Problem 1.

(10 points) Find the least squares polynomial of degree 2 for the data in the following table. Compute the error E. Graph the data and the polynomial.

x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.84	1.96	2.21	2.45	2.94	3.18

Solution

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.1 & 1.1^2 \\ 1 & 1.3 & 1.3^2 \\ 1 & 1.5 & 1.5^2 \\ 1 & 1.9 & 1.9^2 \\ 1 & 2.1 & 2.1^2 \end{bmatrix} b = \begin{bmatrix} 1.84 \\ 1.96 \\ 2.21 \\ 2.45 \\ 2.94 \\ 3.18 \end{bmatrix}$$

$$A^T Aa = A^T b \Rightarrow (A^T A)^{-1}(A^T A)a = (A^T A)^{-1} A^T b \Rightarrow a = (A^T A)^{-1} A^T b$$

$$a = \begin{bmatrix} 66.841 & -90.719 & 28.748 \\ -90.719 & 124.544 & -39.811 \\ 28.748 & -39.811 & 12.832 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1.1 & 1.3 \\ 1.5 & 1.9 & 2.1 \\ 1 & 1.21 & 1.69 \\ 2.25 & 3.61 & 4.41 \end{bmatrix} \begin{bmatrix} 1.84 \\ 1.96 \\ 2.21 \\ 2.45 \\ 2.94 \\ 3.18 \end{bmatrix}$$

$$= \begin{bmatrix} 4.86939 & 1.83451 & -2.51039 & -4.55547 & -1.74620 & 3.10816 \\ -5.98639 & -1.89239 & 3.90693 & 6.52134 & 2.19542 & -4.74490 \\ 1.76871 & 0.48237 & -1.32035 & -2.09647 & -0.56895 & 1.773469 \end{bmatrix} \begin{bmatrix} 1.84 \\ 1.96 \\ 2.21 \\ 2.45 \\ 2.94 \\ 3.18 \end{bmatrix}$$

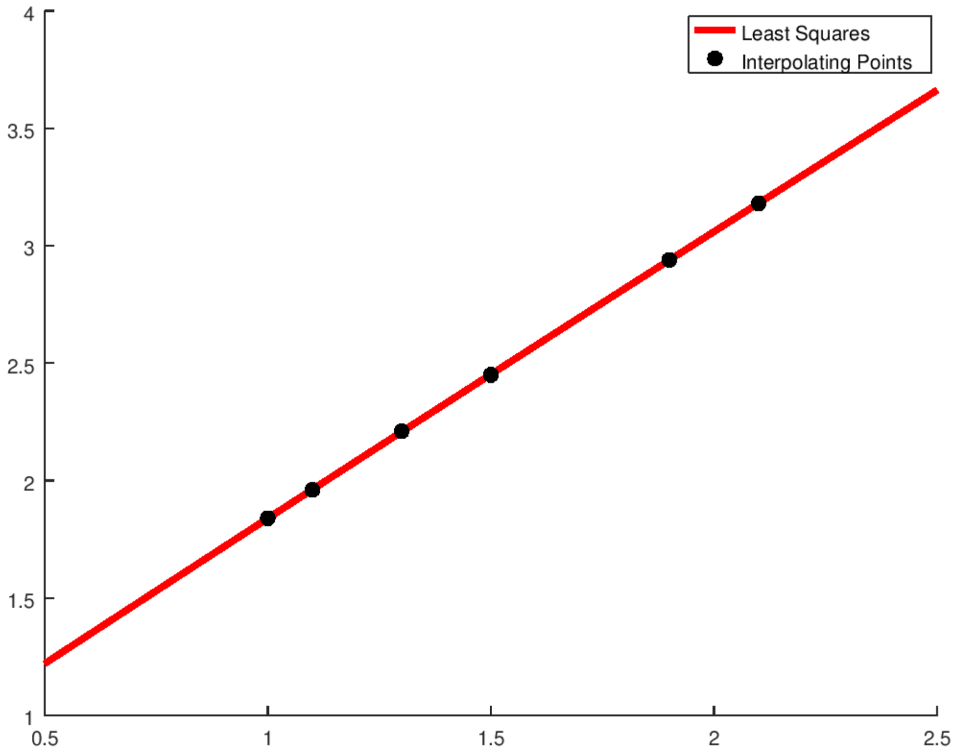
$$= \begin{bmatrix} 0.596581 \\ 1.253293 \\ -0.010853 \end{bmatrix}$$

This gives us the interpolating polynomial as $P_2(x) = -0.010853x^2 + 1.253293x + 0.596581$. Evaluating our initial points using our polynomial produces the results...

x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.84	1.96	2.21	2.45	2.94	3.18
$P(x_i)$	1.839	1.9621	2.2075	2.4521	2.9387	3.1806
$\|P(x_i) - y_i\|$	-9.79e-4	2.0712e-3	2.4797e-3	2.1012e-3	1.3416e-3	6.3457e-4

Taking the sum of the differences finds the total error across all interpolating points as $E = 6.69 \times 10^{-6}$.

We can see that the least squares fit finds a polynomial which is almost linear, but not quite. From both the graph below, and the error above, the polynomial is clearly doing a good job (though not perfect) of fitting to the points.



Problem 2.

(20 points)

(a) Find the least squares polynomial of degree 2 for the following function $f(x)$ on the indicated interval

$$f(x) = e^x, \ [-1, 1]$$

Compute the error E. Graph the function and the polynomial.

(b) Repeat (a) with Legendre Polynomials.

Solution

TODO

Problem 3.

(20 points)

(a) Use the zeros of $\overline{T}_4$ to construct an interpolating polynomial of degree 3 for the following function on the interval $[-1, 1]$:

$$f(x) = e^x$$

Find a bound for the maximum error of the approximation.

(b) Repeat (a) on interval $[0, 2]$.

Solution

TODO

Problem 4.

(20 points) Find all the Chebyshev rational approximations of degree 2 for $f(x) = e^{-x}$. Graph the function and the polynomial. (Use MATLAB routines on classpage)

Solution

TODO

Problem 5.

(30 points) (Use MATLAB routines on classpage)

(a) Find the continous least squares trigonometric polynomial $S_3(x)$ for $f(x) = e^x$ on $[-\pi, \pi]$.

(b) Find the discrete least squares trigonometric polynomials $S_n(x)$ for $f(x) = e^x$ on $[-\pi, \pi]$ with $n = 3, m = 6$.

(c) Find the trigonometric interpolating polynomial $S_n(x)$ for $f(x) = e^x$ on $[-\pi, \pi]$ with $n = 8$.

Solution

TODO