + \frac{5}{6} \times 2 < \frac{5}{6} \times 3 \rightarrow$ so G don't play h at X_2.

The only PBEs are these.

12.C.18)

a) Firm 1 chooses q_1 knowing that given it's choice, firm 2 will produce $b_2(q_1)$, where $b_2(q_1)$ is firm 2's best response function, Firm 1's program, therefore is: $\max_{q_1} \Pi^1(q_1, b_2(q_1))$

The first order condition for this program can be written as

$$\Pi_1^1(q_1, b_2(q_1)) = -\Pi_2^1(q_1, b_2(q_1))\, b_2'(q_1) < 0$$

(since $\Pi_2^1(q_1, b_2(q_1)) < 0$ and $b_2'(q_1) < 0$).

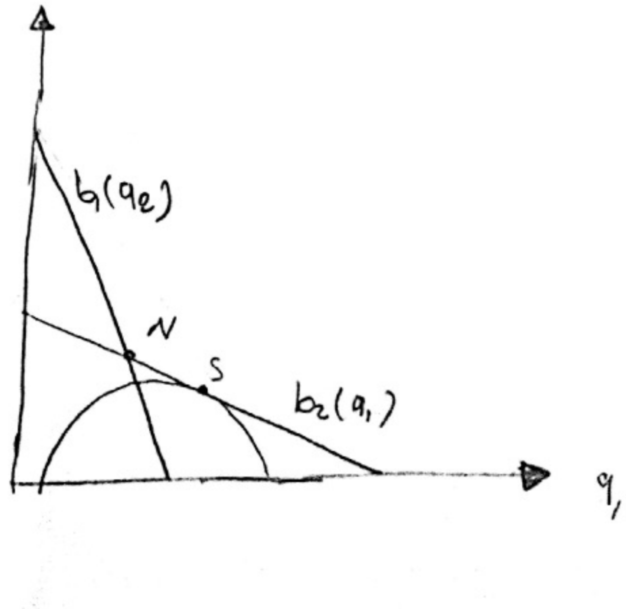
If the firms instead choose quantities simultaneously, the first order condition becomes $\Pi_1^1(q_1, b_2(q_1)) = 0$

since $\Pi_{11}^1(q_1, b_2(q_1)) < 0$, this implies that the stackelberg leader picks a larger quantity in equilibrium than in the cournot game.

Since the best response function of firm 2 is downward_sloping with a slope larger than -1, this implies that the follower picks a smaller quantity and aggregate output increases (and therefore price decreases).

Scince the leader could have chosen the cournot quantity, we know that his profits as a stackelberg leader are higher. The follower produces less and obtains a lower price than the cournot outcome, which implies that his profits are lower.

b) N denotes the Nash equilibrium outcome while S denotes the equilibrium of the stackelberg game.



9.B.11)

Direct calculation show that it is not worth for firm A to stay in the market for $t > 20$, even if it has a monopoly Position. Similarly, firm B will by staying in the market if $t > 25$.

consider period 20. This is the last Period in which A could still be in. If B continues to stay in, it will obtain monopoly profits during Periods 21 through 25 of:

$$(51-2\times21)+(51-2\times22)+(51-2\times23)+(51-2\times24)+(51-2\times25)=25$$

If firm A were to stay in at Period 20, then B will obtain its duopoly profits of $10.5-20=-9.5$ in period. the net profit for firm B of staying in at Period 19 is therefore $25-9.5=15.5>0$

Thus at Period 20 firm B will stay in no matter if firm A is in market or not. This implies that if firm A stays in at Period 20 it will make a loss of $(105-10\times20)=-95$, which in turn implies that firm A will stay out from Period 19 on.

The above argument can be applied backward and we will get the (unique) SPNE of this game. However, the following reasoning applies:

The analysis above shows that with rational play firm 2 will exit after firm 1. therefore, as far as firm 1 is concerned, it will never make monopoly profits. It follows that we only need calculate the last Period in which both firms enter the market. In later Periods, firm 2 will be in the market alone (until Period 25). The unique SPNE is: For $t=1,2,\ldots,10$ both firms stay in; in $t=11,12,\ldots,25$ A is out and B stays in; for $t\geq 26$ both firms are out.

12.D.3.a)

The equilibrium yeilds the cournot outcome in which each firm makes a profit of $(a-c)^2/9b$. The maximum gain from deviation can be obtained by playing the best response to $q^m/2 = (a-c)/4b$, which is $3(a-c)/8b$. This maximum gain is $9/64\ (a-c)^2/b$. Monopoly profit can be calculated to be $(a-c)/4b$. The payoff from deviating is

$$9(a-c)^2/64b + \sum_{t=1}^{\infty} \delta^t (a-c)^2/9b = 9(a-c)^2/64b + \delta/(1-\delta)\ (a-c)^2/9b$$

The payoff from not deviating is $\sum_{t=0}^{\infty} \delta^t (a-c)^2/8b = \frac{1}{(1-\delta)} (a-c)^2/8b$

Therefore, deviation is not profitable if and only if

$\frac{1}{(1-\delta)} (a-c)^2/8b \geq 9(a-c)^2/64b + \delta/(1-\delta)\ (a-c)^2/9b$, or $\delta \geq 9/17$