Innopolis University, 2018
F18 Differential Equations
Programming Assignment Report
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B17-03
Variant 9

Problem

Given first-order nonlinear ordinary differential equation – Bernoulli equation
`f(x, y) = x / y + y / x`
`y’ = f(x, y)`

Analytical solution
Point that `(x != 0)` and `(y != 0)`
Substract `(y / x)` from both sides and multiply both sides by `(2 * y)`:
`2 * (dy / dx) * y - 2 * (y^2 / x) = 2 * x`

Let `(u(x) = y^2)`:
(du / dx) - 2 * (du / x) = 2 * x

Let `(mu = e ^ (integral(-2 / x)dx) = 1 / (x^2)`:
`(du / dx) / (x^2) + (d(1 / (x^2)) / dx) * u = 2 / x`

Apply reverse product rule:
`d(u / (x^2)) / dx = 2 / x`

Integrate both sides with `dx`:
`integral(d(u / (x^2)) / dx)dx = integral(2 / x)dx`
`u / (x^2) = 2 * ln(x) + c`
(`c` – some constant)

`u = (x^2) * (2 * ln(x) + c)`

`y = (+/-)x * sqrt(2 * ln(x) + c)`

Evaluate `(c)` in terms of initial value problem `(x0, y0)`:
`c = (y0^2) / (x0^2) - 2 * ln(x0)`

Computational problem for this solution is trivial:
`y = (+/-)x * sqrt(2 * ln(x) + ((y0^2) / (x0^2) - 2 * ln(x0)))`

Approximation methods

Let (h) be some delta for (x) steps, (xi) and (yi) – x and y values for previous step

Euler method

```
y = yi + h * f(xi, yi)
```

Improved Euler method

```
k1 = f(xi, yi)
k2 = f(xi + h, yi + h * k1)
y = yi + h * (k1 + k2) / 2
```

Runge-Kutta method

```
xz = xi + h / 2
k1 = f(xi, yi)
k2 = f(xz, yi + h * k1 / 2)
k3 = f(xz, yi + h * k2 / 2)
k4 = f(xi + h, yi + h * k3)
y = yi + h * (k1 + 2 * k2 + 2 * k3 + k4) / 6
```

Technical implementation

https://github.com/imajou/F18-DE-Assignment

Platform: Android Native
Pattern: MVC
Language: Kotlin
IDE: Android Studio 3.2.1
Minimal SDK: 21 (Android 5.0)
Third-party libs: com.jjoe64:graphview (plotting lib)

Built apk provided in GitHub repo, if one wants to build it from sources, it is recommended to use Android Studio for building purposes&

The application provides single activity with plotting view with ability to control showed graphic and control panel to adjust problem parameters.

activities/MainActivity.kt
Activity logic, listeners binding, plotting information receiver.

model/EquationVariant.kt
Contains parameters x0 and y0 for initial value problem
Interface to specify needed functions to solve a problem:

> getExactDifferentialSolution(x: Double): Double
> Returns exact analytical solution for the initial value problem
>
> getFunctionValue(x: Double, y: Double): Double
> Returns f(x, y) function result.
>
> getEulerSolutionY(xi: Double, yi: Double, h: Double): Double
> Returns next point y-value corresponding to Euler method approximation
>
> getEulerImprovedSolutionY(xi: Double, yi: Double, h: Double): Double
> Returns next point y-value corresponding to improved Euler method approximation
>
> getRungeKuttaSolutionY(xi: Double, yi: Double, h: Double): Double
> Returns next point y-value corresponding to Runge-Kutta method approximation

model/EquationVariant9.kt
EquationVariant interface implementation for variant 9.

model/Equation.kt
Main computational power of the application stores and computes solutions and approximations/local errors of particular equation variant

> updateData()
> Updating all methods values

model/Solution.kt
Object to compute points list for different approximation approaches.

> exact(variant: EquationVariant, steps: Int, fromX: Double, toX: Double): Array<DataPoint>
> Returns array of points generated by exact equation solution
>
> euler(variant: EquationVariant, steps: Int, fromX: Double, toX: Double): Array<DataPoint>
> Returns array of points generated by Euler approximation

eulerImproved(variant: EquationVariant, steps: Int, fromX: Double, toX: Double): Array<DataPoint>
Returns array of points generated by improved Euler approximation

rungeKutta(variant: EquationVariant, steps: Int, fromX: Double, toX: Double): Array<DataPoint>
Returns array of points generated by Runge-Kutta approximation

model/ErrorLocal.kt
Object to compute points list for different approximation approaches local errors.
Functions are identical as Solution.kt (no exact solution error, of course)

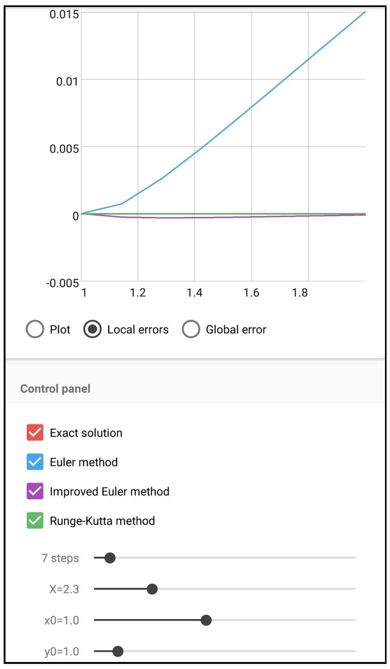
model/ErrorGlobal.kt
Object to compute points list for different approximation approaches global errors.
Functions are identical as Solution.kt (no exact solution error, of course)

res/layout/activity_main.xml
Main activity layout

Overall structure

When application starts and creates main activity, the variant object and the equation object are created. Values for all datasets are being computed for predefined parameters – such as initial value problem and x-boundaries. Graph are showed. When user changes some value with any seekbar, it is updated in variant or equation and then everything that depends of the value is being recomputed. Equation class just aggregates all data for plotting and triggers update. EquationVariant class perform formula computation for any particular point. Solution, ErrorLocal and ErrorGlobal compute corresponding arrays with points for plotting. Changing graphics visibility settings with checkboxes does not affect state of the data, but plotting view only.

