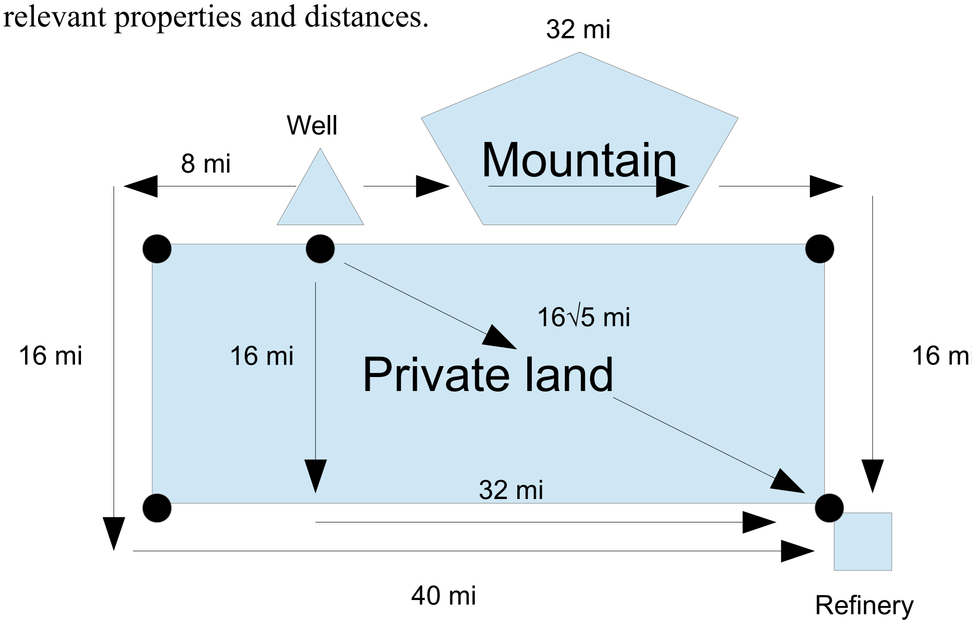
Salt Lake CC, Math 1210 Pipeline Project – Chris Shepard

The goal of this report is to provide a cost analysis of laying a pipeline from a high-producing well on BLM ground to a refinery. First, a review of the current situation. Below is a diagram showing the layout of relevant properties and distances.



There are 5 proposed paths, each originating from the well and terminating at the refinery:

1. A pipeline running entirely through BLM land, 8 miles westward, followed by 16 miles southward, and finally 40 miles eastward towards the refinery. 64 miles total.
2. A pipeline running 32 miles eastward, through BLM land, and a nearby mountain, then, once through, 16 miles south to the refinery. 48 miles total.
3. A pipeline running 16 miles directly south of the well through private land, then 32 miles eastward to the refinery. 48 miles total.
4. A pipeline running approximately 35.78 ($16\sqrt{5}$) miles across private land, directly toward the refinery.
5. A pipeline running across private land at some angle to be determined later in this report to optimize for minimum incurred cost.

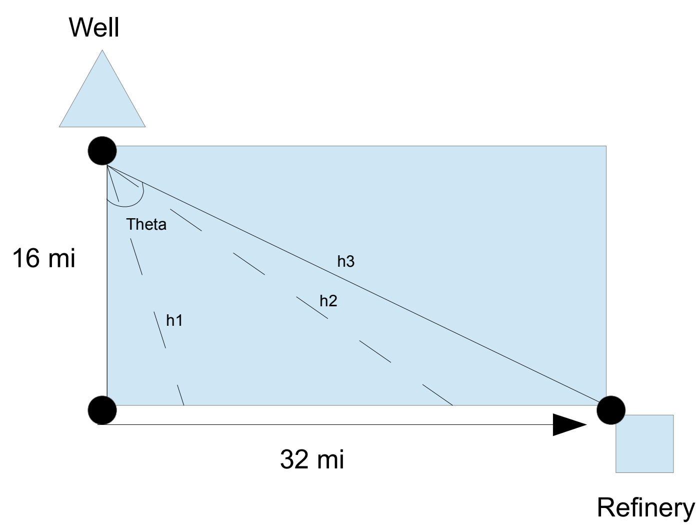
Each of these paths have associated costs.

