

15

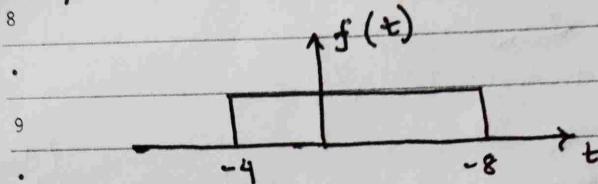
MARCH
Friday

75/291

(N H)

20/08/24

Signals & Systems



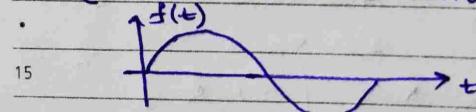
- Dependant variable that depends on some independant variable.
- Physical quantity
- Have some information that can be displayed, manipulated and even conveyed.

→ There are 4 types of analog signal -

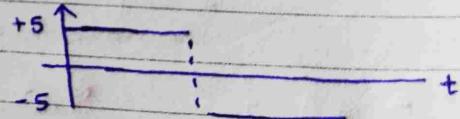
- (i) CTCV (ii) CTDV (iii) DTcv (iv) DTDV (Quantization)

↓

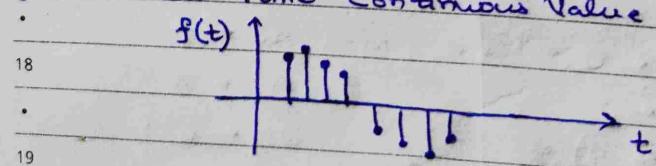
(Continuous Time Continuous Value)



(ii) Continuous Time Discrete Value



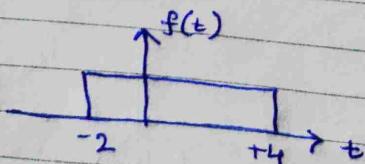
(iii) Discrete Time Continuous Value



→ Operations :

1. Time Shifting -

* $f(t) =$



'+' → Advanced
'-' → Delay

3/2024
18
19
20
21
22
23
24
25
26
27
28
29
30
31

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	1	2	3	4	5	6	7
2	8	9	10	11	12	13	14
3	15	16	17	18	19	20	21
4	22	23	24	25	26	27	28
5	29	-	-	-	-	-	-

4/2024

MARCH
Saturday

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11. week

$$f(t+3) = \begin{array}{c} f(t) \\ \text{---} \\ -5 \quad +1 \end{array}$$

(P.P)

Advanced

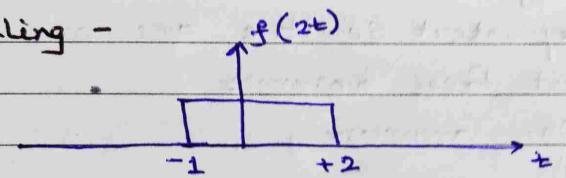
(shifting towards left)

$$f(t-3) = \begin{array}{c} f(t) \\ \text{---} \\ +1 \quad +7 \end{array}$$

Delay

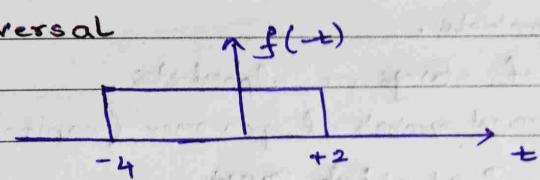
(shifting towards right)

2. Time Scaling -

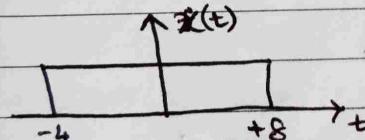


(to wide)

3. Time Reversal

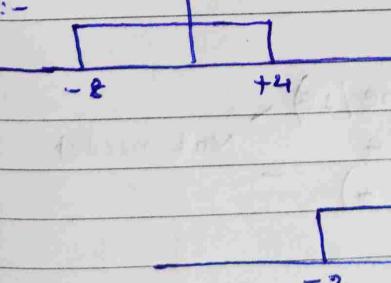


Q:

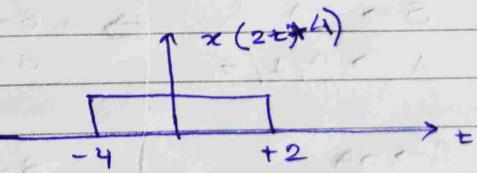


Find $x(-2t+4)$.

Ans:-



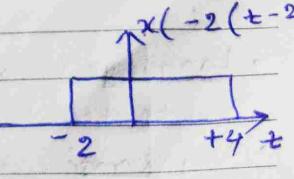
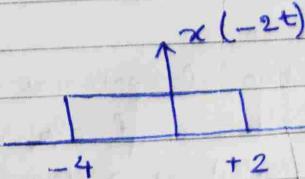
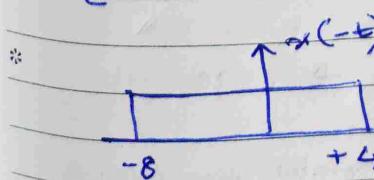
~~$x(-2t+4)$~~



~~$x(-2t+4)$~~



$x[-2(t-2)]$



There is no duty we so much underrate as the duty of being happy. — Robert L. Stevenson

	4/2024				
Monday	1	8	15	22	29
Tuesday	2	9	16	23	-
Wednesday	3	10	17	24	-
Thursday	4	11	18	25	-
Friday	5	12	19	26	-
Saturday	6	13	20	27	-
Sunday	7	14	21	28	-

MARCH
Wednesday

20

80/286

12 week

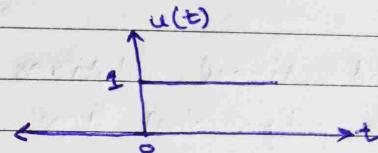
22/08/24

(NH)

- Elementary Signals :

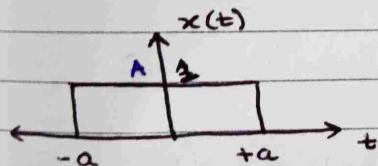
- Unit Step -

$$\begin{aligned} u(t) &= 1 \text{ for } t \geq 0 \\ &= 0 \text{ for } t < 0 \end{aligned}$$



for scaling operation, unit step signal will remain same.

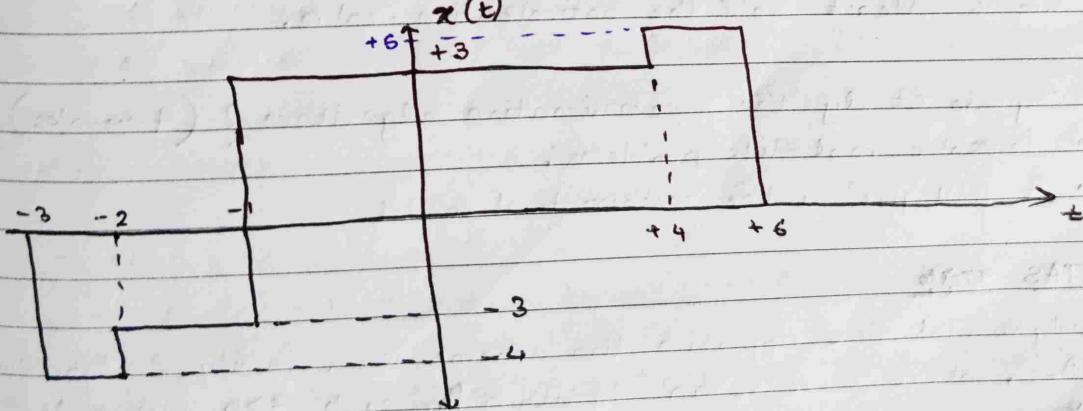
- * # Causal Signal
 - Causality - $x(t) = 0$ for $t < 0$
 - Anti-causality - $x(t) = 0$ for $t > 0$
 - Non-causality - $x(t) \neq 0$ for $t < 0$



$$x(t) = u(t+a) - u(t-a)$$

$$\hookrightarrow x(t) = +A u(t+a) - A u(t-a)$$

Q:



$$x(t) = -4u(t+3) + 0u(t+2) + 6u(t+1) + 3u(t-4) - 5u(t-6)$$

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MARCH
Monday

85/281 27/08/24

(N+)

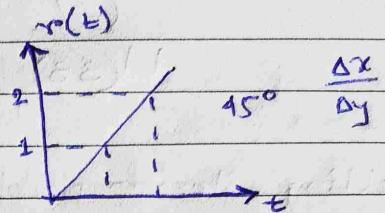
3/2011

Monday	-	4	11	3/2024
Tuesday	-	5	12	18
Wednesday	-	6	13	19
Thursday	-	7	14	20
Friday	1	8	15	22
Saturday	2	9	16	23
Sunday	3	10	17	24

