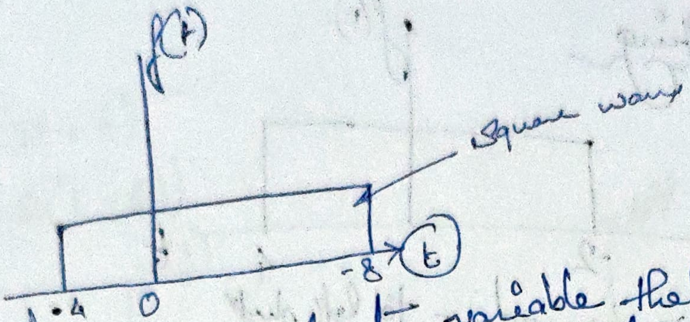
Signal & Systems



Signal

1. Dependent variable that depends on some independent variable (like time, space, etc)
2. Physical quantity
3. Have some information that can be displayed, manipulated and even conveyed.

defn of signal.

System

1. Conversion of one form of signal to another form of signal

Two types of signals

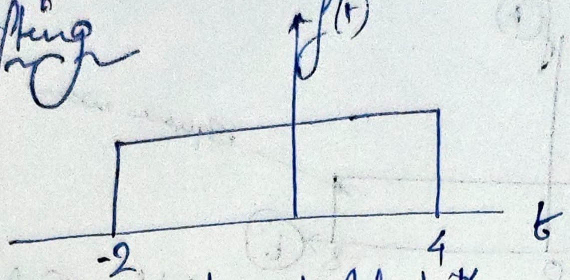
Signals
- Analog signal $f(t)$ → t
  - Continuous in time Continuous value
  - Discrete in time Continuous value
  - Continuous in time Discrete value
  - Discrete time Discrete value.
- Digital signal. (0/1)

Note* A discrete signal is analog signal

Operations on signal
1. Time Scaling
2. Time Shifting
3. Time Reversal

① Time Shifting

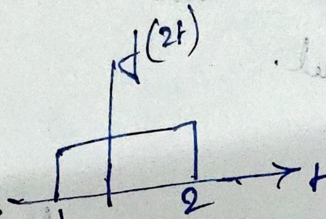
$f(t) =$



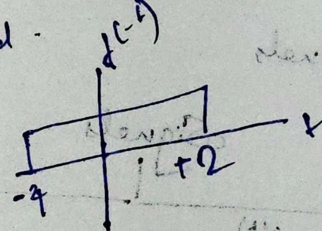
$f(t+3) \rightarrow$ advanced → to left shift

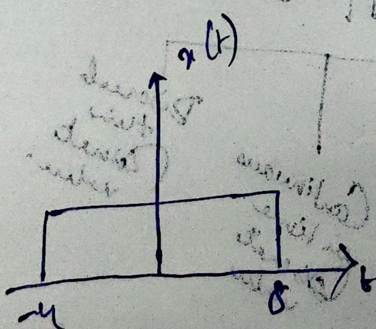
$f(t-3) \rightarrow$ Delayed → to right shift

