اقتصاد خرد ۲
دکتر محمدحسین دهقانی

دانشگاه خاتم
نیم‌سال دوم ۹۹-۱۳۹۸

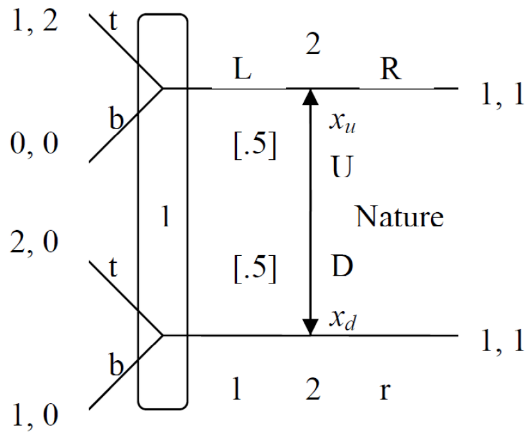
تمرین‌های سری ۵

زمان تحویل: دوشنبه ۱۳۹۹/۱/۱۸ ساعت ۲ بعد از ظهر

Chapter 9: MWG Exercises 9.B.11
Chapter 12: MWG Exercises 12.C.18, 12.D.3a

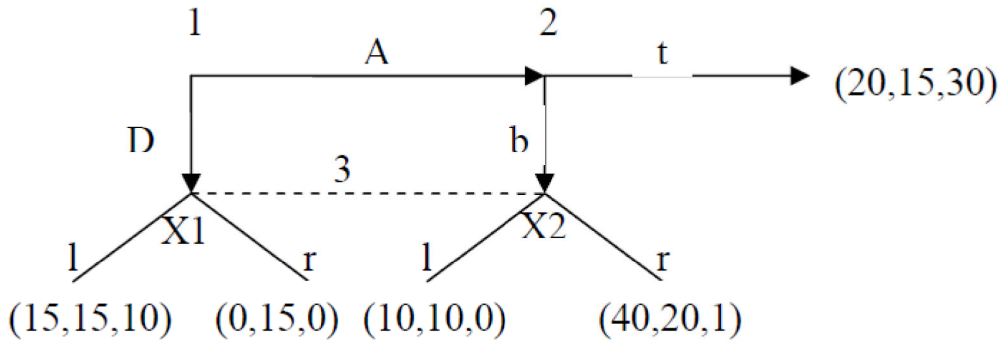
Additional Exercise 1: Consider the following Pirate Game: There are R pirates who must decide how to divide N bottles of rum among themselves. The bottles of rum are indivisible, so a division is feasible only if each pirate gets a whole number of bottles. The mechanism that they use is as follows: Pirate 1 proposes a division. Then Pirate 2 can accept or reject it. If he accepts, the proposed division is implemented and the game is over. If he rejects, Pirate 1 is thrown to the sharks, after being allowed to drink one bottle of rum. Then Pirate 2 proposes a division of the remaining $N-1$ bottles to Pirate 3, and so on. (Assume that $N > R$.) Pirates prefer more rum to less and are indifferent about how much rum any other pirate gets; being fed to the sharks gives a pirate very large negative utility. Watching another pirate being fed to the sharks gives a pirate positive utility, but a pirate always prefers an extra bottle of rum to watching sharks eat. Describe the subgame perfect Nash equilibria of this game. How does your answer depend on the value of R?

Additional Exercise 2: Consider the following game, in which Nature moves first.:



a) Find all the perfect Bayesian equilibria.
b) Find a Nash equilibrium that is not part of a PBE.

Additional Exercise 3: Consider the following game:



a) Write the corresponding normal form.
b) Show that no player has any weakly dominated strategies.
c) Let p be the probability that Player 3 chooses action l. For what values of p is (A, t, p) a Nash equilibrium?
d) Show that whenever (A, t, p) is a Nash equilibrium, then it is also part of a sequential equilibrium.