13. week

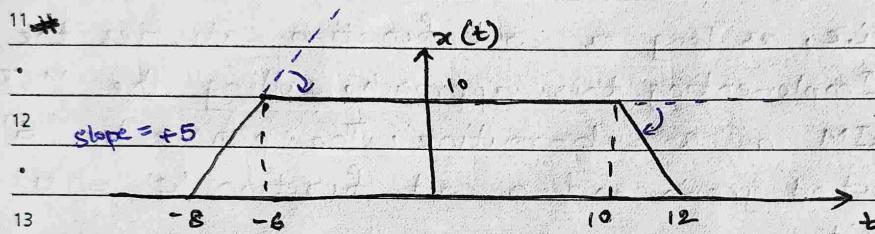
- ## • Ramp Signal -

$$r(t) = t \quad \text{for } t > 0 \\ r(t) = 0 \quad \text{otherwise}$$

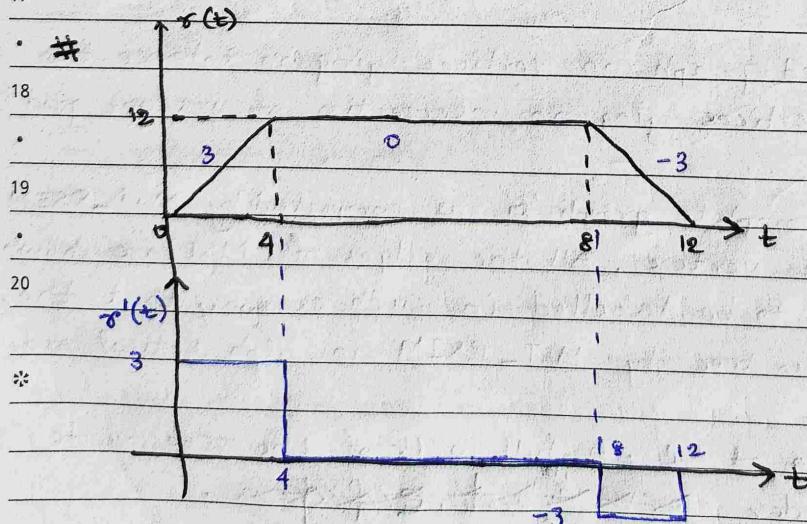
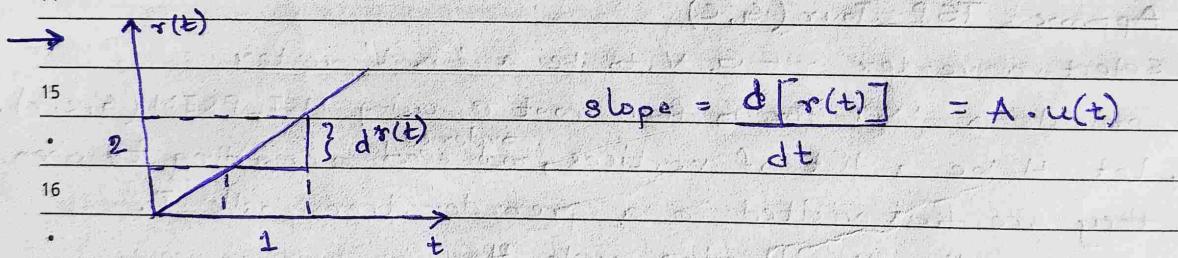


$$r(t) = A \cdot t \Rightarrow A = \text{slope}$$

$$10 \quad \underline{\underline{r(t) = t_1 u(t)}}$$



$$r(t) = 5r(t+8) - 5r(t+6) - 5r(t-10) + 5r(t-12)$$

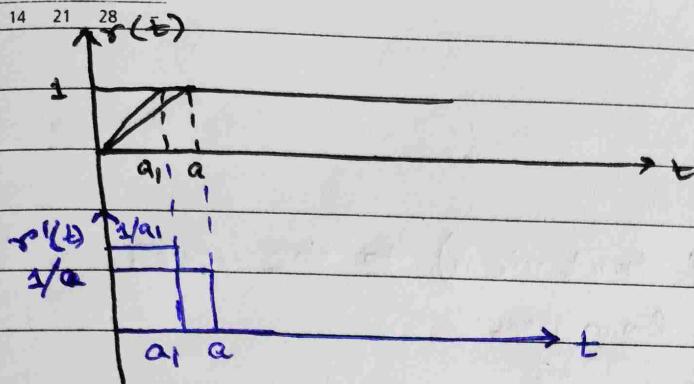


What a man dislikes in his superiors, let him not display in the treatment of his inferiors. — Tsang Sin

4/2024

	1	8	15	22	29
Monday	2	9	16	23	30
Tuesday	3	10	17	24	-
Wednesday	4	11	18	25	-
Thursday	5	12	19	26	-
Friday	6	13	20	27	-
Saturday	7	14	21	28	-
Sunday					

13.week

MARCH
Tuesday

26

86/280

NOTE : Derivative of Ramp signal \rightarrow Step signal
 Derivative of Step signal \rightarrow Impulse signal

$$\delta(t) = \frac{d(u(t))}{dt}$$

$\delta(t) = 1$ (defines the area)

$\delta(t) = \infty$ (defines the value)

06

APRIL
Saturday

	5/2024
8	15 22 29
9	16 23 30
10	17 24 -
11	18 25 -
12	19 26 -
13	20 27 -
14	21 28 -

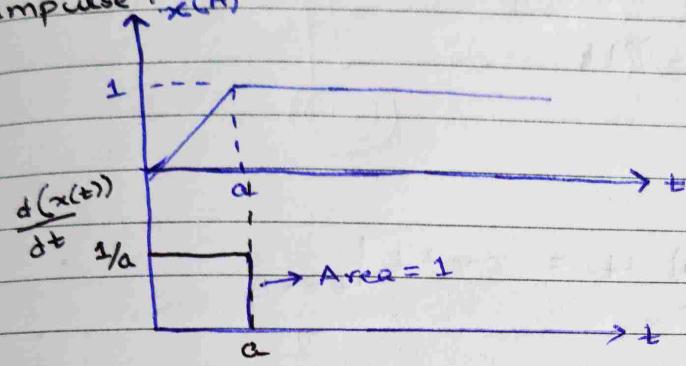
14. week

14/week
03/09/24

97/269

(NH)

→ Impulse $\delta(t)$

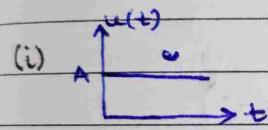


- magnitude at $t=0$
is ∞

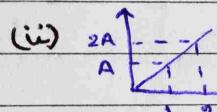
- area at $t=0$
is 1

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

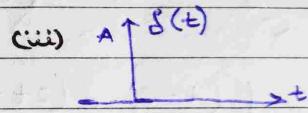
$$\int_0^{\infty} \delta(t) dt = 1$$



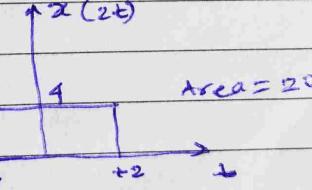
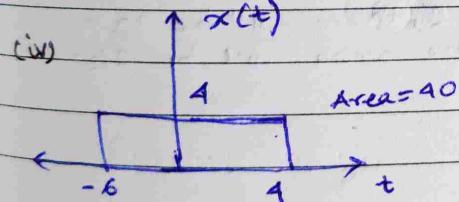
★ $u(t)$
↳ magnitude



★ $A r(t)$
↳ Slope



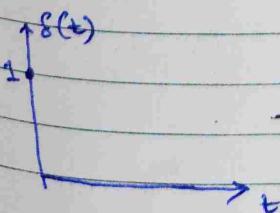
★ $A \delta(t)$
↳ Area



- Scaling changes area, it does not change magnitude.

$$\text{area}[x(\alpha t)] = \frac{1}{|\alpha|} \cdot \text{area}[x(t)]$$

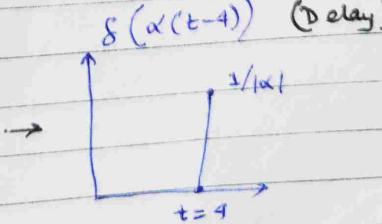
Scaling



~~1/|α|~~ $\delta(\alpha t)$

$1/|\alpha|$

scaling and shifting



$\delta(\alpha(t-4))$ (Delay)

07

APRIL
Sunday

93/268

- NOTE : • $x(t) \cdot \delta(t) = x(0)$ [Value at $t=0$]
 • $x(t) \cdot \delta(t-t_0) = x(t_0)$
 • $x(t) \cdot \delta(t+t_0) = x(-t_0)$.

$$I = \int_{-6}^{+8} (t^2 + 4) \delta(t-3) dt$$

$$\underline{t=3} = 13$$

$$\rightarrow \int_{-\pi}^{\pi} \cos^2 t \delta(t - \pi/4) dt = \cos^2 t \Big|_{t=\pi/4}$$

$$\rightarrow \int_{-3}^{+4} (t^3 + 5) \cdot \delta(t-6) dt = 0$$

$$\rightarrow \int_{-6}^{+5} (t-2) \delta(2t-4) dt$$

For $\delta(2t-4)$

$$= \int_{-6}^{+5} (t-2) \frac{1}{2} \delta(t-2) dt$$

Area = $\frac{1}{2} \delta(t-2)$

$$= 0$$

At $t=2$, there will be an impulse whose area will be $1/2$.

