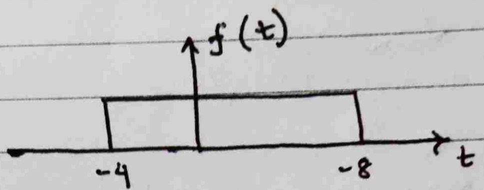
(N H)

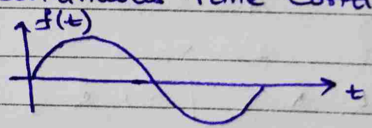
20/08/24

Signals & Systems

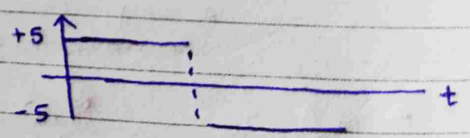


- Dependant variable that depends on some independant variable.
- Physical quantity
- Have some information that can be displayed, manipulated and even conveyed.

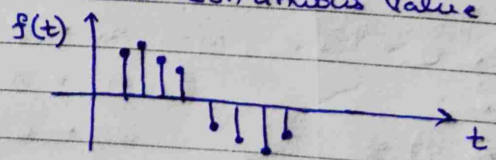
→ There are 4 types of analog signal -
(i) CTCV (ii) CTDV (iii) DTCV (iv) DTDV (Quantization)
↓
(Continuous Time Continuous Value)



(ii) Continuous Time Discrete Value



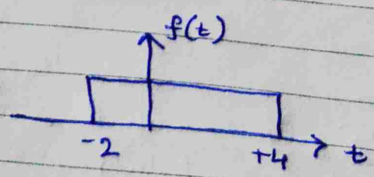
(iii) Discrete Time Continuous Value



→ Operations :

1. Time Shifting -

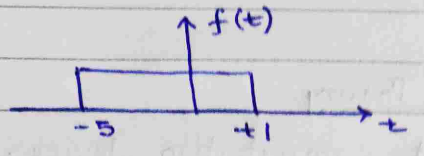
* $f(t) =$



'+' → Advanced
'–' → Delay

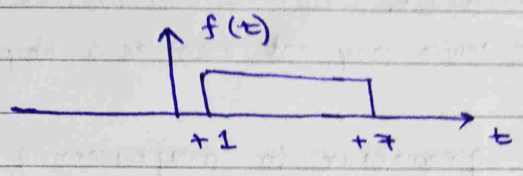
4/2024					
Monday	1	8	15	22	29
Tuesday	2	9	16	23	30
Wednesday	3	10	17	24	–
Thursday	4	11	18	25	–
Friday	5	12	19	26	–
Saturday	6	13	20	27	–
Sunday	7	14	21	28	–

$f(t+3)$ =



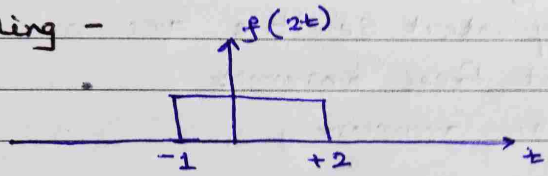
Advanced (shifting towards left)

$f(t-3)$ =



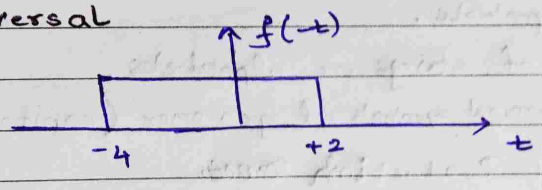
Delay (shifting towards right)

2. Time Scaling -

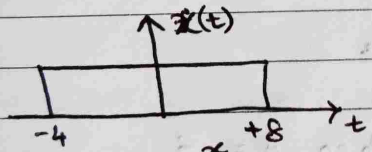


(Divide)

3. Time Reversal

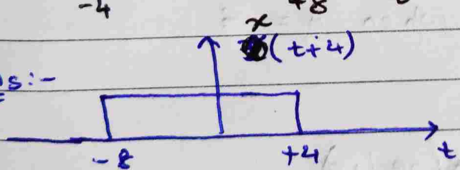


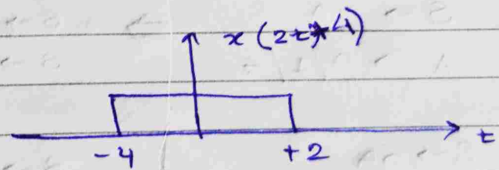
Q:

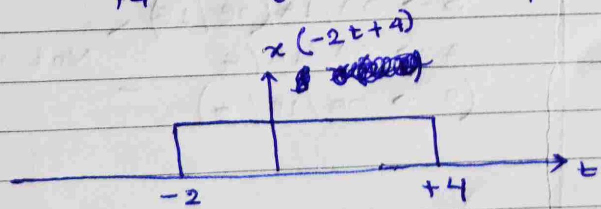


Find $x(-2t+4)$.

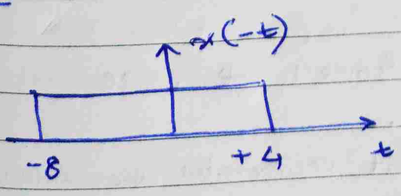
Ans:-

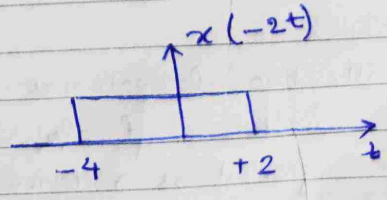


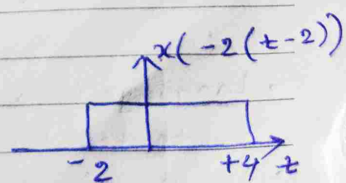




$x[-2(t-2)]$





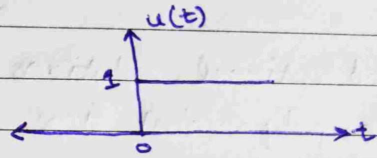


22/08/24 (NH)

→ **Elementary Signals :**

• Unit Step –

$$u(t) = 1 \quad \text{for } t \geq 0$$
$$= 0 \quad \text{for } t < 0$$

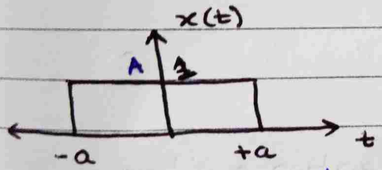


for scaling operation, unit step signal will remain same.

* Causal Signal – Causalty – $x(t) = 0$ for $t < 0$

– Anti-causalty – $x(t) = 0$ for $t > 0$

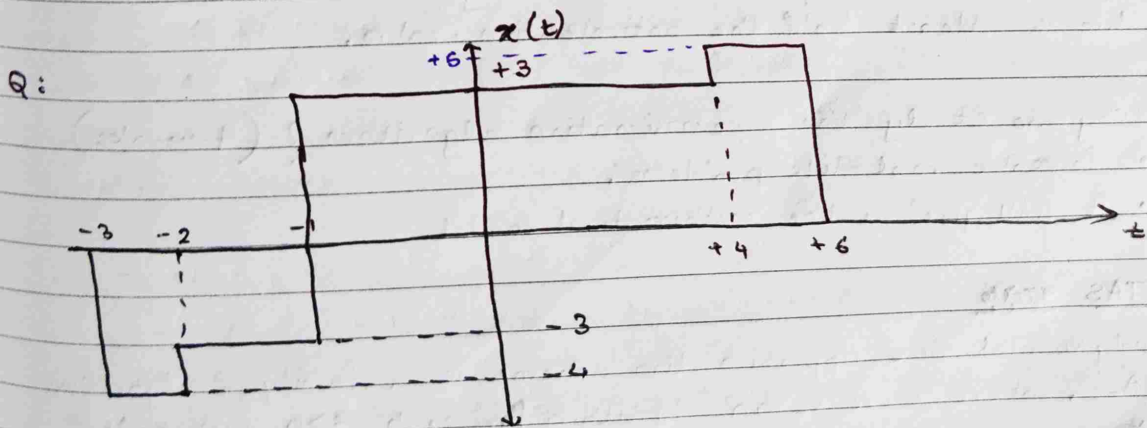
– Non-causalty – $x(t) \neq 0$ for $t < 0$



$$x(t) = u(t+a) - u(t-a)$$

↳ $x(t) = +A\,u(t+a) - A\,u(t-a)$

Q :



