A study into the control of a Quadcopter on one axis of movement

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Abstract

Quadcopters have proven out to be one very interesting piece of technology which can be used for many purposes and can be afforded even by student. Due to this fact, the focus of this project is to get an insight of how a Quadcopter can stabilize on the pitch axis at no inclination on that axis. The first approach in solving this problem was developing a mathematical model which could describe the dynamic behavior of the system. After this model has been determined, a transfer function of the system has been developed. The system proven out to be unstable, but the stabilization of it can be achieved in the closed-loop system with a PID controller. The fact that a PID controller is needed to achieve the purpose of this project has been determined in the open-loop analysis using Bode plots for each controller in order to check the stability in the closed-loop system. In the closed-loop analysis, the PID tuning has been realized using 3 different tuning methods: PID tune feature in Simulink, the Root Locus and Ziegler-Nichols tuning methods. The limitations of the system have been determined in closed-loop using Bode Plots in order to see what the gain and phase margins are. The PID has never been implemented in the actual prototype due to time constraints, so the PID and thus the validity of the mathematical model developed were not verified experimentally.

Chapter 1: Introduction

Quadcopters, or Quadrotors are simply 4 rotor helicopters, the rotors being “directed upwards and they are placed in a square formation with equal distance from the center of mass of the quadcopter. The quadcopter is controlled by adjusting the angular velocities of the rotors which are spun by electric motors.” [1]

The idea of a Quadcopter date since the early 20’s[2] and have constantly been developed in order to become a very interesting piece of technology that this century has embraced.

Nowadays, they are used in a very big range of appliances. These appliances vary from food and package delivery to surveillance and medicine delivery.[3] It is yet to see what this piece of technology can be used for, nowadays people finding more creative ways to use it such as a pace keeper for joggers and for racing purposes.

Although this technology has its advantages, the debate on the advancement of Quadcopters/drones is more and more tense as it can be seen as either a mean on making life easier and bring commodity and safety for people who work in dangerous environments or are facing with critical situations, or it can be seen as a threat as more and more people tend to mock other people with this technology by either harassing them in their private property or used in order to create disastrous situations as it was the case with the drone strikes in the Middle East, as seen on various news channels.

Figure 1-Quadcopter[1]

In Figure 1 a picture of a Quadcopter is presented. By looking at the picture, it can be seen that this UAV includes quite a few components. Some of them can even be manufactured at home without the need of buying them, for the enthusiast who wishes to have a more DIY approach of building a Quadrotor. These components the Quadcopter is comprised of are the frame, battery, propellers, electric motors, electronic speed controller and last but not least, the flight controller.

The components mentioned above are the basic building blocks of a Quadrotor. Depending on the purpose of the Quadrotor, it can contain even a gimbal to which GoPro cameras or DSLRs can be attached in order to perform aerial photography or just for surveillance purposes. In the case in which the flight controller is built from scratch using a microcontroller, some external sensors must be incorporated to the microcontroller in order to obtain a control system. Usually modern flight controllers have incorporated some sensing device, be it a more basic sensing or advanced sensing depending on the quality of the flight controller.

The battery stores energy and usually the amount of energy stored in it is measured in mAh. Usually the batteries used for Quadcopters are Li-Po batteries (Lithium Polymer). When the user of the Quadcopter gives an input command through a RC (remote control), the signal is sent to the Quadcopter from the transmitter to the RC receiver which works as a link between the user input and the flight controller. After the input signal has been received, the flight controller processes this signal along with the readings of the sensors and outputs a signal as a PWM. PWM is a way of representing an analog value in digital form by switching the flow of current on-off for certain periods of time (duty cycle).[4] The PWM signal is then processed by the ESC (electronic speed controller) which will then supply the correct amount of current to the motors as it was required. The electrical energy will be converted to mechanical energy by the motors, which will make the propeller spin thus creating thrust. The thrust generated by the spin of the propellers will counteract the force of gravity and will enable the Quadcopter to move through air.

This piece of technology has a cost ranging from around 20 dollars (the miniature ones which weigh around 13g) to a couple of thousands of dollars the ones with more advances features and specifications.

The most fascinating thing about Quadcopters is the fact that due to this technological boom, this piece of technology has become so accessible, that even students can grasp it and work with it to acquire knowledge about what it actually takes to control a real physical system, work with real-time systems or other university related projects.

Due to this fact, the group has decided to work with Quadrotors with the overall goal to learn more about control theory and how it applies to different systems and how to implements this control design in order to find how a system behaves and how and what one can do more to influence the behavior of the chosen system (control the electro-mechanical part of the system in this case) in order for it to behave in a particular way.

Summary:

Quadcopters are 4 rotor helicopters which have a bit range of appliances in the real world. They are comprised of several parts which are the frame, battery, propellers, motors, ESC and last but not least, the flight controller. Although these are the main components a Quadcopter is comprised of, it can contain even more depending on the purpose of it. Quadcopters have a price range from a couple of dollars to thousands of dollars but for the average models, it can be considered an accessible piece of technology.

1.1: Purpose and structure of the report

The purpose of this research paper is to analyze the control of a Quadcopter on only one axis of movement, the pitch axis. The way of approaching this issue and general analysis of how this can be obtained will be discussed in the subchapter called Problem Analysis.

The main reason why the group has decided to implement a control design only for one axis of movement is due to the fact that a Quadcopter has six degrees of freedom (three rotational and three translational) case in which the system will have MIMO(multiple-input multiple-output). In the case of a MIMO system, the group would have to go deeper into modern control theory and the use of state-space model which is not among the purposes of this semester requirements. As the theme of the project is “Basic Control Engineering”, the group has considered that controlling a Quadcopter only on one axis of movement (one degree of freedom) would be a sufficient task for the group in this semester project in order to get a grasp of how to approach systems and create control designs in order to obtain the desired outcome.

Other chapters of this research paper are called Methods, in which the group will approach the problem, analyze the dynamics of the system and construct a control design. In the chapter Results, the results will be discussed. Last but not least, in the chapter called Discussions, the overall impression will be discussed, what the limitations of the control design are, what are the specifications obtained and if they match the desired specifications, if the learning goals of this project have been met and further work on Quadcopter which can be applied in next semesters projects.

Summary:

The purpose of this report is to analyze the control of a Quadcopter on the pitch axis of movement which will be overall analyzed in all the 4 chapters of this report.

1.2: Learning Goals

The learning goals of the project are an essential aspect which has to be taken into consideration when making a research paper. These learning goals for this semester project include obtaining a correct mathematical model of the system which will offer a good insight into how the dynamic system behaves, the ratio between the output and the input of the system in the Laplace domain in order to obtain a Transfer function of the system. Moreover, a linearization of the system using Taylor Series (or small angle approximation) must be also included if needed, but the linear or non-linear behavior of the system will be analyzed in later chapters.

The stability of the system will be discussed and if it is unstable/marginally stable, by what means this can be corrected in order to obtain the desired outcome. Another thing to consider while thinking about the learning goals of this project is a correct implementation of a controller in the closed-loop system, either a P controller, PD, PI or PID and an analysis of the influence and comparison between each and every controller over the system and a rationale description of why the controller chosen will be the best option in the case of the project. Specifications of the controlled system will also be included in this research paper in order to have an exact idea of what the actual outcome should be.

Summary:

A correct mathematical model must be obtained in order to appreciate the dynamic behavior of the system. If the respective mathematical model contains any non-linear terms, a linearization of the equations must be performed using Taylor expansion or small angle approximation in order to find the adjacent Transfer Function of the system. After analyzing the linear or non-linear behavior of the system, the equations describing its dynamics will be converted from the time domain into the complex Laplace domain in order to find the ratio between the output and the input (Transfer Function). After the transfer function has been obtained, the stability of the system will be analyzed by finding the poles in the denominator of the Transfer Function. After the transfer function of the system has been obtained, a correct implementation of a controller will be the next step while implementing a closed-loop control system. An analysis of why the controller chosen for the respective closed-loop control system will be discussed as well as why is the best choice with respect to this project

Chapter 1.3: Problem analysis

This chapter has the purpose of analyzing how the Quadcopter can be controlled on one axis of movement.

In the case of this project, the pitch axis has been chosen to be controlled although any of the three axes could have been chosen. The Quadcopter prototype should also stabilize at a 0 angle on the pitch axis whenever a disturbance is introduced that can cause a deviation from the desired angle. The Quadcopter should have as little overshoot as possible, a good rise time which will not be too aggressive but rather robust in order to avoid overshoots. The specifications of the control design will be discussed in another subchapter. In order to stabilize itself at a 0 degree angle on the pitch axis, all the rotational speeds of the 4 motors should be equal.

In order to implement a control design for the whole system on the control of one axis, one should define the motion of the Quadcopter and how one can achieve this control on the pitch axis. Implementation issues will also be discussed and analyzed.

The main issue with this approach is how to design a proper control system based on the readings of a sensor (the gyroscope is chosen as the sensor which can detect any deviation on the pitch axis from the desired output which is 0 degrees), and a rather smooth automatic stabilization of the Quadcopter which will require a fine tuning of the controller.

As it has been mentioned in the previous subchapter, Quadcopters have six degrees of freedom, three translational and the other three are rotational. This means that the Quadcopter can perform both rotational and translational (around the axis and along the axis) motion on the pitch, roll and yaw axis. The yaw and roll axis of movement must be blocked in order to be able to test and appreciate the quality of the design and response of the Quadcopter on only the pitch axis.

Quadrotors have four motor. These four motors are divided into two pairs, on pair has clockwise rotational motion and the other has counter-clockwise rotational motion. Without this opposite rotational motion from the rotors of the Quadcopter, it could not fly smoothly because the torque induced by the motors would make it spin around its own body axis without having any control over it. Thus, Quadcopters use torque system by oscillating the rotations of the motors in order to compensate for the torque and eliminate its effect.

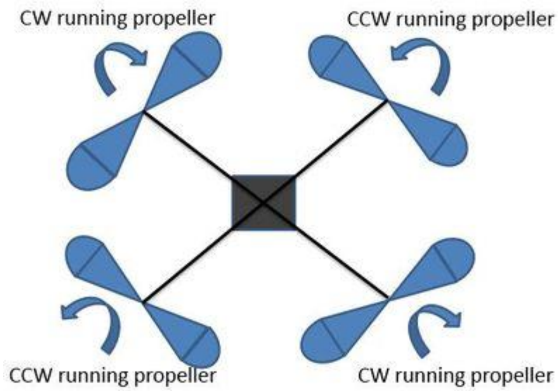

Figure 2-Differential torque system of a Quadcopter[2]

In Figure 2, the differential torque system of a Quadcopter is presented. The two diagonal pairs of motors have inverse rotational motion (clockwise and counter-clockwise) in order to cancel out the torque induced by the motors. In the actual construction of the Quadcopter, this fact must be taken into account.

In order for a Quadcopter to perform a motion on the on the pitch axis, from front motors should have a smaller rotational motion, thus thrust, than the rear pair of motors or vice versa. The same motor speed manipulation can be used in order to control the movement of the Quadcopter on other axis of movement as well, but these will not be discussed as it is not the focus of the project.

Controlling a Quadcopter on only one axis of movement outdoors would be a horrific task if not impossible, so a special stand should be designed in order to block the movement of the Quadcopter on the other two axes, so that testing sessions could be held. With a specially designed stand, the possibility of damaging the Quadcopter or injuring people is diminished, but accidents can still happen if one is not cautious enough. The stand will be built from solid wood which will be strong enough to withstand eventual situations where the control of the Quadcopter will be lost while on the stand and react erratically (in the testing session).

In order to control a Quadcopter on one axis of movement, a control design must be initially made in order to predict the behavior of the system. As a first step in order to solve this issue, a mathematical model of the system must be made in order to obtain the adjacent Transfer Function. This transfer function will give an insight about the relationship between the input and the output of the system (difference in thrust of the two pairs of motors-angle change on the pitch axis) and moreover, by analyzing the characteristic equation in the denominator of the transfer function, the group will be able to determine the stability of the system which is a crucial step with regard to decision making with respect to the actual implementation of a controller within the closed-loop control system. The mathematical model must be analyzed on the motion of the Quadcopter on one axis of movement, the pitch axis, excluding the movement on the yaw and roll axis. To achieve a good mathematical model, classic mechanics as well as concepts of aerodynamics must be applied in order to analyze the forces acting on the body of the Quadcopter while in motion. This analysis has the sole purpose of predicting the motion of the Quadcopter on the pitch axis.

The logic behind controlling this actual Quadcopter on one axis is to always feedback the angle difference between the desired angle and the actual angle and thus obtaining an error. When this error will occur, if the angle is pointing in the negative part of the pitch axis, then the microcontroller should send a certain PWM to two of the motors (front pair) in order to increase their angular velocity and thus thrust, and for the other two motors (rear pair) a certain PWM to decrease the angular velocity and thrust in order to correct the error. When the angle error is in the positive part of the pitch axis, then the process is vice-versa, in the front pair of motors should be given less power and in the rear pair of motors should be given more. This transition from the angle error to the desired angle should be made as smooth as possible in order to avoid any big overshoots. There is a trade-off between rise time and overshoots, a faster rise time due to hardware limitations would almost certainly cause a substantial overshoot of the Quadcopter on the stand, while a much smoother rise time would make a more cleaner transition towards the desired state of the system.

After a control design has been created, several simulations must be run in order to see the behavior of the system. The actual implementation will take place in the Arduino Uno microcontroller, which is the microcontroller chosen to be used for this prototype. Arduino Uno is an inexpensive microcontroller and due to the fact that it was introduced in one of the courses of the 4^{th} semester, the group has decided to use it and apply some knowledge from that specific course if possible (with respect to the software design).Other aspects of this specific microcontroller that convinced the group to use it are the facts that it has enough computational power and memory to withstand the requirements of this project and it was previously used in project exams so the group has some knowledge of how to use it.

In the implementation stage of the project, the group is conscious that some slight or big differences may occur from the control design response and the actual system; these differences might need a slight or big correction. The implementation and testing of the prototype will be a quite lengthy process as a certain quality of the prototype is expected so in this stage, a thorough analysis of the behavior of the actual system will be taken into account in order to obtain the desired quality level.

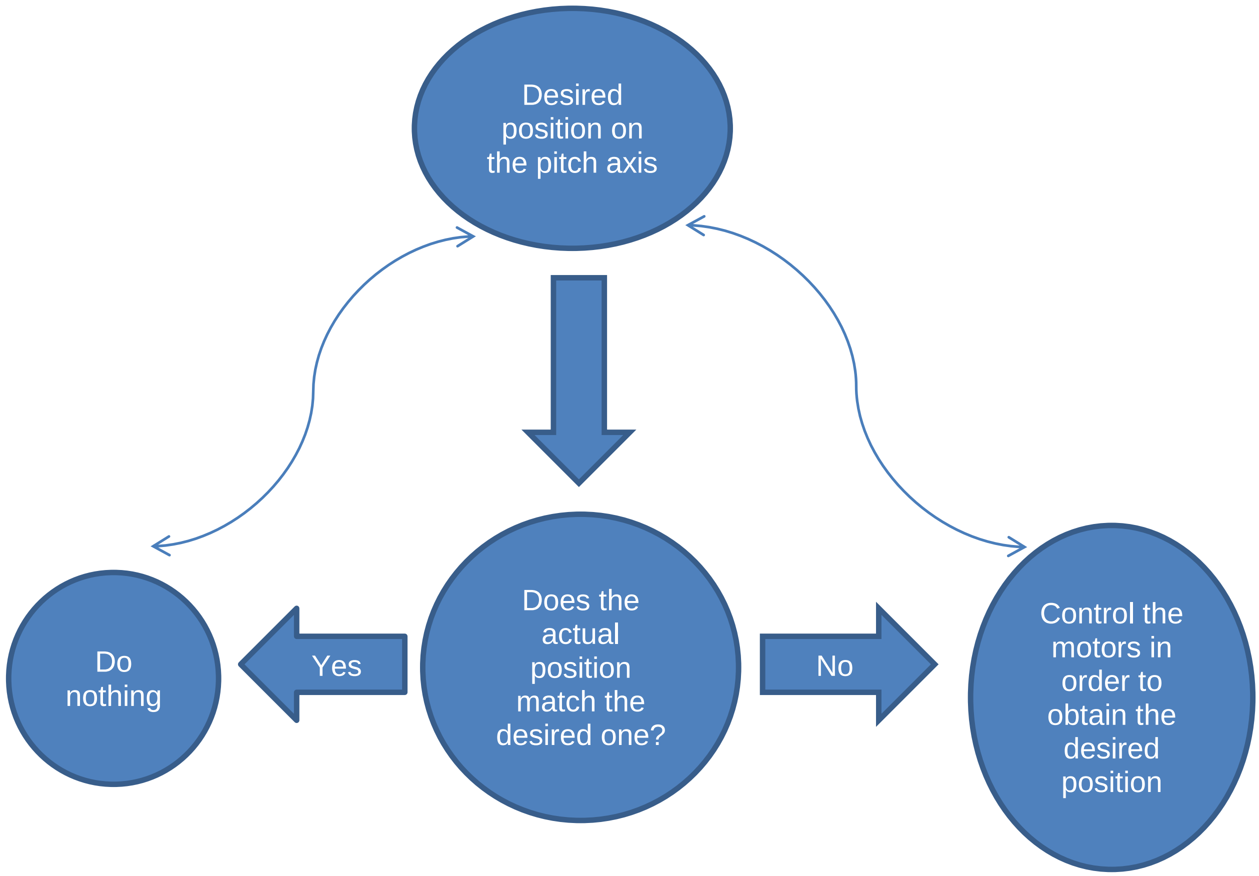

Figure 3-Flowchart representing the expected behavior of the system

The flowchart represented in Figure 3 represents the expected behavior of the Quadcopter prototype.