4/2024
8 15 22 29
9 16 23 -
0 17 24 -
1 18 25 -
2 19 26 -
3 20 27 -
4 21 28 -

5/2024

-	6	13	20	27
-	7	14	21	28
1	8	15	22	29
2	9	16	23	30
3	10	17	24	31
4	11	18	25	-
5	12	19	26	-

APRIL
Wednesday

10

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15 week

12/09/24

(NH)

$$\rightarrow I = \int_{-\infty}^{\infty} x(t) \cdot \delta \{ g(t) \} dt$$

(i) $\delta [g(t)] = 0$ if g' is nowhere zero. For any value of t , $g'(t) \neq 0$.

(ii) $\delta [g(t)] = \frac{\delta (t-t_0)}{|g'(t_0)|}$ if g' has one real root at $t=t_0$.

(iii) If 'g' has more than one real root:

$$\delta [g(t)] = \sum_i \frac{\delta (t-t_i)}{|g'(t_i)|} = \frac{\delta (t-t_0)}{|g'(t_0)|} + \frac{\delta (t-t_1)}{|g'(t_1)|} + \dots$$

$$Q: I = \int_{-10}^{10} (t^2 + 16) \delta (t^2 + 16) dt.$$

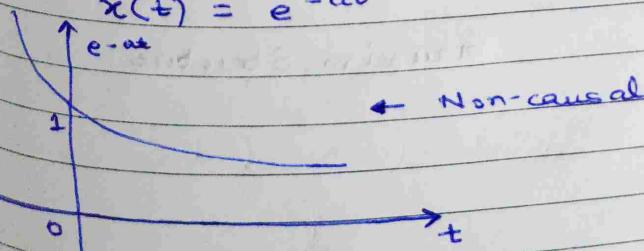
$$\text{Ans:- } g(t) = t^2 - 16 \quad t_0 = 4, t_1 = -4$$

$$= \frac{\delta (t-4)}{|8|} + \frac{\delta (t+4)}{|-8|} = \frac{1}{8} [\delta (t-4) + \delta (t+4)]$$

$$I = \int_{-10}^{10} (t^2 + 16) \frac{1}{8} [\delta (t-4) + \delta (t+4)] dt \\ = \frac{13}{2}.$$

→ Exponential function:

$$x(t) = e^{-at}$$



You can give without loving, but you cannot love without giving. - Amy Carmichael

MAY

JUN

11

APRIL
Thursday

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Monday	1	8
Tuesday	2	9
Wednesday	3	10
Thursday	4	11
Friday	5	12
Saturday	6	13
Sunday	7	14

Monday	15
Tuesday	16
Wednesday	17
Thursday	18
Friday	19
Saturday	20
Sunday	21

$$|t| = +t, t \geq 0$$

$$-t, t < 0$$

$$e^{-at|t|} \cdot u(t) = e^{-at}$$

$$\left. \begin{array}{l} e^{-a|2|} = e^{-2a} \\ e^{-a|-2|} = e^{-2a} \end{array} \right\}$$

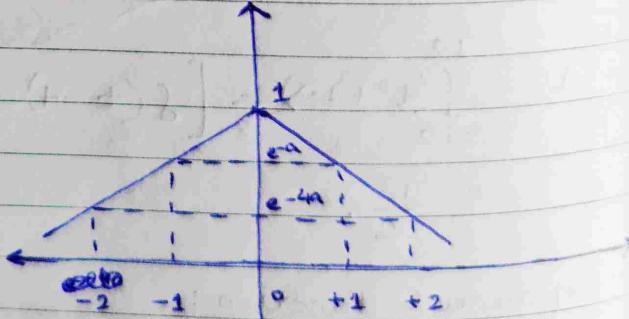
$$\# e^{-at|t|} = e^{-a(t)} \cdot u(t) + e^{-a(-t)} u(-t)$$

$$= e^{-at} \cdot u(t) + e^{+at} \cdot u(-t)$$

t	$e^{-at} \cdot u(t)$	$e^{at} \cdot u(-t)$	In both cases, decaying rate is same.
0	• 1	-1	e^{-a}
1	e^{-a}	-2	e^{-2a}
2	e^{-2a}	-3	e^{-3a}
3	e^{-3a}		

$$\rightarrow x(t) = e^{-at^2}$$

t	$x(t)$
0	1
1	e^{-a}
2	e^{-4a}
-1	e^{-a}
-2	e^{-4a}



Gaussian Structure

15 week 19/09/24

APRIL
Sunday

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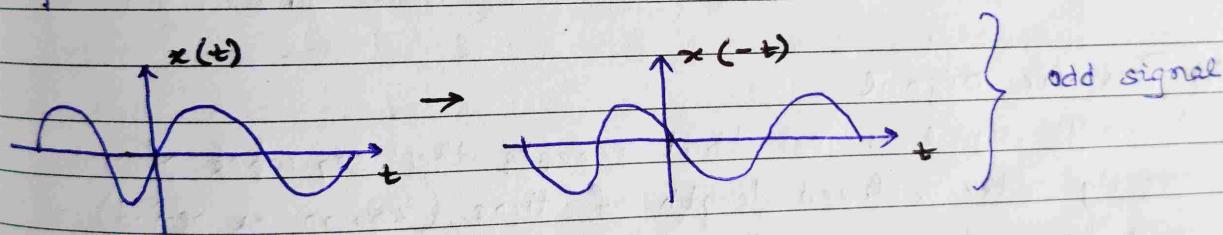
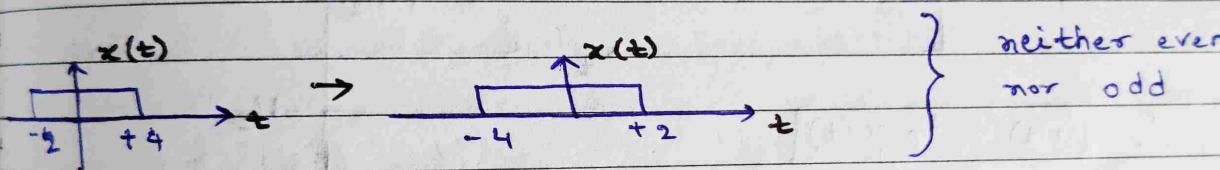
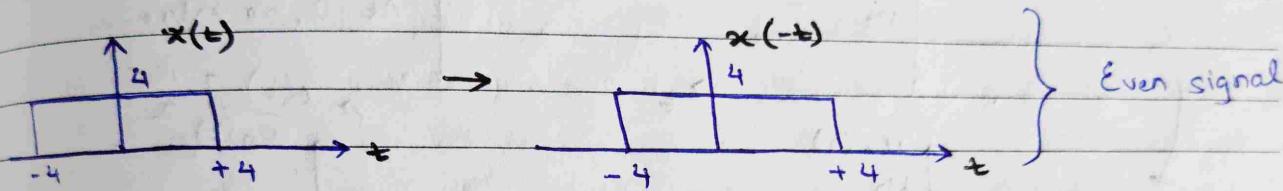
(NH)

→ Classification of Signals:

(i) Even and Odd Signals -

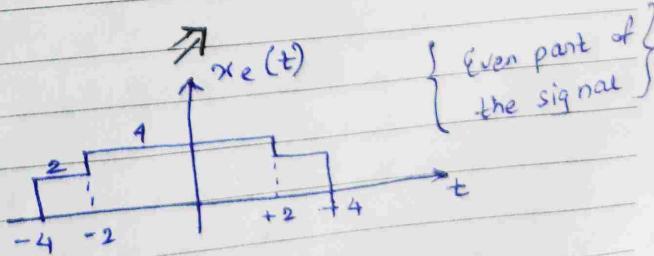
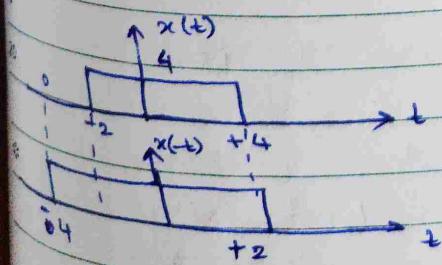
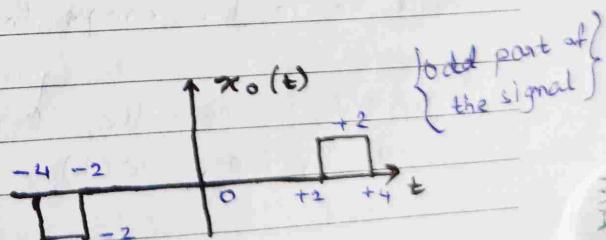
$$x(-t) = x(t) \rightarrow \text{even}$$

$$x(-t) = -x(t) \rightarrow \text{odd}$$



$$\cdot x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$\cdot x_o(t) = \frac{x(t) - x(-t)}{2}$$



