Innopolis University, 2018
F18 Differential Equations
Programming Assignment Report
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B17-03
Variant 9

Problem

Given first-order nonlinear ordinary differential equation – Bernoulli equation
`f(x, y) = x / y + y / x`
`y’ = f(x, y)`

Analytical solution
Point that `(x != 0)` and `(y != 0)`
Substract `(y / x)` from both sides and multiply both sides by `(2 * y)`:
`2 * (dy / dx) * y - 2 * (y^2 / x) = 2 * x`

Let `(u(x) = y^2)`:
(du / dx) - 2 * (du / x) = 2 * x

Let `(mu = e ^ (integral(-2 / x)dx) = 1 / (x^2)`:
`(du / dx) / (x^2) + (d(1 / (x^2)) / dx) * u = 2 / x`

Apply reverse product rule:
`d(u / (x^2)) / dx = 2 / x`

Integrate both sides with `dx`:
`integral(d(u / (x^2)) / dx)dx = integral(2 / x)dx`
`u / (x^2) = 2 * ln(x) + c`
(`c` – some constant)

`u = (x^2) * (2 * ln(x) + c)`

`y = (+/-)x * sqrt(2 * ln(x) + c)`

Evaluate `(c)` in terms of initial value problem `(x0, y0)`:
`c = (y0^2) / (x0^2) - 2 * ln(x0)`

Computational problem for this solution is trivial:
`y = (+/-)x * sqrt(2 * ln(x) + ((y0^2) / (x0^2) - 2 * ln(x0)))`

Approximation methods

Let (h) be some delta for (x) steps, (xi) and (yi) – x and y values for previous step

Euler method

```
y = yi + h * f(xi, yi)
```

Improved Euler method

```
k1 = f(xi, yi)
k2 = f(xi + h, yi + h * k1)
y = yi + h * (k1 + k2) / 2
```

Runge-Kutta method

```
xz = xi + h / 2
k1 = f(xi, yi)
k2 = f(xz, yi + h * k1 / 2)
k3 = f(xz, yi + h * k2 / 2)
k4 = f(xi + h, yi + h * k3)
y = yi + h * (k1 + 2 * k2 + 2 * k3 + k4) / 6
```

Technical implementation

https://github.com/imajou/F18-DE-Assignment

Platform: Android Native
Pattern: MVC
Language: Kotlin
IDE: Android Studio 3.2.1
Minimal SDK: 21 (Android 5.0)
Third-party libs: com.jjoe64:graphview (plotting lib)

Built apk provided in GitHub repo, if one wants to build it from sources, it is recommended to use Android Studio for building purposes&

The application provides single activity with plotting view with ability to control showed graphic and control panel to adjust problem parameters.

activities/MainActivity.kt
Activity logic, listeners binding, plotting information receiver.

model/EquationVariant.kt
Contains parameters x0 and y0 for initial value problem
Interface to specify needed functions to solve a problem:

> <u>getExactDifferentialSolution(x: Double): Double</u>
> Returns exact analytical solution for the initial value problem
>
> <u>getFunctionValue(x: Double, y: Double): Double</u>
> Returns f(x, y) function result.
>
> <u>getEulerSolutionY(xi: Double, yi: Double, h: Double): Double</u>
> Returns next point y-value corresponding to Euler method approximation
>
> <u>getEulerImprovedSolutionY(xi: Double, yi: Double, h: Double): Double</u>
> Returns next point y-value corresponding to improved Euler method approximation
>
> <u>getRungeKuttaSolutionY(xi: Double, yi: Double, h: Double): Double</u>
> Returns next point y-value corresponding to Runge-Kutta method approximation

model/EquationVariant9.kt
EquationVariant interface implementation for variant 9.

model/Equation.kt
Main computational power of the application stores and computes solutions and approximations/local errors of particular equation variant

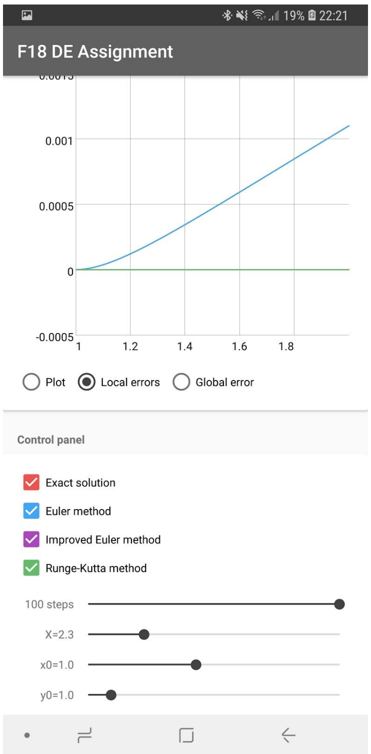
model/Solution.kt
Object to compute points list for different approximation approaches.