Summary:

If a disturbance input in introduced in the system, the Quadcopter should always stabilize itself on the pitch axis at a certain angle (0 degrees on the pitch axis) based on the reading of the gyroscope. In order to compensate for the torque induced by the motors, Quadcopters use a differential torque system, this fact being taken into account in the actual prototype. To control the motion of the Quadcopter on the pitch axis, the front and rear pairs of motors must be controller in order to obtain an angular velocity difference. This angular velocity difference will create different thrust in the two motors enabling them to move on the pitch axis. In order to be able to test the prototype and block the other two axis of movement, a special wooden frame will be built. A more robust control of the Quadcopter is attempted to have a cleaner response of the system instead of an aggressive one.

Chapter 1.4: Prototype parts specifications

Parts	Product Specifications	Mass
Turnigy Nano-tech battery	-6000mAh capacity -25C(continuous) and 50C(burst) discharge rate -11.1V nominal voltage -3 cells, 3.7V/cell	0.481kg
Z700-V2 Quadcopter Frame	-glass fiber frame with polyamide nylon arms -attachable landing gear	0.570kg
Arduino Uno	-5V operating voltage -14 Digital I/O pins -6 Analog pins	0.025kg
Turnigy Multistar Brushless motor	-800Kv(800 RPM/V) -0.169Ω internal resistance	0.066kg
Turnigy Multistar Brushless ESC	-20A constant current -has no BEC(battery elimination circuit)	0.025kg
MPU 6050 3-axis Gyroscope & 3-axis Accelerometer		0.006kg
12x4.5'' Propellers	~30cm length ~11.4cm pitch inclination	0.02kg each propeller ~0.1kg for 4 propellers
Power supply board	-custom made -8X3.5 female bullet connectors	~0.07kg
Time spent on building prototype: 2 days	**Using a drive calculator to determine the parameters of the motor, it has been determined that the motor used can generate 1801g thrust (17.66N)**	**Total mass:1.616kg**

All the specifications mentioned above are taken from the website from where the parts were bought, Hobbyking.

Analyzing the total mass of the Quadcopter can give an insight of how much thrust the actual prototype needs in order to be able to lift. If the mass of the actual prototype is roughly 1.6kg, this means that the Quadcopter should develop around 400g (~3.92N) thrust for each motor in order to be able to lift it. Although the unit for thrust is Newton, the unit gram force is also used because it gives a clearer hint about the thrust needed in order to lift the Quadcopter. Using a drive calculator, in which one specifies what components are used, will generate the corresponding motor parameters and performance based on the chosen components. Using this drive calculator and specifying all the components used, it has been determined that one motor can generate a static thrust of 1801g (17.66N) which is more than enough to lift the Quadcopter.

A practical experiment in which the group has tried to measure the thrust of one motor using a Newton-meter has been performed. At about 6500 RPM, the motor was generating 5N thrust. Due to the fact that this experiment was dangerous, the group stopped trying to measure the thrust generated by the motor at its maximum rotational speed (8880 RPM). Furthermore, no graphs have been generated due to the fact that it was really difficult to conduct the full experiment and gather data in the same time. The experiment has not been completed, and the final result of the experiment is hard to predict, but as far as the experiment went through, it is not likely that one motor at a maximum rotational speed of 8800RPM can generate 17.66N (if one of the motors develops 5N thrust at about 6500RPM).

Figure 4-Experimental determination of thrust

Chapter 2: Methods

This chapter has the purpose of analyzing the behavior of the system using a mathematical model, a control design will be also proposed in this chapter and the specific responses of the control design will be discussed and analyzed. After the control design has been created and the response has been analyzed, the implementation stage is the next step in the prototype phase. In the final subchapter of this chapter, a description about the differences obtained from the responses of the closed-loop system and the actual model will be discussed in order to assess the quality of the design made.

Chapter 2.1: Mathematical model

The mathematical model of a system is important in control theory as it gives an insight into the behavior of the dynamic system.

Using the adjacent mathematical model, Laplace transform will be used in order to convert the model from the time domain into the complex Laplace domain in order to obtain a transfer function. This transfer function will give the group an idea of the stability of the system by analyzing the characteristic equation in the denominator and finding the roots of this characteristic equation, also called poles.

Before defining the equations describing the dynamics of the motors, the motor parameters and units will be presented.[5]

Notation	Motor parameters	Units
J	Moment of inertia	Kg·m^2
B	Damping ratio of the mechanical system	Nms
I	Current	A
T	Torque	Nm
R	Resistance	Ω
L	Inductance	H
Ke	Electromotive force constant	V/rad/S
Kt	Motor Torque constant	Nm/A

Motor parameters	Measurements/Datasheet
R	0,169Ω
L	28,64µH
Kt	0,12Nm/A
J	0,404µkgm^2
Ke	0,00126V/rad/s
B	0,00121Nms

In order to be able to analyze the response of the motor in order to appreciate its behavior, the motor parameters must be determined. As the datasheet for the motors bought does not provide any information about the motor constants, these will be acquired from the datasheet of a motor with similar specifications in order to get an approximate response of the motor.[6] Some of the motor characteristics have been measured in the laboratory such as winding resistance and inductance using an LCR meter.

Most of the specifications of the motor are taken from a similar motor[7] (only the resistance and inductance of the motor were measured in the laboratory using an LCR meter) so the analysis of the response of the motor used for the Quadcopter will be an approximate one.

The first model presented will be the model of the actuator, the DC brushless motor. The DC brushless motor has two ODEs that describe the dynamic behavior of it. The first ODE represents the electrical part of the motor whereas the second one represents the actual motion and forces generated through the motion of the motor.

The differential equations describing the motor are [8]:

$L_a \frac{di}{dt} + R_a I_a = v_a - K_e \dot{\theta}_m$ (1)

$J_m \ddot{\theta}_m + b\dot{\theta}m = K_t I_a$ (2)

Where $\omega(t) = \dot{\theta}(t)$

Where the connections between electrical/mechanical parts are[9]:

$T = K_t I_a$ (3)

$e = K_e \dot{\theta}_m$ (4)

In order to obtain equations (1) and (2), an analysis of the circuit of the motor using Kirchhoff Voltage law and a free body analysis of the rotor using Newton's law is necessary[10].

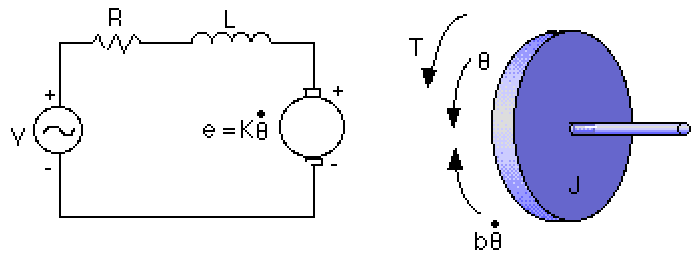

Figure 5-Electrical circuit of the armature & free body diagram of rotor[3]

In order to obtain the transfer function of the motor, the Laplace transform must be applied to the equations describing the dynamics of the system in order to convert them from the time domain into the complex Laplace domain. As the equations describing the motors dynamic behavior do not present any non-linear terms, linearization of the equations is not required.

The Laplace transform of the equations for a DC motor is:

$L\{L_a \frac{di}{dt}\} + L\{R_a I_a\} = L\{v_{a\}}$ –L{ e} (electrical part of a motor)

$L\{J_m \dot{\omega}$(t) + L{b ω(t)} =L{ T} (mechanical part of a motor)

$I(s)(\ sL_a + R_a)$ = V(s) – e(s)

$\omega(s)(sJ_m +$ b) = T(s)

$$\omega(s) = \frac{T(s)}{(\ sJ_m + \ b)}$$

I(s) = $\frac{V(s) - e(s)}{(\ sL_a + R_a)}$

$I(s) = \frac{V(a) - K_e\,\omega(s)}{(sL_a + R_a)}$

$\omega(s) = \frac{K_t\ i(s)}{(sJ_m + b)}$

$I(s)=\frac{V(a)-K_e\,\omega(s)}{(sL_a+R_a)}$

$I(s)=\frac{\omega(s)(sJ_m+b)}{K_t}$

$\frac{V(a)-K_e\,\omega(s)}{(sL_a+R_a)}=\frac{\omega(s)(sJ_m+b)}{K_t}$

$\omega(s)\ (sJ_m + b)(\ sL_a + R_a)= K_t\ V(a)\text{-}K_e\ \omega(s)$

$\omega(s)\ ((sJ_m + b)(\ sL_a + R_a)+ K_e\ ) = K_t\ V(a)$

$\frac{\omega(s)}{V(a)}=\frac{K_t}{(sJ_m+b)(\ sL_a+R_a)+\ K_e}$

In the end, the transfer function of the motor is:

H(s) = $\frac{\omega(s)}{V(a)} = \frac{K_t}{(sJ_m+b)(\ sL_a+R_a)+\ K_e}$ with units in $\frac{rad/s}{V}$ (5)

Replacing all the parameters of the motor with numerical values will give the following equation:

H(s) = $\frac{\omega(s)}{V(a)}=\frac{0{,}12}{(0{,}00000404s+0{,}00121)(0{,}00002864s+0{,}169)+0{,}00126}$

H(s) = $\frac{\omega(s)}{V(a)}=\frac{0{,}12}{0{,}000000000115s^2+0.0000000682s+0{,}0000000346s+0{,}0002+0{,}00126}$

H(s) = $\frac{0{,}12}{0{,}000000000115s^2+0{,}0000001029s+0{,}00146}$ (6)

The obtained Transfer function is a second order transfer function which has 2 poles and no zeros. As it can be seen, both the poles of the actuator are in the LHP meaning that the motor is stable (it has a bounded output for every bounded input). One of the poles of the motor is at s_1= -0.4448 + 1.0122i and the other is at s_2= -0.4448 - 1.0122i (due to the fact that the computations resulted in small numerical coefficients of the quadratic equation, the computation of the roots has been carried out using Matlab).

As the poles of the actuator have both a real and an imaginary part, this means that the actuator will have some oscillations before reaching its steady state.

Using Matlab, the step response of the motor for a step of 1 has been generated. This step response represents the response of the motor for 1 applied volt with a steady state of 81.9 rad/s. If the conversion from radians/second is made into RPM (rotations per minute), then 81.9rad/s is approximately 782 RPM (the actuator used for the prototype has 800Kv, which means that for each applied volt it will generate a rotational motion of 800 RPM).

Due to the fact that the parameters of the motor have been taken from a motor with similar characteristics with the one used for the prototype of this semester project, the response has a certain offset of 18RPM which is caused by the fact that the exact motor parameters have not been used, but only approximate ones in order to have an understanding of the behavior of the actuator. Although there is a slight offset from its actual steady state for one volt applied, the model and response allows one to predict the behavior of the actuator for each applied volt which comes in handy to analyze the response time of the actuator, any overshoots it may have and the transient towards the steady state.

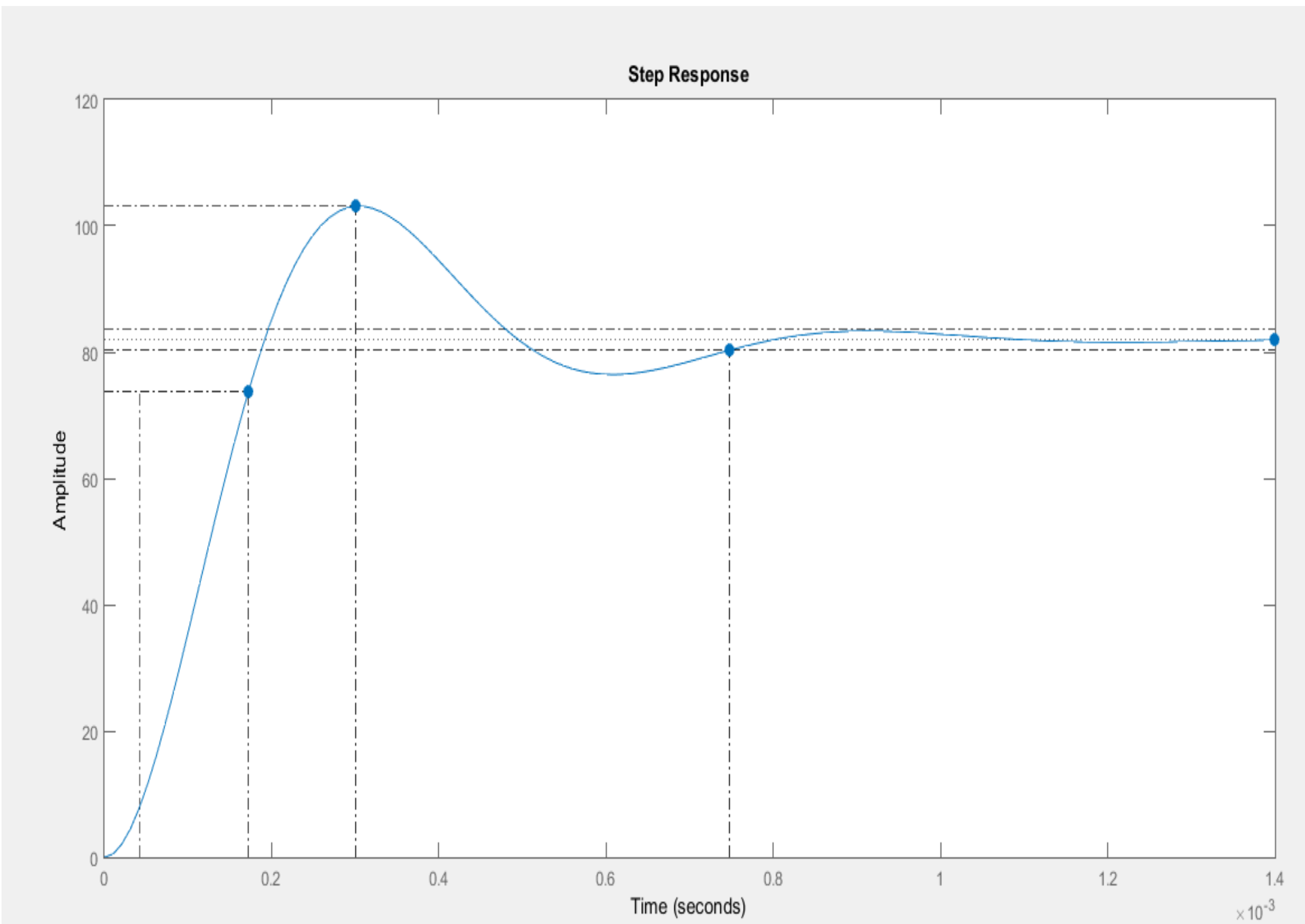

Figure 6-Step response of motor

The characteristics of the step response of the motor are the following:

- Rise time of 0.00013 seconds

- Overshoot of 25.8% and peak amplitude of 103rad/s

- Settling time of 0.00074 seconds

- Steady state of 81.9 rad/s (approximately 782 RPM)

Summary of the chapter until now:

Modeling the DC motor used for the prototype gives an insight of the possible response of the actuator. The motor parameters have been taken from the datasheet of a motor with similar specifications as no datasheet was available for the Turnigy 800Kv motor. Due to the fact that there might be some slight differences between the actual motor parameters and the ones used, there are some inconsistencies in the response of the motor as for each volt applied it has a steady state of 81.9 rad/s (782RPM), whereas the motor used has 800Kv (it has a rotational speed of 800RPM for each applied volt). The transfer function is a second order transfer function; it has two poles and no zeros. The poles are negatively complex valued (they have both a real and imaginary part meaning that the poles are in the LHP and the actuator will exhibit some oscillations before it will stabilize at its steady state). The rise time and settling time happens at an extremely fast rate, which means that the motor has a fast response.

Chapter 2.2: System model

The equations of the motor will be used in order to obtain the transfer of the system, which will be the ratio between the angle change on the pitch axis over difference in thrust for the two pair of motors. But in order to obtain the transfer function, the equations of motion of the system must be obtained in order to analyze the motion of the Quadcopter on the pitch axis. If the equations derived describe a non-linear behavior, then the Taylor expansion formula will be used in order to linearize these equations.

The equations of motion of an airplane are composed of translational and rotational equations and are called the 6 degrees of freedom (6DOF).[11]

Due to the fact that the purpose of this project is control a Quadcopter on only one axis of movement, the components describing the motion of the Quadcopter on the yaw and roll axis will be neglected. Only the rotational equation of the Quadcopter on the pitch axis will be analyzed, meaning that the equation will describe only one degree of freedom (1DOF).