② Time Scaling



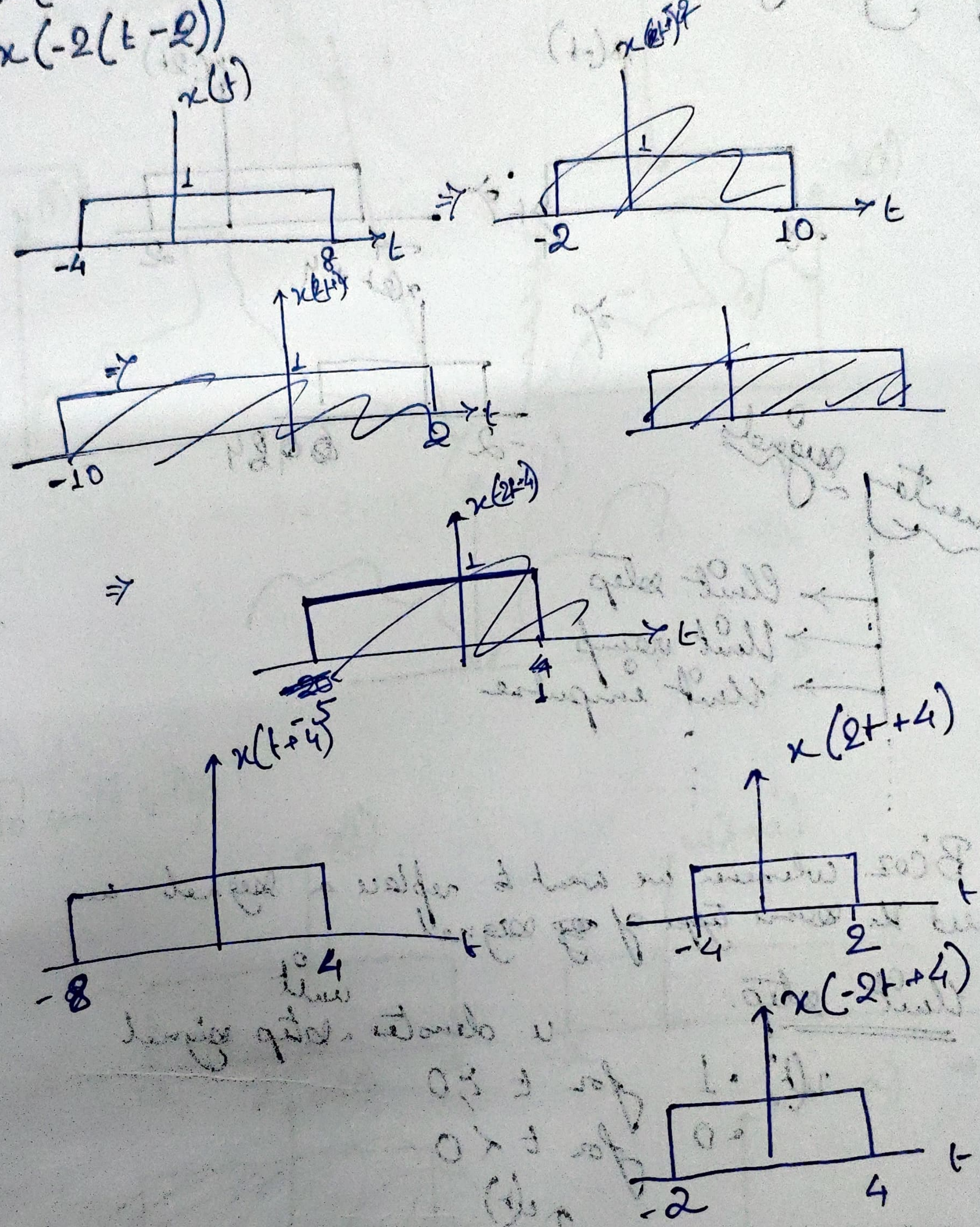
③ Time reversal



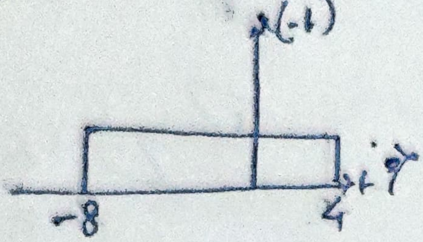


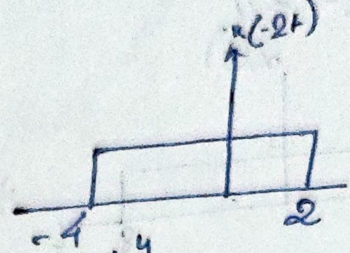
find $x(-2t+4)$

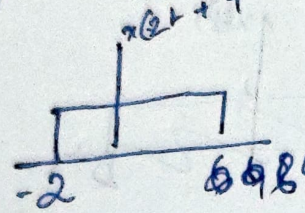
find
$x(-2t+4)$
$x(-2(t-2))$



from right to left.







Elementary signal

- Unit step
- Unit ramp
- Unit impulse
- ⋮

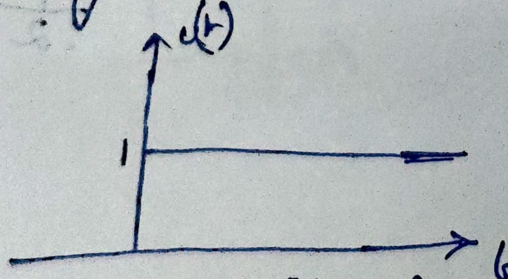
B'coz whenever we want to replace a signal we need the same type of signal.

Unit step

u denotes unit step signal

$u(t) = 1$ for $t \geq 0$

$\quad\;\; = 0$ for $t < 0$

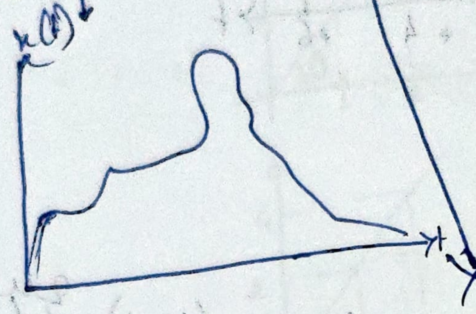


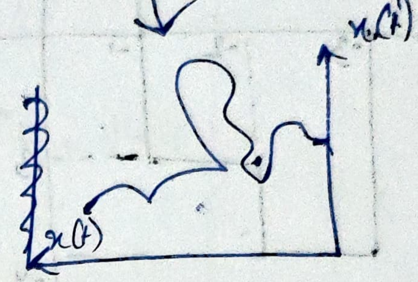
Scaling operation gives same answer. but for it starts a value other than 0 then that value will be divided