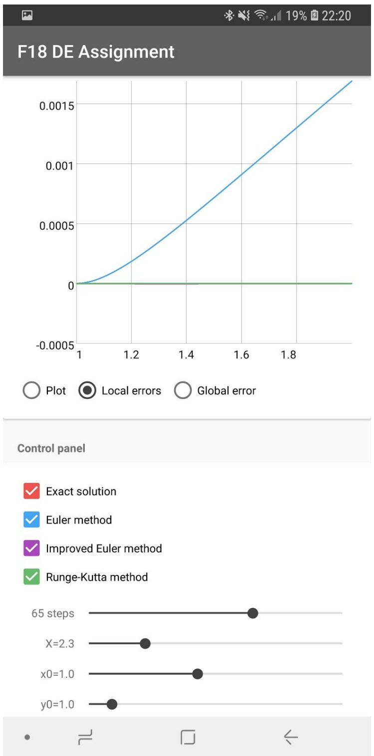
model/ErrorLocal.kt
Object to compute points list for different approximation approaches local errors.

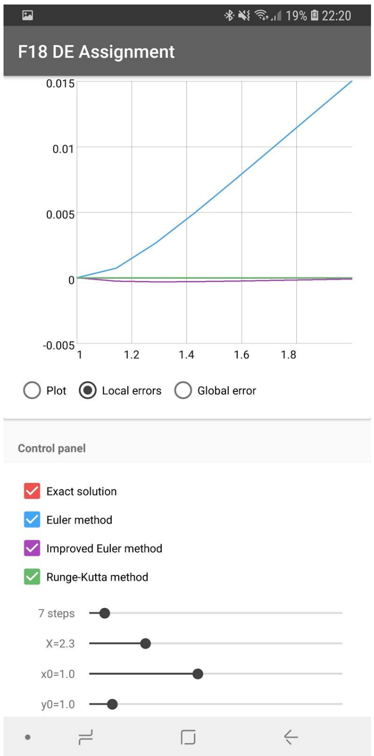
model/ErrorGlobal.kt
Object to compute points list for different approximation approaches global errors.

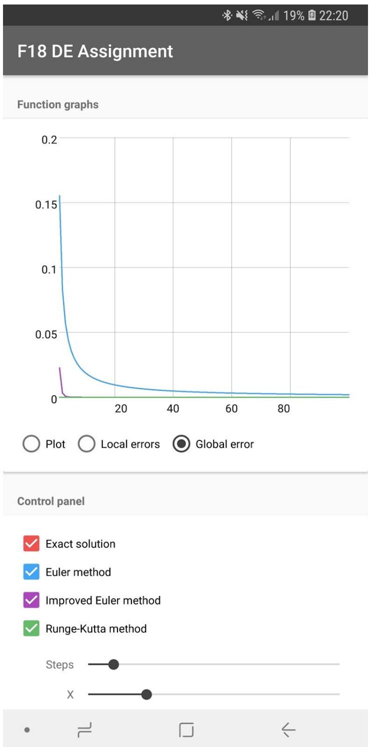
res/layout/activity_main.xml
Main activity layout

Graphics


19% 22:21
F18 DE Assignment
0.001
0.0005
0
-0.0005
1
1.2
1.4
1.6
1.8
Plot
Local errors
Global error
Control panel
Exact solution
Euler method
Improved Euler method
Runge-Kutta method
100 steps
X=2.3
x0=1.0
y0=1.0


19% 22:20
F18 DE Assignment
0.0015
0.001
0.0005
0
-0.0005
1
1.2
1.4
1.6
1.8
Plot
Local errors
Global error
Control panel
Exact solution
Euler method
Improved Euler method
Runge-Kutta method
65 steps
X=2.3
x0=1.0
y0=1.0


19% 22:20
F18 DE Assignment
0.015
0.01
0.005
0
-0.005
1
1.2
1.4
1.6
1.8
Plot
Local errors
Global error
Control panel
Exact solution
Euler method
Improved Euler method
Runge-Kutta method
7 steps
X=2.3
x0=1.0
y0=1.0


19% 22:20
F18 DE Assignment
Function graphs
0.2
0.15
0.1
0.05
0
20
40
60
80
Plot
Local errors
Global error
Control panel
Exact solution
Euler method
Improved Euler method
Runge-Kutta method
Steps
X

Graphics analysis

Let us analyze graphics for basic given initial value problem – (1.0, 1.0)

The function is steadily increasing on the given interval – no special points

As one can possibly see, that local error for Euler method is maximal one, improved Euler method going next and Runge-Kutta is the best of all. Overall, local error tends to increase while we decreasing steps count.

Euler method uses trapeze to approximate and have average error of O(h)
Improved Euler method uses mean values of trapeze – error O(h^2)
Runge-Kutta method error – O(h^4).

Increasing number of steps on steady increasing function improves accuracy by decreasing means of error.