An airplane moves relative to two frames of reference: with respect to the Earth's frame of reference and with respect to its own.

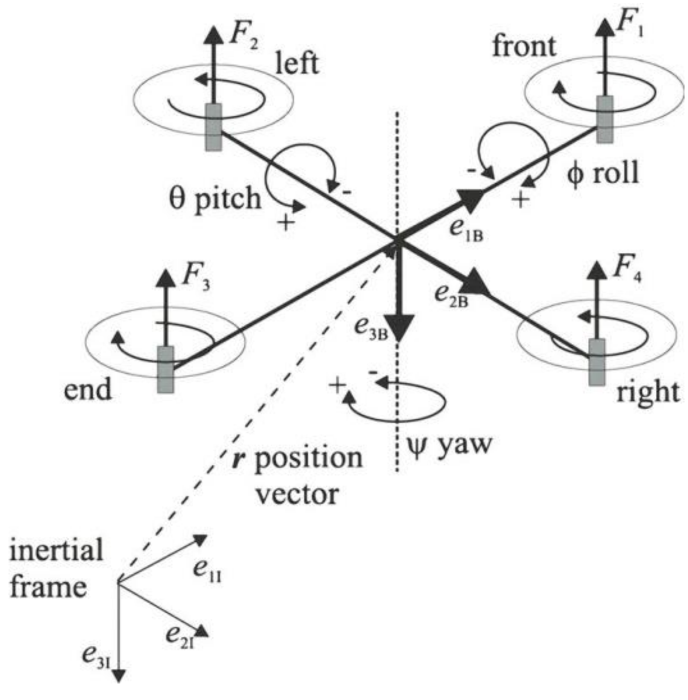

Figure 7-Figure showing the 2 frames of reference[4]

The motion with respect to the Earth's reference frame will be neglected as the Quadcopter will be fixed on a wooden frame which will prohibit its motion in open space on other axis of movement, so only the rigid-body frame will be taken into account and the motion of the prototype on the pitch axis.

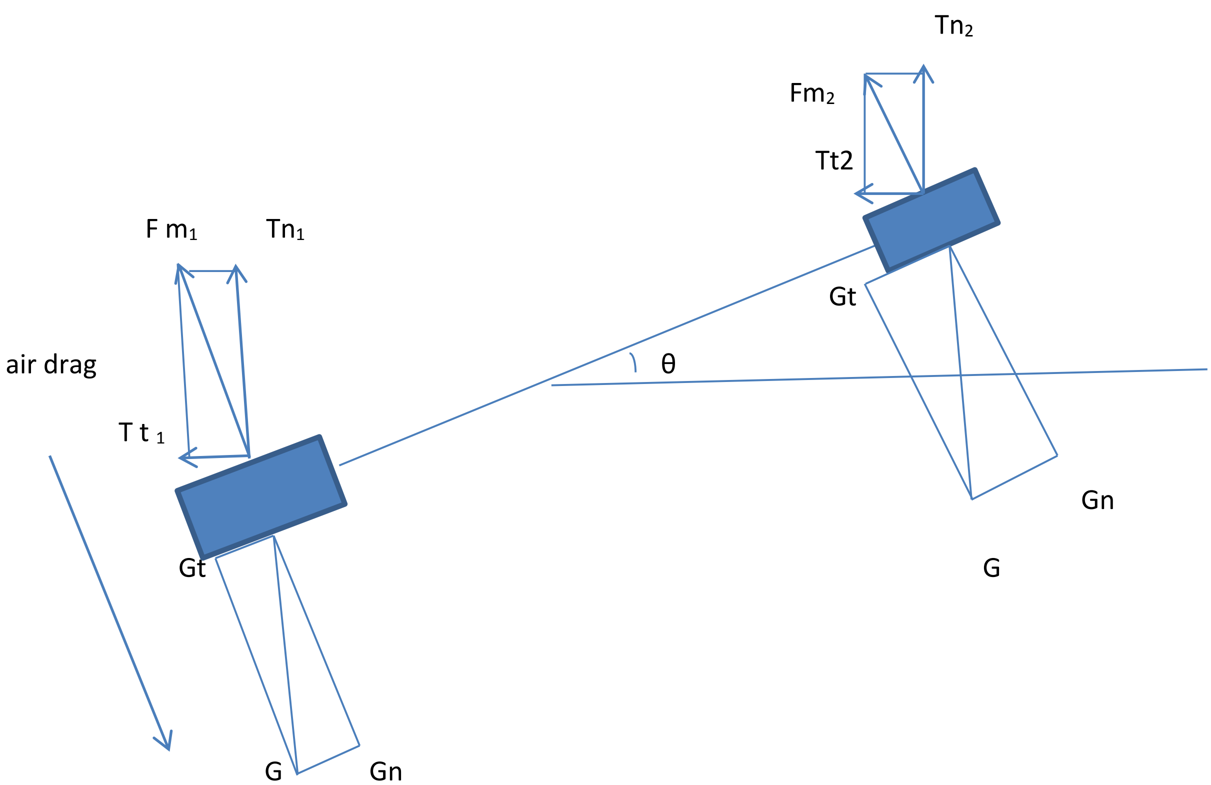

Figure 8-Free body diagram of a Quadcopter for only one axis

Equations describing the dynamic behavior of the system (first attempt of determining a system model):

$T + ml^2 – mg\cos\theta - \mu_{wood} (m_{bar} + m_{total})\, g = m_{total}\, \dot{\omega}$ (6)

$T = \rho \frac{\pi d^2}{4} \left(\frac{\omega}{2\pi} p \right)^2$ [12] (7)

Where,

T is the thrust produced by a pair of motors and is measured in N

m is the mass of one pair of motors and is measured in kg

l is the length of the frame and is measured in m

g is the gravitational acceleration which is roughly 9,81 m/s^2

θ is the deviation angle of the Quadcopter on the pitch axis and is measured in radians.

μ_{wood} is the friction coefficient with wood and is a dimensionless coefficient.

m_{bar} is the mass of the bar blocking the motion of the Quadcopter on the roll and yaw axis and is measured in kg

m_{total} is the total mass of the Quadcopter and is measured in kg

$\dot{\omega}$ is the first derivative of the angular velocity, which is also called angular acceleration, measured in $radians/second^2$

ρ is the air density and is roughly 1.229 kg/m^3 measured at sea level [13]

d is the diameter of the motor measured in m

ω is the angular velocity measured in radians/second

p is the propeller pitch measured in m

Equations (6) and (7) were the first attempt of the group of deriving a mathematical model of the Quadcopter system on only one axis of movement. The approach was to decompose the thrust and weight components of both the pair of motors into their normal and tangential components. After this decomposition of forces has been performed, the next step was to apply classic mechanics in trying to express the equation. This first attempt of determining the equations of motion was unsuccessful. The equations were vague and not describing the actual dynamics of the system.

Equations describing the dynamic behavior of the system(second attempt):

These equations have been derived using the help of one of the research assistants. In deriving the equations of motion of the Quadcopter on one axis, the free body diagram of the system has been used (represented in Figure 7).

Moment of inertia:

"The moment of inertia of an object about a given axis describes how difficult it is to change its angular motion about that axis." [14] The moment of inertia is measured in kgm^2.

For the given problem, the total moment of inertia can be described as:

$I_{total} = 2I_{motor} + I_{frame}$

Where,

$I_{motor} = m_m r^2$ and $I_{frame} = \frac{1}{12} m_{frame} (2r)^2$. [15]

m_m represents the mass of one pair of motors measured in kg(a certain extra mass has been added to the mass of the pair of motors consisting a small part of the frame).

r represents half of the length of the frame measured in m.

m_{frame} represents the total mass of the frame measured in kg.

Calculating the moment of inertia for the motor, frame and the total moment of inertia, the following values are obtained:

$I_{motor} = m_m r^2 = 0,\ 0079$

$I_{frame} = 0,\ 0099$

$I_{total} = 0,\ 025$

Thrust

The thrust of the Quadcopter has been divided as the thrust produced by one pair of motors and the thrust produced by the other pair of motors.

The formulas describing the torque induced by each pair of motors in relationship with the thrust developed by the two pairs of motors can be defined as following:

$T_1 = Fm_1 r$

$T_2 = -Fm_2 r$

$Tn_1 = m_m g \, r \sin\theta$

$Tn_2 = m_m g \, r \sin\theta$

$T_{total} = T_1 + T_2 + Tn_1 + Tn_2$

$T_{total} = \Delta Fm + 2m_m g \, r \sin\theta$: (8)

$T_{total} = I_{total} \, \alpha$ (9)

$\alpha = \ddot{\theta}$ (10)

Where,

T_1 is the torque produced by one pair of motors, measured in Nm

Fm_1 is the thrust produced by on pair of motors, measured in N

r is half of the length of the frame, measured in m

T_2 is the torque produced by the other pair of motors, measured in Nm

Fm_2 is the thrust produced by the other pair of motors, measured in N

Tn_1 is the gravitational component of the torque for one pair of motors, measured in Nm

Tn_2 is the gravitational component of the torque for the other pair of motors, measured in Nm

m_m is the mass of one pair of motors(it will be the same for the other pair of motors as the motors are identical)

g is called the gravitational acceleration which is approximately 9.81, measured in m/s^2

θ is the deviation angle on the pitch axis, measured in radians

T_{total} is the total torque produced by the Quadcopter

$\ddot{\theta}$ is called the angular acceleration of the Quadcopter, measured in $radians/s^2$

The equations describing the dynamic behavior of the system are equations (8), (9) and (10):

$T_{total} = \Delta Fm + 2m_m\, g\, r\, \sin\theta$:

$T_{total} = I_{total}\, \alpha$

$\alpha = \ddot{\theta}$

Equation (8) is non-linear as it contains the sin component. This equations can be linearize either by using the small angle approximation (knowing that θ is measured in radians) or by the Taylor Expansion formula, f (a) + f' (a) (x-a).

Using the small angle approximation, equation (8) will become:

$T_{total} = \Delta Fm + 2m_m\, g\, r\, \theta$ (11)

In order to obtain the transfer function of the system, the equations describing the dynamic behavior of the system must be converted from the time domain into the complex Laplace domain using the Laplace transform.

Applying Laplace transform to equations (9), (10) and (11), the following equations are obtained:

$T_{total}(s) = \Delta Fm(s) + 2m_m\, g\, r\, \theta(s)$ (12)

$\alpha = s^2\, \theta(s)$ (13)

$T_{total}(s) = I_{total}\, s^2\, \theta(s)$ (14)

Expressing equations (14) in a different form, the following equation is obtained:

$\theta(s) = \frac{\mathrm{Ttotal}(s)}{I_{total}\, s^2}$ (15)

Replacing the θ(s) in equation (12) with equation (15), the following is obtained:

$\Delta Fm(s) = \frac{T_{total}\,(s) I_{total}\, s^2 - 2m_m\, g\, r\, T_{total}}{I_{total}(s) s^2}$ (16)

In the end, the transfer function of the system will be the ratio between the angle deviation on the pitch axis and the delta thrust (thrust difference between the first pair of motors and the second pair of motors) in the complex Laplace domain:

$$H(s) = \frac{\theta(s)}{\Delta Fm(s)} = \frac{\text{Ttotal}(s)}{I_{total}\, s^2} : \frac{T_{total}\,(s) I_{total}\; s^2 - 2m_m\, g\, r\, T_{total}}{I_{total}(s) s^2} \qquad (17)$$

Simplifying equation (17), the following transfer function will result:

$$H(s) = \frac{\theta(s)}{\Delta Fm(s)} = \frac{1}{(I_{total}\, s^2 - 2m_m\, g\, r)} \qquad (18)$$

Replacing with numerical coefficients in equation (18), the final transfer function of the system will result:

$$H(s) = \frac{\theta(s)}{\Delta Fm(s)} = \frac{1}{(0{,}016\, s^2 - 0{,}67)} \qquad (19)$$

The resulting transfer function is a second order transfer function, which can be determined by analyzing the characteristic equation in the denominator of the transfer function. The system has no zeroes and 2 poles.

Equalizing the characteristic equation of the denominator of the transfer function with 0, the poles can be determined:

$0{,}016\, s^2 - 0{,}67 = 0$

$0{,}016\, s^2 = 0{,} 67$

$s^2 = \frac{0{,}67}{0{,}016}$

$s^2 = 41{,}8$

$s = \pm\sqrt{41{,}8}$

$s_1 = + 6{,}46$

$s_2 = -6{,}46$

The two poles of the system are located at -6,46 and +6,46 in the complex s-plane. The poles of a system can give an insight into the stability of the system. Knowing that the poles are at those two specific locations in the s-plane, it can be said that the system is unstable. If the poles of a system are located in the LHP, then the system is stable; if the poles of a system are located in the RHP, then the system is unstable. If one pole is in the LHP and the other is in the RHP, then the system is unstable.

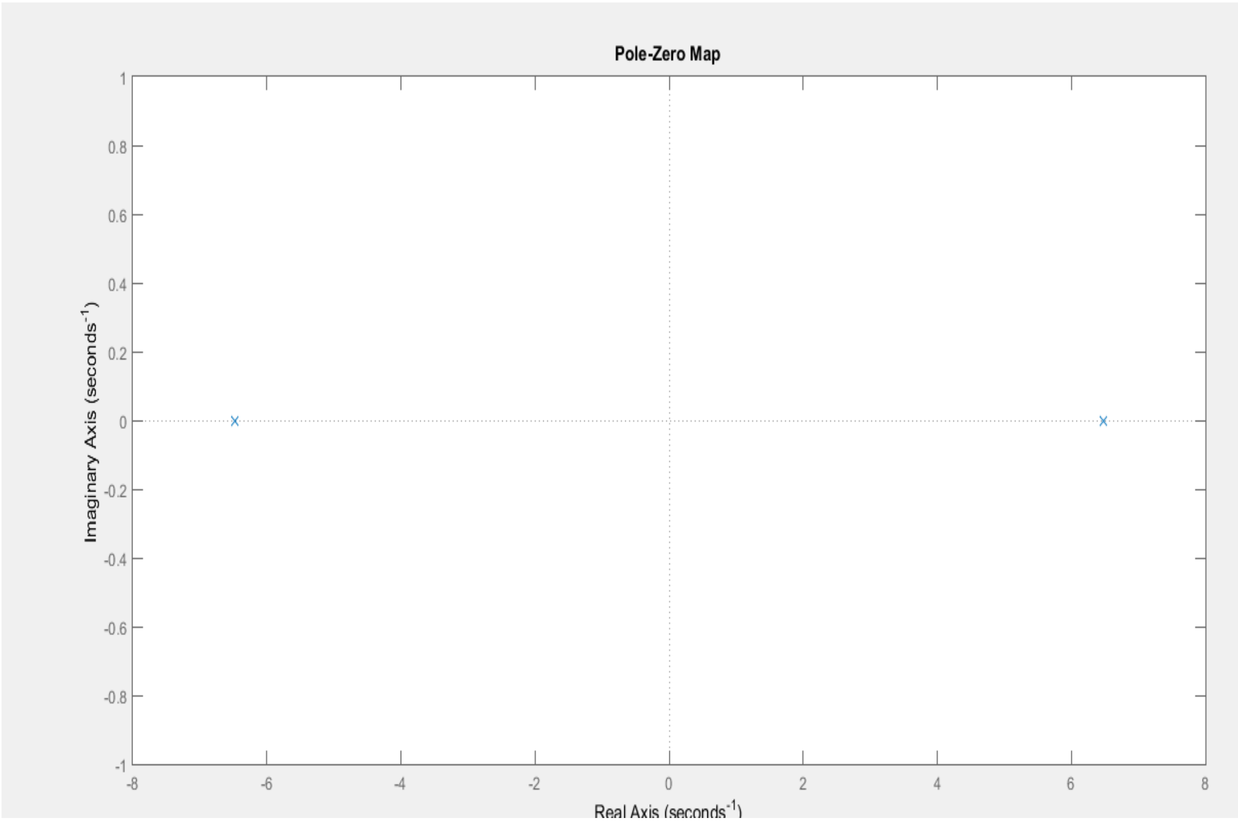

Figure 9-pole-zero map showing the location of the poles in the s-plane

Applying a step input to the transfer function of the system, the unstable behavior of the system in open loop can be seen:

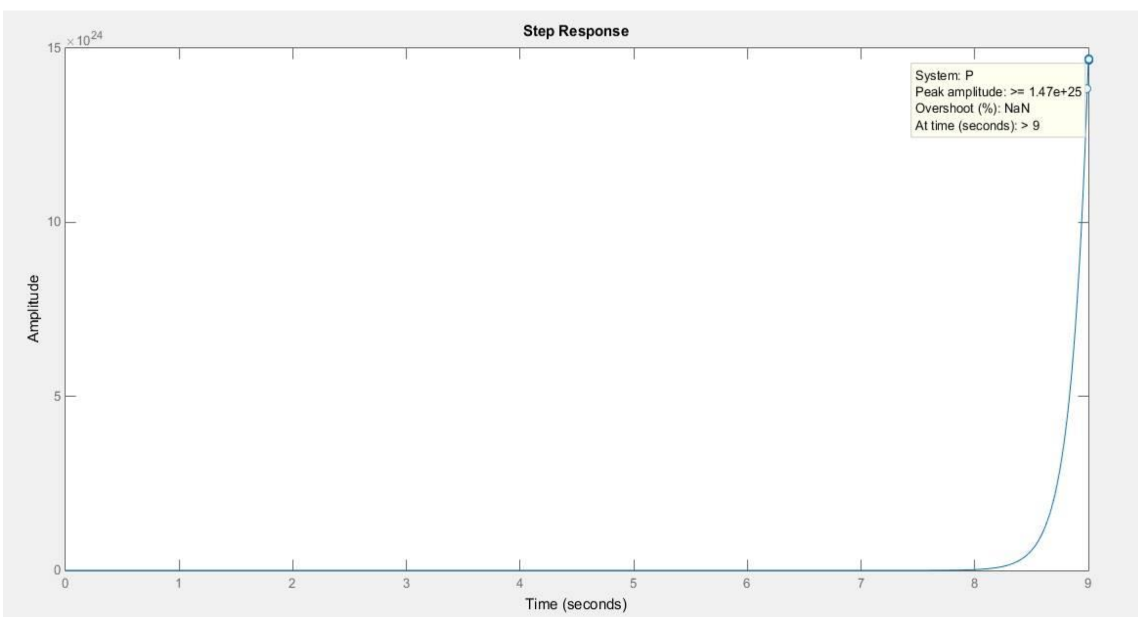

Figure 9-Step response

As it can be seen in Figure 9, the system has no bounded output for every bounded input; it has an exponential growth without ever reaching a steady state.