* $x(t) = -4\,u(t+3) + [illegible]\,u(t+2) + 6\,u(t+1) + 2\,u(t-4) - 5\,u(t-6)$

25 MARCH Monday

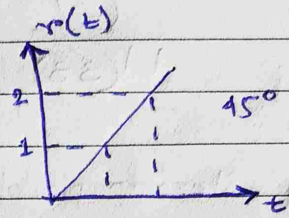
3/2024				
Monday	–	4	11	18
Tuesday	–	5	12	19
Wednesday	–	6	13	20
Thursday	–	7	14	21
Friday	1	8	15	22
Saturday	2	9	16	23
Sunday	3	10	17	24

85/281 27/08/24 (NH) 13.week

• Ramp Signal –

$r(t) = t$ for $t \geq 0$

$= 0$ otherwise

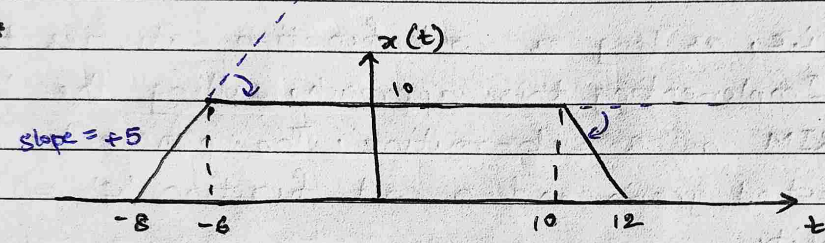


45° $\frac{\Delta x}{\Delta y}$

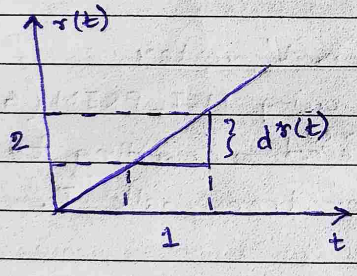
$r(t) = A.t \Rightarrow A = \text{slope}$

$\boxed{r(t) = t.u(t)}$

#

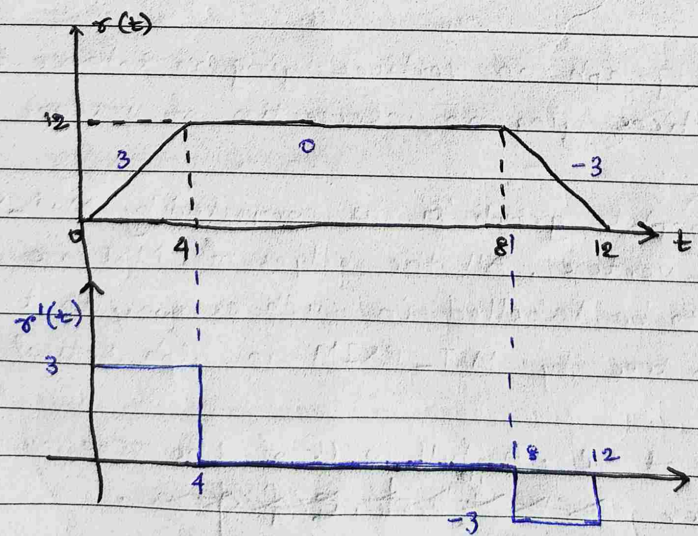


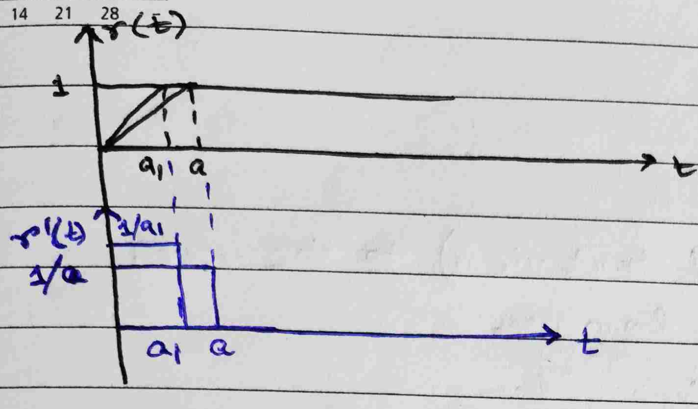
$r(t) = 5r(t+8) - 5r(t+6) - 5r(t-10) + 5r(t-12)$



$\text{slope} = \frac{d[r(t)]}{dt} = A.u(t)$

#





NOTE : Derivative of Ramp signal → Step signal
Derivative of step signal → Impulse signal

$$\delta(t) = \frac{d(u(t))}{dt}$$

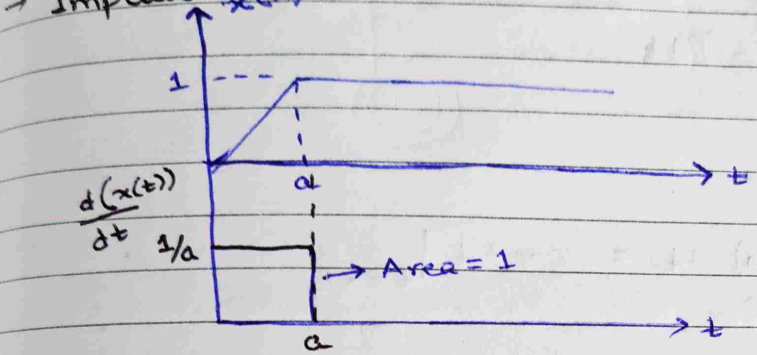
$\delta(t) = 1$ (defines the area)

$\delta(t) = \infty$ (defines the value)

03/09/24

(NH)

→ Impulse :

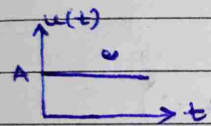


- magnitude at $t=0$ is ∞
- area at $t=0$ is 1

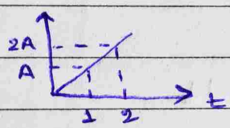
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$$\int_{0^-}^{0^+} \delta(t)\, dt = 1$$

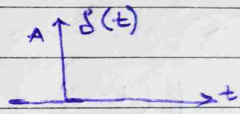
(i)



(ii)



(iii)

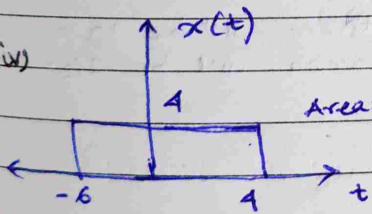


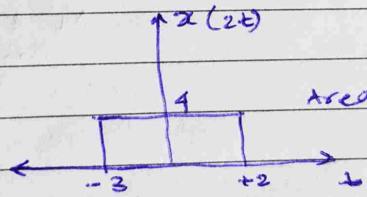
$A\,u(t)$ → magnitude

$A\,r(t)$ → Slope

$A\,\delta(t)$ → Area

(iv)