- Every mile of pipeline costs $480,000 in materials, labor and fees to run
- Drilling through the mountain requires a $4,500,000 one-time cost, a $600,000 environmental impact study, and an expected 8 month delay, which will cost an additional total $800,000 over the time period, for a total additional expense of $5,900,000 on top of the cost of laying pipeline.
- Every mile over private land incurs an additional $360,000 expense in right-of-way fees, $840,000 per mile total.

This puts the total cost of each pipeline path at:

1. \$30,720,000 in pipe-laying costs to route westward around private land.
2. \$23,040,000 in pipe-laying costs, and \$5,900,000 in routing through the mountain, eastward, around private land, for a total cost of \$28,940,000.
3. \$23,040,000 in pipe-laying costs, and an additional \$5,760,000 in right-of-way fees to cross private land directly southward, before heading eastward to the refinery, for a total cost of \$28,800,000.
4. \$17,173,002.07 in pipe-laying costs, and an additional \$12,879,751.55 in right-of-way fees to cross private land directly toward the refinery, for a total cost of \$30,052,753.62.

Below is a diagram illustrating possible paths across private land, which will be used to create the cost function needed to determine the lowest cost for the fifth possible pipeline path:



In order to figure the cost of the fifth possible pipeline path, we must find the optimal angle from the well across private land to minimize costs. The associated distance (h on the diagram above) can be calculated with:

$$h=\frac{y}{\cos\theta}$$

where y is the vertical distance from the well to the refinery, which is 16 miles. Where the line h intersects the southern edge of private land will determine the remaining distance left over BLM land towards the refinery. This distance can be calculated with:

$$f=32-x$$

where x is:

$$h\sin\theta$$

The total distance, therefore, is

$$d=h+f$$

or

$$d=\frac{y}{\cos\theta}+32-h\sin\theta$$

or

$$d = \frac{y}{\cos\theta} - \frac{y}{\cos\theta}\sin\theta + 32$$

or

$$d = y\,sec\,\theta(1-\sin\theta)+32$$

with y being equal to 16 miles, and the cost of laying pipe on private and BLM land taken into account, we find the cost function:

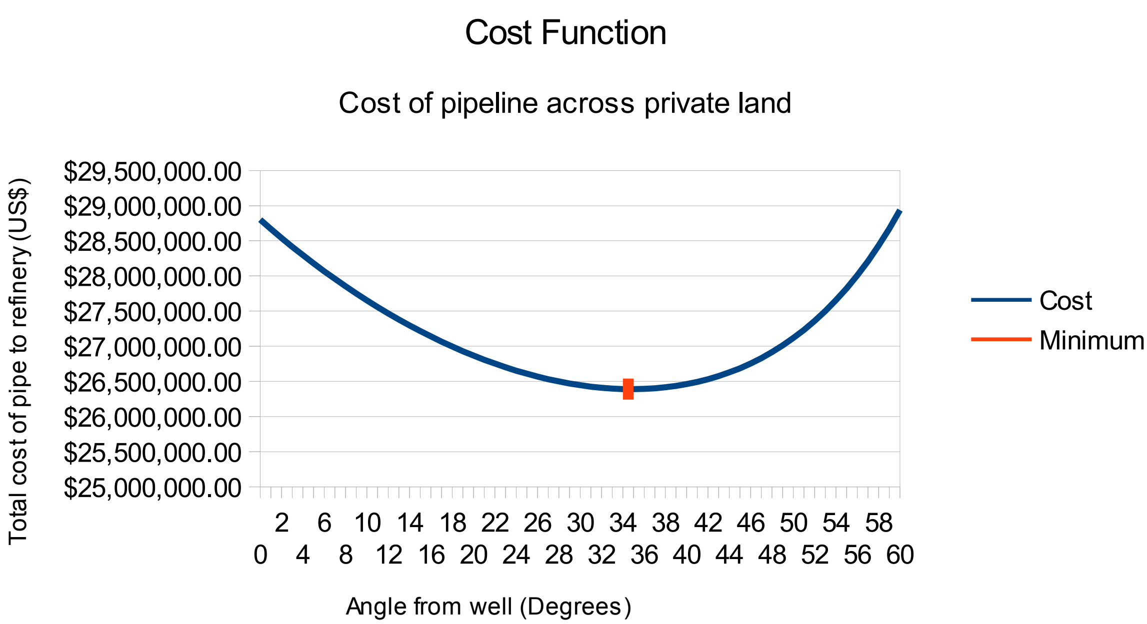
$$C(\theta)=16\,sec\,\theta(840{,}000-480{,}000\sin\theta)+32\cdot 480{,}000$$

which simplifies to:

$$C(\theta)=1{,}920{,}000\,sec\,\theta(7-4\sin\theta)+15{,}360{,}000\ \ for\ 0\le\theta\le\frac{\pi}{3}$$