Summary:

After many attempts to obtain a correct mathematical model from textbooks, the group has decided to try de derive the equations describing the dynamic behavior of the system by using a free body diagram of the Quadcopter. In the first attempt of determining these equations, the model obtained was vague and not accurate enough. The second model derived was more successful as the group was helped by one of the research assistants. The obtained transfer function of the system is a second order transfer function. The two poles of the system are located at -6.46 and +6.46 meaning that the system is unstable.

Chapter 2.3: Open-loop analysis

As the system is unstable in the open-loop, the purpose of this chapter is to determine which controller could stabilize the system in the closed-loop.

The approach of this subchapter will be to just introduce different controllers with random gains and just graph the Bode and Nyquist diagram in order to see the closed-loop stability of different controllers and gains. As accuracy, steady-state error control and response time are not the purpose of this analysis, these will not be tackled in this subchapter.

A Bode plot is characteristic to the frequency response of the system and is a graphical technique which shows the difference in magnitude and phase of the output of a system across the entire frequency spectrum. The Nyquist Diagram displays the same information as the Bode Plot, but the magnitude and phase are expressed as complex numbers and are shown by their location in the complex s-plane. [16]

The gains selected to perform this open-loop analysis are: Kp = 100, Ki = 1 and Kd = 1. The first controller introduced in the open-loop is a P controller with gain 100.

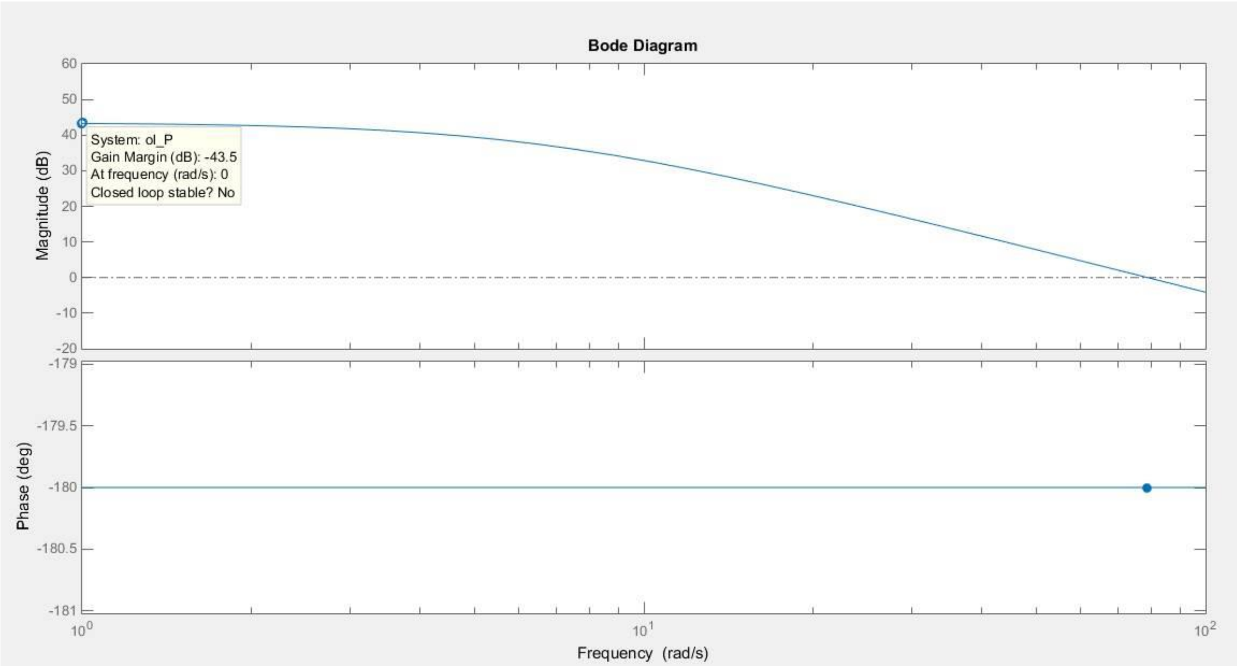

Figure 10-Bode plot of the system with a P controller

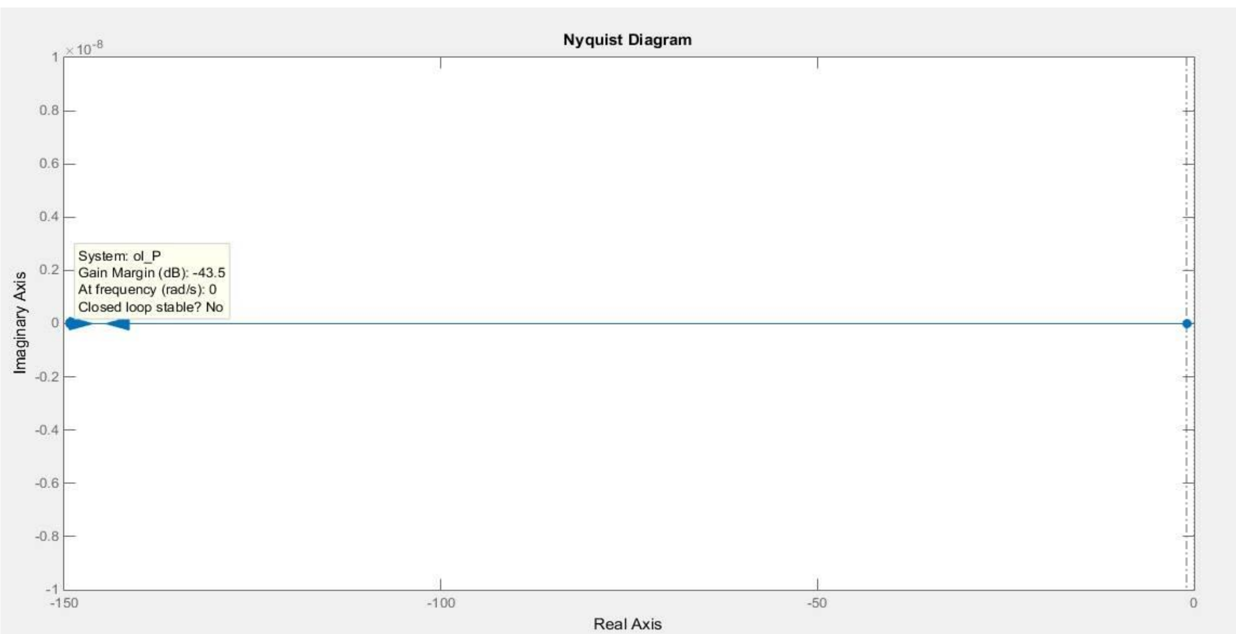

Figure 11-Nyquist Diagram of the system with a P controller

As it can be seen from Figure 10 and 11, the system cannot be stabilized with just a P controller. The P controller is responsible for making the response of the system faster and to correct but never eliminate the steady state error[17].

The next controller that will be introduced in the system is a PI controller of the aforementioned gains.

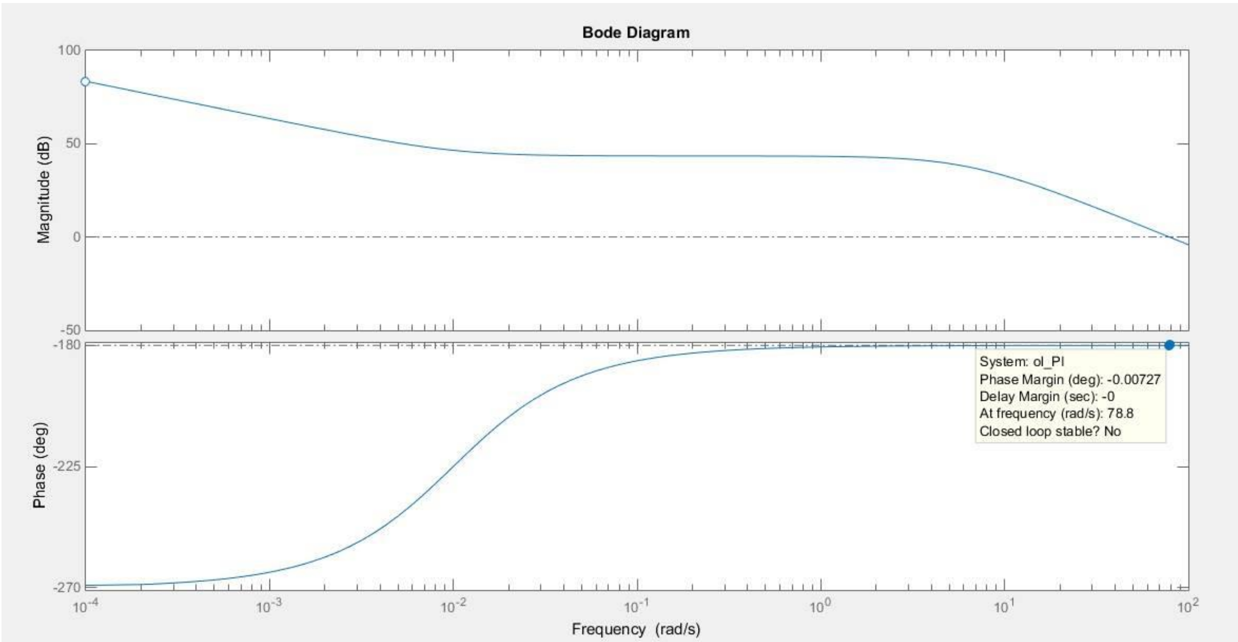

Figure 12-Bode Plot of the system with a PI controller

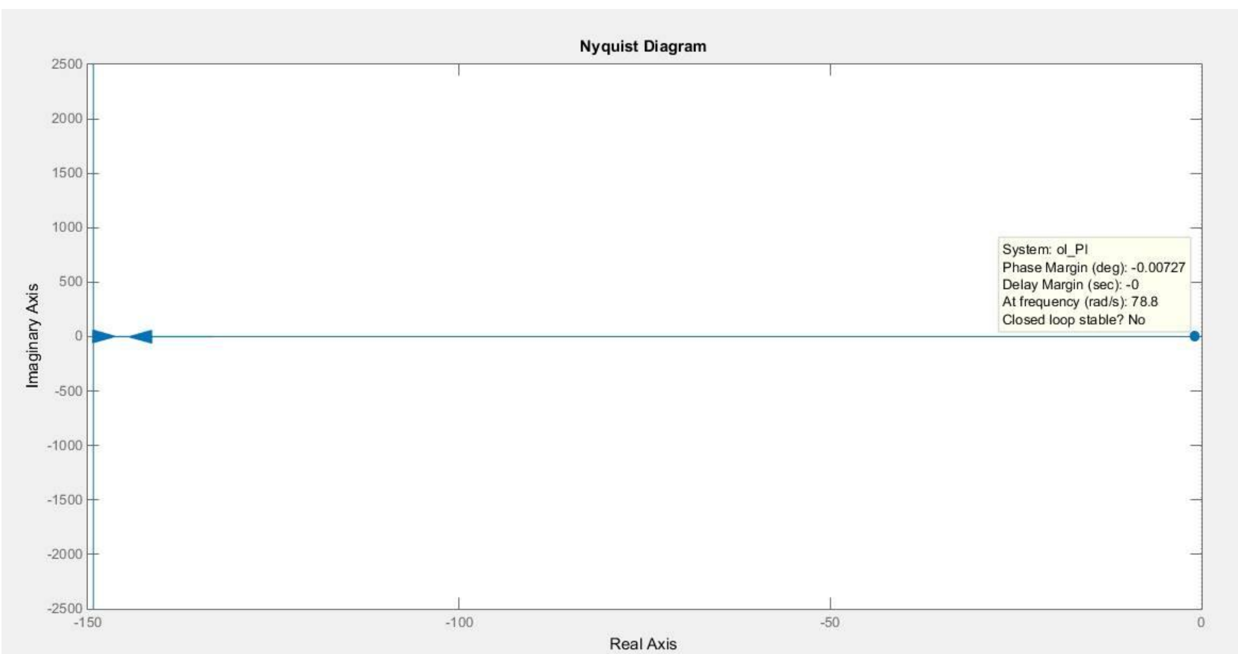

Figure 13-Nyquist Diagram

A PI controller is still not the best choice when it comes to stabilizing the system as the purpose of an I controller is to eliminate the steady state error, and as a side-effect it will make the transient response worse.[18]

The third controller that will be introduced in the open-loop system in order to determine if it can stabilize the system in the closed-loop is a PD controller.