[Point to point calculation for getting the even signal]

True peace is not merely the absence of tension; it is the presence of justice. — Martin Luther King

15

APRIL
Monday

Monday	1	8	15
Tuesday	2	9	16
Wednesday	3	10	17
Thursday	4	11	18
Friday	5	12	19
Saturday	6	13	20
Sunday	7	14	21

Monday
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Saturday
Sunday

16 week

8 Th
• (i)
9 (ii)

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11 (iii)

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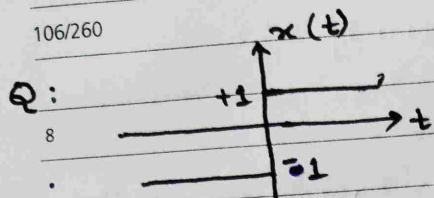
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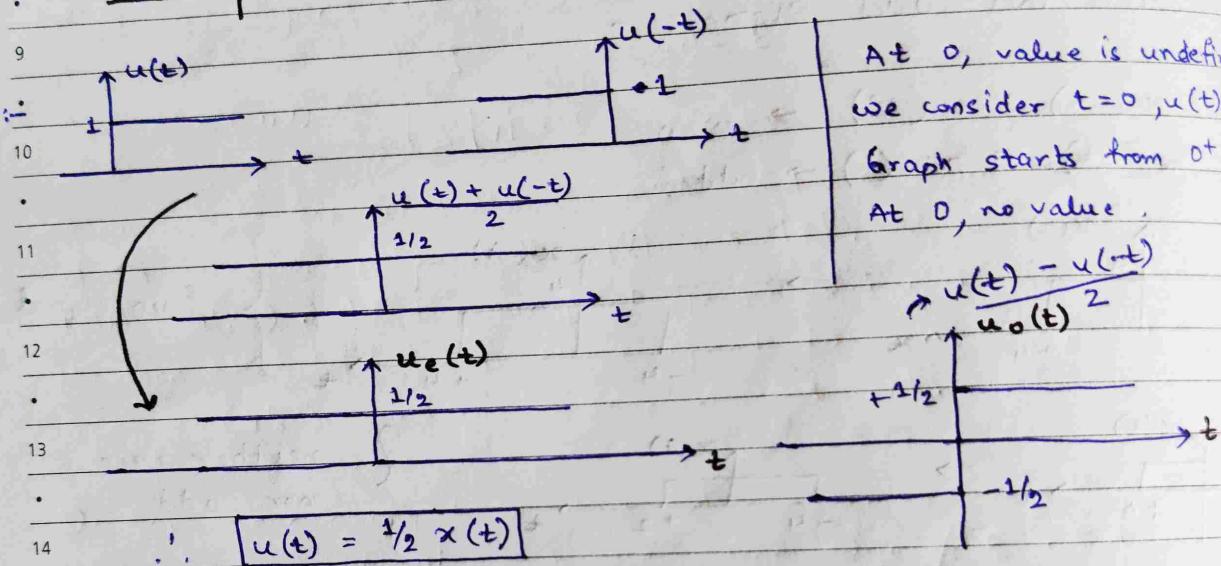
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Show that even and odd parts of $u(t)$ can be represented in parts of $x(t)$.

Ans:-



At 0, value is undefined.
we consider $t=0, u(t)=1$.

Graph starts from 0^+ and 0^- .

At 0, no value.

$$\frac{u(t) - u(-t)}{2}$$

15 (ii) Periodic Signal

- A periodic signal is one that repeats the sequence of values exactly after a fixed length of time (known as period) and it must be in the range from $-\infty$ to $+\infty$.

- A signal is periodic if it repeats itself after a certain period T , i.e., $x(t) = x(t+T)$.

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APRIL
Thursday

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22/10/24

(NH)

8

→ Periodic and Aperiodic :

i) Sinusoidal

$$x(t) = A \sin(\omega t \pm \phi)$$

↓ ↓
Amplitude Phase Angle

Phase shift

$\omega \rightarrow$ Angular frequency.

$$T = \frac{2\pi}{\omega}$$

11

Q: $x(t) = 5 \cos(300\pi t)$. Periodic or aperiodic?

If periodic calculate fundamental period.

Ans:- $\omega_0 = 300\pi$

13

$$T = \frac{2\pi}{300\pi} = \frac{1}{150} \text{ second}$$

Fundamental period.

14

$$15 \rightarrow x(t) = 4 \cos \pi t + 3 \sin 2\pi t + 2 \sin 3\pi t$$

$$\omega_{01} = \pi \quad \omega_{02} = 2\pi \quad \omega_{03} = 3\pi$$

$$16 \quad T_1 = 2s \quad T_2 = 1s \quad T_3 = \frac{2}{3}s$$

17

$$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_1}{T_3}$$

Check if the ratio is rational or not

→ Ratio of p/q where p & q are integers

18

$$T = \text{LCM}[T_1, T_2, T_3]$$

→ Decimal values must be terminating or repeating.

$$19 \quad \text{for Ratio} \rightarrow T = \frac{\text{LCM}[\text{Numerator}]}{\text{HCF}[\text{Denominator}]}$$

20 NOTE: $T = \pi \rightarrow$ Signal is aperiodic.

*

$$\therefore T = \frac{2}{\pi} \text{ second}$$

→ Phase shift does not hamper periodicity.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
1	1	2	3	4	5	6	7
2	8	9	10	11	12	13	14
3	15	16	17	18	19	20	21
4	17	18	19	20	21	22	23
5	18	19	20	21	22	23	24
6	20	21	22	23	24	25	26
7	21	22	23	24	25	26	27

17. We

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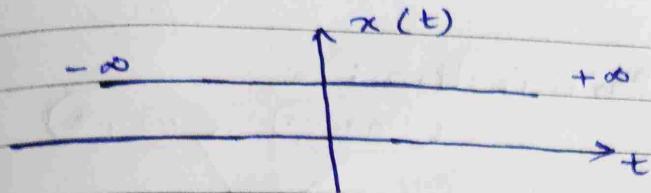
5/2024
6 13 20 27
7 14 21 28
8 15 22 29
9 16 23 30
10 17 24 31
11 18 25 -
12 19 26 -

APRIL
Friday

26

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ii) Constant or DC signal



- Periodic signal without any periodic value.

$$x(t) = 5 \cos(20\pi t + \pi/2)$$

$$\hookrightarrow T = \frac{2\pi}{20\pi} = 1/10$$

$$x(t) = 4 + 5 \cos(20\pi t + \pi/2)$$

$$\hookrightarrow T = 1/10$$

$x(t) = \text{Even of } [\sin 4\pi t \cdot u(t)]$ Not a periodic signal.

$$x_1(t) = \sin 4\pi t \cdot u(t)$$

$$x_1(-t) = -\sin 4\pi t \cdot u(-t)$$

$$\text{Even } [x_1(t)] = \frac{x_1(t) + x_1(-t)}{2}$$

$$= [\sin 4\pi t \cdot u(t) - \sin 4\pi t \cdot u(-t)]/2$$

$$= \sin 4\pi t \left[\frac{u(t) - u(-t)}{2} \right]$$

(Result is not periodic.)

01

MAY
Wednesday

122/244

05/11/24

(NH)

8

- Complex exponential signal :

$$x(t) = e^{j\omega t} = \cos \omega t + j \sin \omega t$$

↑ real magnitude
↓ imaginary magnitude

9

$$\text{phase} = t^{-1} \left(\frac{\sin \omega t}{\cos \omega t} \right) = \omega t$$

→ derivative of ωt
is phase angle

10

$$|x(s)| = 1 \text{ (magnitude)}$$

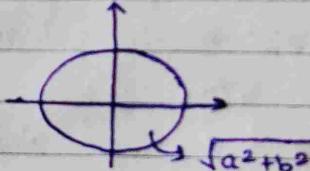
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Complex rectangular signal is of two type / format -

12

- (i) Polar format :

$$r \angle \theta \quad \text{radius} \quad \text{angle}$$



13

- (ii) Rectangular format :

$$a + jb$$

14

NOTE : Combination of Periodic signal is also periodic .

