ComSS 331 **Fall 2017** **Name:**____

HW 6 Due: 06 oct 2017

1. Define a context-free grammar for the language $L = \{0^n 1^m 0^m 1^n : n, m \in \mathbb{N}\}$. 30

2. Define a context-free grammar for the language $L = \{a^n b^m : n \leq 3m\}$. 30

3. The truth value of a logical expression is defined recursively as:

   - The truth value of t is t.
   - The truth value of f is f.
   - The truth value of $(x_1 \wedge x_2)$ is t if both x_1 and x_2 have truth value t, it is f otherwise.
   - The truth value of $(x_1 \vee x_2)$ is f if both x_1 and x_2 have truth value f, it is t otherwise.
   - The truth value of $\neg(x)$ is f if x has truth value t, it is t otherwise.

   Define a CFG that generates the following language over $\{t, f, \wedge, \vee, \neg, (,), =\}$:

   $$L = \{w{=}x : w \text{ is a logical expression over } \{t, f\},\ x \in \{t, f\}, \text{ and } x \text{ is the truth value of } w\}$$

   Thus, "$t = t$", "$((t \wedge f) \vee f) = f$", and "$\neg(((t \wedge f) \vee f)) = t$" are in L, but "$((t \wedge f) \vee f) = t$" and "$(t \wedge f) \vee f = f$" are not: the former because $((t \wedge f) \vee f)$ is false and not true, the latter because the expression lacks the outermost set of parentheses. 100