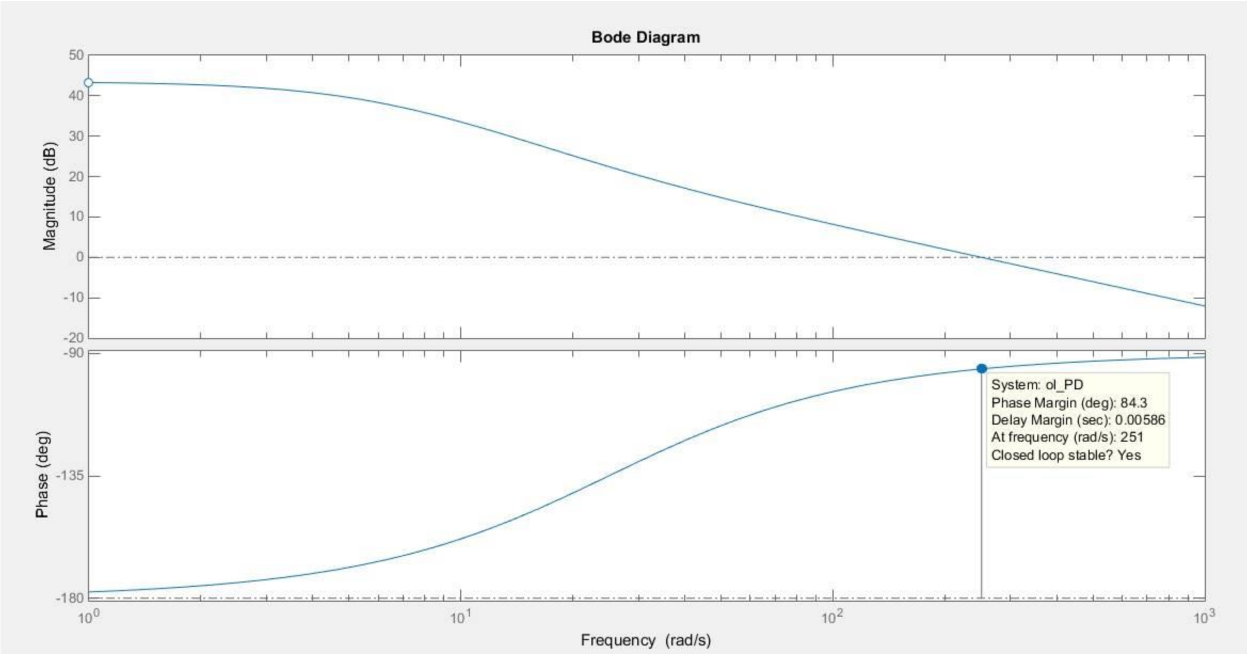

Figure 14-Bode plot of the system with a PD controller

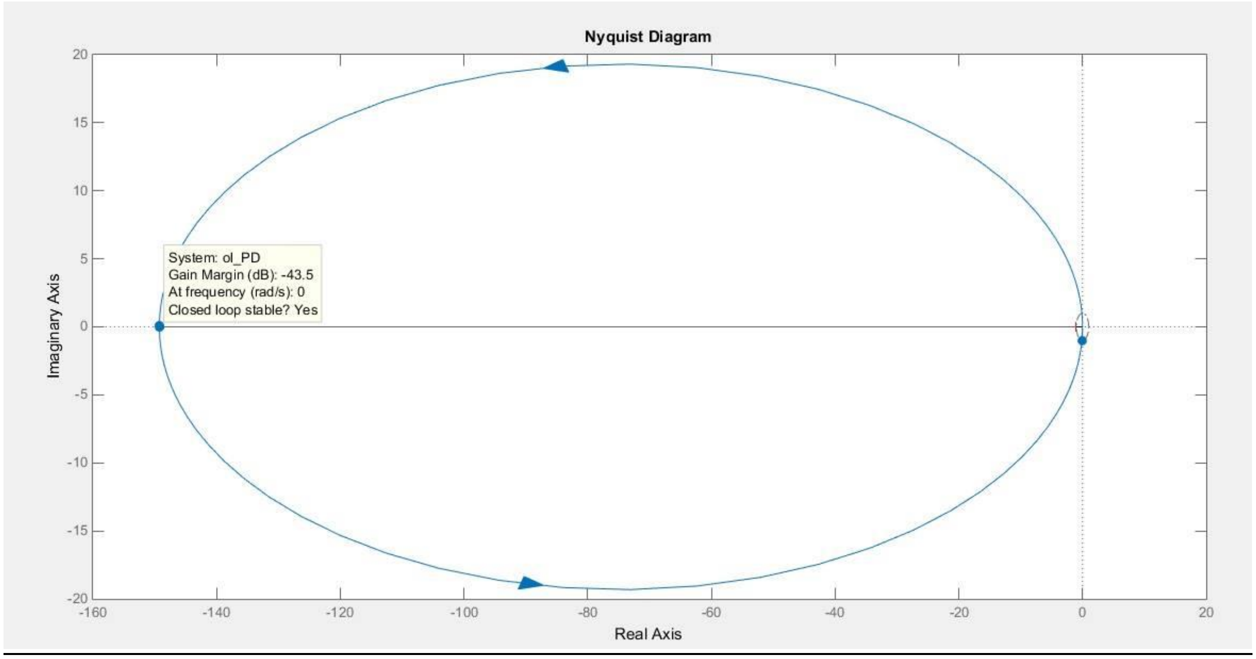

Figure 15-Nyquist Diagram of the system with a PD controller

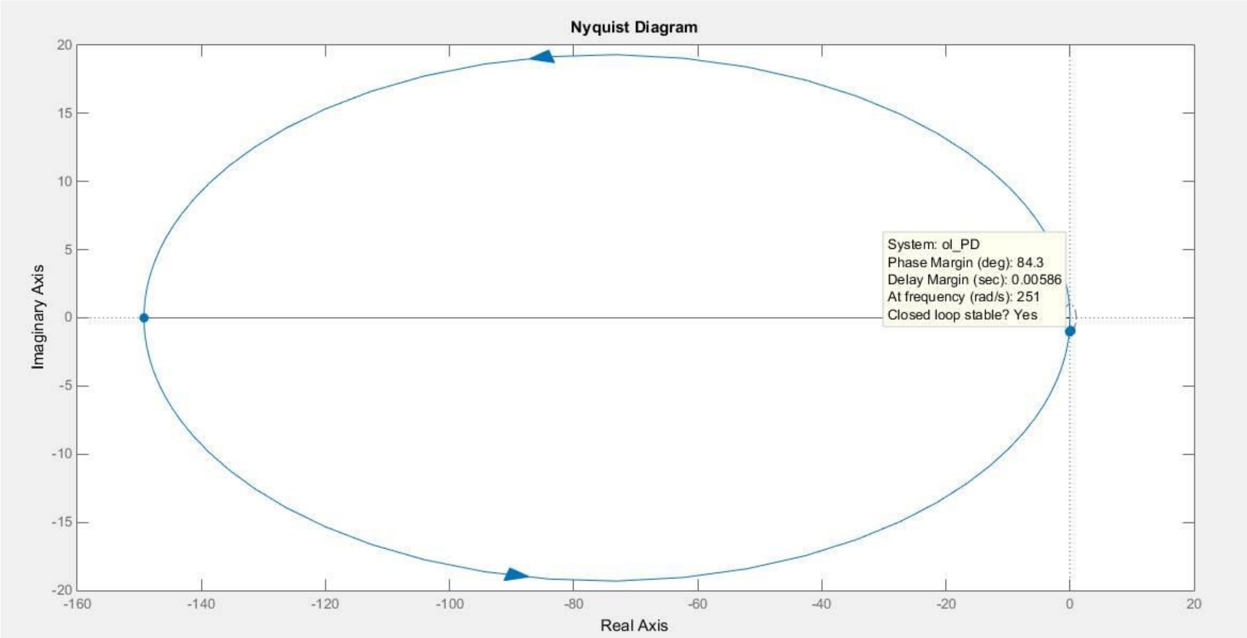

Figure 16-Nyquist Diagram of the system with a PD controller

As it can be seen from Figures 14, 15 and 16, a PD controller is able to stabilize the system in the closed loop system as the D controller acts as a dampener in the system. The purpose of the D controller is to improve the transient response, reduce the overshoot and increase the stability of the system.[19]

The final controller which will be introduced in the open-loop system is a PID controller.

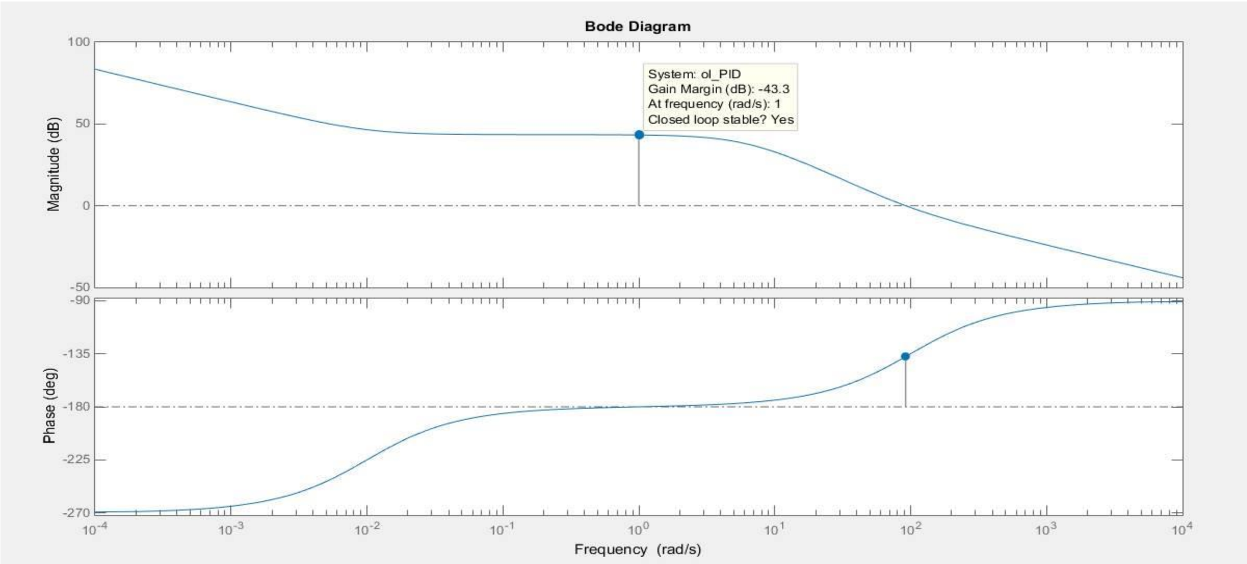

Figure 17-Bode plot of the system with a PID controller

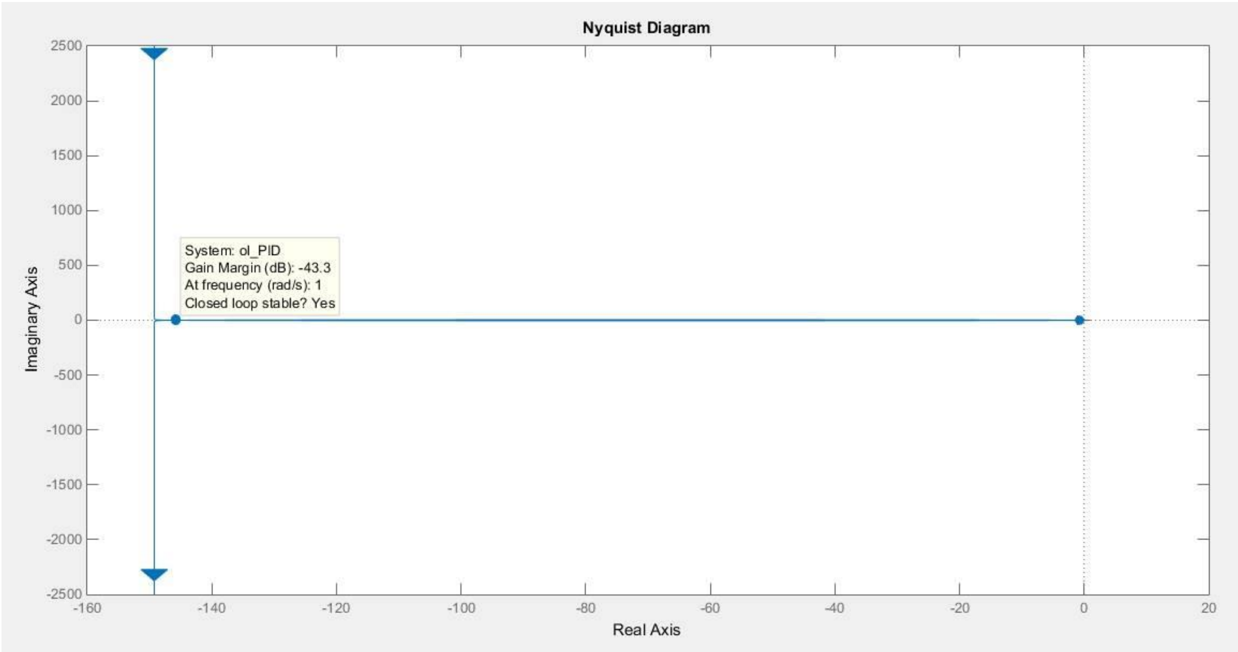

Figure 18-Nyquist Diagram of the system with a PID controller

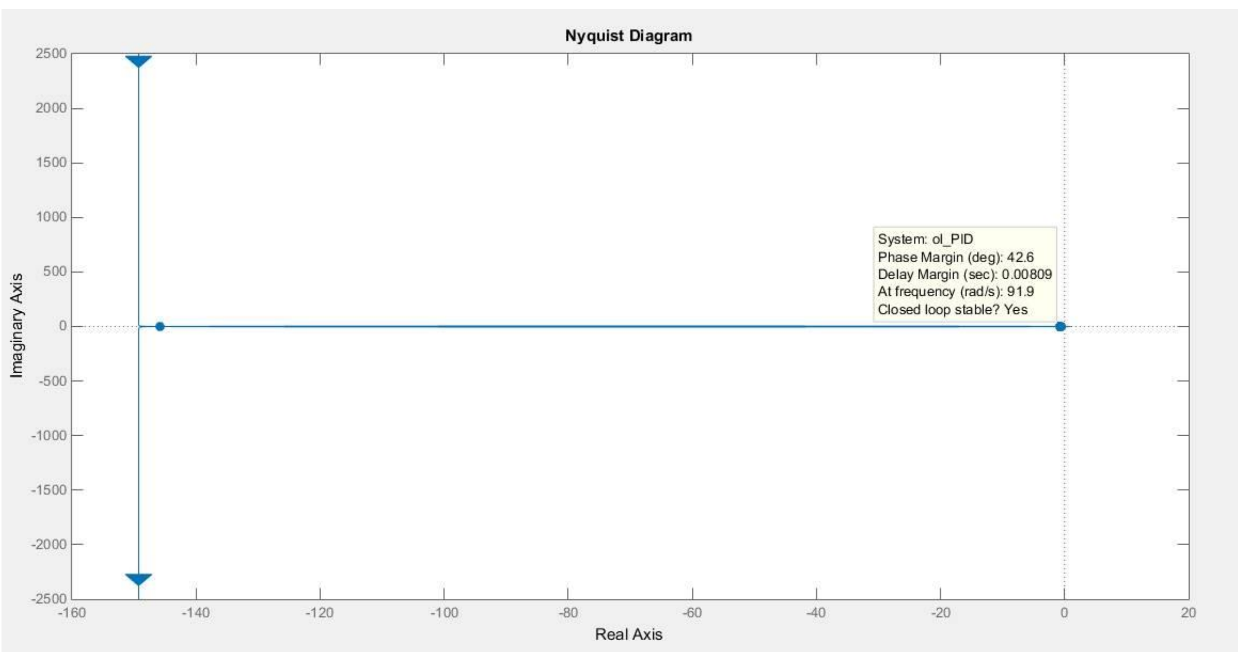

Figure 19-Nyquist Diagram of the system with a PID controller

It can be seen from Figure 17 that a PID controller would stabilize the system when feedback is introduced.

It can be concluded that the system can be stabilized in the closed-loop using either a PD or PID controller. A further analysis of the closed-loop system and the differences in the response of the system with these 2 controllers will be discussed in the next subchapter.

Summary:

Open-loop analysis is an important step in choosing a controller that can stabilize the given system in the closed-loop. Open-loop analysis can give an insight into the gain margins, phase margins and bandwidth, but as the final controller gains have not been determined yet, these will be determined in later subchapters. In the end of the subchapter it has been concluded that a PD controller or PID controller can stabilize the system in closed-loop.

Chapter 2.5: Closed-loop analysis

The purpose of this subchapter is to determine which controller would suit best the purpose of this project, either a PD or PID controller. In this subchapter, different tuning methods will be presented with which the gains of the controller will be determined as well as the gain and phase margins of the closed-loop system. These tuning methods that will be presented are the PID tuning function in Matlab, the Root Locus and last but least, Ziegler-Nichols tuning method.

Before determining which controller would be the best choice in order to stabilize the Quadcopter on the pitch axis at 0 degrees, some controller design specifications will be mentioned in order to have an idea what the final outcome of the system should be.

Control specifications:

-Overshoot less than 10%

-No steady-state error

-Rise time less than 2 seconds

-Settling time less than 2 seconds

The aforementioned specifications are realistic enough to be achieved in a real life situation.

The first controller that will be implemented in the closed-loop system is the PD controller. The overall goal is to see the response of the system if the design specifications are met.

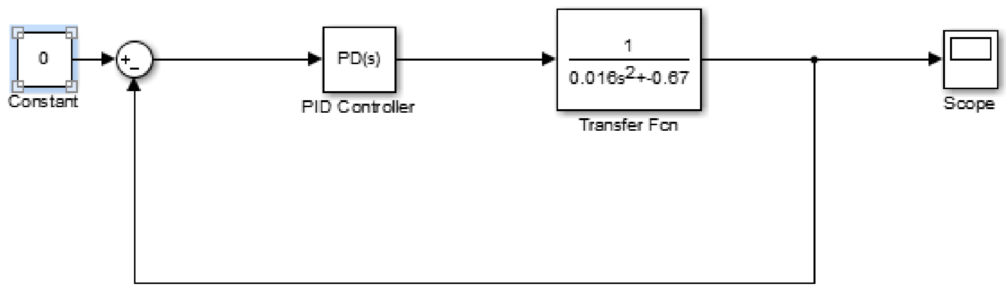

Figure 20-Block diagram of the closed-loop system with a PD controller

In Figure 20, the block diagram of the closed loop system with a PD controller can be seen. Using the Simulink PD tuning feature, the controller will be tuned in order for the system to match the design specifications.