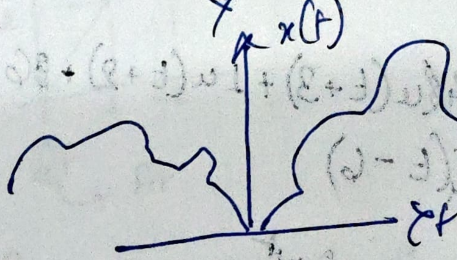
Causality → $x(t) = 0$ for $t < 0$

AntiCausality → $x(t) = 0$ for $t > 0$

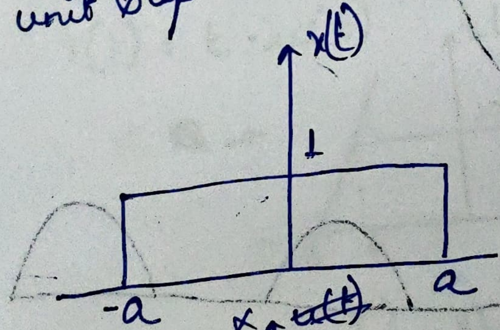
Non-causality → $x(t) \neq 0$ for both

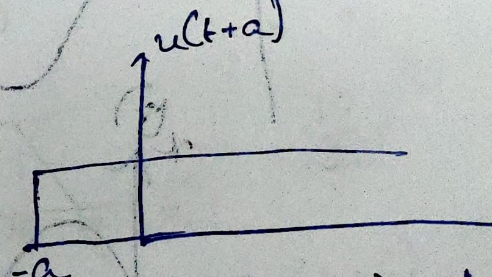


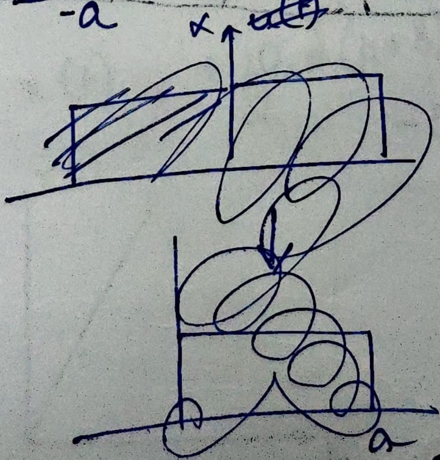


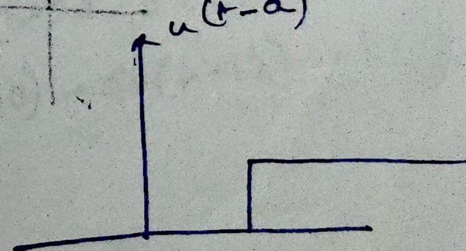


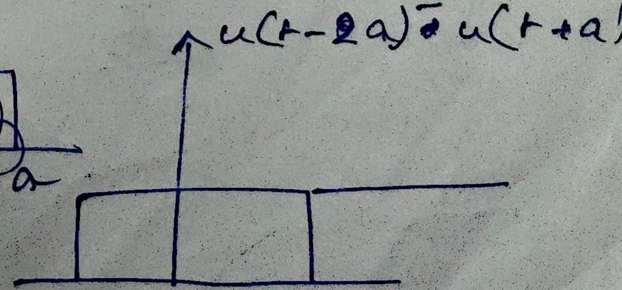
① To unit step



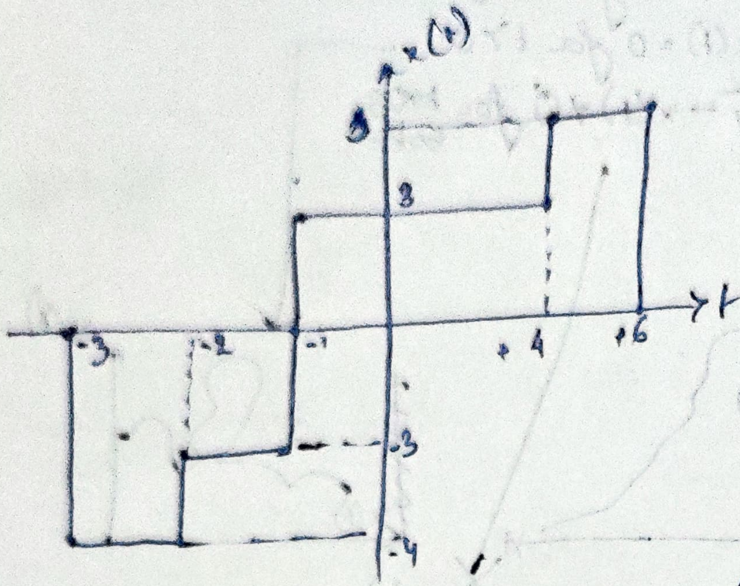




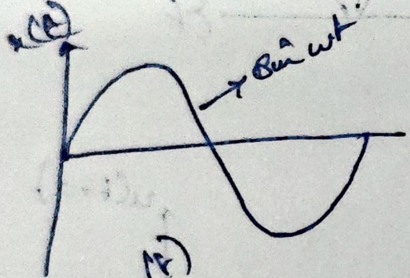




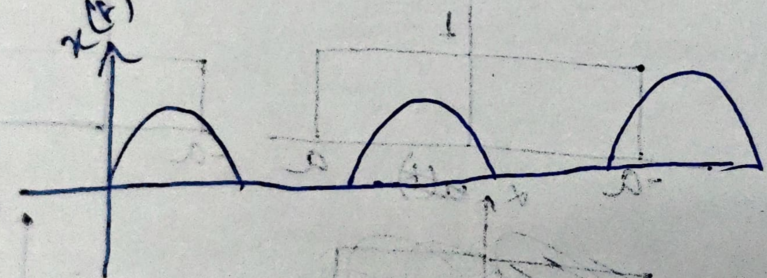
$Au(t+a) = Au(t+\frac{a}{a})$



$-4u(t+3)+1u(t+2)+6u(t+1)+2u(t-4)$
$-5u(t-6)$



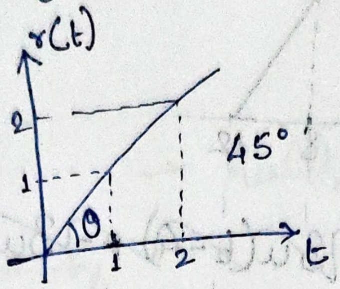
$x = (\sin wt) = ?$



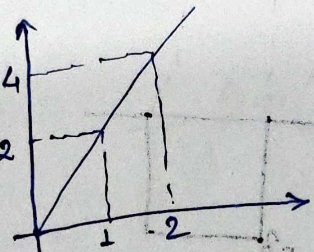
ramp signal.

