Comparing Metropolis and Gibbs Sampling Method

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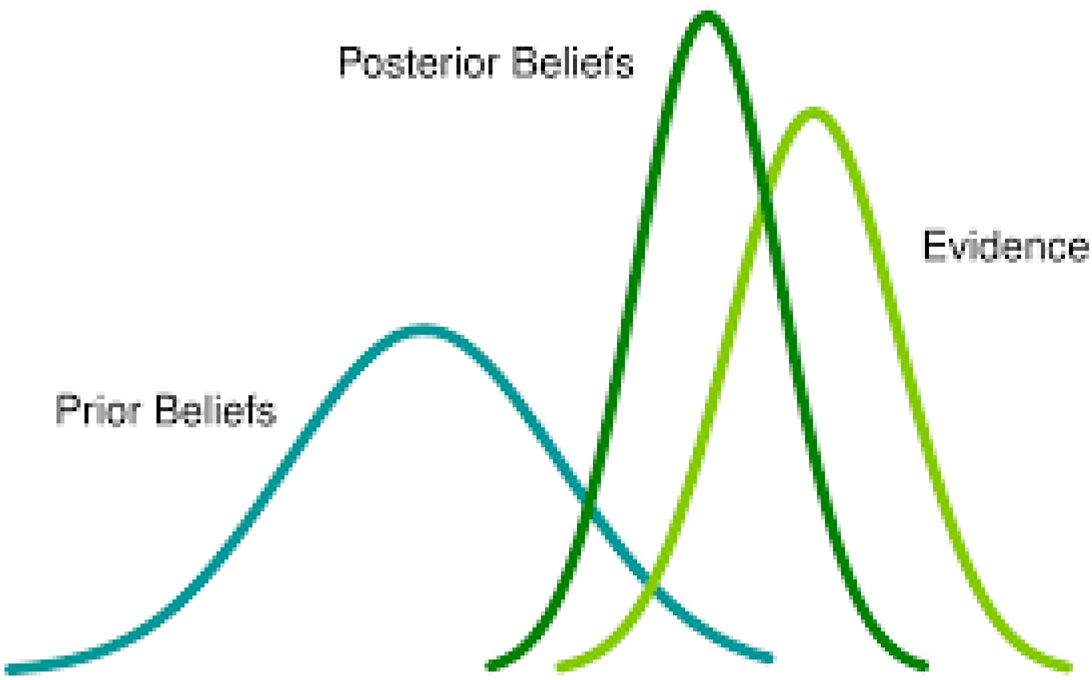

Introduction

The purpose of this project is to explore sampling methods derived from algorithms in the Monte Carlo Markov Chain (MCMC) family called the Metropolis sampling and Gibbs sampling methods. These MCMC algorithms are used in a bayesian setting where direct sampling from the posterior distribution is difficult. By sampling from the posterior we will be able to estimate mean and other parameters to further analyze the data. In order to check for performance, speed and accuracy of those sampling methods, we shall use a known posterior distribution so the distribution obtained from Metropolis and Gibbs sampling methods can be compared. Therefore, the setting of the sampling distribution is Multinomial - Dirichlet conjugate with K = 3 categories with two-dimensional parameters to update.

Difference between Gibbs and Metropolis Algorithm

Metropolis-Hastings sampler is the umbrella algorithm for both Gibbs and Metropolis sampling methods. These sampling methods are used when direct sampling is deemed difficult to perform on a given posterior distribution. Gibbs sampling is a special case of Metropolis hastings algorithm with a probability acceptance rate of 1. Metropolis sampling is also a special case of Metropolis hastings, requiring a symmetric proposal distribution. Although the algorithms are slightly different, the purpose is to sample from the posterior distribution.
Some issues to consider are:

- determine a good burn-in amount to decrease dependence on starting values
- determine a good thinning amount to decrease autocorrelation
- determine a good proposal distribution for Metropolis sampler
- partial correlation between parameters
- convergence diagnostics

We will explore these issues throughout this simulation process.

Set-Up

- Data:
  - True parameters: $\theta_1 = 0.1$, $\theta_2 = 0.7$, $\theta_3 = 0.2$
  - Hyperparameters: $\alpha_1 = 2$, $\alpha_2 = 2$, $\alpha_3 = 2$
- Prior: $X_i|\theta \overset{iid}{\sim} Multinom(n, \theta_1, \theta_2, \theta_3); i = 1, \ldots, n$
- Likelihood function: (singular) $Multinom(\theta_1, \theta_2, \theta_3)$.
- Proposal: $Dirichlet(\alpha_1, \alpha_2, \alpha_3)$
- Posterior: $Dirichlet(\alpha_1 + \sum X_{1i},\ \alpha_2 + \sum X_{2i} + \alpha_3 + \sum X_{3i})$

Algorithms

Metropolis algorithm:

- Choose a starting point for θ's from a starting distribution
- For t = 1, 2, ...
  - Sample a proposed $\theta^\star$ from a symmetric jumping/proposal distribution at time t.
  - Calculate the density ratio:

$$ratio = \frac{p(\theta^\star)|y}{\theta^{t-1}|y} \tag{1}$$

  - Set:

$$\theta^t = \begin{cases} \theta^\star, & \text{with propability min(ratio, 1)} \\ \theta^{t-1}, & \text{otherwise} \end{cases} \tag{2}$$

Gibbs algorithm:

- Set initial values for θ's
- Set $Y = \frac{\theta_1}{1-\theta_2}|\theta_2$
  $Y \sim Beta(\alpha_1 + \sum X_{1i},\ \alpha_2 + \sum X_{2i} + \alpha_3 + \sum X_{3i})$
- Set $Z = \frac{\theta_2}{1-\theta_1}|\theta_1$
  $Z \sim Beta(\alpha_2 + \sum X_{2i},\ \alpha_1 + \sum X_{1i} + \alpha_3 + \sum X_{3i})$
- Obtain updated sample of θ's
- Simulate steps 1-4 with burning and thinning