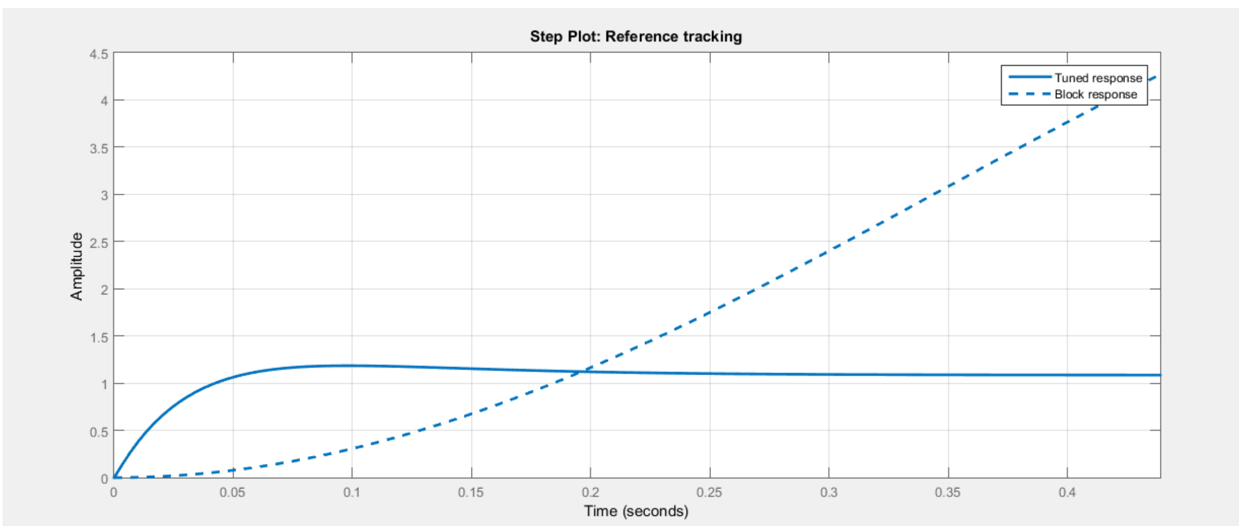

Figure 21-Step response of the system with the tuned PD controller

Controller Parameters

	Tuned	Block
P	8.4922	1
I		
D	0.71877	0
N	5209.6703	100

Performance and Robustness

	Tuned	Block
Rise time	0.0374 seconds	NaN seconds
Settling time	0.23 seconds	NaN seconds
Overshoot	9.14 %	NaN %
Peak	1.18	Inf
Gain margin	-22.1 dB @ 0 rad/s	-3.48 dB @ 0 rad/s
Phase margin	75 deg @ 45.6 rad/s	0 deg @ 4.54 rad/s
Closed-loop stability	Stable	Stable

Figure 22-Performance and robustness of the system with the tuned PD controller

As it can be seen from Figures 21 and 22, the system is stable in the closed loop with a PD controller. Furthermore, the PD controller was tuned in such a way that it will meet the design specifications. Although the system is stable and the design specifications are met, the PD controller is not the best choice for accomplishing the goals of this project as the system will have a steady-state error of 1.12(as it can be seen in Figure 21), which cannot be eliminated with a PD controller. The P or D components cannot correct the steady state-error by themselves, so in order to get rid of the steady-state error, an I is necessary.

The next controller that will be introduced in the closed-loop system will be the PID controller.

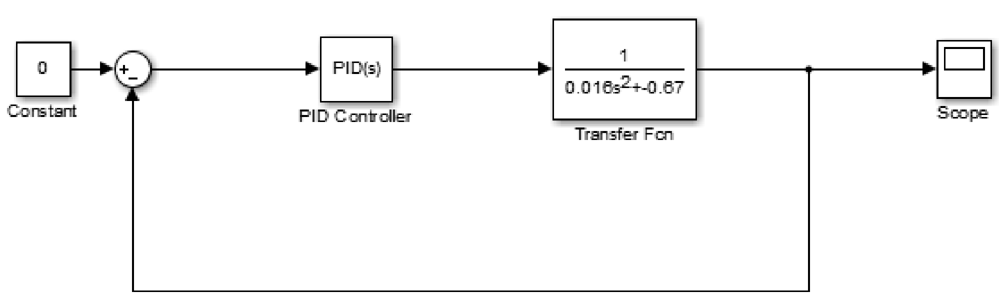

Figure 23-Block diagram representing the closed-loop system with a PID controller

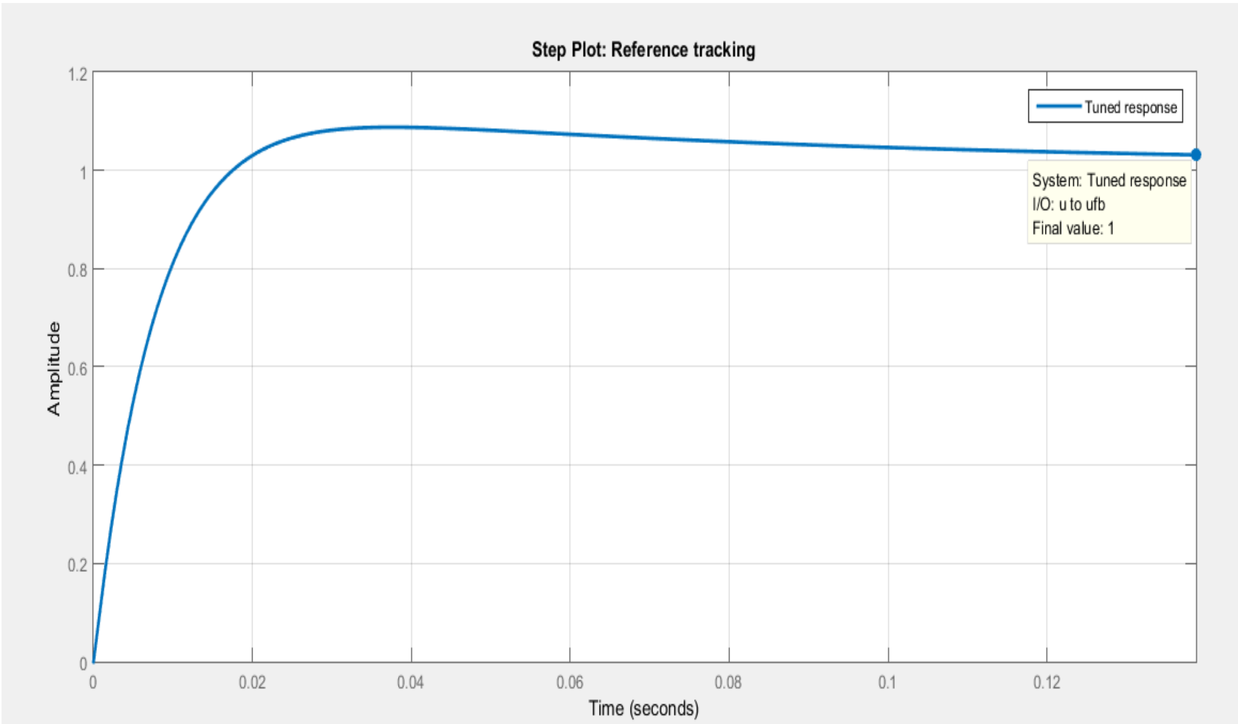

Figure 24-Step response of the system after the tuning of the PID controller

As it can be seen from Figure 24, a PID controller will eliminate the steady-state error of the system.

Controller Parameters

	Tuned
P	37.7034
I	83.7652
D	2.2981
N	16474.4241

Performance and Robustness

	Tuned
Rise time	0.0118 seconds
Settling time	0.195 seconds
Overshoot	8.81 %
Peak	1.09
Gain margin	-29.6 dB @ 6.04 rad/s
Phase margin	83 deg @ 144 rad/s
Closed-loop stability	Stable

Figure 25-Performance and robustness of the system after the PID controller tuning

As it can be seen from Figures 24 and 25, the PID controller will fit best the purpose of this project as it can eliminate the steady-state error. The design specifications have also been met after the tuning of the PID controller.

If the gains of the PD controller are Kp=1 and Kd=1, then the step response of the system will have a steady-state error of 3.03.

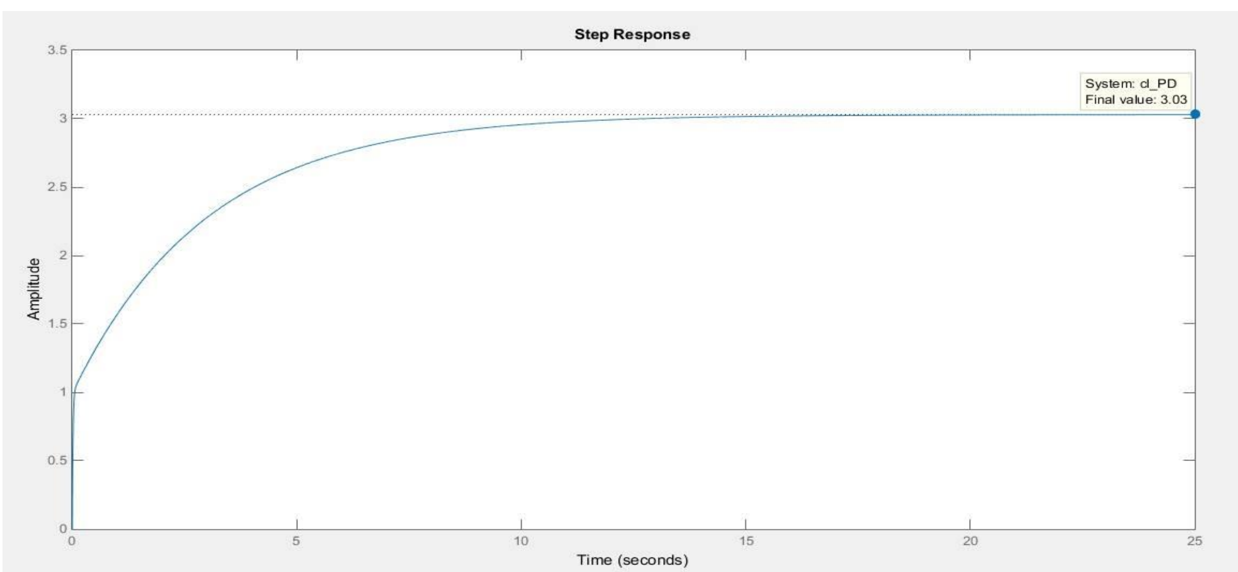

Figure 26-Step response of the system with a PD controller of gains Kp=1 and Kd=1

If a PID controller with gains of Kp=1, Ki=1 and Kd=1, then the step response of the system will have a big settling time (which can be corrected by adjusting the derivative controller gain) but no steady-state error.

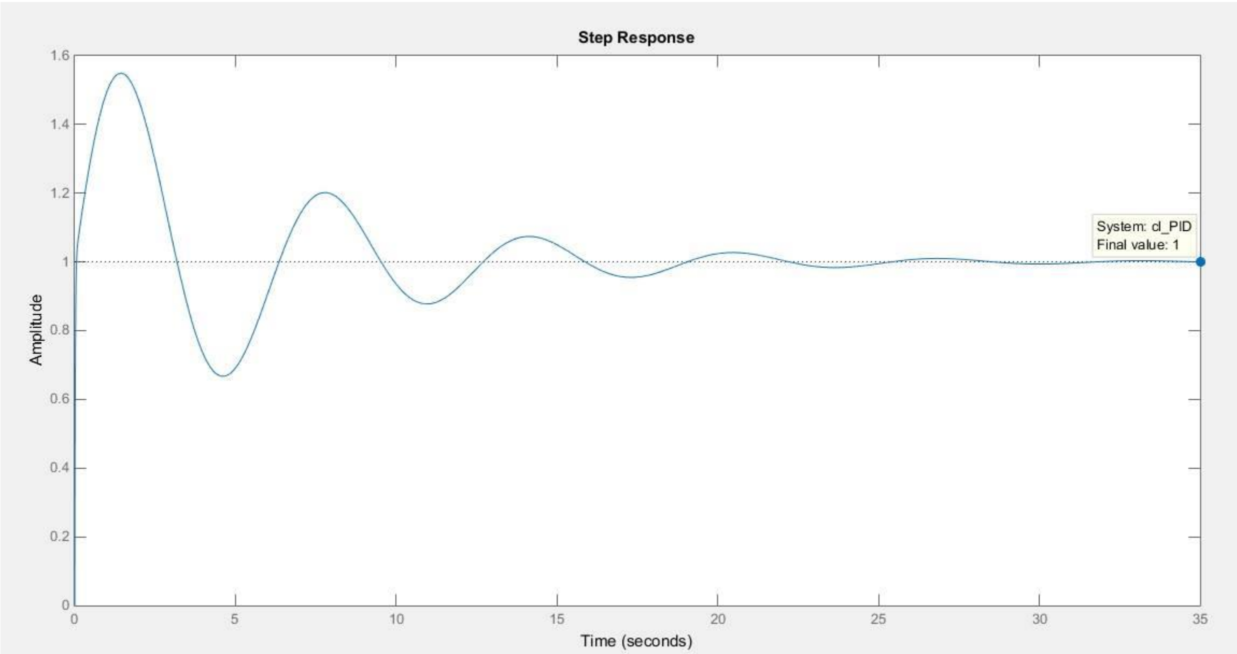

Figure 27-Step response of the system when the PID controller has gains of Kp=1, Ki=1 and Kd=1

The closed-loop system limitations will be determined using Bode plot in order to see how "robust" is the system and how must can the gain and phase of the system be changed until it becomes unstable.

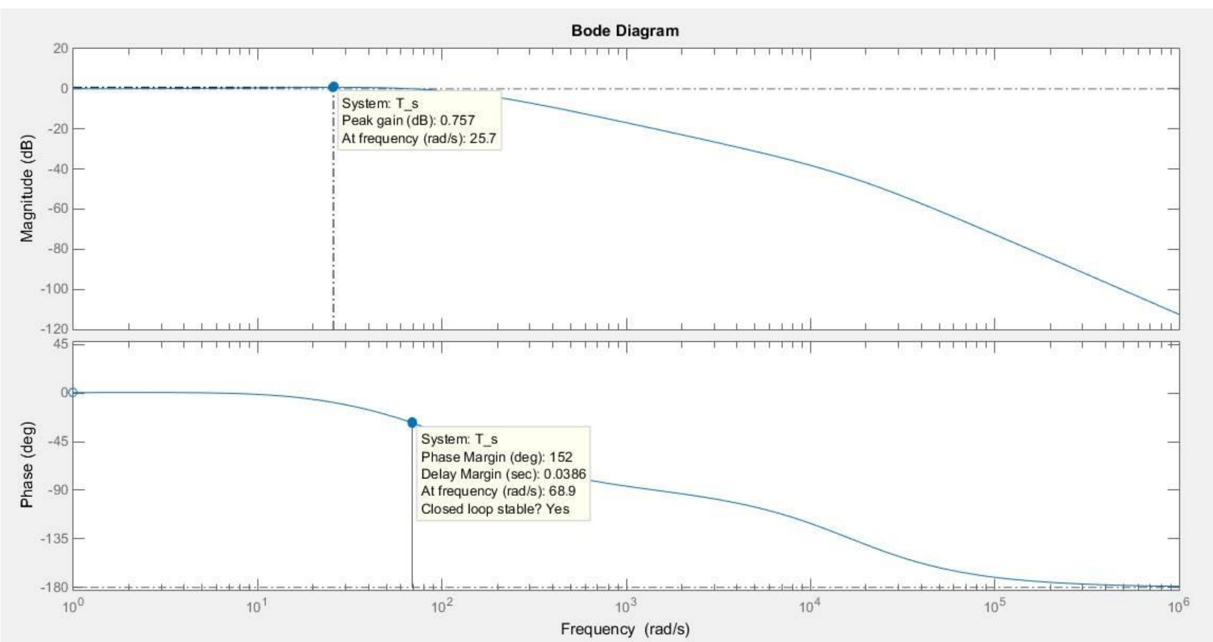

Figure 28-Bode of the closed-loop system with the PID tuned using the tune feature in Matlab

As it can be seen, the closed-loop system has a phase margin of 152 and infinite gain margin.

Tuning methods for the PID controller:

One of the tuning methods mentioned at the beginning of this subchapter has been already used in order to meet the design specifications. This method is the PID tuning feature Simulink offers. The gains of the PID controller obtained through this tuning method are: Kp=37.7, Ki=83.76 and Kd=2.2

The next tuning method that will be presented is the Root Locus method. After all the tuning methods of the PID mentioned at the beginning of this subchapter will be presented, a small comparison of the gains obtained through all the methods will be made.

Root Locus tuning method:

Root Locus is a plot which "indicates how the closed loop poles of a system vary with a system parameter (typically a gain, K)." [20]

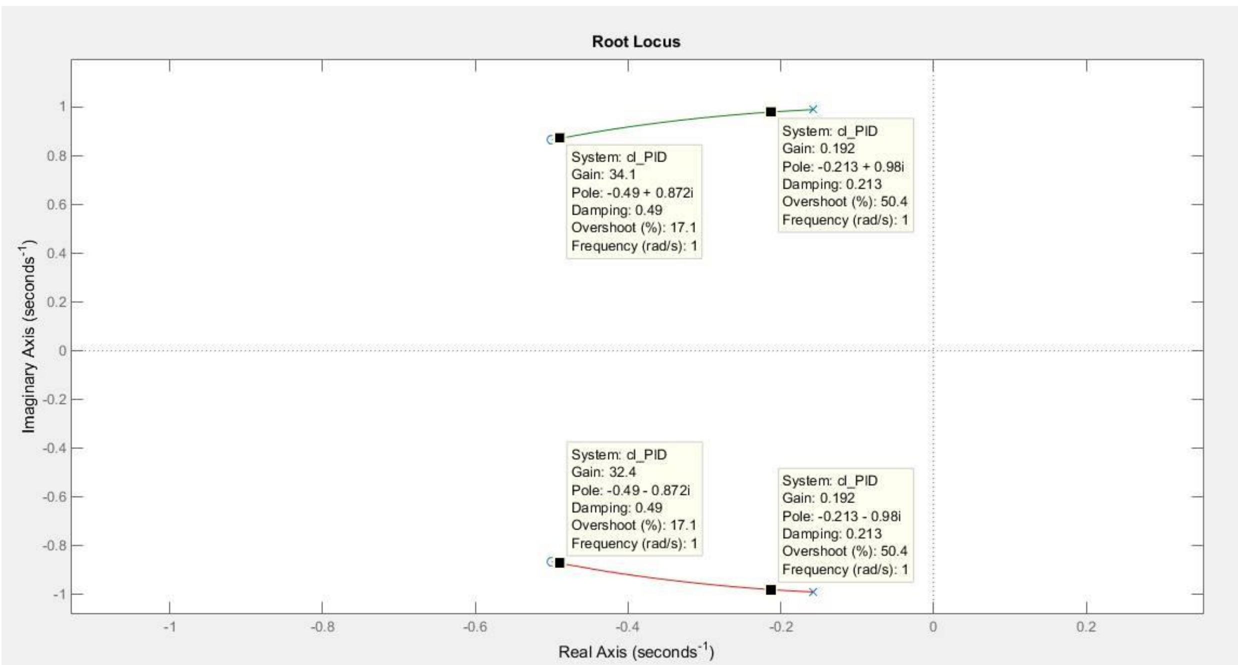

Figure 29-Root locus of the closed-loop system with a PID controller of gains Kp=1, Ki=1 and Kd=1

As the gain of the root locus will be increase more and more, the poles of the system will move more and more in the LHP of the S-plane, increasing the damping of the system. The gain of the root locus that can be seen in Figure 28 must be multiplied by the controller, which will alter the response of the system.