$r(t) = t$ for $t \geq 0$

$= 0$ otherwise



$\frac{\Delta y}{\Delta x} = 1$

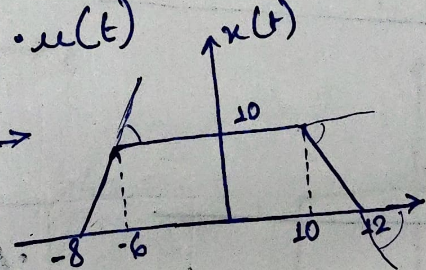


$r(t) = At \to At$

$u(-t-1) = 0$
then signal is always ~~anti~~ anticausal

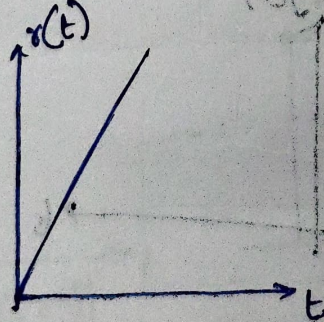
$r(t) = t \cdot u(t)$

→



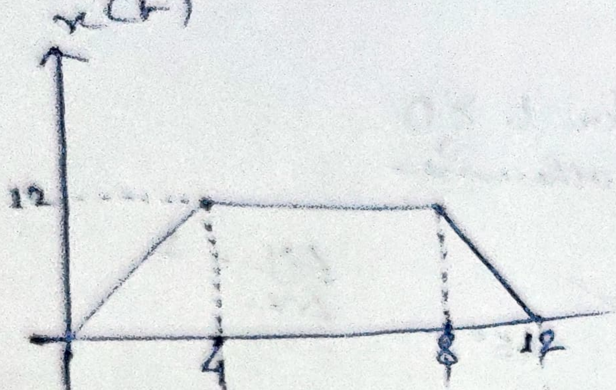
$\frac{10}{2}$

$x(t) = + \frac{10}{2} \cdot r(t+8) \cancel{+ 5r(t-10)} - 5r(t+6) - 5r(t-10) + 5r(t-12)$

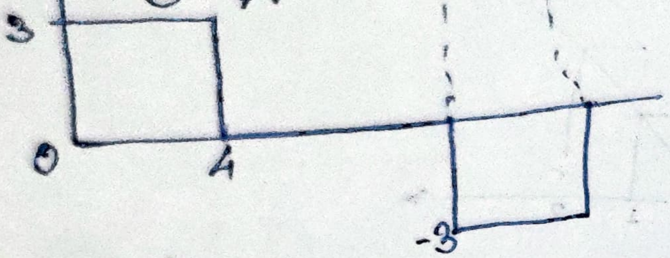


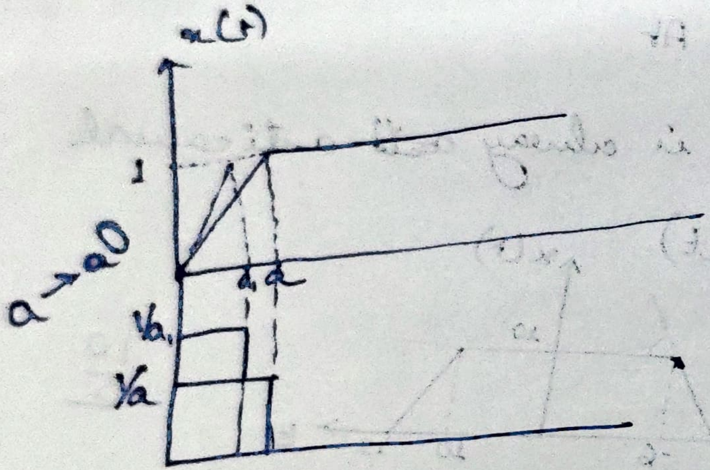
$\xrightarrow{\frac{d}{dt}}$

$\frac{d}{dt}[r(t)] = A \cdot u(t)$



$$3u(t) - 3u(t-4) - 3u(t-8) + 3u(t-12)$$

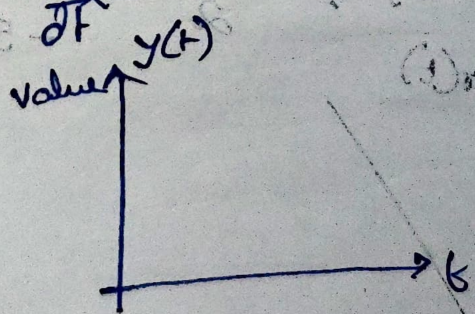




~~derivative~~ derivative of a step signal we will have impulse signal $\delta(t) = \frac{d}{dt}(u(t))$

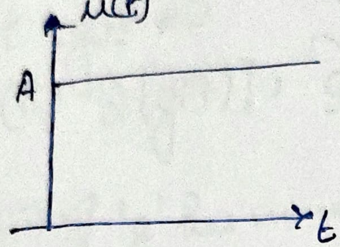
For

value = ∞
~~impulse~~
max area = 1 = $\delta(t)$
~~$\delta(t)$~~



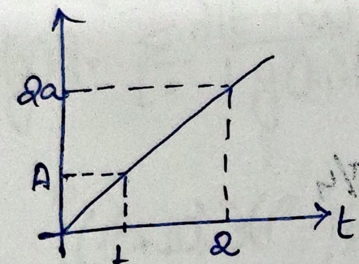
$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

(i)



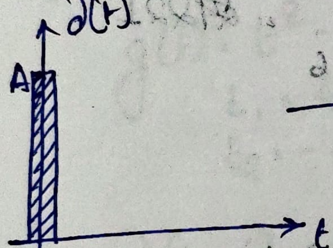
$\rightarrow Au(t) \rightarrow A \rightarrow$ magnitude

(ii)



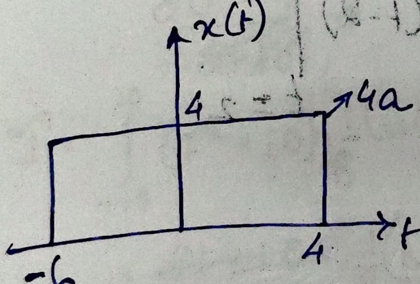
$\rightarrow Ar(t) \rightarrow A \rightarrow$ slope.