Note the domain of C is over $0\le\theta\le\sin^{-1}\frac{32}{16\sqrt{5}}$ which represents all values of theta as they cross the entirety of the 32 mile horizontal distance from the point directly south of the well to the refinery. For simplicity's sake, we round the domain to $\pi/3$ as it is clear from the graph below that the minimum cost is not found at the edges of the domain.

Now that we have a cost function, we must find the optimal angle to minimize the cost. Looking at the graph below, it is clear that the minimum could be expected somewhere around 35 degrees ($7\pi/36$ radians).



We will find the exact value via finding the derivative of the cost function, and solving for when it is equal to zero:

$$C'(\theta)=\frac{d}{d\theta}(1{,}920{,}000\,sec\,\theta(7-4\sin\theta)+15{,}360{,}000)$$

$$C'(\theta)=1{,}920{,}000\frac{d}{d\theta}(sec\,\theta(7-4\sin\theta))+\frac{d}{d\theta}15{,}360{,}000$$

$$C'(\theta)=1{,}920{,}000(\frac{d}{d\theta}sec\,\theta(7-4\sin\theta)+sec\,\theta\frac{d}{d\theta}(7-4\sin\theta))+0$$

$$C'(\theta)=1{,}920{,}000(sec\,\theta\tan\theta(7-4\sin\theta)-4\sec\theta\cos\theta)$$

$$C'(\theta)=1{,}920{,}000(\frac{1}{\cos\theta}\frac{\sin\theta}{\cos\theta}(7-4\sin\theta)-4\cdot\frac{1}{\cos\theta}\cos\theta)$$

$$C'(\theta)=1{,}920{,}000(\frac{7\sin\theta}{\cos^2\theta}-\frac{4\sin^2\theta}{\cos^2\theta}-4)$$

$$C'(\theta)=1{,}920{,}000(\frac{7\sin\theta-4\sin^2\theta}{\cos^2\theta}-4)$$

$$C'(\theta)=\frac{13{,}440{,}000\sin\theta-7{,}680{,}000}{\cos^2\theta}$$

Setting C' to 0 and solving for theta, we find:

$$C'(\theta)=0$$

$$\frac{13{,}440{,}000\sin\theta-7{,}680{,}000}{\cos^2\theta}=0$$

$$13{,}440{,}000\sin\theta-7{,}680{,}000=0$$

$$13{,}440{,}000\sin\theta=7{,}680{,}000$$

$$\sin\theta=\frac{7{,}680{,}000}{13{,}440{,}000}$$

$$\sin\theta=\frac{4}{7}$$

$$\theta=\sin^{-1}\frac{4}{7}\approx 34.85^\circ$$

This tells us that we should expect a minimum or maximum value at approximately 34.85 degrees, but to distinguish whether it is a minimum or maximum without relying on only a visual examination of our graph, we test the value using the second derivative of our cost function:

$$C''(\theta)=\frac{d}{d\theta}\frac{13{,}440{,}000\sin\theta-7{,}680{,}000}{\cos^2\theta}$$

$$C''(\theta)=\frac{\cos^2\theta\cdot\frac{d}{d\theta}[13{,}440{,}000\sin\theta-7{,}680{,}000]-\frac{d}{d\theta}[\cos^2\theta]\cdot(13{,}440{,}000\sin\theta-7{,}680{,}000)}{(\cos^2\theta)^2}$$

$$C''(\theta)=\frac{13{,}440{,}000\cos^2\theta\frac{d}{d\theta}\sin\theta-\frac{d}{d\theta}[\cos^2\theta]\cdot(13{,}440{,}000\sin\theta-7{,}680{,}000)}{\cos^4\theta}$$

$$C''(\theta)=\frac{13{,}440{,}000\cos^3\theta+3{,}840{,}000\cos\theta\sin\theta(7\sin\theta-4)}{\cos^4\theta}$$

$$C''(\theta)=\frac{13{,}440{,}000\cos^2\theta+3{,}840{,}000\sin\theta(7\sin\theta-4)}{\cos^3\theta}$$

$$C''(\theta)=\frac{13{,}440{,}000\cos^2\theta+26{,}880{,}000\sin^2\theta-15{,}360{,}000\sin\theta}{\cos^3\theta}$$

$$C''(\theta)=\frac{7\cos^2\theta+14\sin^2\theta-8\sin\theta}{1{,}920{,}000\cos^3\theta}$$