15

$$x = a + jb$$

16

$$r = \sqrt{a^2 + b^2}$$

$$\angle \theta = \tan^{-1}(b/a)$$

17

Others can stop you temporarily; only you can do it permanently. - Don Ward

		6/2024
3	10	17
4	11	18
5	12	19
6	13	20
7	14	21
1	8	15
2	9	16
		23
		30

MAY
Thursday

02

123/243

Q: $x = 3 + j4$

Ans:- $r = \sqrt{3^2 + 4^2} = 5$

$$\theta = \tan^{-1}(b/a) \\ = \tan^{-1}(4/3)$$

$$j = 90^\circ$$

$$j^2 = 90^\circ + 90^\circ = 180^\circ$$

$$j^3 = 90^\circ + 90^\circ + 90^\circ = 270^\circ$$

$$x = 5 \tan^{-1}(4/3)$$

$$\sqrt{x} = \sqrt{5} \cdot \sqrt{\frac{\tan^{-1}(4/3)}{2}}$$

Q: $x = \frac{(4 + j5)(3 + j6)}{(8 + j4)(9 + j3)}$

Ans:-

$$x = \frac{\sqrt{41} \angle \tan^{-1}(5/4)}{\sqrt{80} \angle \tan^{-1}(1/2)} \cdot \frac{\sqrt{45} \angle \tan^{-1}(2)}{\sqrt{90} \angle \tan^{-1}(2/3)}$$

$$= \frac{\sqrt{41} \cdot \sqrt{45}}{\sqrt{80} \cdot \sqrt{90}} \cdot \frac{[\tan^{-1}(5/4) + \tan^{-1}(2)] \text{ degree}}{[\tan^{-1}(1/2) + \tan^{-1}(2/3)] \text{ degree}}$$

07

MAY
Tuesday

128/238

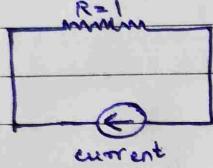
12/11/24

(NH)

				5/2024
Monday	-	6	13	20
Tuesday	-	7	14	21
Wednesday	1	8	15	22
Thursday	2	9	16	23
Friday	3	10	17	24
Saturday	4	11	18	25
Sunday	5	12	19	26

19 week

8 → Energy and Power Signal :



$$P_x = \frac{[i(t)]^2}{R} R = \frac{V_{RMS}^2}{R}$$

$$= (I_{RMS})^2$$

$$= (V_{RMS})^2$$

A graph showing the function $\frac{1}{T} \int_{-T/2}^{T/2} |i(t)|^2 dt$. The x-axis is labeled from $-T/2$ to $T/2$, and the y-axis is labeled $\frac{1}{T}$.

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |i(t)|^2 dt \quad \leftarrow \text{Aperiodic Signal}$$

$$E_x = \text{Power} \times \text{Time}$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \cdot T \int_{-T/2}^{T/2} |i(t)|^2 dt$$

$$E_x = \int_{-\infty}^{\infty} |i(t)|^2 dt$$

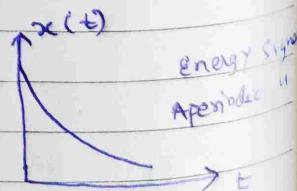
NOTE :- Finite Energy $\rightarrow \infty$ Power $\rightarrow 0$
Power \rightarrow Finite Energy $\rightarrow \infty$

Q:

$$(i) x(t) = e^{-4t} \cdot u(t) \Rightarrow \text{Energy Signal}$$

$$E_x = \int_{-\infty}^{\infty} e^{-8t} dt = \frac{e^{-8t}}{-8} \Big|_0^{\infty} = \frac{1}{8} \quad (\text{finite})$$

$$P_x = 0$$



6/2024

Monday	3	10	17	24
Tuesday	-	4	11	18
Wednesday	-	5	12	19
Thursday	-	6	13	20
Friday	-	7	14	21
Saturday	1	8	15	22
Sunday	2	9	16	23

MAY
Wednesday

08

19.week

129/237

(i) $x(t) = A \cdot \sin \omega t$

$P_x = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{A^2}{2}$

For an sinusoidal signal \rightarrow RMS value $= A/\sqrt{2}$.

For Power $= A^2/2$

$$\int \frac{1}{T} \int_{-T/2}^{T/2} |A \sin \omega t|^2 d\omega t$$

(iii) $x(t) = u(t)$
 $E_x = \int_{-\infty}^{\infty} |x(t)| dt = \infty \int_{-\infty}^{\infty} 1 dt = \infty$

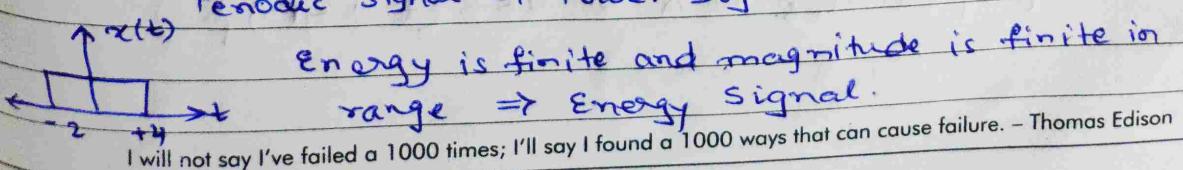
$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

 $= 1/2$

(iv) $x(t) = \sin t \cdot u(t) \Rightarrow$ Power Signal

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \sin^2 t dt$$

 $= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{1 - \cos 2t}{2} dt$
 $= 1/4$

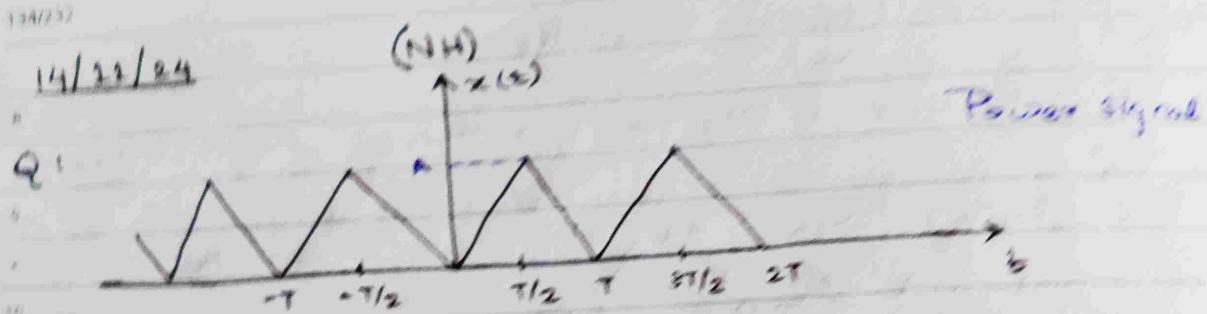
(i) $\rightarrow x(t) = 0$ at $t \rightarrow \infty$ and if value of $x(t)$ decreases with time \Rightarrow Energy Signal.(ii) \rightarrow w.r.t time value is neither zero nor infinite \Rightarrow PowerPeriodic Signal \Rightarrow Power Signal.Energy is finite and magnitude is finite in that range \Rightarrow Energy Signal.

13

MAY
Monday

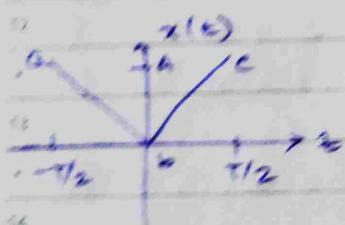
13/05/23

14/05/23



Pulse Signal

$$P_x = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$



$$Y_{00} = -\frac{2A}{\pi} \cdot \frac{1}{2} + 0 \quad \text{for } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$Y_{00} = \frac{2A}{\pi} \cdot \frac{1}{2} + 0 \quad \text{for } 0 \leq \theta < \pi/2$$

$$P_x = \frac{1}{T} \int_{-T/2}^{0} \frac{4A^2}{T^2} \cdot t^2 dt + \frac{1}{T} \int_{0}^{T/2} \frac{4A^2}{T^2} \cdot t^2 dt$$

$$= \frac{4A^2}{T^2} \left[\frac{t^3}{3} \right]_{-T/2}^0 + \frac{4A^2}{T^2} \left[\frac{t^3}{3} \right]_0^{T/2}$$

$$= \frac{4A^2}{3T^2} \left[\frac{T^3}{8} + \frac{T^3}{8} \right] = \frac{A^2}{3} (\text{Ans})$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

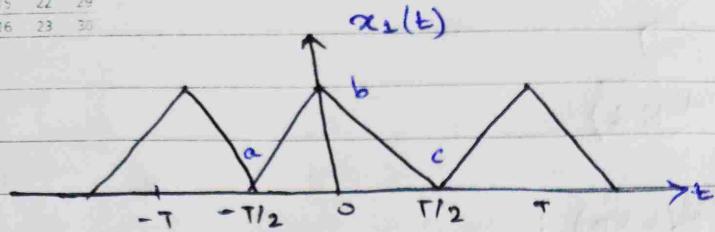
$$= P_x \cdot \infty = \infty$$

Ans,

$$x_2(t) = x(t - T/2)$$

Saturday	4	11	18	25
Sunday	5	12	19	26
Monday	6	13	20	27
Tuesday	7	14	21	28
Wednesday	1	8	15	22
Thursday	2	9	16	23
Friday	3	10	17	24

20. week Q:



$$Y_{ab} = +\frac{2A}{T}t + A$$

$$Y_{bc} = -\frac{2A}{T}t + A$$

$$\therefore P_x = \frac{1}{T} \left[\int_{-T/2}^0 \left(\frac{4A^2}{T^2} \cdot t^2 + A^2 + \frac{4At}{T} \right) dt \right]$$

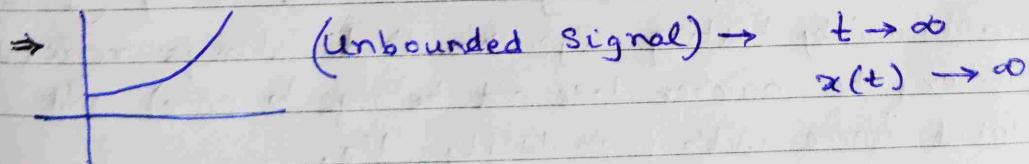
$$+ \int_0^{T/2} \left(A^2 - \frac{4At}{T} + \frac{4A^2t^2}{T^2} \right) dt \Big]$$

$$= \frac{1}{T} \left[\frac{4A^2}{T^2} \cdot \frac{t^3}{3} \Big|_{-T/2}^0 + A^2 \cdot \frac{T}{2} + \frac{4A}{T} \left(\frac{t^2}{2} \Big|_{-T/2}^0 \right) \right.$$

$$\left. + A^2 \cdot \frac{T}{2} - \frac{4A}{T} \frac{t^2}{2} \Big|_0^{T/2} + \frac{4A^2}{T^2} \cdot \frac{t^3}{3} \Big|_0^{T/2} \right]$$

$$P_x = \frac{A^2}{3}$$

⇒ Thus signals which are not bounded ~~by~~ are neither energy nor power signals.