(iii)

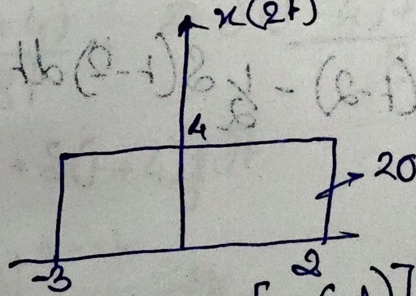


$\rightarrow$ ~~Aδ(t)~~ $A\delta \rightarrow A \rightarrow$ Area at t.

(iv)

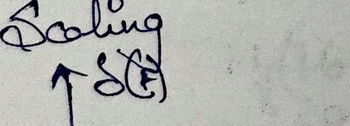


$\rightarrow$

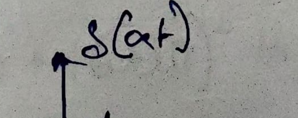


$$\text{area}[x(\alpha t)] = \frac{1}{\alpha}\,\text{area}[x(t)]$$

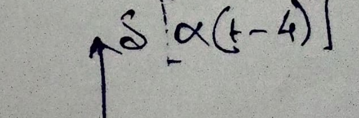
Scaling

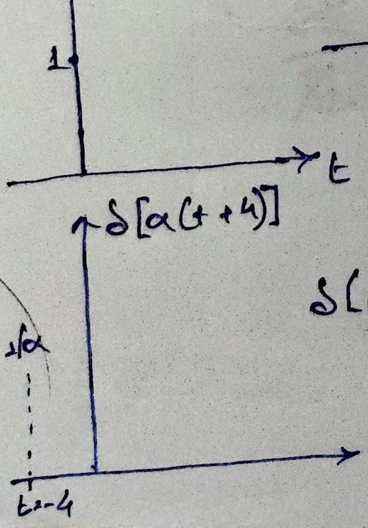


$\rightarrow$



$\rightarrow$





$\delta(t+t_0)$

$x(t) \cdot \delta(t - t_0) = x(t_0)$

$x(t) \cdot \delta(t + t_0) = x(-t_0)$

$$I = \int_{-6}^{+8} (t^2+4)\, \delta(t-3)\, dt$$

$$= t^2+4 \Big|_{t=3} = 13$$

$$\int_{-\pi}^{\pi} \cos^2 t\, \delta(t - \pi/4)\, dt = \cos^2 t \Big|_{t=\pi/4}$$

$$= 1$$

$$\int_{-3}^{4} (t^3+5)\, \delta(t-6)\, dt = t^3+5 \Big|_{t=6} = \cancel{221}\ 221$$

$$\int_{-6}^{+5} (t-2)\, \delta(2t-4)\, dt =$$

$$\int_{-6}^{} (t-2) \cdot \frac{1}{2} \delta(t-2)\, dt = \frac{1}{2}(t-2) \Big|_{t=2}$$

$$= 0$$

Properties of impulse response

$$I = \int_{-\infty}^{\infty} x(t) \cdot \partial\{g(t)\} dt$$

Conditions

(i) $\partial[g(t)] = 0$ if 'g' is nowhere 'zero' for any value of t, $g(t) \neq 0$

(ii) $\partial[g(t)] = \dfrac{\partial(t-t_0)}{g'(t_0)}$ if 'g' has one real root at $t = t_0$

(iii) If 'g' has more than one ~~relation~~ real root:

$$\partial[g(t)] = \sum_{i} \frac{\partial(t - t_i)}{|g'(t_i)|} = \sum \frac{\partial(t-t_0)}{|g'(t_0)} + \frac{\partial(t-t_1)}{|g'(t_1)|} + \dots\dots$$

$$I = \int_{-10}^{10} (t^2 + 10)\, \partial(t^2 - 16)\, dt$$

$g(t) = t^2 - 16$

$\left.\begin{array}{l} t_1 = 4 \\ t_2 = -4 \end{array}\right\}$ roots of t

$$\frac{\partial(t-4)}{\cancel{2t}|8|} + \frac{\partial(t+4)}{|-8|} = \cancel{2}\,\frac{\partial(t-4) + \partial(t+4)}{8}$$

$$I = \int_{-10}^{10} (t^2 + 10) \cdot \frac{1}{8}\{\delta(t-4) + \delta(t+4)\}\, dt$$

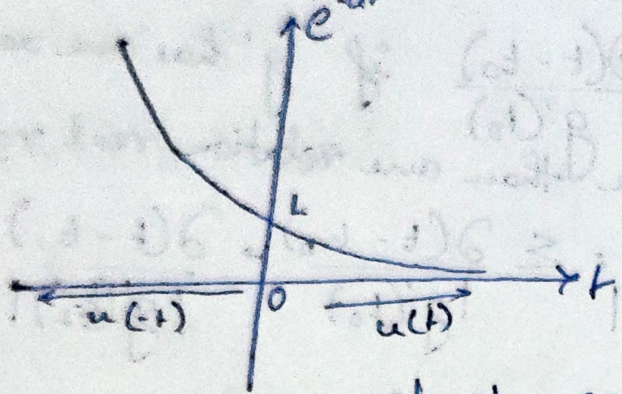
$$= \cancel{2t+10}\ \frac{1}{8}(t^2 + 10)\Big|_{t=-4}^{t=+4} \cancel{dt}.$$

$$= \frac{1}{8}(16 + 10 + 16 + 10)$$

$$= \frac{\cancel{52}}{\cancel{8}_2} = \frac{13}{2} = 6.5.$$

Exponential function

$x(t) = e^{-at}$



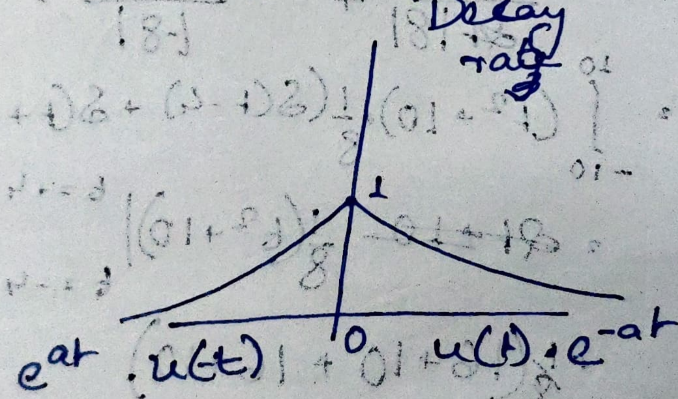
e^{-at} - Non-causal to causal/anticausal