Graphics

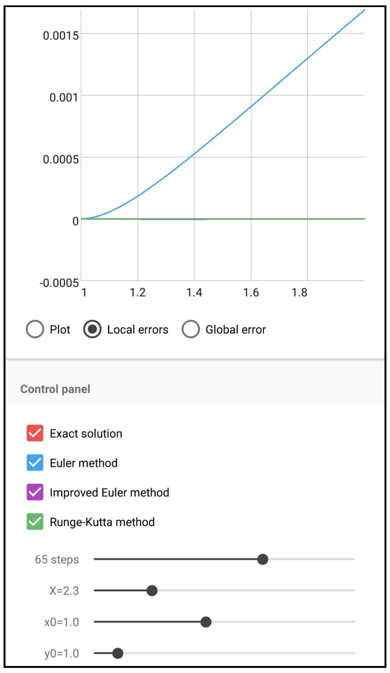


○ Plot [● Local errors] ○ Global error

Control panel

✓ Exact solution
✓ Euler method
✓ Improved Euler method
✓ Runge-Kutta method

7 steps
X=2.3
x0=1.0
y0=1.0

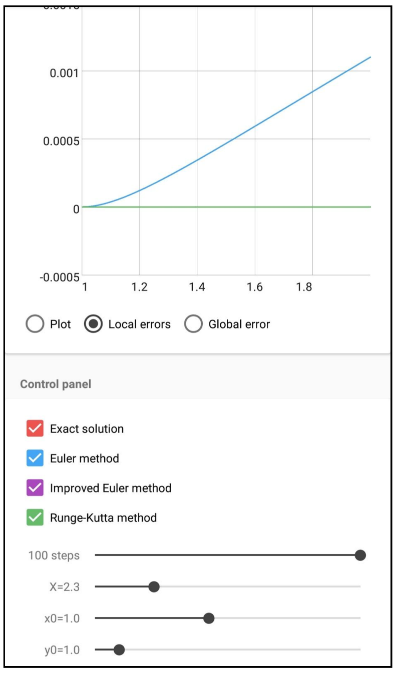


○ Plot [● Local errors] ○ Global error

Control panel

✓ Exact solution
✓ Euler method
✓ Improved Euler method
✓ Runge-Kutta method

65 steps
X=2.3
x0=1.0
y0=1.0

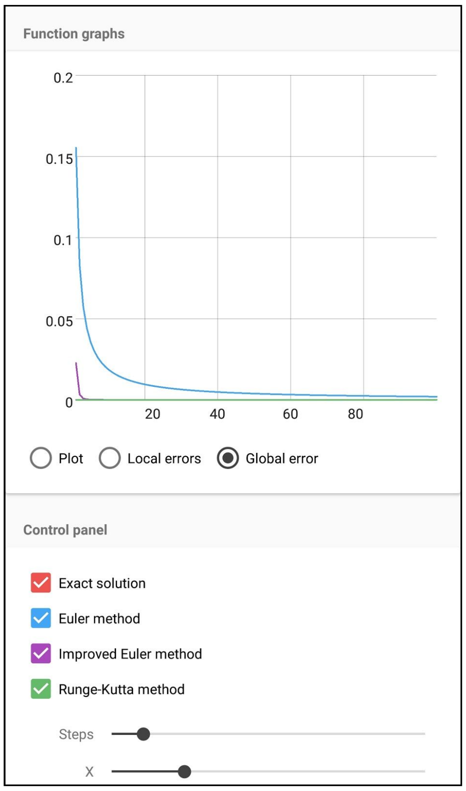


○ Plot [● Local errors] ○ Global error

Control panel

✓ Exact solution
✓ Euler method
✓ Improved Euler method
✓ Runge-Kutta method

100 steps
X=2.3
x0=1.0
y0=1.0


Function graphs

○ Plot ○ Local errors [● Global error]

Control panel

✓ Exact solution
✓ Euler method
✓ Improved Euler method
✓ Runge-Kutta method

Steps
X

Graphics analysis

Let us analyze graphics for basic given initial value problem – `(1.0, 1.0)`
on `x` interval `[1.0, 2.3]`
I took steps values as 7, 65 and 100.

The function is steadily increasing on the given interval – no special points or something interesting.

As one can possibly see, that local error for Euler method is maximal one, improved Euler method going next and Runge-Kutta is the best of all. Overall, local error tends to increase while we decreasing steps count.

Euler method uses trapeze to approximate and have average error of `O(h)`
Improved Euler method uses mean values of trapeze – error `O(h^2)`
Runge-Kutta method error – `O(h^4)`

So, when `h` is decreased by increasing number of steps upper bound of error numerically decreases too, thus, global error for given interval is decreasing on step count.