With second derivative in hand, we set it to the potential minimum or maximum value we found for the zero of our first derivative, and check if the second derivative's sign is positive (indicating a minimum value) or negative (indicating a maximum).

$$C''(\sin^{-1}\frac{4}{7})=\frac{7\cos^2(\sin^{-1}\frac{4}{7})+14\sin^2(\sin^{-1}\frac{4}{7})-8\sin(\sin^{-1}\frac{4}{7})}{1{,}920{,}000\cos^3(\sin^{-1}\frac{4}{7})}$$

$$C''(\sin^{-1}\frac{4}{7})=\frac{7\cos^2(\sin^{-1}\frac{4}{7})+14(\frac{4}{7})^2-8(\frac{4}{7})}{1{,}920{,}000\cos^3(\sin^{-1}\frac{4}{7})}$$

We note that

$$\sin^{-1}\frac{4}{7}>0$$

and

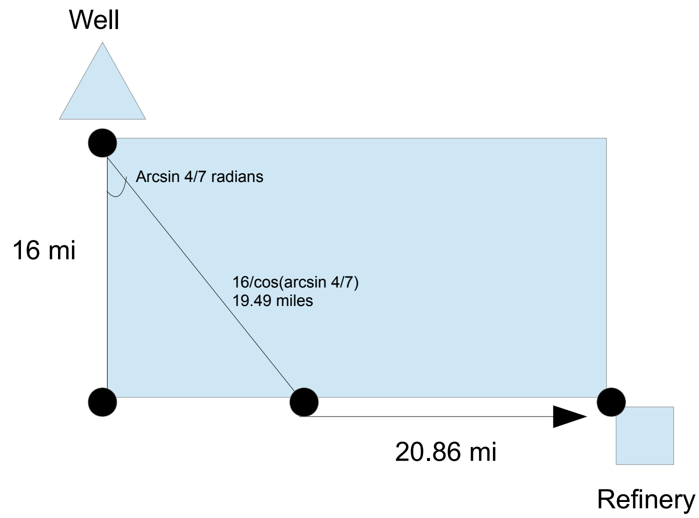
$$14(\frac{4}{7})^2>8(\frac{4}{7})$$

So

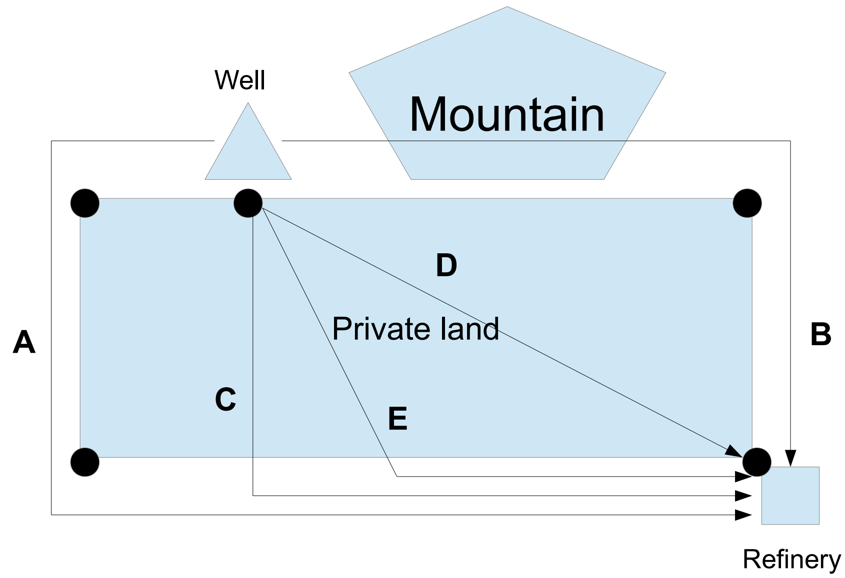
$$C''(x)>0$$

therefore $C(\sin^{-1}\frac{4}{7})$ is a minimum value.