$e^{-at} \cdot u(t) \longleftarrow$

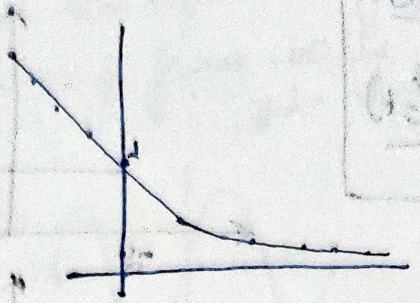
$x = e^{-a|t|}$

$e^{-a|t|} = e^{-a(t)} \cdot u(t) + e^{-a(-t)} \cdot u(-t)$

$= e^{-at} \cdot u(t) + e^{at} \cdot u(-t)$

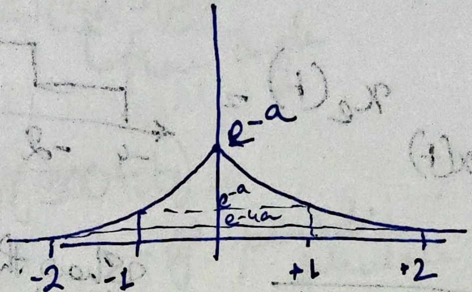


② $x(t) = e^{-at^2}$



$e^{-at^2} \cdot u(t)$

$e^{-a|t^2|} = e^{-a(t^2)} \cdot u(t) + e^{-a(t^2)} \cdot u(-t)$



It is not perfectly exponential & the fig. is known gaussian structure

⇒ Start at 1 if coefficient to unit

⇒ Both side change with t^2

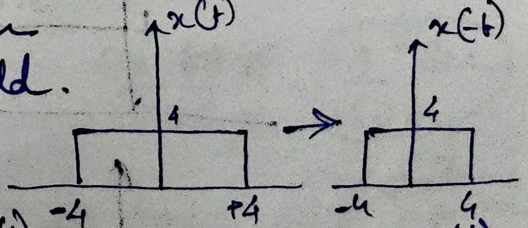
Classification of Signals

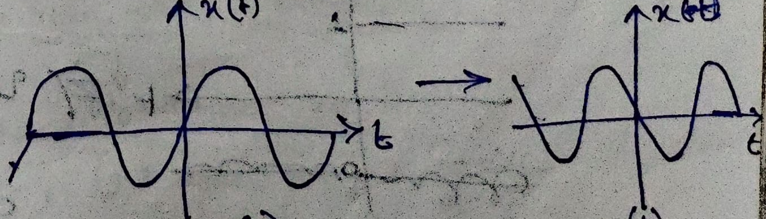
① Even and add signals.

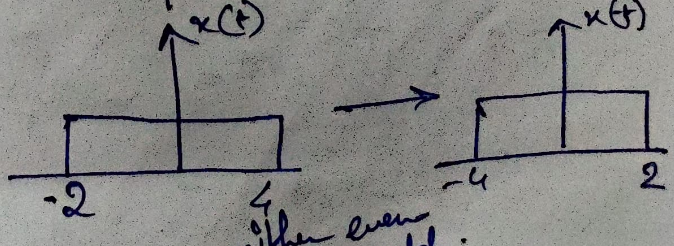
Theorem

⊙ $x(-t) = x(t) \longrightarrow$ even

⊙ $x(-t) = -x(t) \longrightarrow$ add.

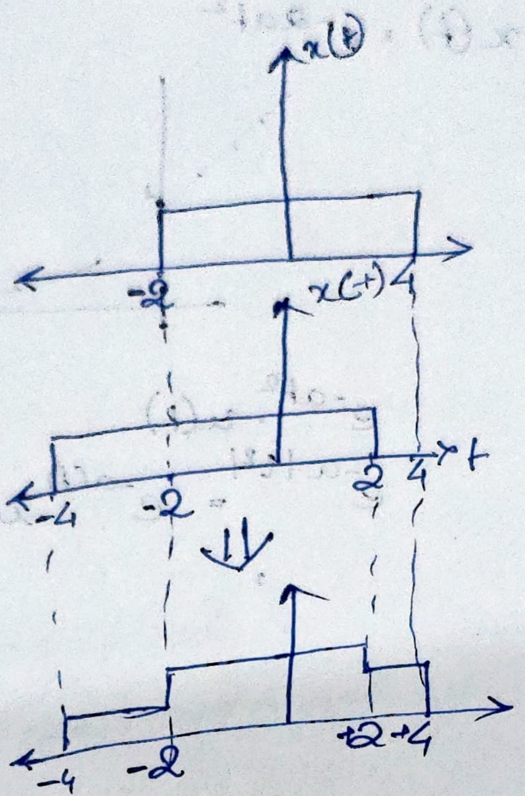


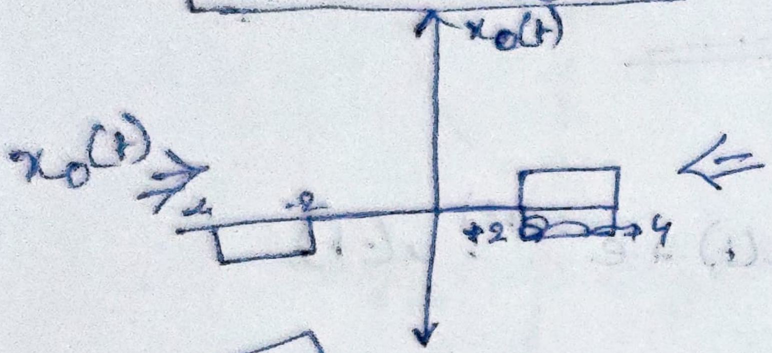




neither even nor add.