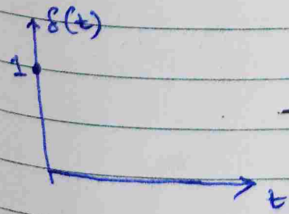




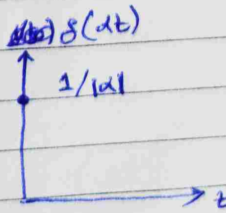
- Scaling changes area, it does not change magnitude.

$$\text{area}\left[x(\alpha t)\right] = \frac{1}{|\alpha|} \cdot \text{area}\left[x(t)\right]$$

Scaling



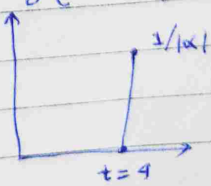
→



→

scaling and shifting

$\delta(\alpha(t-4))$ (Delay)



NOTE : • $x(t) \cdot \delta(t) = x(0)$ [Value at $t=0$]

• $x(t) \cdot \delta(t-t_0) = x(t_0)$

• $x(t) \cdot \delta(t+t_0) = x(-t_0)$.

$$I = \int_{-6}^{+8} (t^2+4)\, \delta(t-3)\, dt$$

$\underline{t=3}$ $= 13$

→ $\int_{-\pi}^{\pi} \cos^2 t\, \delta(t-\pi/4)\, dt = \cos^2 t \Big|_{t=\pi/4}$

→ $\int_{-3}^{4} (t^3+5) \cdot \delta(t-6)\, dt = 0$

→ $\int_{-6}^{+5} (t-2)\, \delta(2t-4)\, dt$

$= \int_{-6}^{+5} (t-2)\, \frac{1}{2}\, \delta(t-2)\, dt$

$= 0$

For $\delta(2t-4) \xrightarrow{} 2(t-2)$

Area $= \frac{1}{2}\, \delta(t-2)$

At $t=2$, there will be an impulse whose area will be $1/2$.

(NH)

12/09/24

→ $$I = \int_{-\infty}^{\infty} x(t) \cdot \delta\{g(t)\}\, dt$$

(i) $\delta[g(t)] = 0$ if g' is nowhere zero. For any value of t, $g(t) \neq 0$.

(ii) $\delta[g(t)] = \dfrac{\delta(t-t_0)}{g'(t_0)}$ if g' has one real root at $t = t_0$.

(iii) If 'g' has more than one real root:

$$\delta[g(t)] = \sum_i \frac{\delta(t-t_i)}{|g'(t_i)|} = \frac{\delta(t-t_0)}{|g'(t_0)|} + \frac{\delta(t-t_1)}{|g'(t_1)|} + \dots$$

Q: $$I = \int_{-10}^{10} (t^2+10)\, \delta(t^2+16)\, dt.$$

Ans:- $g(t) = t^2 - 16$ $\quad t_0 = 4,\ t_1 = -4$

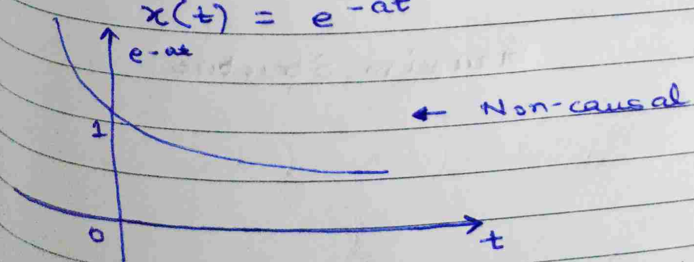
$$= \frac{\delta(t-4)}{|8|} + \frac{\delta(t+4)}{|-8|} = \frac{1}{8}\left[\delta(t-4) + \delta(t+4)\right]$$

$$I = \int_{-10}^{10} (t^2+10)\, \frac{1}{8}\left[\delta(t-4) + \delta(t+4)\right] dt$$

$$= \frac{13}{2}.$$

→ Exponential function:

$$x(t) = e^{-at}$$



$|t| = +t, \quad t \geq 0$

$\quad\;\; -t, \quad t < 0$

$e^{-a|t|} \cdot ut = e^{-at}$

$$\left.\begin{aligned} e^{-a|2|} &= e^{-2a} \\ e^{-a|-2|} &= e^{-2a} \end{aligned}\right\}$$

* $e^{-a|t|} = e^{-a(t)} \cdot u(t) + e^{-a(-t)}\, u(-t)$

$= e^{-at} \cdot u(t) + e^{at} \cdot u(-t)$

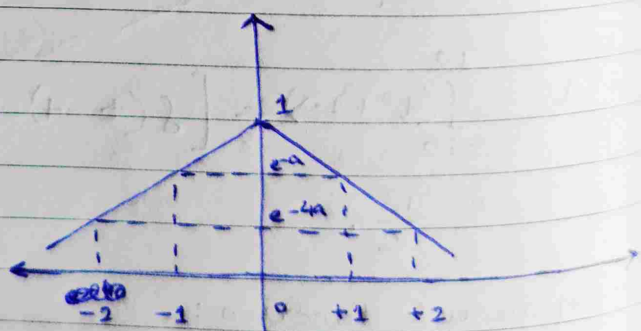
t	$e^{-at} \cdot u(t)$
0	1
1	e^{-a}
2	e^{-2a}
3	e^{-3a}

t	$e^{at} \cdot u(-t)$
-1	e^{-a}
-2	e^{-2a}
-3	e^{-3a}

In both cases, decaying rate is same.

→ $x(t) = e^{-at^2}$

t	$x(t)$
0	1
1	e^{-a}
2	e^{-4a}
-1	e^{-a}
-2	e^{-4a}



Gaussian Structure

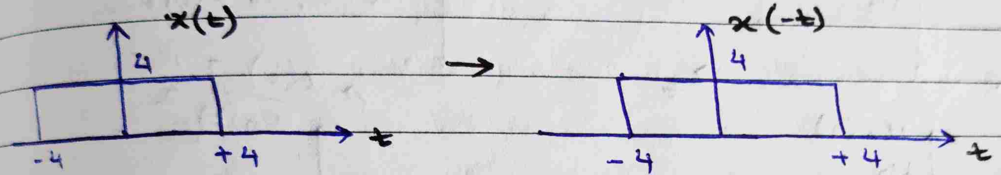
19/09/24 (NH)

→ Classification of Signals :