Example - $e^{2t} u(t)$

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} e^{4t} dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \left[\left[\frac{e^{4t}}{4} \right]_0^{T/2} \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \frac{e^T}{4} = \infty \times \frac{\infty}{\infty}$$

$E_x = \infty$
$P_x = \infty$

A good politician is quite as unthinkable as an honest burglar. - H. L. Mencken

21

MAY
Tuesday

142/224

28/12/24

(NH)

convert time signal to
frequency signal.

8

→ Continuous Time Fourier Transform:

Why?? - frequency domain gives more features and analysed signal.

10 - If the signal is periodic then only it can be represented in
frequency domain for its time domain signal.

11

* ~~Linear combination of harmonically related complex exponential.~~

12

$$\downarrow F[a_1x_1(t) + a_2x_2(t)] = a_1F[x_1(t)] + a_2F[x_2(t)]$$

13

$$\Rightarrow a y(t) = a_1y_1(t) + a_2y_2(t)$$

14

operated together or splitted and then operated together, the
result will be same.

15

- If we consider a periodic signal which is not a sinusoidal wave with a time period T . Now, if we represent a sinusoidal signal with the same time period, it is 1st Harmonic.
- Half the time period ($T/2$) → 2nd Harmonic
- [double the frequency]

~~Harmonically related~~

Any periodic signal can be represented by harmonic signal and
the frequency of the signal will be same as the given signal
and the frequency increases ~~or~~ or related by a simple ratio.

* $e^{j\omega t}, e^{j2\omega t}, e^{j3\omega t}$

← Harmonically related

Complex exponential

Monday	-	6	13	20	520
Tuesday	-	7	14	21	
Wednesday	1	8	15	22	
Thursday	2	9	16	23	
Friday	3	10	17	24	
Saturday	4	11	18	25	
Sunday	5	12	19	26	

Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
Sunday

21. week

8

9

10

11

12

13

14

15

16

17

18

19

20

		6/2024
Monday	3	10
Tuesday	4	11
Wednesday	5	12
Thursday	6	13
Friday	7	14
Saturday	8	15
Sunday	9	16
	23	30

Step 1

- Combining magnitude and phase to get frequency domain signal.

MAY
Wednesday

22

21. week

143/223

Synthesis

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j k \omega_0 t}$$

$k = 1 \rightarrow 1^{\text{st}}$ Harmonic

$2 \rightarrow 2^{\text{nd}}$ Harmonic

a_k = Fourier series coefficient

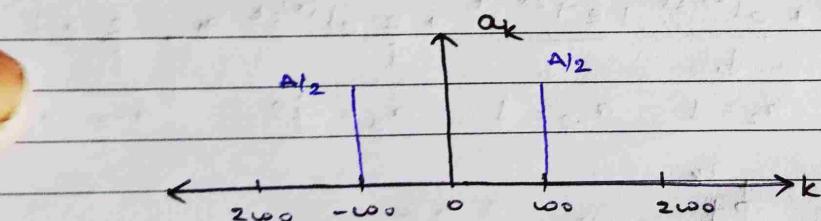
[Using synthesis equation only]

Q: $x(t) = A \cos \omega_0 t$

Ans: $x(t) = A \cdot \frac{e^{j \omega_0 t} + e^{-j \omega_0 t}}{2}$

$$= \frac{A}{2} e^{j \omega_0 t} + \frac{A}{2} e^{-j \omega_0 t}$$

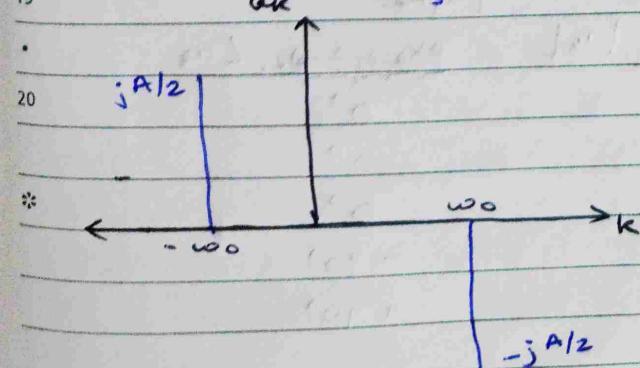
$a_1 = A/2 \quad a_{-1} = A/2 \rightarrow$ Fourier series coefficients



Q: $x(t) = A \sin \omega_0 t$

Ans: $x(t) = A \cdot \left(\frac{e^{j \omega_0 t} - e^{-j \omega_0 t}}{2j} \right)$

$$a_1 = \frac{A}{2j} = -j \frac{A}{2}, \quad a_{-1} = \frac{+A}{2j} = j \frac{A}{2}$$



23

MAY
Thursday

$$Q: x(t) = 2 + \sin \omega_0 t + 2 \cos \omega_0 t + \cos [2\omega_0 t + \pi/4]$$

Monday	-	6	13	20	27	5/2024
Tuesday	-	7	14	21	28	
Wednesday	1	8	15	22	29	
Thursday	2	9	16	23	30	
Friday	3	10	17	24	31	
Saturday	4	11	18	25	-	
Sunday	5	12	19	26	-	

144/222

Q: $x(t) = 4 + 2 \cos \frac{2\pi}{3} t + 4 \sin \frac{5\pi}{3} t$

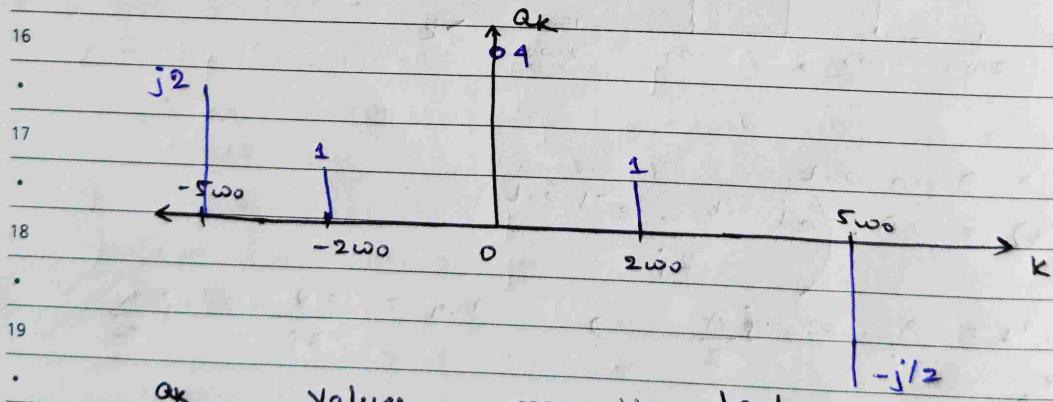
Ans:- $\omega_{01} = 2\pi/3$ $T_1 = 3$

$\omega_{02} = 5\pi/3$ $T_2 = 6/5$

$T = \text{LCM} \left[\frac{3}{2}, \frac{6}{5} \right] = 6 \text{ second}$ $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$

Fundamental frequency $\Rightarrow \boxed{\omega_0 = \frac{\pi}{3}}$ Also the first harmonic frequency.

$$\begin{aligned} x(t) &= 4 \cdot e^{j\omega_0 t} + 2 \frac{e^{j2\pi/3 t} + e^{-j2\pi/3 t}}{2} + 4 \frac{e^{j5\pi/3 t} - e^{-j5\pi/3 t}}{2j} \\ &= 4 \cdot e^{j\omega_0 t} + \underbrace{e^{j2\pi/3 t}}_{\downarrow} + \underbrace{e^{-j2\pi/3 t}}_{\downarrow} + \underbrace{\frac{2}{j} e^{j5\pi/3 t}}_{\downarrow} - \underbrace{\frac{2}{j} e^{-j5\pi/3 t}}_{\downarrow} \\ a_0 &= 4 \quad a_2 = 1 \quad a_{-2} = 1 \quad a_5 = -j2 \quad a_{-5} = j2 \end{aligned}$$