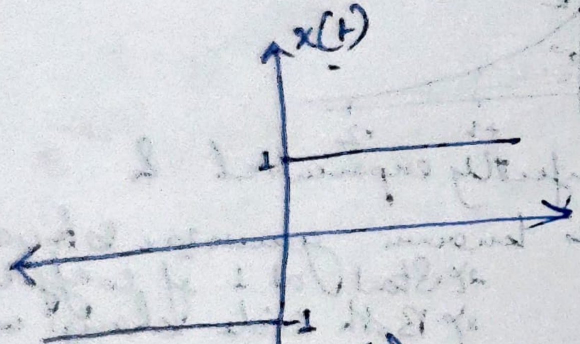
$$\boxed{\begin{aligned} x_e(t) &= \frac{x(t)+x(-t)}{2} \\ x_o(t) &= \frac{x(t)-x(-t)}{2} \end{aligned}}$$



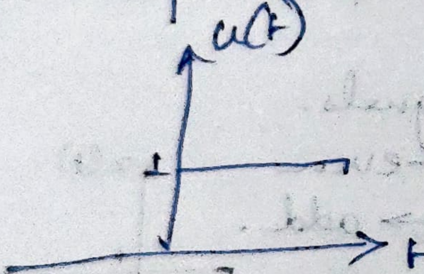


$\boxed{\text{Q. pos. is imp}}$

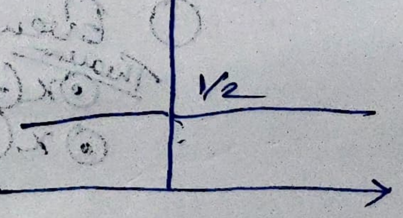
$x_e(t) =$

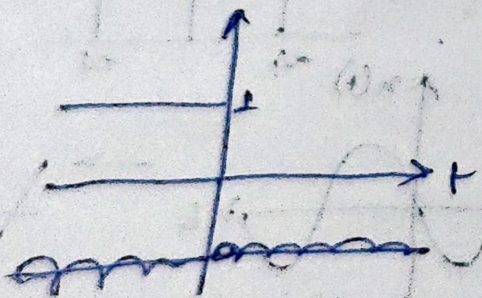


Show that even & odd of $u(t)$ can be represented in terms of $x(t)$

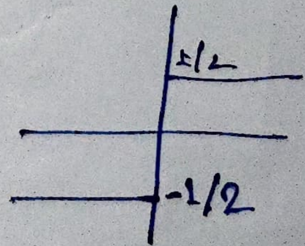


$\Rightarrow u_e(t) =$





$\Rightarrow u_o(t) = \frac{1}{2} x(t)$



periodic signals

Any signal that repeats itself ~~from~~ after certain time

must maintain range from $-\infty$ to $+\infty$

time shifting has no effect on periodicity

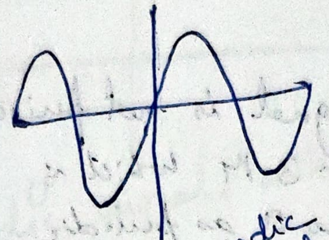


Fig. periodic signal.

amplitude

phase shift

(i) $x(t) = A \cdot \sin(\omega_0 t \pm \phi)$

phase angle

phase angle related to angular frequency

(ii) $x(t) = 5 \cdot \cos(300\pi t)$

→ If periodic calculate fundamental period.

(iii) $x(t) = 5 \cdot \cos(20\pi t + \pi/2)$

Ans (ii) Step 1: Compare of generalized form i.e. $x(t) = A \sin(\omega_0 t \pm \phi)$

$\omega_0 = 300\pi$

$T = \frac{2\pi}{\omega_0} = \frac{2 \times \cancel{180}\,\cancel{\pi}}{300\,\cancel{\pi}} = \boxed{\frac{1}{150}}$ sec

fundamental period.

Suppose, $x(t) = 4\cos \pi t + 3\sin 2\pi t + 2\sin 3\pi t$

$\omega_0 = \pi$, $\omega_0 = 2\pi$, $\omega_0 = 3\pi$

$T_1 = 2$ sec, $T_2 = 1$ sec, $T_3 = \frac{2}{3}$ sec

$\frac{T_2}{T_1} = \frac{1}{2}$ or $\frac{T_3}{T_2} = \frac{2}{3}$ or $\frac{T_1}{T_3} = 3$

$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_1}{T_3} =$ Rational

Rational numbers

1. Ratio of P/q where p, q are integer
2. Decimal values must be terminating/repeating

Note

π value is π then signal is not periodic
But system π means $\pi = 3.14$ which is periodic
So the sys. considers π as periodic