With this information in hand, we can finally determine the lowest cost for path 5, which would be traveling across private land at an angle of $\sin^{-1}\frac{4}{7}\approx 34.85^\circ$ from due-south from the well until private land has been fully crossed $\left(16/\cos\left(\sin^{-1}\frac{4}{7}\right)\approx 19.49\text{ miles}\right)$, and then eastward toward the refinery for the rest of the way $\left(32-16\tan\left(\sin^{-1}\frac{4}{7}\right)\approx 20.86\text{ miles}\right)$. The cost of this path would be \$26,389,560.28 in total.



To summarize, there are 5 proposed paths, labeled A, B, C, D, E on the diagram below. Their total distances and costs follow.



A) 8 miles west, 16 miles south, 40 miles east, 64 miles total, all through BLM land. Total cost of path: $30,720,000.
B) 32 miles east, 16 miles south, 48 miles total, all through BLM land. $23,040,000 pipe-laying cost. Additional costs: $4,500,000 to drill through the mountain. $600,000 environmental impact study. $100,000 per month for each month delayed, with an estimated 8 month total delay for the study. Total additional costs: $5,900,000. Total cost of path: $28,940,000.
C) 16 miles south, 32 miles east, 48 miles total. Southward is through private land, incuring additional $360,000 per mile for right-of-way fees. $23,040,000 pipe-laying cost, $5,760,000 right-of-way fees. Total cost of path: $28,800,000.
D) $16\sqrt{5}$ (~35.78) miles directly toward refinery, all through private land. $17,173,002.07 pipe-laying costs, and $12,879,751.55 in right-of-way fees, for a total cost of $30,052,753.62.
E) ~19.49 miles south of the well at a ~34.85 degree angle, through private land, followed by ~20.86 miles east toward the refinery through BLM land. Total distance of approximately 40.35 miles. Pipe-laying costs are $19,370,741.05, and right-of-way fees are $7,018,819.23. Total cost of path: $26,389,560.28. This value is rounded to the nearest cent.

Path	Total Distance	Total Cost	Cost Rank (1 = Cheapest)
A	64 miles	$30,720,000	5
B	48 miles	$28,940,000	3
C	48 miles	$28,800,000	2
D	~35.78 miles	$30,052,753.62	4
E	~40.35 miles	$26,389,560.28	1

As can be seen from the summary above, path E (angled travel through private land) is the cheapest available route among the proposed options.

Finally, I am to include a reflection on the value of calculus, and the things I've learned this semester.

I've learned many ways to find derivatives, and several useful things that can be done with these derivatives. Examples include finding minimum and maximum values of functions (as shown above with finding the cost function), optimizing for single conditions (a use of finding maximums and minimums, really), figuring out if functions are continuous, differentiable, or contain some form of discontinuity or pivot point. I've learned a few different means of approximations, such as approximating the value of a function (using linear approximation), or approximating the root of a function (using Newton's method). I have also learned a great deal about graphing functions, and how to find many characteristics of a function using calculus (such as where it is increasing or decreasing with its first derivative, or which direction its concavity lies at various points with its second derivative), and how to do the reverse in some cases (taking a graph and discovering things about its equation).

All of these things are vaguely abstract, but have useful practical value in the real world. Optimization problems with minimums and maximums permit finding the cheapest way to package a widget, or the most cost effective path among many, or secure the largest possible container with a given surface area, all of which are useful in navigation and manufacturing. What's more, calculus is integral to more advanced fields, such as aerodynamics, physics, statistics, probability theory, and more. Anywhere where there's a thing that changes, and there's an interest in the details of that change, calculus has a place, be that planning how to make a spacecraft reach orbit, or just trying to spend the least amount of money on materials for a new kitchen table.

Specific to my own interests, calculus has proven useful for building my own software projects. Rather than massive amounts of hand tweaking variables, I can now find maximums or minimums of speeds, velocities, and accelerations to prevent objects from violating the physics model I implement. I can easily find if a particular model of difficulty increase over time is too extreme, or too sedate, by using derivatives. I can simplify complicated equations that a computer could do instantly into something I can do by hand with a known error margin (handy for working ideas out on paper, or when the software is meant to integrate with something outside of the computer like a pen-and-paper game, where computer-assisted computation isn't guaranteed).

I am excited to increase my knowledge of calculus, and broaden its available uses in my life.