<u>ak</u>	<u>Values</u>	<u>magnitude ak </u>	<u>Phase Angle ∠ak</u>
a ₀	4	4	0°
a ₂	1	1	0°
a ₋₂	1	1	0°
a ₅	-j2	2	-90°
a ₋₅	+j2	2	+90°

024
27
28
29
30
31
ek

	6/2024			
Monday	3	10	17	24
Tuesday	-	4	11	18
Wednesday	-	5	12	19
Thursday	-	6	13	20
Friday	-	7	14	21
Saturday	1	8	15	22
Sunday	2	9	16	23
				30

MAY
Thursday

30

151/215

22. week

05/12/24

(NH)

→ Properties of Fourier Series Coefficients :

1. Linearity : $x(t) \xleftrightarrow{FS} a_k, y(t) \xleftrightarrow{FS} b_k$

$$z(t) = [\alpha x(t) + \beta y(t)] \xleftrightarrow{FS} c_k$$

$$\text{then } c_k = \alpha a_k + \beta b_k.$$

2. Time shifting : $x(t) \xleftrightarrow{FS} a_k, x(t \pm t_0) \xleftrightarrow{FS} b_k$

$$\text{then } b_k = e^{j k \omega_0 (t \pm t_0)} a_k$$

3. Time Reversal : $x(t) \xleftrightarrow{FS} a_k, x(-t) \xleftrightarrow{FS} b_k$

$$\text{then } b_k = a_{-k}$$

$$\text{if } x(t) \text{ even : } b_k = a_k = a_k$$

$$\text{if } x(t) \text{ odd : } b_k = a_{-k} = -a_k$$

for ①

$$Q: x(t) = \cos \omega_0 t$$

$$y(t) = \sin \omega_0 t$$

$$\underline{\text{Ans:}} \quad a_1 = 1/2, a_{-1} = 1/2$$

$$b_1 = -j/2, b_{-1} = j/2$$

$$\text{Let } \alpha = 1, \beta = 1 \Rightarrow z(t) = \cos \omega_0 t + \sin \omega_0 t$$

$$c_1 = a_1 + b_1 = 1/2 - j/2$$

$$c_2 = 1/2 + j/2 \quad \circ \quad [= a_{-1} + b_{-1}]$$

for ②

$$Q: x(t) \xleftrightarrow{FS} a_k, x(t+t_0) + x(t-t_0) \xleftrightarrow{FS} b_k. \text{ Calculate } b_k.$$

$$\underline{\text{Ans:}} \quad x(t+t_0) \leftrightarrow e^{j k \omega_0 t_0} a_k$$

$$x(t-t_0) \leftrightarrow e^{-j k \omega_0 t_0} a_k.$$

$$\therefore b_k = \frac{1}{2} a_k \left(e^{j k \omega_0 t_0} + e^{-j k \omega_0 t_0} \right)$$

$$= 2 a_k \cdot \cos(k \omega_0 t_0).$$

31

MAY
Friday

152/214

	5/2024		
	Monday	Tuesday	Wednesday
1	-	6	13
2	-	7	14
3	1	8	15
4	2	9	16
5	3	10	17
6	4	11	18
7	5	12	19
8			20
9			21
10			22
11			23
12			24
13			25
14			26

22 week

$$|b_k| = \cos [k\omega_0(\pm t_0)] + j\sin [k\omega_0(\pm t_0)] \\ = 1 = |a_k|$$



This means that using the time shifting property there is no change in the magnitude.

$$\angle b_k = \angle e^{j k \omega_0(\pm t_0)} + \angle a_k$$

$$= \tan^{-1} \frac{\sin k\omega_0(\pm t_0)}{\cos k\omega_0(\pm t_0)} + \angle a_k$$

$$\angle b_k = \angle a_k + k\omega_0(\pm t_0) \Rightarrow \text{There will be change in phase angle after time shifting.}$$

For ③

$$Q: x(t) \leftrightarrow a_k, x(1-t) + x(t-1) \leftrightarrow b_k. \text{ Calculate } b_k.$$

Ans:-

$$x(-t) \leftrightarrow a_{-k} \quad (\text{Here } (-) \text{ sign come because of } -k)$$

$$x(-t+1) \leftrightarrow e^{-j k \omega_0} a_{-k}$$

$$x(t-1) \leftrightarrow e^{-j k \omega_0} a_k$$

$$b_k = e^{-j k \omega_0} a_{-k} + e^{-j k \omega_0} a_k$$

$$b_k = e^{-j k \omega_0} [a_k + a_{-k}]$$

$$\text{If even} \rightarrow e^{-j k \omega_0} [2a_k]$$

$$\text{If odd} \rightarrow \cancel{e^{-j k \omega_0}} 0$$

Since for odd part value is zero, the signal is purely even signal.

01

JUNE
Saturday

153/213

Monday	3	10
Tuesday	4	11
Wednesday	5	12
Thursday	6	13
Friday	7	14
Saturday	8	15
Sunday	9	16

22 Week

4. Time Scaling:

$$x(t) \leftrightarrow a_k$$

$$x(at) \leftrightarrow b_k$$

then $b_k = a_k$.

* $x(t) = \cos t$

$x(at) = \cos at$

⇒ there will be no change in coefficients.

Q: $x(t) = \cos 2\omega_0 t$

$$x(t) = \frac{e^{j2\omega_0 t} + e^{-j2\omega_0 t}}{2}$$

$$= \frac{1}{2} e^{j2\omega_0 t} + \frac{1}{2} e^{-j2\omega_0 t}$$

$$a_1 = 1/2, a_{-1} = 1/2$$

$$x(t) = \cos \omega_0 t \Rightarrow a_1 = 1/2, a_{-1} = 1/2$$

7/2024

	1	8	15	22	29
Monday	2	9	16	23	30
Tuesday	3	10	17	24	-
Wednesday	4	11	18	25	-
Thursday	5	12	19	26	-
Friday	6	13	20	27	-
Saturday	7	14	21	28	-
Sunday					

JUNE
Tuesday

04

156/210

23 week

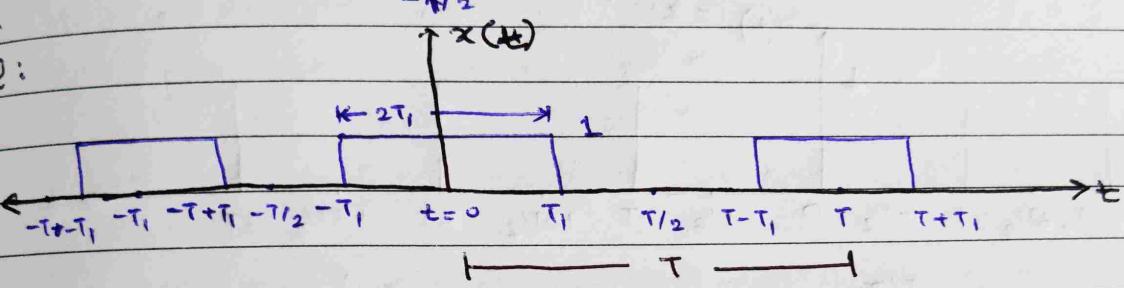
(NH)

10/12/24

→ Analysis Equation :

$$a_k = \frac{1}{T} \int_{-\pi/2}^{\pi/2} x(t) e^{-j k \omega_0 t} dt$$

Q:



$$a_k = \frac{1}{T} \int_{-T_1}^{T_1} x(t) e^{-j k \omega_0 t} dt$$

$$= \frac{1}{T} \int_{-T_1}^{T_1} e^{-j k \omega_0 t} dt$$

$$= \frac{1}{T} \left[\frac{e^{-j k \omega_0 t}}{-j k \omega_0} \right]_{-T_1}^{T_1}$$

$$= \frac{1}{T} \frac{e^{-j k \omega_0 T_1} - e^{-j k \omega_0 (-T_1)}}{-j k \omega_0}$$

$$= \frac{1}{T} \frac{2 e^{-j k \omega_0 T_1}}{2 j} = \frac{e^{-j k \omega_0 T_1}}{j k \omega_0}$$

$$= \frac{2}{j k \omega_0} \sin(k \omega_0 T_1)$$

Here, $\omega_0 = 2\pi/T$ $a_k = \frac{1}{k \pi} \sin(k \omega_0 T_1)$ For periodic pulse train

Pulse width = $2T_1$ Time Period = T [For $k=0$, value is undefined.]

$$a_0 = \lim_{k \rightarrow 0} \frac{d/dk \sin(k \omega_0 T_1)}{d/dk (\pi/k)}$$

[Using L'Hospital Rule]

