MATH 373 – Homework #1

Last Name: _____ First Name: _____

UID: _____ Siganuture: _____

For computer programming problems, copy of computer codes and copy of computation results must be submitted. If a method is used for different problems, you only need to submit one copy of this method.

1. (10 points)
   (a) Show that $x - (\ln x)^x = 0$ has at least one solution on $[4,\ 5]$;
   (b) Use the Intermediate Value Theorem and Rolle's Theorem to show that the graph of $f(x) = x^3 + 2x + k$ crosses x-axis exactly once, regardless of the value of the constant k.

2. (10 points) Let $f(x) = 2x\cos(2x) - (x-2)^2$ and $x_0 = 0$.
   (a) Find the third Taylor polynomial $P_3(x)$, and use it to approximate $f(0.4)$;
   (b) Use the error formula in Taylor's Theorem to find an upper bound for the error $|f(0.4) - P_3(0.4)|$. Compute the actual error.

3. (15 points) Let $f \in C[a,\ b]$ be a function whose derivative exits on $(a,\ b)$. Suppose f is to be evaluated at x_0 in $(a,\ b)$, but instead of computing the actual value $f(x_0)$, the approximate value, $\tilde{f}(x_0)$, is the actual value of f at $x_0 + \epsilon$, that is $\tilde{f}(x_0) = f(x_0 + \epsilon)$.
   (a) Use the Mean Value Theorem to estimate the absolute error $|f(x_0) - \tilde{f}(x_0)|$ and the relative error $|f(x_0) - \tilde{f}(x_0)|/|f(x_0)|$, assuming $f(x_0) \neq 0$.
   (b) If $\epsilon = 5 \times 10^{-6}$ and $x_0 = 1$, find bounds for the absolute and relative errors for
   (i) $f(x) = e^x$;
   (ii) $f(x) = \sin x$.

4. (20 points) The equation $4x^2 - e^x - e^{-x} = 0$ has four solutions $\pm x_1$ and $\pm x_2$. Use Newton's method to approximate the solution to within 10^{-5} with the following values of p_0: (a) $p_0 = -10$, (b) $p_0 = -5$, (c) $p_0 = -3$, (d) $p_0 = -1$, (e) $p_0 = 0$, (f) $p_0 = 1$, (g) $p_0 = 3$, (h) $p_0 = 5$, (i) $p_0 = 10$. Comment on the results.

5. (15 points) Use Newton's method and Modified Newton's method to find the solution accurate to within 10^{-5} for

$$1 - 4x\cos x + 2x^2 + \cos 2x = 0, \quad \text{for } 0 \leq x \leq 1.$$

   Comment on the performance of the methods.

6. (20 points) Use each of the following methods to find a solution in $[0.1,\ 1]$ accurate to within 10^{-4} for

$$600x^4 - 550x^3 + 200x^2 - 20x - 1 = 0.$$

(a) Bisection method; (b) Newton's method; (c) Secant method; and (d) Müller's method. Comment on the performance of these methods.

7. (10 points) An object falling vertically through the air is subjected to viscous resistance as well as to the force of gravity. Assume that an object with mass m is dropped from a height s_0 and that the height of the object after t seconds is

$$s(t) = s_0 - \frac{mg}{k}t + \frac{m^2g}{k^2}(1 - e^{-kt/m}),$$

where $g = 32.17$ ft/s^2 and k represents the coefficient of air resistance in lb-s/ft. Suppose $s_0 = 300$ft, $m = 0.25$lb, and $k = 0.1$ lb-s/ft. Find, to within 0.01 s the time it takes this quarter-pounder to hit the ground. Use Fixed-point iteration, Steffensen's method and Newton's method to find the solution.