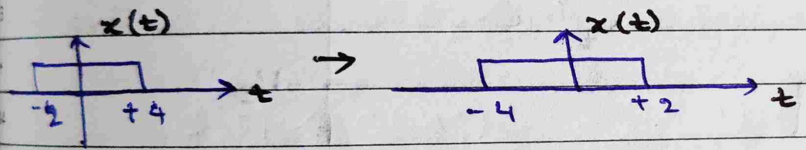
(i) Even and Odd Signals -

$x(-t) = x(t) \rightarrow$ even

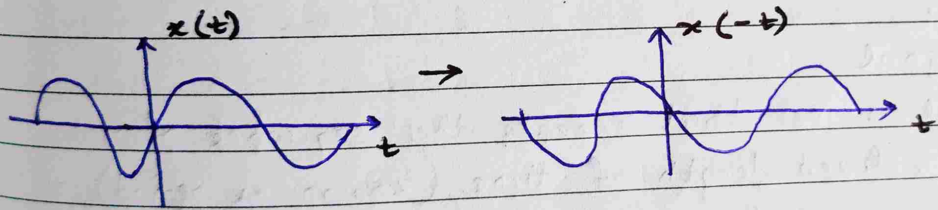
$x(-t) = -x(t) \rightarrow$ odd



Even signal

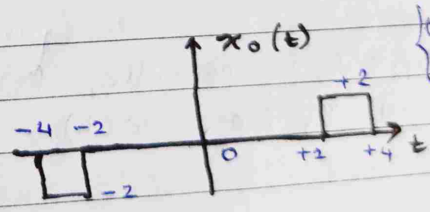


neither even nor odd

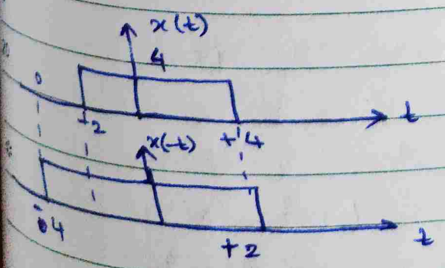


odd signal

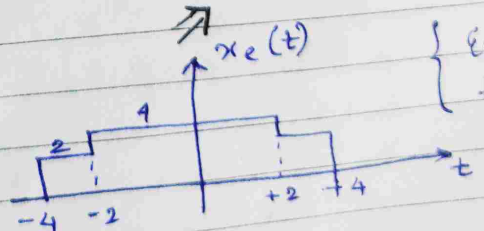
- $x_e(t) = \dfrac{x(t) + x(-t)}{2}$
- $x_o(t) = \dfrac{x(t) - x(-t)}{2}$



{odd part of the signal}



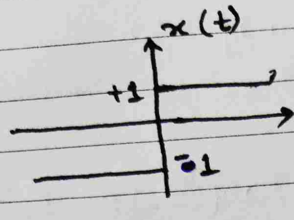
⇒



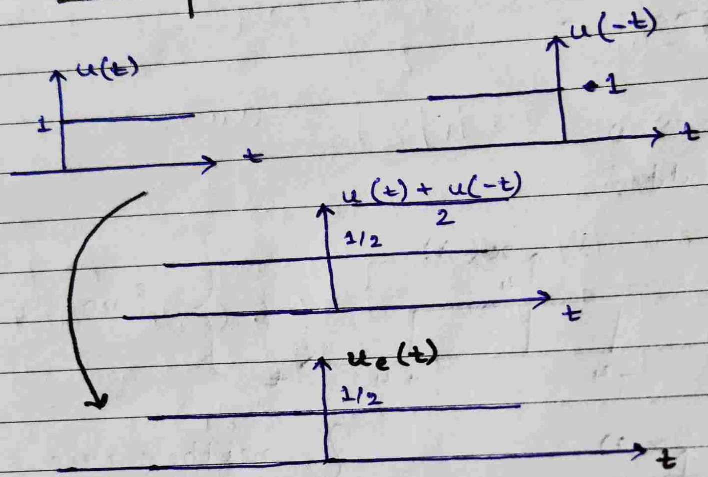
{Even part of the signal}

[Point to point calculation for getting the even signal]

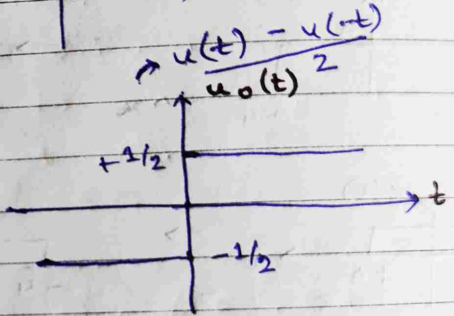
Q: Show that even and odd parts of $u(t)$ can be represented in parts of $x(t)$.



Ans:-



At 0, value is undefined.
we consider $t=0$, $u(t)=1$.
Graph starts from 0^+ and 0^-.
At 0, no value.



$\therefore$ $\boxed{u(t) = \frac{1}{2}\, x(t)}$

(ii) Periodic Signal

- A periodic signal is one that repeats the sequence of values, exactly after a fixed length of time (known as period) and it must be in the range from $-\infty$ to $+\infty$.

- A signal is periodic if it repeats itself after a certain period T, i.e, $x(t) = x(t+T)$.

22/10/24 (NH)

→ Periodic and Aperiodic :

i) Sinusoidal

$x(t) = \underbrace{A}_{\text{Amplitude}} \sin(\omega_0 t \pm \phi)$ — ϕ → Phase shift; $\omega_0 t \pm \phi$ → Phase Angle

ω → Angular frequency.

$T = \frac{2\pi}{\omega}$

Q: $x(t) = 5\cos(300\pi t)$. Periodic or aperiodic? If periodic calculate fundamental period.

Ans:- $\omega_0 = 300\pi$

$T = \frac{2\pi}{300\pi} = 1/150$ second ← Fundamental period.

→ $x(t) = 4\cos\pi t + 3\sin 2\pi t + 2\sin 3\pi t$

$\omega_{0_1} = \pi$ $\quad \omega_{0_2} = 2\pi$ $\quad \omega_{0_3} = 3\pi$

$T_1 = 2s$ $\quad T_2 = 1s$ $\quad T_3 = 2/3\ s$

$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_1}{T_3}$ Check if the ratio is rational or not.

→ Ratio of p/q where p & q are integers

→ Decimal values must be terminating or repeating.

$T = LCM[T_1, T_2, T_3]$

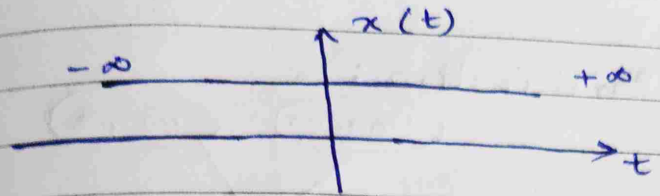
for Ratio → $T = \frac{LCM[\text{Numerator}]}{HCF[\text{Denominator}]}$

NOTE : $T = \pi$ → Signal is aperiodic.

∴ $T = \frac{2}{1}$ second

→ Phase shift does not hamper periodicity.

ii) Constant or DC signal



- Periodic signal without any periodic value.

$$x(t) = 5\cos(20\pi t + \pi/2)$$

$$\hookrightarrow T = \frac{2\pi}{20\pi} = 1/10$$

$$x(t) = 4 + 5\cos(20\pi t + \pi/2)$$

$$\hookrightarrow T = 1/10$$

- $x(t)$ = Even of $[\sin 4\pi t \cdot u(t)]$ Not a periodic signal.

$$x_1(t) = \sin 4\pi t \cdot u(t)$$

$$x_1(-t) = -\sin 4\pi t \cdot u(-t)$$

$$\text{Even}[x_1(t)] = \frac{x_1(t) + x_1(-t)}{2}$$

$$= [\sin 4\pi t\ u(t) - \sin 4\pi t\ u(-t)]/2$$

$$= \sin 4\pi t \left[\frac{u(t) - u(-t)}{2}\right]$$

(Result is not periodic.)

(NH)

05/11/24

→ Complex exponential signal :

$x(t) = e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$

↳ real magnitude

↳ imaginary magnitude

$\text{phase} = t^{-1}\left(\frac{\sin \omega_0 t}{\cos \omega_0 t}\right) = \omega_0 t$

⇒ derivative of $\omega_0 t$ is phase angle

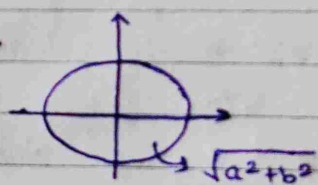
$|x(f)| = 1$ (magnitude)

Complex rectanguler signal is of two type / format -

(i) Polar format :

$r \angle \theta$ ⇒

↙ radius ↘ angle



(ii) Rectangular format :

$a + jb$

NOTE : Combination of Periodic signal is also periodic .