05

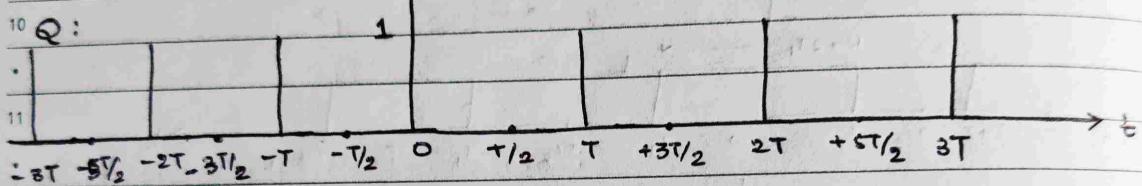
JUNE
Wednesday

15/209

$$= \lim_{k \rightarrow 0} \left(\cos k \omega_0 T_1 \right) \frac{\omega_0 T_1}{\pi}$$

$$a_0 = \frac{\omega_0 T_1}{\pi}$$

$$P(t) = \sum \delta(t - kT) = \frac{a_0}{\pi}$$



Ans:-

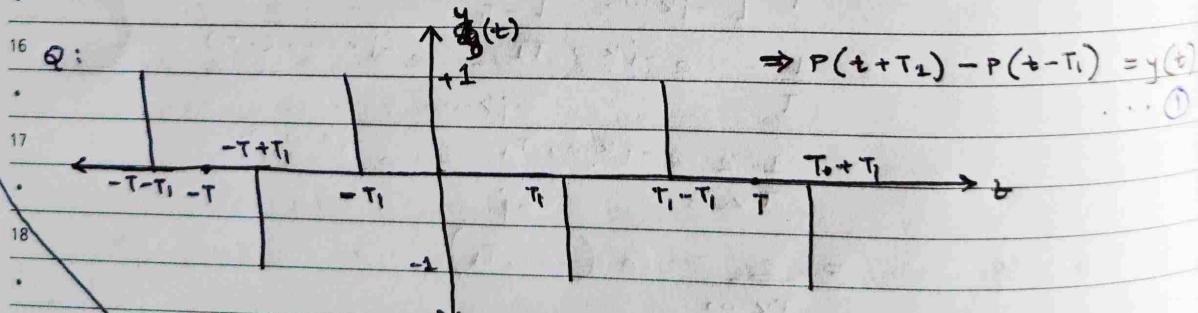
$$a_k = \int_{-T/2}^{T/2} \delta(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T} \left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0} \right]_{-T/2}^{T/2}$$

This signal is periodic
but not sinusoidal

$$a_k = \frac{1}{T} \quad \leftarrow \text{For impulse train}$$

Independent of 'k'



Ans:-

$P(t - T_1)$	$-2T_1 + T_1$	$-T + T_1$	T_1	$T + T_1$	$2T + T_1$
$P(t + T_1)$	$-2T - T_1$	$-T - T_1$	$-T_1$	$T - T_1$	$2T - T_1$

Using shifting
and linearity
property

$$P(t) \xleftrightarrow{FSC} \frac{1}{T}$$

$$P(t + T_1) \xleftrightarrow{FSC} e^{jk\omega_0 T_1} \cdot \frac{1}{T}$$

$$P(t - T_1) \xleftrightarrow{FSC} e^{-jk\omega_0 T_1} \cdot \frac{1}{T}$$

From ①

$$\Rightarrow \frac{1}{T} (e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}) = \frac{2j}{2} \cdot \sin(j\omega_0 T_1)$$

The reward of one duty done is the power to fulfill another. — George Eliot

Monday	
Tuesday	3
Wednesday	4
Thursday	5
Friday	6
Saturday	7
Sunday	8

Monday	1
Tuesday	2
Wednesday	3
Thursday	4
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Saturday	6
Sunday	7

23.week
13/12

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	7/2024						
Monday	1	8	15	22	29		
Tuesday	2	9	16	23	30		
Wednesday	3	10	17	24	31		
Thursday	4	11	18	25	-		
Friday	5	12	19	26	-		
Saturday	6	13	20	27	-		
Sunday	7	14	21	28	-		

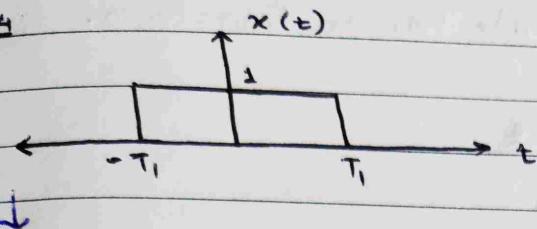
* * *
Q: How can we get Fourier transform from Fourier series coefficients? JUNE Wednesday

12

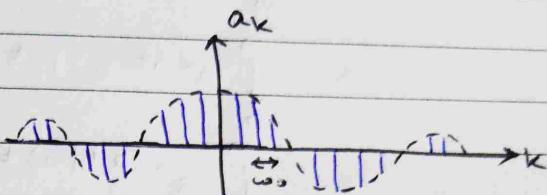
24.week

164/202

19/12/24



Fourier Transform



$$a_k = \frac{\sin k \omega_0 T_1}{\pi k}$$

$$a_0 = \frac{\omega_0 T_1}{\pi}$$

To make the above graph continuous,

$$\omega_0 \rightarrow 0$$

∴ For $T \rightarrow \infty$, calculate Fourier transform. $\hookrightarrow T = \frac{2\pi}{\omega_0} \Rightarrow T = \infty$

$$\text{Also, } k\omega_0 \rightarrow \omega$$

By analysis equation,

$$a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-j k \omega_0 t} dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} x(t) e^{-j k \omega_0 t} dt$$

$$\lim_{T \rightarrow \infty} a_k = \int_{-\infty}^{\infty} x(t) e^{-j k \omega_0 t} dt \quad \left| \begin{array}{l} \text{Fourier Transform} \\ \text{From this we can} \\ \text{get continuous} \\ \text{waveform.} \end{array} \right.$$

$$x(\omega) = \lim_{T \rightarrow \infty} a_k = \int_{-\infty}^{\infty} x(t) e^{-j \omega t} dt$$

$$F.T \rightarrow x(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j \omega t} dt \rightarrow x(0) = \int_{-\infty}^{\infty} x(t) dt$$

$$\text{Inverse FT} \rightarrow x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) e^{j \omega t} d\omega \rightarrow x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) d\omega$$

[Area in frequency domain]

Area of a signal in time domain - represent the frequency domain in 0 and integrate the signal in a given range.

The fruit of love is service, The fruit of service is peace, And peace begins with a smile. - Mother Teresa

13

JUNE
Thursday

		6/2024
Monday	3	10 17
Tuesday	4	11 18
Wednesday	5	12 19
Thursday	6	13 20
Friday	7	14 21
Saturday	1	8 15 22
Sunday	2	9 16 23

165/201

24. Week

Q: $x(t) = e^{-at} u(t)$. Calculate Fourier Transform.

Ans:-

$$x(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{e^{-(a+j\omega)t}}{-a-j\omega} \Big|_0^{\infty} \Rightarrow x(\omega) = \frac{1}{a+j\omega}$$

Magnitude

$$|x(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$$

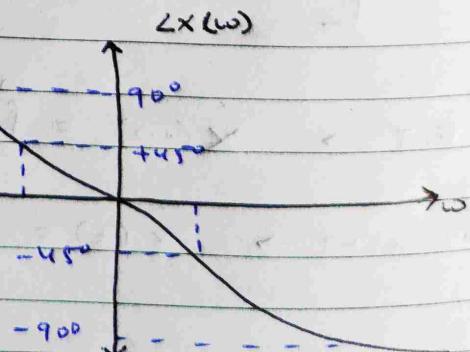
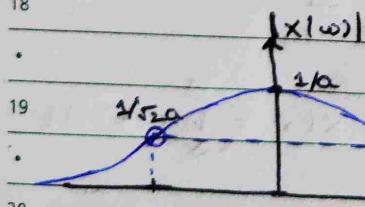
Phase

$$\angle x(\omega) = \frac{-1}{\tan^{-1} \omega/a}$$

$$= -\tan^{-1} \omega/a$$

$$(L_1 = 0)$$

<u>ω</u>	<u>$x(\omega)$</u>	<u>$\angle x(\omega)$</u>
0	$1/a$	0°
$-a$	$1/\sqrt{2}a$	45°
$+a$	$1/\sqrt{2}a$	-45°
$-\infty$	0	$+90^\circ$
$+\infty$	0	-90°



We could never learn to be brave and patient, if there were only joy in the world. — Helen Keller

7/2024

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Sunday	7	14	21	28	-

JUNE
Friday

14

24. week

166/200

Q : $x(t) = e^{-at} u(t)$

Ans :-

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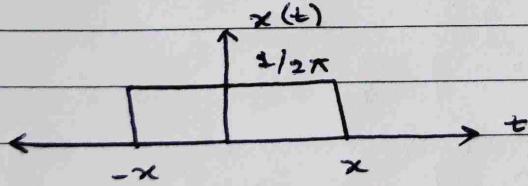
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15 Q :



Ans :-

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-j\omega t} dt$$

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