Additional Exercise 4: A seller has a good of quality Q, which only she knows. To others, Q takes values 1 and 2 with probabilities $1 - \pi$ and π, respectively. A good of quality Q breaks at time t, where t is a random variable distributed uniformly on $[0, Q]$. A buyer receives a utility of 2 each moment he has the good before it breaks. Thus, if she purchases the good for price p and it breaks at time t, his utility is $2t - p$ and the seller's is p. Both their payoffs are 0 if no trade occurs.

Suppose that the seller can sell the good with a warranty. The warranty is characterized in terms of a date D: if the good breaks before date D, the warranty commits the seller to donating the purchase price p to penguins' flood relief in Antarctica. This donation does not directly affect the buyer's utility, but it does affect the seller's: her net utility is 0 if the good breaks before D.

The seller makes a take-it-or-leave-it offer (D, p). The buyer then accepts or rejects, whereupon the corresponding payoffs are realized.

a) What is the expected utility of the seller if she sells a good of quality Q for price p and warranty D?
b) Draw the indifference curves of the two seller types that correspond to utility 1 in (D, p) space. (Put D on the horizontal axis.) Is the single crossing property satisfied?
c) What is the maximum warranty length in a pooling perfect Bayesian equilibrium? What are the restrictions on the warranties that the two seller types offer in any separating PBE?

Additional Exercise 5: Doctor Duck has invented a new kind of machine that will produce an uncertain income. Call this machine a "risky asset." Dr. Duck is considering selling this asset to one of two very wealthy investors by holding a first-price, sealed-bid auction with no reserve price. Consider the following model: At time $t = 1$ Dr. Duck decides whether or not to hold the auction. If he decides to hold the auction, it is held at $t = 2$. At $t = 3$ the asset's return is realized and paid to whoever owns it then. The asset is indivisible.

All three agents are expected utility maximizers. Dr. Duck's Bernoulli utility function for income is $u(y) = 2y^{0.5}$. The two investors both have Bernoulli utility function $v(y) = y$. There is no discounting. The asset returns $z > 0$ dollars with probability ½, and it returns 0 dollars with probability ½. The structure of the game is common knowledge.

a) Describe the subgame perfect equilibria of the game.

Now assume that the value of the parameter z is known to Dr. Duck, but not to the investors, at $t = 0$. The value of z is either $z_h = 20$ (with probability a) or $z_l = 8$ (with probability $1-a$). That information structure is common knowledge.

b) Describe all pooling perfect Bayesian equilibria. Are further assumptions needed for these equilibria to exist? If so, what are they?
c) Describe all separating perfect Bayesian equilibria. Are further assumptions needed for these equilibria to exist? If so, what are they?

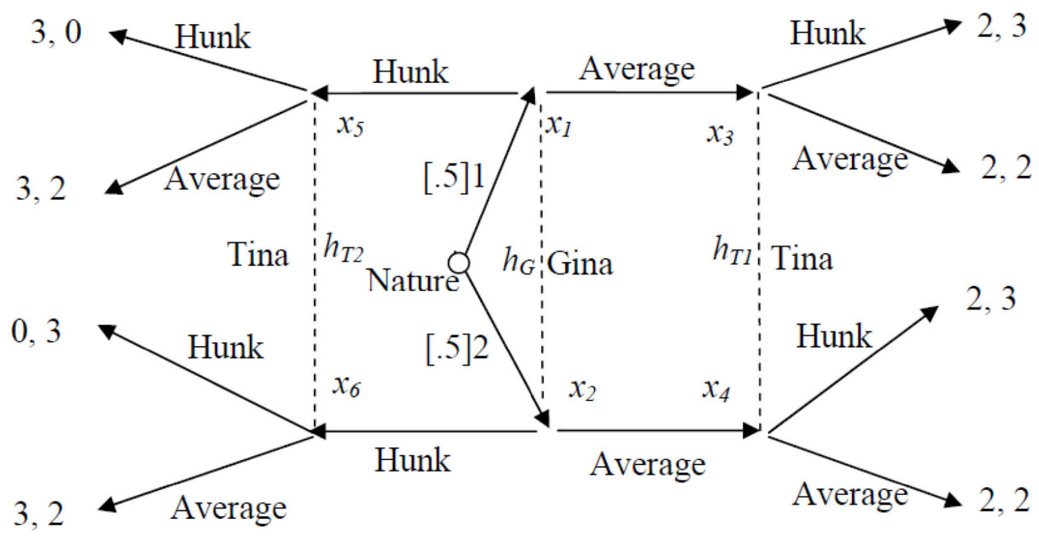
<u>Additional Exercise 6:</u> Gina Gnash and her friend Tina Tnash, who are both expected utility maximizers, are sitting in a bar in Philadelphia. Three guys walk in: One is a great-looking blond Hunk, and the other two are okay-looking Average guys. Going on a date with the Hunk would give Gina or Tina a utility of 3. Going on a date with an Average guy gives utility 2. Going on no date gives utility 0.