$x = a + jb$

$\sqrt{x}$

$r = \sqrt{a^2+b^2}$

$\angle\theta = \tan^{-1}(b/a)$

Q: $x = 3 + j4$

Ans:- $r = \sqrt{3^2+4^2} = 5$

$\angle\theta = \tan^{-1}(b/a)$

$= \tan^{-1}(4/3)$

$j = 90^\circ$

$j^2 = 90^\circ + 90^\circ = 180^\circ$

$j^3 = 90^\circ + 90^\circ + 90^\circ = 270^\circ$

$x = 5 \tan^{-1}(4/3)$

$\sqrt{x} = \sqrt{5} \cdot \angle \frac{\tan^{-1}(4/3)}{2}$

Q: $x = \dfrac{(4+j5)(3+j6)}{(8+j4)(9+j3)}$

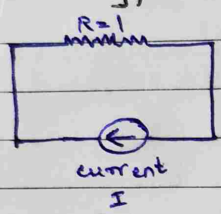
Ans:-

$$x = \frac{\sqrt{41} \angle \tan^{-1}(5/4) \cdot \sqrt{45} \angle \tan^{-1}(2)}{\sqrt{80} \angle \tan^{-1}(1/2) \cdot \sqrt{90} \angle \tan^{-1}(1/3)}$$

$$= \frac{\sqrt{41} \cdot \sqrt{45}}{\sqrt{80} \cdot \sqrt{90}} \cdot \frac{[\tan^{-1}(5/4) + \tan^{-1}(2)] \text{ degree}}{[\tan^{-1}(1/2) + \tan^{-1}(1/3)] \text{ degree}}$$

12/11/24 (NH)

→ Energy and Power Signal :



$$P_x = [i(t)]^2_{RMS} R = \frac{V^2_{RMS}}{R}$$

$$= (I_{RMS})^2$$

$$= (V_{RMS})^2$$

$$\frac{1}{T}\int_{-T/2}^{T/2} |i(t)|^2 \, dt$$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |i(t)|^2 \, dt$$ ← Aperiodic Signal

$$E_x = \text{Power} \times \text{Time}$$

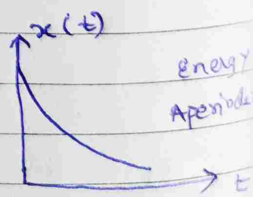
$$= \lim_{T\to\infty} \frac{1}{T} \cdot T \int_{-T/2}^{T/2} |i(t)|^2 \, dt$$

$$E_x = \int_{-\infty}^{\infty} |i(t)|^2 \, dt$$

NOTE :- Energy → Finite ∞, Power → 0
Power → Finite, Energy → ∞

Q :

(i) $x(t) = e^{-4t} \cdot u(t)$ ⇒ Energy Signal



$$E_x = \int_{0}^{\infty} e^{-8t} \, dt = \left.\frac{e^{-8t}}{8}\right|_0^{\infty} = \frac{1}{8} \quad \text{(finite)}$$

$$P_x = 0$$

(ii) $x(t) = A \cdot \sin\omega t$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt = \frac{A^2}{2}$$

For an sinusoidal signal, RMS value $= A/\sqrt{2}$.

For Power $= A^2/2$ $\quad \sqrt{\frac{1}{T}\int_{-T/2}^{T/2} |A\sin\omega t|^2\,d\omega t}$

(iii) $x(t) = u(t)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\infty}^{\infty} 1 \cdot dt = \infty$$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |u(t)|^2\,dt$$

$$= 1/2$$

(iv) $x(t) = \sin t \cdot u(t)$ $\Rightarrow$ Power Signal

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \sin^2 t\,dt \qquad E_x = \infty$$

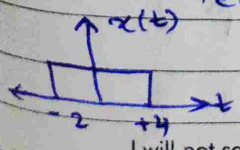
$$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} \frac{1-\cos 2t}{2}\,dt$$

$$= 1/4$$

(i) $\rightarrow$ $x(t) = 0$ at $t \to \infty$ and if value of $x(t)$ decreases with time $\Rightarrow$ Energy Signal.

(ii) $\rightarrow$ w.r.t time value is neither zero ~~or~~ or infinite $\Rightarrow$ Power

Periodic Signal $\Rightarrow$ Power Signal

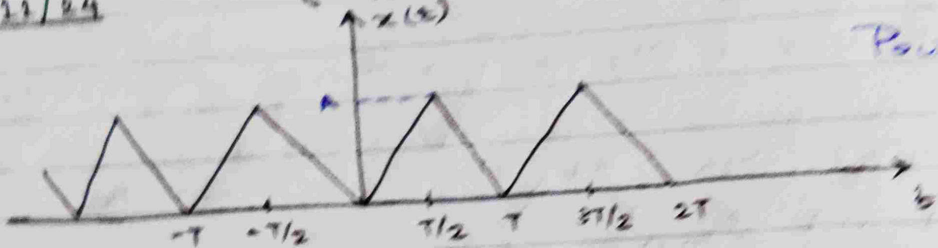


Energy is finite and magnitude is finite in that range $\Rightarrow$ Energy Signal.

14/11/24

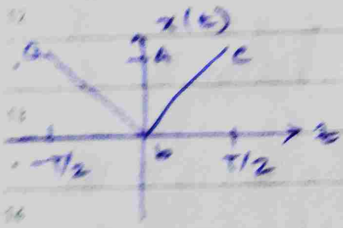
(NH)

Q :



Power signal

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 \, dt$$



$y_{ab} = -\frac{2A}{T} t + 0$ for $-\frac{T}{2} < t < 0$

$y_{bc} = \frac{2A}{T} \cdot t + 0$ for $0 < t < T/2$

$$P_x = \frac{1}{T}\int_{-T/2}^{0} \frac{4A^2}{T^2} \cdot t^2 \, dt + \frac{1}{T}\int_{0}^{T/2} \frac{4A^2}{T^2} \cdot t^2 \, dt$$

$$= \frac{4A^2}{T^3}\left[\frac{t^3}{3}\Big|_{-T/2}^{0} + \frac{t^3}{3}\Big|_{0}^{T/2} \right]$$

$$= \frac{4A^2}{3T^3}\left[\frac{T^3}{8} + \frac{T^3}{8} \right] = \frac{A^2}{3} \text{ (Ans)}$$

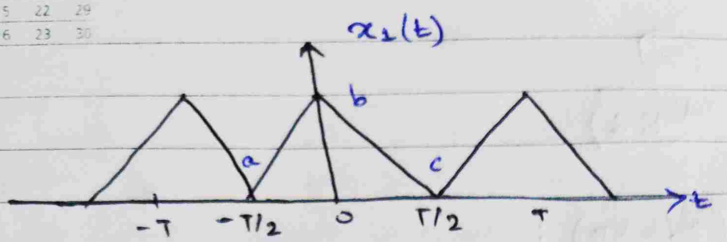
$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= P_x * time = \infty$$

Now,

$$x_1(t) = x(t - T/2)$$

Q :



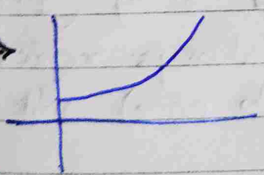
$Y_{ab} = +\frac{2A}{T} t + A$ $\qquad$ $Y_{bc} = -\frac{2A}{T} \cdot t + A$

$$\therefore P_x = \frac{1}{T}\left[\int_{-T/2}^{0} \left(\frac{4A^2}{T^2} \cdot t^2 + A^2 + \frac{4At}{T}\right) dt + \int_{0}^{T/2}\left(A^2 - \frac{4At}{T} + \frac{4A^2t^2}{T^2}\right) dt \right]$$

$$= \frac{1}{T}\left[\frac{4A^2}{T^2}\cdot \frac{t^3}{3}\Big|_{-T/2}^{0} + A^2 \cdot T/2 + \frac{4A}{T}\left(\frac{t^2}{2}\Big|_{-T/2}^{0}\right) + A^2\cdot\frac{T}{2} - \frac{4A}{T}\frac{t^2}{2}\Big|_{0}^{T/2} + \frac{4A^2}{T^2}\cdot\frac{t^3}{3}\Big|_{0}^{T/2}\right]$$

$$\boxed{P_x = \frac{A^2}{3}}$$

⇒ Thus signals which are not bounded ~~by~~ are neither energy nor power signals.