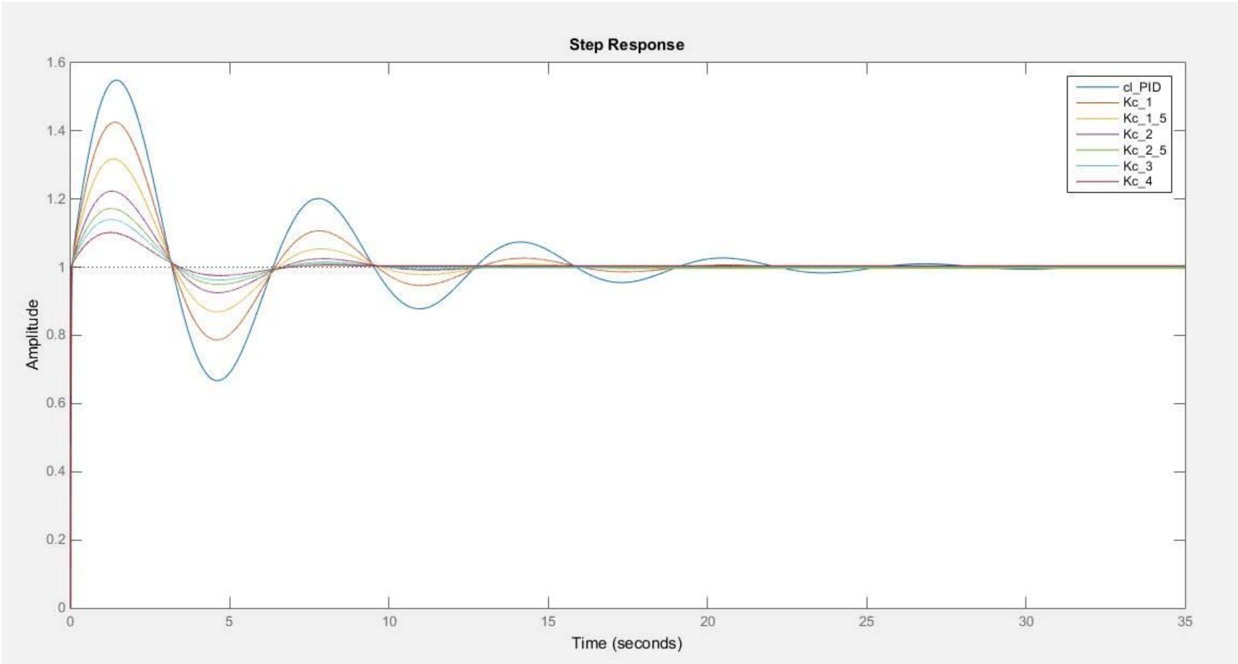

Figure 30-Step response of the system when the root locus gain is increased

As it can be seen in Figure 29, the damping of the system is increased as the poles move more and more towards the LHP of the system. After a root locus gain of 4, the system will have over damping, behavior that can be seen in Figure 30.

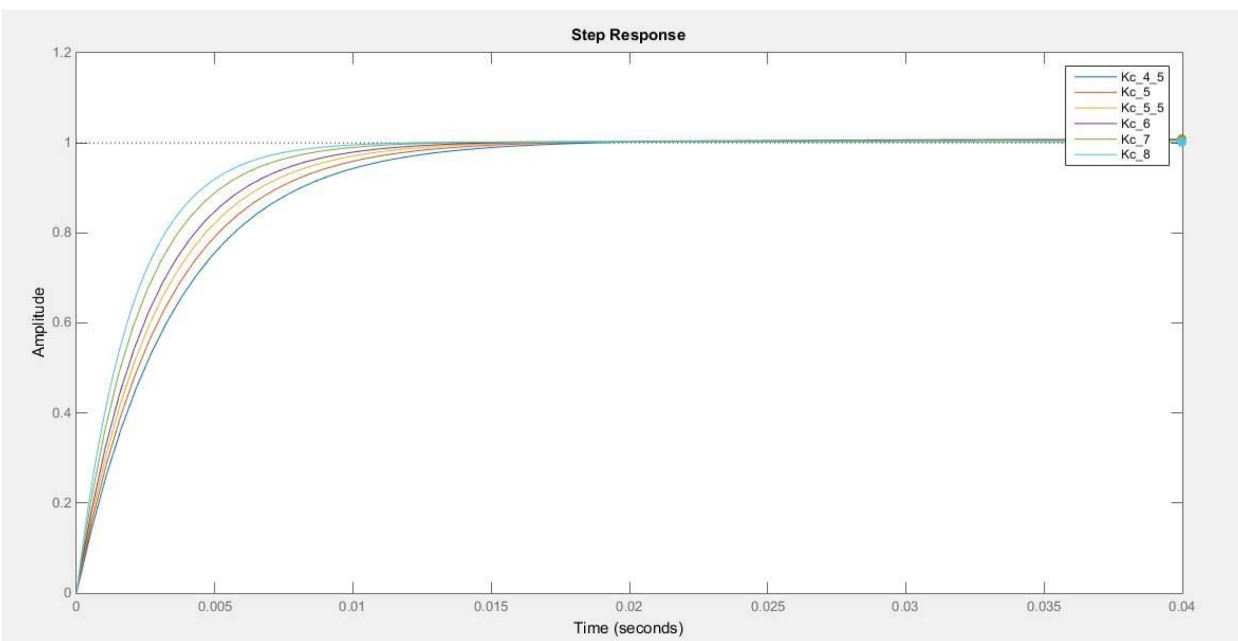

Figure 31-Step response of the system when the root locus gains are increased even more

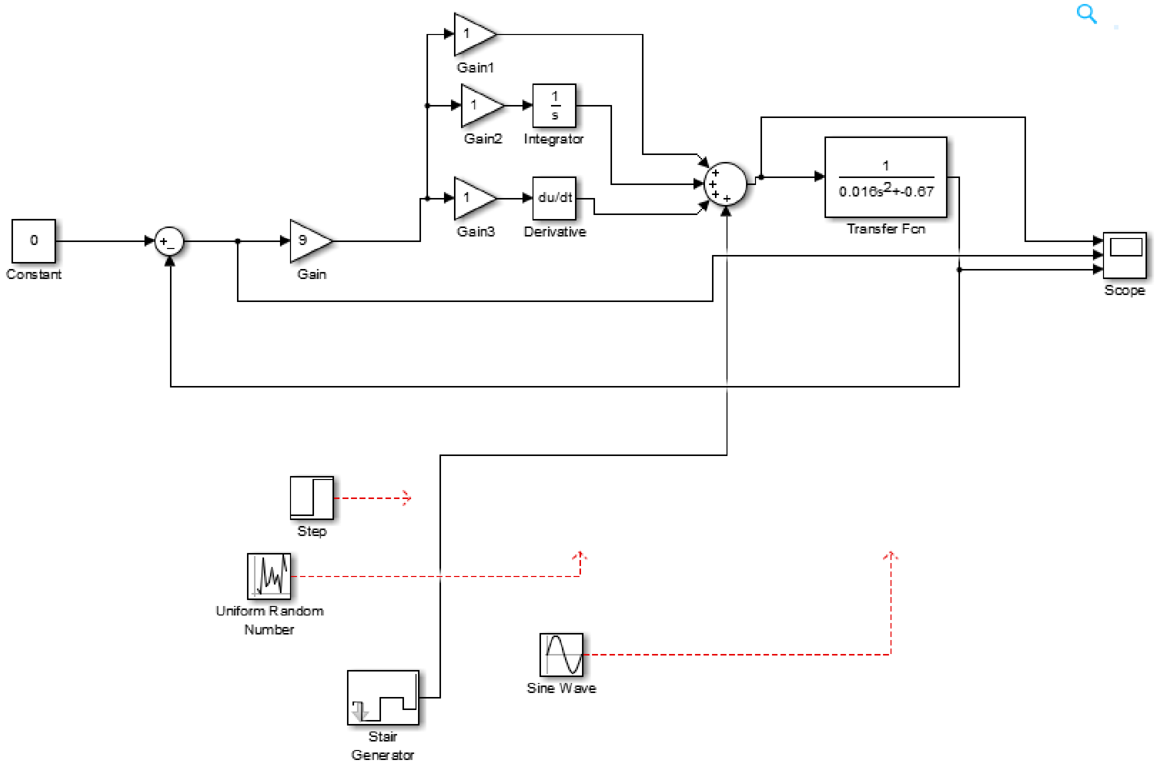

Figure 32-Block diagram representing the closed loop system with a gain of 9 determined through the root locus method

When a stair generator disturbance is introduced in the system, the system is able to correct the error and track the reference point pretty accurate, always eliminating the effect of the disturbance and reaching the set point(as it can be seen in Figure 32).

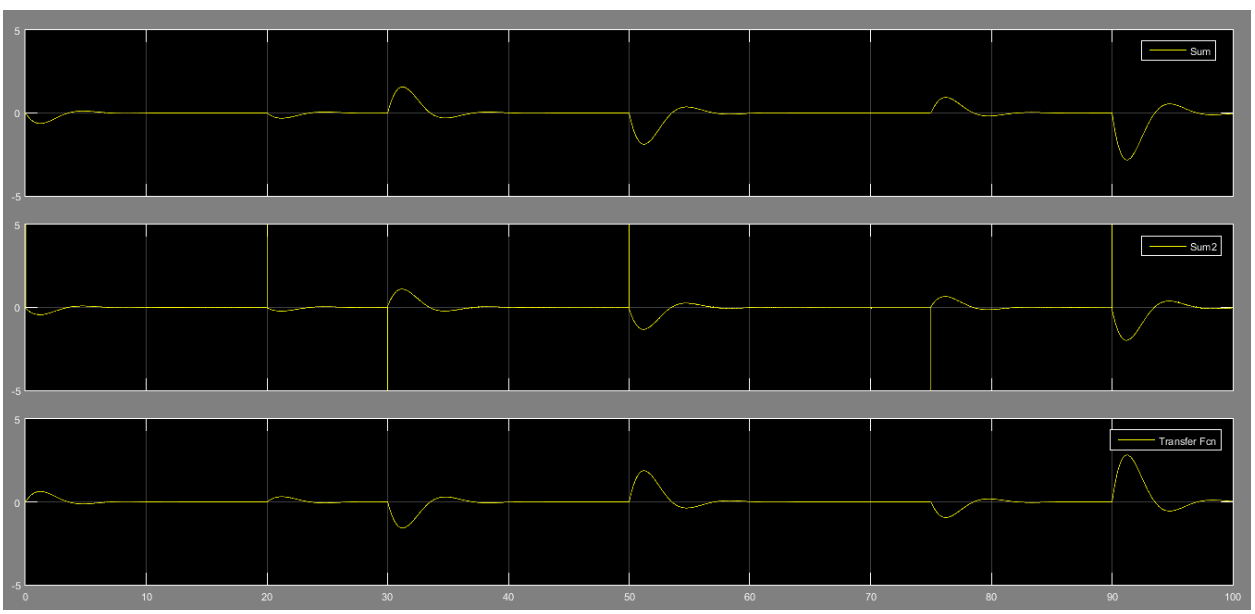

Figure 33-Disturbance rejection by the system

Now that the gains of the PID have been tuned using root locus, the limitations of the closed-loop system must be determined. These limitations refer to phase and gain margin. Knowing the gain and phase margin, one can determine how much the gain and phase of the system can be changed until the system will shift into instability.

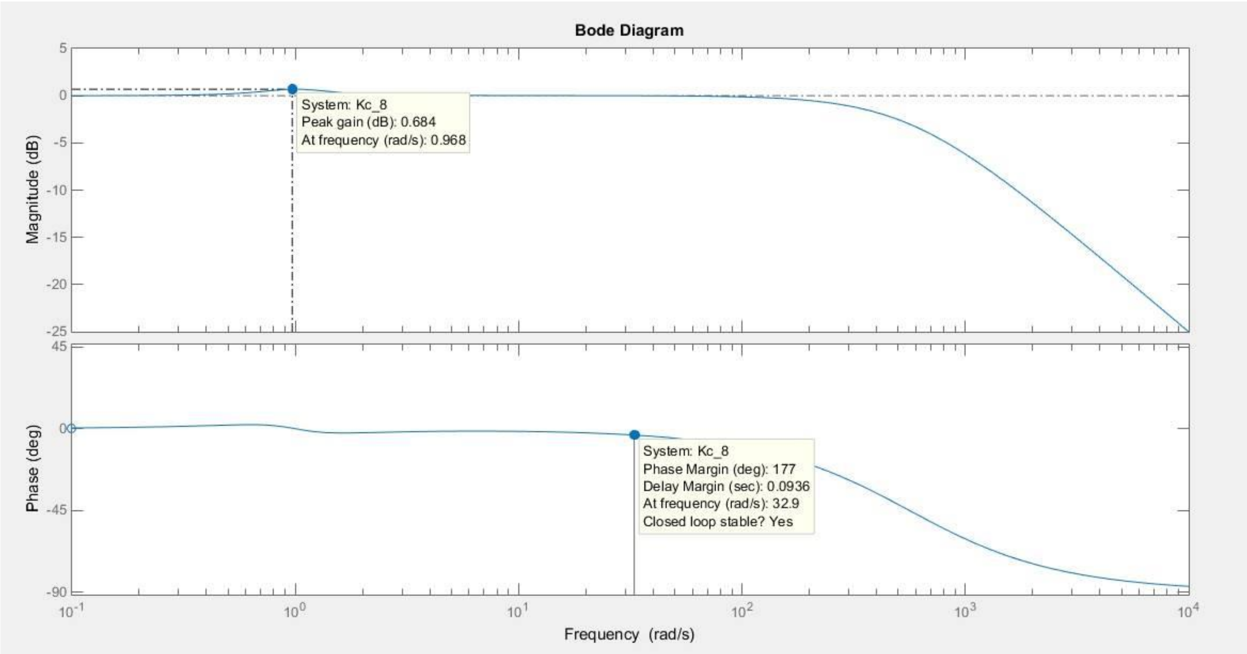

Figure 34-Bode plot of the closed-loop system with the PID gains tuned using the root locus method

As it can be seen in Figure 33, the closed-loop system has a phase margin of 177 degrees and infinite gain margin.