Average guys will accept any offer of a date. (There are enough Average guys to go around, so if both Gina and Tina decide to ask out Average guys, then both are accepted.) If the Hunk receives only one offer, he will accept it. If both Tina and Gina ask him out, his decision depends on the state of Nature. In State 1, which occurs with probability 1/2, the Hunk will choose Gina over Tina. In State 2, which also occurs with probability 1/2, he chooses Tina.

For the first part of the question, suppose that Gina and Tina must simultaneously decide who to ask out (the Hunk or an Average guy). If they are accepted, they get the resulting utility. If they are rejected, they go on no date and get utility 0.

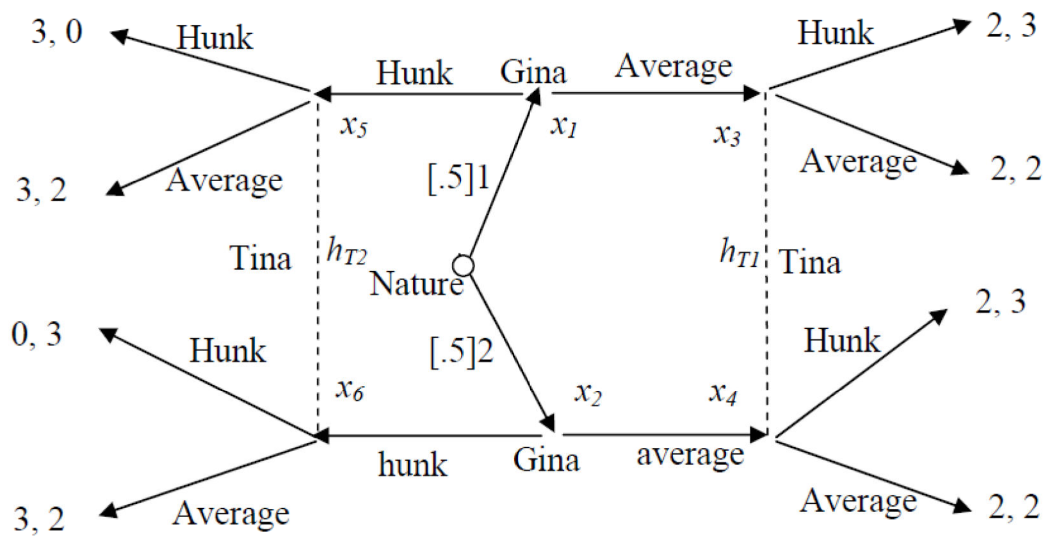
a) (12 points) Find all the Nash equilibria of the normal form game described above, including those in mixed strategies.

Now suppose that Gina is slightly quicker than Tina, and makes her move first. That situation is represented by the following extensive form game (Gina's payoffs are given first):



b) (12 points) Find all the pure strategy subgame perfect equilibria of the extensive form game above.

Now suppose that Gina has such a keen understanding of men that she can tell whose offer the Hunk would accept by the look in his eye. (That is, Gina observes Nature's choice.) Tina, on the other hand, still cannot tell. That situation is represented by the following extensive form game:



c) (12 points) Find all the perfect Bayesian equilibria of the extensive form game above, including those in mixed strategies.

Finally, suppose that Gina has a slight cold and isn't looking her best, so that the probability that the Hunk would choose her over Tina (that is, the probability of State 1) is only 1/6.

d) (12 points) Repeat part c), using the new probabilities.