⇒  (Unbounded Signal) → $t \to \infty$, $x(t) \to \infty$

Example - $e^{2t} u(t)$

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} e^{4t}\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{e^{4t}}{4}\right]_{0}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T}\frac{e^{T}}{4} = \infty \times \qquad \frac{\infty}{\infty}$$

$\boxed{\begin{array}{l} E_x = \infty \\ P_x = 0 \end{array}}$

28/12/24 (NH)

→ Continuous Time Fourier Transform : → convert time signal to frequency signal.

Why?? = - frequency domain gives more features and analysed signal.

- If the signal is periodic then only it can be represented in frequency domain for its time domain signal.

** Linear combination of harmonically related complex exponential.

$$F[a_1 x_1(t) + a_2 x_2(t)] = a_1 F[x_1(t)] + a_2 F[x_2(t)]$$

$$\Rightarrow a\, y(t) = a_1 y_1(t) + a_2 y_2(t)$$

↓

operated together or splitted and then operated together, the result will be same.

- If we consider a periodic signal which is not a sinusoidal wave with a time period T. Now, if we represent a sinusoidal signal with the same time period, it is 1^{st} Harmonic.
- Half the time period $(T/2)$ ⇒ 2^{nd} Harmonic
  └ [double the frequency]

Harmonically related — Any periodic signal can be represented by harmonic signal and the frequency of the signal will be same as the given signal and the frequency increases or related by a simple ratio.

$e^{j\omega t}$, $e^{j2\omega_0 t}$, $e^{j3\omega_0 t}$ ⇐ Harmonically related

Complex exponential

Step 1

- Combining magnitude and phase to get frequency domain signal.

Synthesis

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Analysis

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt.$$

$k = 1 \rightarrow 1^{st}$ Harmonic

$2 \rightarrow 2^{nd}$ Harmonic

a_k = fourier series coefficient

[Using synthesis equation only]

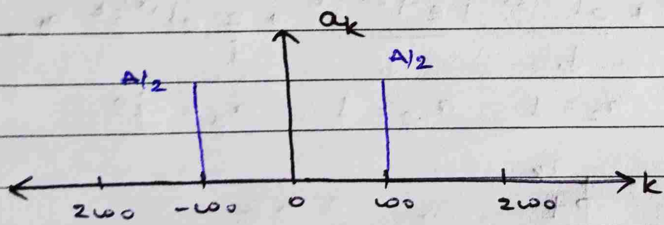
Q: $x(t) = A\cos\omega_0 t$

Ans:- $x(t) = A \cdot \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$

$= \dfrac{A}{2} e^{j\omega_0 t} + \dfrac{A}{2} e^{-j\omega_0 t}$

~~this~~ for signal

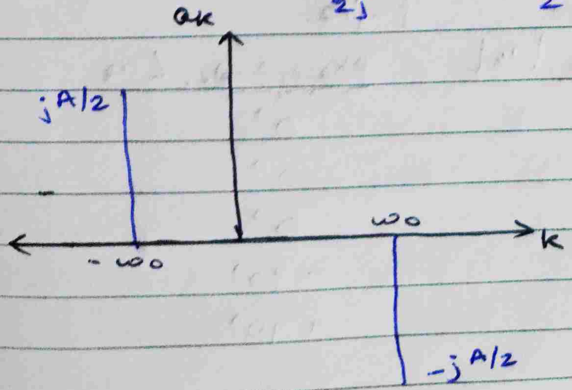
$a_1 = A/2 \qquad a_{-1} = A/2 \rightarrow$ fourier series coefficients



Q: $x(t) = A\sin\omega_0 t$

Ans:- $x(t) = A \cdot \left(\dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right)$

$a_1 = \dfrac{A}{2j} = -j\dfrac{A}{2}$, $a_{-1} = \dfrac{-A}{2j} = j\dfrac{A}{2}$



Q : $x(t) = 2 + \sin \omega_0 t + 2\cos \omega_0 t + \cos\left[2\omega_0 t + \pi/4\right]$

Q : $x(t) = 4 + 2\cos \frac{2\pi}{3} t + 4 \sin \frac{5\pi}{3} t$

Ans:- $\omega_{0_1} = 2\pi/3$ $\quad T_1 = 3$

$\omega_{0_2} = 5\pi/3$ $\quad T_2 = 6/5$

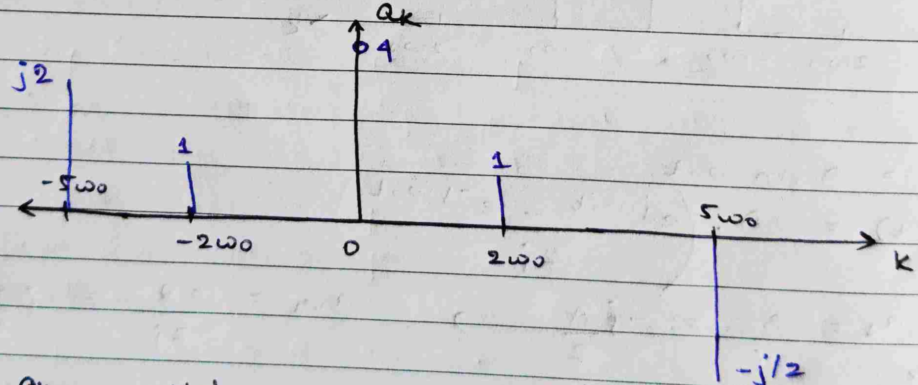
$T = LCM\left[\frac{3}{1}, \frac{6}{5}\right]$ $\quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6}$

$= 6$ second

Fundamental frequency $\Rightarrow$ $\boxed{\omega_0 = \frac{\pi}{3}}$ Also the first harmonic frequency.

$$x(t) = 4 \cdot e^{j0\omega_0 t} + 2\,\frac{e^{j2\pi/3\, t} + e^{-j2\pi/3\, t}}{2} + 4\,\frac{e^{j5\pi/3\, t} - e^{-j5\pi/3\, t}}{2j}$$

$$= \underbrace{4 \cdot e^{j0\omega_0 t}}_{a_0 = 4} + \underbrace{e^{j2\pi/3\, t}}_{a_2 = 1} + \underbrace{e^{-j2\pi/3\, t}}_{a_{-2} = 1} + \underbrace{\frac{2}{j} e^{j5\pi/3\, t}}_{a_5 = -j2} - \underbrace{\frac{2}{j} e^{-j5\pi/3\, t}}_{a_{-5} = j2}$$



a_k	Values	magnitude $\lvert a_k \rvert$	Phase Angle $\angle a_k$
a_0	4	4	0°
a_2	1	1	0°
a_{-1}	1	1	0°
a_5	$-j2$	2	-90°
a_{-5}	$+j2$	2	$+90^\circ$

05/12/24

(NH)

→ Properties of Fourier Series Coefficients :

1. **Linearity** : $x(t) \xleftrightarrow{FS} a_k$, $y(t) \xleftrightarrow{FS} b_k$

$z(t) = [\alpha x(t) + \beta y(t)] \xleftrightarrow{FS} c_k$

then $c_k = \alpha a_k + \beta b_k$.

