# Problem 1.

(30 points)

(a) Use the forward-difference and backward-difference formulas to determine each missing entry in the following table with $f(x) = \sin x$, compute the errors and find error bounds using error formulas.

(b) Choose your favorite function $f$, nonzero number $x$. Generate approximations of $f_n'(x)$ to $f'(x)$ by

$$f_n'(x) = \frac{f(x + 10^{-n}) - f(x)}{10^{-n}}$$

for $n = 1, 2, \dots, 10$ and describe what happens.

**Solution**

**Part (a)**

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 0.5 | 0.4794 | |
| 0.6 | 0.5646 | |
| 0.7 | 0.6442 | |

**Part (b)**

## Problem 2.

(60 points)

(a) Approximate the following integral using the Trapezoidal rule, find a bound for the error using error formula and compare this to the actual error:

$$\int_{0.5}^{1} x^4 dx.$$

(b) Repeat part (a) using Simpson's rule.

(c) Repeat part (a) using Composite Trapezoidal rule with $n = 4$.

(d) Repeat part (a) using Composite Simpson rule with $n = 4$.

(e) Write a code to implement part (c) and (d) in MATLAB.

(f) Write a function $v = \text{CompositeTrapezoidalRule}(f, a, b, n)$ to implement Composite trapezoidal rule with a given $n$ for

$$\int_{a}^{b} f(x)dx,$$

verify your code with part (c).

**Solution**

## Problem 3.

(20 points)

(a) Show that the following quadrature formula has a degree of precision equal to 3,

$$\int_{-1}^{1} f(x)dx = f(-\frac{\sqrt{3}}{3}) + f(\frac{\sqrt{3}}{3}).$$

(b) Approximate the following integral using Gaussian quadrature with $n = 2, 3, 5$,

$$\int_{1}^{1.5} x^2 \ln x dx$$

**Solution**

## Problem 4.

(20 points)

(a) Use Composite Simpson rule with $n = 4$ to approximate the following integral

$$\int_0^1 x^{-1/4} \sin x dx$$

(b) Use Composite Simpson rule with $n = 6$ to approximate the following integral

$$\int_1^\infty \frac{\cos x}{x^3} dx$$

**Solution**

## Problem 5.

(30 points)

(a) Use the Gaussian Elimination with Backward substitution to solve the following linear system (must show intermediate steps).

$$\begin{aligned} 2x_1 - 1.5x_2 + 3x_3 &= 1 \\ -x_1 \qquad\quad + 2x_3 &= 3 \\ 4x_1 - 4.5x_2 + 5x_3 &= 1 \end{aligned}$$

(b) Repeat (a) using the Gaussian Elimination with partial pivoting and Backward substitution (must show intermediate steps).

(c) Repeat (a) using Gaussian Elimination with scaled partial pivoting and Backward substitution (must show intermediate steps).

**Solution**

## Problem 6.

(20 points)

(a) Solve the following linear system with forward and backward substitution,

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

(b) Use the LU factorization (no permutation) with forward and backward substitution to solve the following linear system (Use the sample MATLAB codes posted on classpage as discussed in class),

$$\begin{aligned} 2x_1 + 3x_2 \ -x_3 &= 2 \\ 4x_1 + 4x_2 \ -x_3 &= -1 \\ -2x_1 - 3x_2 + 4x_3 &= 1 \end{aligned}$$

**Solution**

# Problem 7.

(20 points)

(a) Use the $A = LDL^t$ factorization to solve the following linear system,

$$\begin{aligned} 2x_1 - x_2 \qquad &= 2 \\ -x_1 + 2x_2 \ -x_3 &= -1 \\ - x_2 \ +2x_3 &= 1 \end{aligned}$$

(b) Repeat (a) with Cholesky factorization $A = LL^t$.

**Solution**