Ziegler-Nichols tuning method:

The Ziegler-Nichols tuning method will be performed in closed-loop (feedback). This method consists of introducing only a P and the gain is increased until the system will become marginally stable with oscillations that have constant amplitude. [21] The next step is to register the ultimate gain (K_v) and the period time. [22]

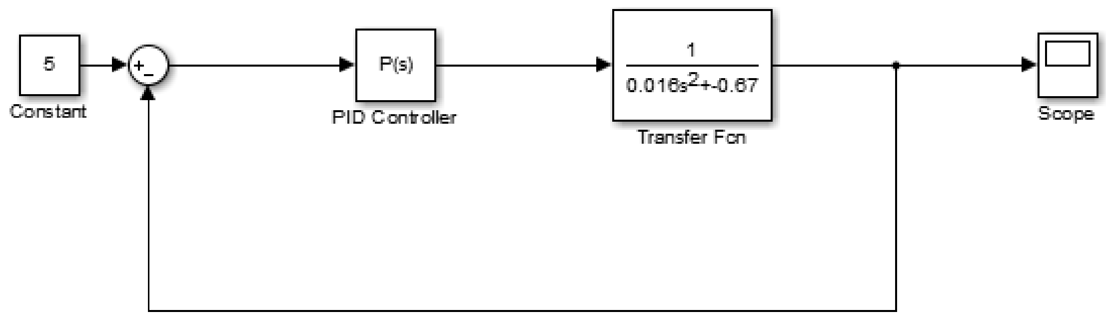

Figure 35-Closed-loop system with different set point and only a P controller

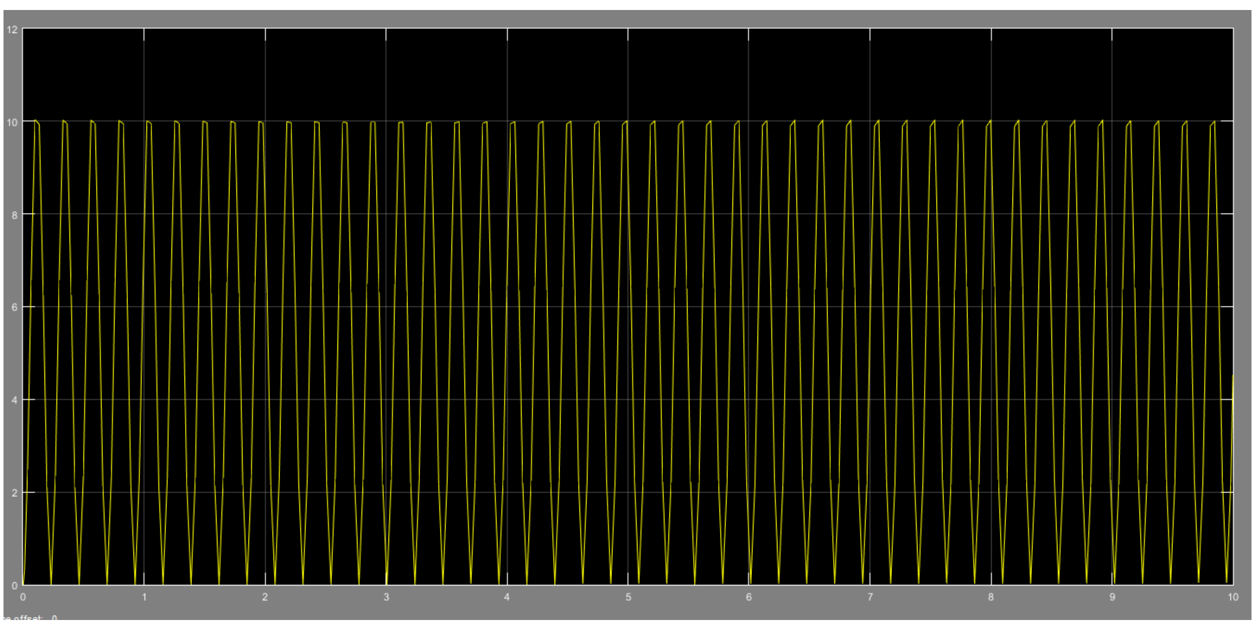

Figure 36-The response of the system with a change of set point and only a P controller

As it can be seen in Figure 36, the amplitude of the response is constant and so does the period. Both can be determined in order to find the gain of the proportional controller (Kc), the parameter that scales the integral controller (Ti) and the parameter that scales the derivative controller (Td). [23]

The ultimate gain is K_v=12.648 and the period is P_v ≈0.3 seconds. In order to find the Kc, Ti and Td, a special table of calculation is offered.

	K_c	T_I	T_D
P	$K_u/2$		
PI	$K_u/2.2$	$P_u/1.2$	
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

Figure 37-Closed-loop calculation of Kc, Ti and Td[5]

Using the table in Figure 37, it can be concluded that the Proportional gain is 7.44, the scaling factor for the integrator controller is Ti=0.150 and the scaling factor for the derivative controller is Td=0.0375.

Now, the I gain and D gain can be determined. If a certain gain is selected by the group, it must be multiplied by the respective scaling factor in order to get the final gain. The group has decided to scale the I and D components 100 times, giving a final I gain of Ki = 15 and a Kd = 3.75. Inserting these gains in the PID, the system will have the following step response.

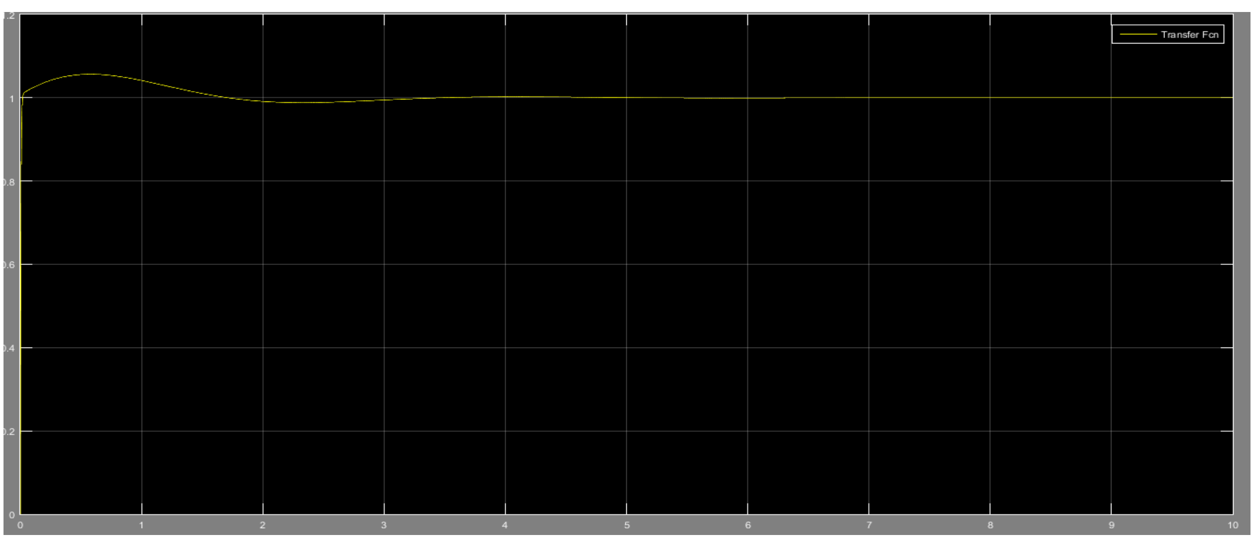

Figure 38-Step response after the PID tuning using the Ziegler-Nichols method

As it can be seen in Figure 38, after the tuning of the PID coefficients using the Ziegler-Nichols method, the system will be stable and no steady-state error (the input is a step of 1), but the rise time is unrealistic, as it will have a value around 0.05 seconds.

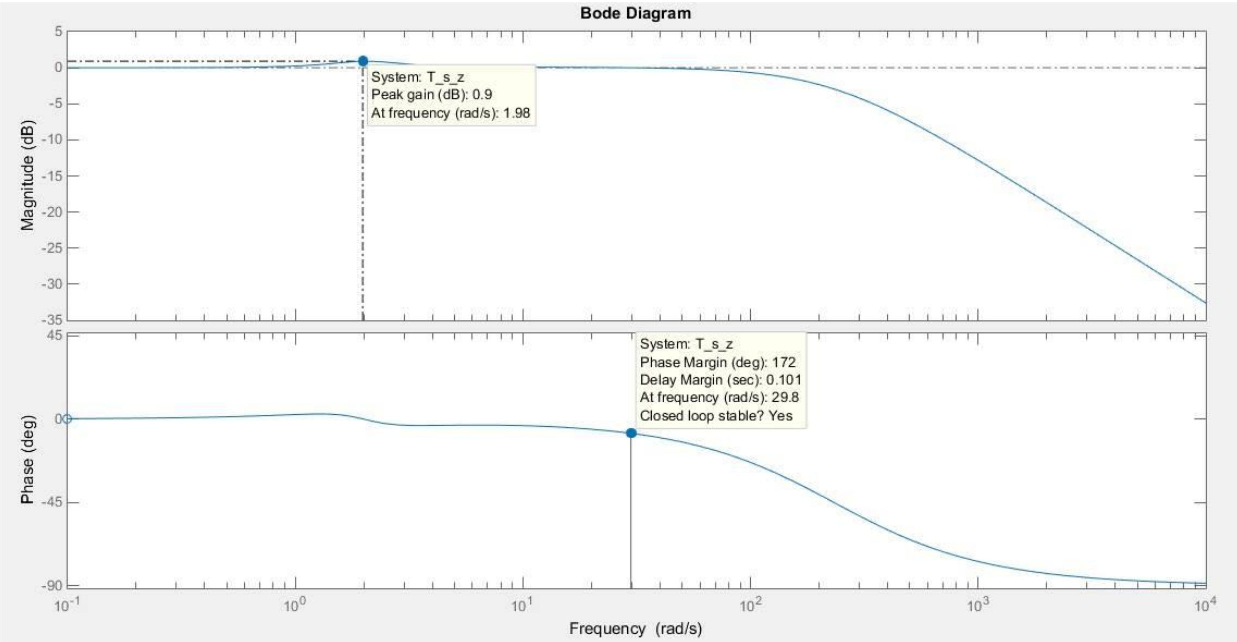

Figure 39-Bode plot of the closed-loop system with the PID gains tuned using the Ziegler-Nichols method

As it can be seen from Figure 39, the closed loop system has a phase margin of 172 degrees and infinite gain margin.

Summary:

The purpose of this chapter was to determine which controller should be implemented in order to achieve the goals of this project (either a PD or a PID controller). Due to the fact that the PD controller will never eliminate the steady-state error of the system, the PID controller is the best choice for implementation purposes. After the controller which will be implemented in the prototype has been determined, the gains of the specific controller must be tuned. Before tuning the PID, the design specifications have been defined in order to get an idea what the response of the system should look like. The tuning methods used in order to determine the PID gains are: PID tune feature offered by Simulink, the Root Locus method and the Ziegler-Nichols method. There are some uncertainties if the design specifications have been met using the Root Locus and Ziegler-Nichols methods. After the PID gains have been determined with 3 different tuning methods, the limitations of the closed-loop system have been determined using the Bode plot. Each of the 3 closed loop systems with PID gains determined through different methods have infinite gain margin. Out of the 3 closed-loop systems, the most robust are the systems with the PID gains determined through Ziegler-Nichols and Root Locus methods, as they have higher phase margins.

Due to various reasons, the group did not have time for implementation purposes, the main emphasis being on the theoretical part of how a Quadcopter can be stabilized on the pitch axis. The differences between theory and practice in this project cannot be told, as the system is robust in closed-loop with a PID controller, but the actual system might not be this robust. Furthermore, when working with the actual system, a lot of things can go wrong as the system is inherently unstable.

Chapter 3: Results

In the previous, the group has used a free body diagram in order to describe the forces acting on the body of the Quadcopter. Using this free body diagram, the equations describing the dynamic behavior of the system on the pitch axis have been derived. These equations have been converted from the time domain into the complex Laplace domain in order to obtain the transfer function of the system. The resulting transfer function was a second order one, meaning that it had 2 poles. By analyzing the location of the poles in the s-plane, the group concluded that the system is unstable in the open-loop (it has no bounded output for every bounded input).

In order to see which controller would stabilize the system in the closed-loop, an open loop analysis has been performed. This analysis consists of introducing different controllers in the open-loop system (each controller has a gain of 1 in this stage) in order to analyze the stability of the system in closed-loop through Bode plots. At the end of this analysis, it has been concluded that a PD or PID controller could stabilize the system in closed-loop.

In the closed-loop analysis, it has been concluded that a PID would suit best the purpose of this project, to stabilize the Quadcopter on the pitch axis without having any deviation, because the system had a steady-state error that could not be correct with only a PD controller. The tuning of the controller is the next step in the evolution of the analysis and in order to have a reference of what the output of the system should be, some control design specifications have been created. The system must have no steady-state error, an overshoot of less than 10%, a rise time and a settling time less than 2 seconds. Following these design specifications, the PID has been tuned using 3 different tuning methods: the PID tune function offered by Simulink, the Root Locus and Ziegler-Nichols methods. The resulted gains for the PID using these 3 tuning methods are different and only PID tune feature from Simulink method offers a relative tracking of the design specifications. In the other two methods, there is some degree of uncertainty about some of the design specifications, meaning overshoot. A closed-loop Bode plot had been generated for each closed-loop system which includes a PID with the gains determined through the tuning of the controller with the 3 methods. The system proved to be very robust as it has infinite gain margin in all cases and really high phase margin. The system proved to be efficient in rejecting disturbances and following the set point (as it can be seen in Figure 33).

From the theoretical point of view, the group managed to stabilize the system in closed-loop so it can be said that the purpose of this project has been met.

From the experimental point of view, due to time issues caused by poor time management, the group was unable to implement the PID in Arduino and see what the response of the system was using the gains determined through the tuning methods in order to see the validity of the theoretical approach.

Chapter 4: Discussions

The analysis and control of a Quadcopter on the pitch axis has turned out to be harsh from the point of view of the mathematics involved. The equations describing the dynamic behavior of a plane, described in different books, are high level mathematics that cannot be grasped with ease. Due to this fact, the group has developed a free body diagram of the system from which the equations describing the motion of the Quadcopter on the pitch axis could be derived. This could lead to some inaccuracies between the equations describing the motion of the system on the pitch axis and the actual motion of the system on this axis as there were many parameters neglected such as air drag and friction with the stand in order to simplify the equations.

Although the purpose of this project is to have a complete theoretical analysis of how the control of a Quadcopter on the pitch axis can be achieved as well as a strong practical approach, due to poor time management and lack of understanding of the mathematics involved in the description of the system, the group did not have time to implement the PID controller in the actual system and make a comparison between the theoretical approach and the real time situation.

The learning goals of this project have been met to a certain degree, as most of the theoretical analysis has been performed and the purpose of this project has been met. The main issue with this project was the lack of implementation. Implementing a PID into the actual system would have given the group a good insight if the theoretical approach was on the right track and what is the difference between theory and practice. Furthermore, the limitations of the system are hard to evaluate as the system seems very robust in closed-loop with a PID controller (in theory) whereas in practice is hard to predict its behavior or if the PID gains obtained through tuning are correct for the actual system.

The time management of this project was poor due to various reasons. The supervisor was not used almost at all; mostly the group followed the advice from the actual exam and the learning goals which were designed for the 4th semester project.

Further implementation could consist of extending the control system from one axis to all axes in order to make a fully operational Quadcopter. Another further implementation could be making a Quadcopter which is able to map terrains, buildings and so on. The further implementations of this project are very broad.

Annex

UAV-Unmanned Aerial Vehicle

DSLR-Digital single-lens reflex camera

Li-Po-Lithium Polymer

RC-Remote Control

PWM-Pulse-Width Modulation

ESC-Electronic Speed Controller

MIMO-Multiple-Input Multiple-Output

I/O-Input/Output

RPM-Revolutions/Rotations Per Minute

BEC-Battery Elimination Circuit

LCR meter-Inductance-Capacitance-Resistance Meter

DC-Direct Current

ODE-Ordinary Differential Equation

LHP-Left-Half Plane

Kv-Rotations per Minute per Applied Volt

DOF-Degree of Freedom

RHP-Right-Half Plane

References

1) http://sal.aalto.fi/publications/pdf-files/eluu11_public.pdf
2) http://ffden-2.phys.uaf.edu/webproj/212_spring_2014/Clay_Allen/clay_allen/history.html
3) http://ffden-2.phys.uaf.edu/webproj/212_spring_2014/Clay_Allen/clay_allen/history.html
4) http://digital.ni.com/public.nsf/allkb/294E67623752656686256DB800508989
5) http://ctms.engin.umich.edu/CTMS/index.php?example=MotorPosition&section=SimulinkModeling
6) http://www.farnell.com/datasheets/55280.pdf
7) http://www.farnell.com/datasheets/55280.pdf
8) Feedback Control Of Dynamic Systems, Gene F.Franklin, J.David Powell, Abbas Emami-Naeini, Third Edition, pages 48-49
9) Feedback Control Of Dynamic Systems, Gene F.Franklin, J.David Powell, Abbas Emami-Naeini, Third Edition, pages 48-49
10) Feedback Control Of Dynamic Systems, Gene F.Franklin, J.David Powell, Abbas Emami-Naeini, Third Edition, pages 48-49
11) http://airspot.ru/book/file/73/hull_airplane_flight_mechanics.pdf, Chapter 2
12) https://engineering.tamu.edu/media/2443120/aggiechallenge-fireflight-motor.pdf
13) https://www.grc.nasa.gov/www/k-12/airplane/airprop.html
14) https://www.moodle.aau.dk/pluginfile.php/452860/mod_resource/content/1/mm2-15.pdf
15) https://www.moodle.aau.dk/pluginfile.php/452860/mod_resource/content/1/mm2-15.pdf
16) http://www.facstaff.bucknell.edu/mastascu/econtrolhtml/Freq/Freq6.html
17) http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlPID
18) http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlPID
19) http://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlPID
20) http://lpsa.swarthmore.edu/Root_Locus/RootLocusWhy.html
21) https://controls.engin.umich.edu/wiki/index.php/PIDTuningClassical
22) https://controls.engin.umich.edu/wiki/index.php/PIDTuningClassical
23) https://controls.engin.umich.edu/wiki/index.php/PIDTuningClassical

Figure references

1)http://www.quadhangar.com/how-to-correct-a-drifting-quadcopter/
2)http://www.socialledge.com/sjsu/index.php?title=S14:_Quadcopter
3)http://www.ni.com/white-paper/7054/en/
4)http://file.scirp.org/Html/12-7900231_35654.htm
5) https://controls.engin.umich.edu/wiki/index.php/PIDTuningClassical