2. **Time shifting** : $x(t) \xleftrightarrow{FS} a_k$, $x(t \pm t_0) \xleftrightarrow{FS} b_k$

then $b_k = e^{jk\omega_0(\pm t_0)} a_k$

3. **Time Reversal** : $x(t) \xleftrightarrow{FS} a_k$, $x(-t) \xleftrightarrow{FS} b_k$

then $b_k = a_{-k}$

if $x(t)$ even : $b_k = a_k = a_k$

if $x(t)$ odd : $b_k = a_{-k} = -a_k$

For ①

Q: $x(t) = \cos \omega_0 t$ $\quad y(t) = \sin \omega_0 t$

Ans:- $a_1 = 1/2$, $a_{-1} = 1/2$ $\quad b_1 = -j/2$, $b_{-1} = j/2$

Let $\alpha = 1, \beta = 1$ ↳ $z(t) = \cos \omega_0 t + \sin \omega_0 t$

$c_1 = a_1 + b_1 = 1/2 - j/2$

$c_2 = 1/2 + j/2$ . .

$[= a_{-1} + b_{-1}]$

for ②

Q : $x(t) \xleftrightarrow{FS} a_k$. $x(t+t_0) + x(t-t_0) \xleftrightarrow{FS} b_k$. Calculate b_k.

Ans :- $x(t+t_0) \longleftrightarrow e^{jk\omega_0 t_0} a_k$

$x(t-t_0) \longleftrightarrow e^{-jk\omega_0 t_0} a_k$.

$\therefore b_k = 2a_k \left(\frac{e^{j\omega_0 k t_0} + e^{-jk\omega_0 t_0}}{2} \right)$

$= 2a_k \cdot \cos(k\omega_0 t_0)$.

$|b_k| = \cos[k\omega_0(\pm t_0)] + j\sin[k\omega_0(\pm t_0)]$

$= 1 = |a_k|$

$\downarrow$

This means that using the time shifting property there is no change in the magnitude.

$\angle b_k = \angle e^{jk\omega_0(\pm t_0)} + \angle a_k$

$= \tan^{-1}\dfrac{\sin k\omega_0(\pm t_0)}{\cos k\omega_0(\pm t_0)} + \angle a_k$

$\angle b_k = \angle a_k + k\omega_0(\pm t_0)$ $\Rightarrow$ There will be change in phase angle after time shifting

For (3)

Q: $x(t) \longleftrightarrow a_k$. $x(1-t) + x(t-1) \longleftrightarrow b_k$. Calculate b_k.

Ans:-

$x(-t) \longleftrightarrow a_{-k}$ (Here (–) sign come because of $-k$)

$x(-t+1) \longleftrightarrow e^{-jk\omega_0} \cdot a_{-k}$

$x(t-1) \longleftrightarrow e^{-jk\omega_0} a_k$

$b_k = e^{-jk\omega_0} a_{-k} + e^{-jk\omega_0} a_k$

$b_k = e^{-jk\omega_0}[a_k + a_{-k}]$

If even $\rightarrow$ $e^{-jk\omega_0}[2a_k]$

If odd $\rightarrow$ ~~[illegible]~~ 0

$\downarrow$

Since for odd part value is zero, the signal is purely even signal.

4. Time Scaling:

$$x(t) \longleftrightarrow a_k \qquad x(\alpha t) \longleftrightarrow b_k$$

then $b_k = a_k$.

$\left.\begin{array}{l} x(t) = \cos t \\ x(\alpha t) = \cos \alpha t \end{array}\right] \Rightarrow$ there will be no change in coefficients.

Q: $x(t) = \cos 2\omega_0 t$

$$x(t) = \left(e^{j2\omega_0 t} + e^{-j2\omega_0 t}\right)/2$$

$$= \frac{1}{2} e^{j2\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$$a_1 = 1/2 \quad , \quad a_{-1} = 1/2$$

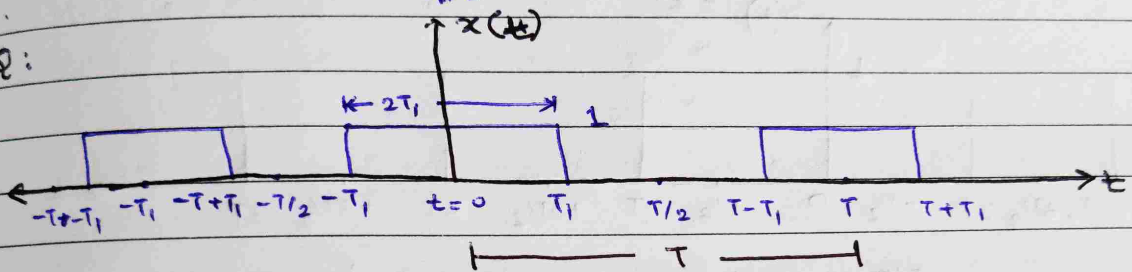
$$x(t) = \cos \omega_0 t \Rightarrow a_1 = 1/2 \, , \, a_{-1} = 1/2$$

10/12/24 (NH)

→ Analysis Equation :

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt$$

Q :



$$a_k = \frac{1}{T} \int_{-T_1}^{T_1} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T} \int_{-T_1}^{T_1} e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T} \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_{-T_1}^{T_1}$$

$$= \frac{1}{T} \frac{e^{-jk\omega_0 T_1} - e^{-jk\omega_0 (-T_1)}}{-jk\omega_0}$$

$$= \frac{1}{T} \frac{2}{k\omega_0} \frac{e^{-jk\omega_0 T_1} - e^{-jk\omega_0 T_1}}{2j}$$

$$= \frac{2}{Tk\omega_0} \sin(k\omega_0 T_1)$$

Here, $\omega_0 = 2\pi/T$ → $\boxed{a_k = \frac{1}{k\pi} \sin(k\omega_0 T_1)}$ For periodic pulse train

Pulse width $= 2T_1$

Time Period $= T$

* [For $k = 0$, value is undefined].

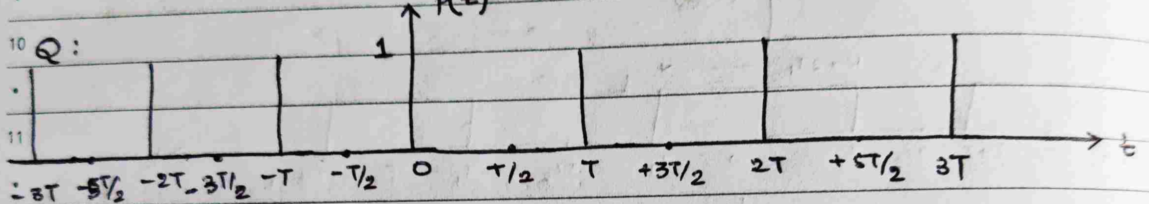
$$a_0 = \lim_{k \to 0} \frac{d/dk\ \sin(k\omega_0 T_1)}{d/dk\ (\pi/k)}$$ [Using L'Hospital Rule]

$$= \lim_{k \to 0} \frac{(\cos k\omega_0 T_1)\ \omega_0 T_1}{\pi}$$

$$\boxed{a_0 = \frac{\omega_0 T_1}{\pi}}$$

$$P(t) = \sum \delta(t - kT) = \frac{1}{T}$$

Q:



This signal is periodic but not sinusoidal

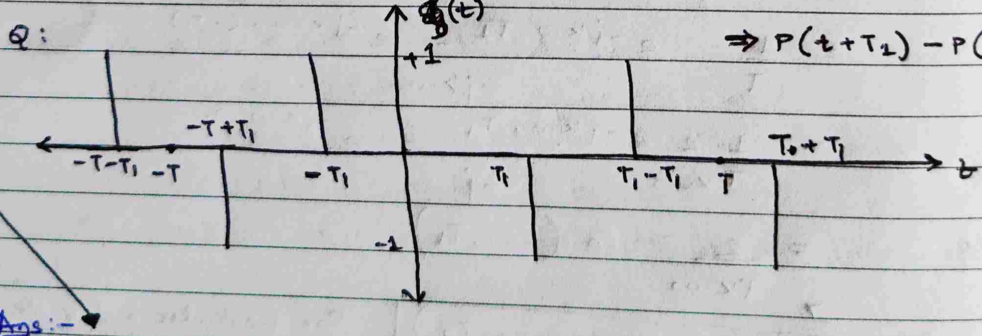
Ans:-

$$a_k = \int_{-T/2}^{T/2} \delta(t)\ e^{-jk\omega_0 t}\ dt$$

$$= \frac{1}{T} \left. \frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right|_{-T/2}^{T/2}$$

$$\boxed{a_k = \frac{1}{T}}$$ ← For impulse train Independent of 'k'

Q:



$$\Rightarrow P(t+T_1) - P(t-T_1) = y(t) \quad \ldots (1)$$

Ans:-

$P(t-T_1) \rightarrow$	$-2T + T_1$	$-T + T_1$	T_1	$T + T_1$	$2T + T_1$
$P(t+T_1) \rightarrow$	$-2T - T_1$	$-T - T_1$	$-T_1$	$T - T_1$	$2T - T_1$

→ Using shifting and linearity property.

$$P(t) \xleftrightarrow{FSC} 1/T$$

$$P(t+T_1) \xleftrightarrow{FSC} e^{jk\omega_0 T_1} \cdot 1/T$$

$$P(t-T_1) \xleftrightarrow{FSC} e^{-jk\omega_0 T_1} \cdot 1/T$$

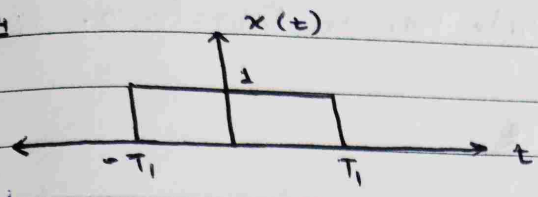
From (1)

$$\Rightarrow \frac{1}{T}\left(e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}\right) = \boxed{\frac{2j}{2} \cdot \sin(k\omega_0 T_1)}$$

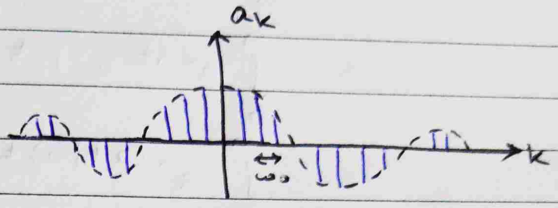
Q : How can we get Fourier transform from fourier series coefficients?

19/12/24

Q



Fourier Transform



$$a_k = \frac{\sin k\omega_0 T_1}{\pi k}$$

$$a_0 = \frac{\omega_0 T_1}{\pi}$$

To make the above graph continuous, $\omega_0 \to 0$

$T = \frac{2\pi}{\omega_0} \Rightarrow T = \infty$

Also, $k\omega_0 \to \omega$

$\therefore$ For $T \to \infty$, calculate fourier transform.

By analysis equation,

$$a_k = \frac{1}{T} \int_{-T_1}^{T_1} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} x(t)\, e^{-j\omega_0 t}\, dt$$

$$\boxed{\lim_{T \to \infty} T a_k = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$ Fourier Transform

From this we can get continuous waveform.

$$\boxed{X(\omega) = \lim_{T \to \infty} T a_k = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

F.T $\longrightarrow$ $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$ $\to X(0) = \int_{-\infty}^{\infty} x(t)\, dt$

Inverse FT $\to$ $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$ $\to x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, d\omega$

[Area in frequency domain]

Area of a signal in time domain - represent the frequency domain in 0 and integrate the signal in a given range.

13 JUNE Thursday

Monday		3	10	17	24
Tuesday	–	4	11	18	25
Wednesday	–	5	12	19	26
Thursday	–	6	13	20	27
Friday	–	7	14	21	28
Saturday	1	8	15	22	29
Sunday	2	9	16	23	30

6/2024

165/201 24.week

Q : $x(t) = e^{-at}\,u(t)$. Calculate Fourier Transform.

Ans:-

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left.\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right|_{0}^{\infty} \Rightarrow \boxed{X(\omega) = \frac{1}{a+j\omega}}$$

Magnitude

$$|X(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

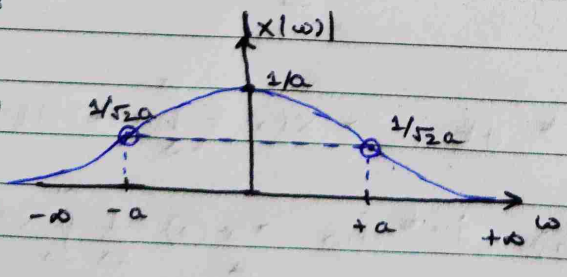
Phase

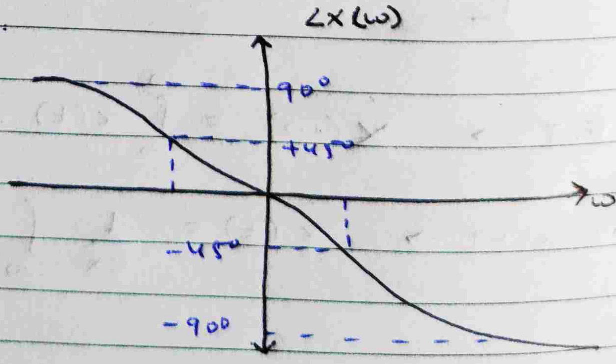
$$\angle X(\omega) = \frac{\angle 1}{\tan^{-1}\omega/a}$$

$$= \angle 1 - \tan^{-1}\omega/a$$

$$= -\tan^{-1}\omega/a \qquad (\angle 1 = 0)$$

ω	$\lvert X(\omega)\rvert$	$\angle X(\omega)$
0	$1/a$	0°
$-a$	$1/\sqrt{2}a$	45°
$+a$	$1/\sqrt{2}a$	-45°
$-\infty$	0	$+90^\circ$
$+\infty$	0	-90°





7/2024					
Monday	1	8	15	22	29
Tuesday	2	9	16	23	30
Wednesday	3	10	17	24	31
Thursday	4	11	18	25	–
Friday	5	12	19	26	–
Saturday	6	13	20	27	–
Sunday	7	14	21	28	–

24.week

JUNE
Friday
14

166/200

Q : $x(t) = e^{-a|t|}$.

8 Ans:-

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10

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11

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14

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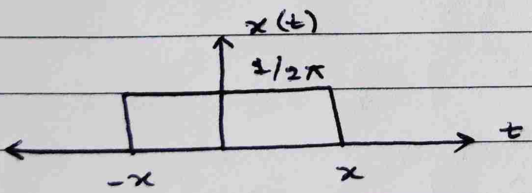
15 Q :



16

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Ans:- $\int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-j\omega t} \, dt$

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19

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20