$x(t) = 4\cos\pi t + 3\sin 2\pi t + 2\sin 3\pi t$

fundamental period $= LCM\{T_1, T_2, T_3\}$

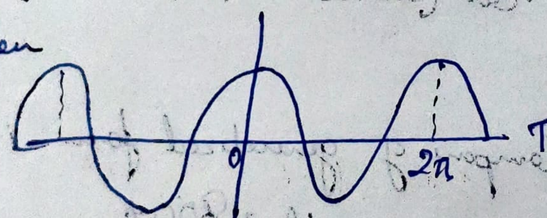
$= LCM[2, 1, \frac{2}{3}]$

$= \frac{LCM(2, 1, 2)}{HCF(1, 1, 3)} = \frac{2}{1} = 2$

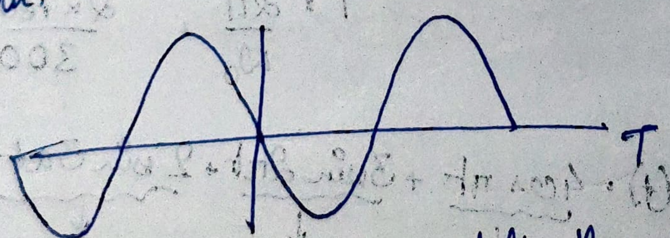
$\therefore$ Fundamental period $= 2$

Q $x(t) = 5\cos(20\pi t + \pi/2)$

given



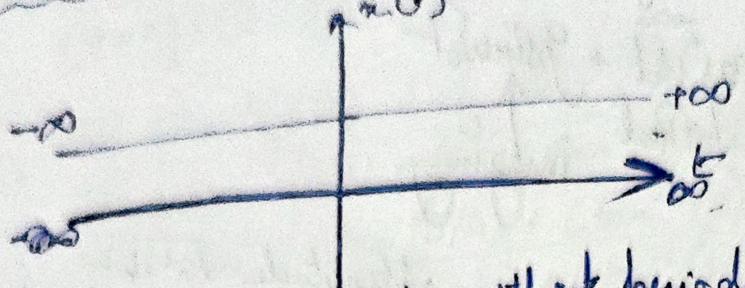
Soln.



If there is phase shift then it doesn't hamper the periodicity of signal.

$\omega_0 = 20\pi$

Constant / DC signal



periodic signal without periodic value

$w(t) = 4 + 5\cos(20\pi t + \pi/2)$

If this is periodic then whole signal is periodic

$x(t) = \text{Even of } [\sin 4\pi t \cdot u(t)]$

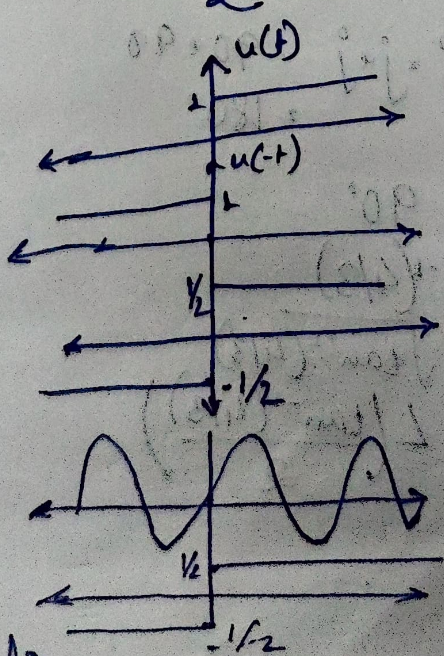
$x_1(t) = \sin(-4\pi t) \cdot u(-t) = -\sin 4\pi t \cdot u(-t)$

$x_2(t) = \sin(4\pi t) \cdot u(t)$

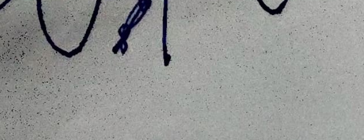
$= \dfrac{\sin 4\pi t \cdot u(t) - \sin 4\pi t \cdot u(-t)}{2}$

$= \sin 4\pi t \dfrac{(u(t) - u(-t))}{2}$ ← even part of $x(t)$

=



⇒



Proved Not periodic.

Complex exponential Signal

$$x(t) = e^{j\omega_0 t} = \overset{\nearrow a}{\cos\omega_0 t} + \overset{\nearrow b}{j\sin\omega_0 t}$$

$\cos\omega_0 t \to$ Real

$j\sin\omega_0 t \to$ imaginary

Magnitude = 1

phase = $\tan^{-1}\left(\frac{\sin\omega_0 t}{\cos\omega_0 t}\right)$

Magnitude = $\sqrt{a^2+b^2}$

phase angle = $\tan^{-1}(b/a)$

$$x(t) = e^{j\omega_0 t} + e^{j\omega_1 t} + e^{j\omega_2 t}$$

$\downarrow T_1 \quad \downarrow T_2 \quad \downarrow T_3$

$$e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$$

$T = LCM(T_1, T_2, T_3)$

Rectangular type

$a + jb$

$\tan^{-1}(b/a)$

$a = \cos\omega_0 t$

$b = \sin\omega_0 t$

$3 + j4$

$r = \sqrt{3^2+4^2}$

$= 5$

$\theta = \tan^{-1}(b/a)$

$= \tan^{-1} 4/3$

$j = 90^\circ$

$j^2 = 180^\circ$

$j^3 = 270^\circ$

$j \cdot j = j^2 = 90 + 90$

$= 180^\circ$

Similarly $\sqrt{j^2} = 90^\circ$

$x = 5\tan^{-1}(4/3)$

$\sqrt{x} = \sqrt{5} \cdot \sqrt{\tan^{-1}(4/3)}$

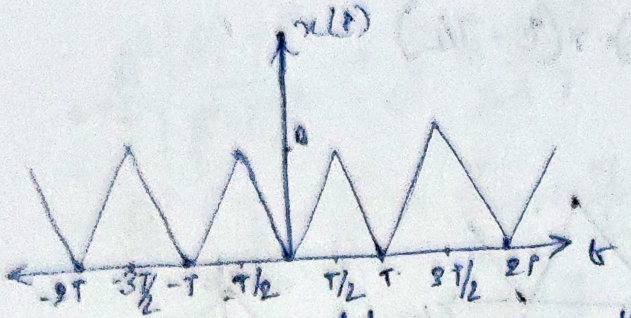
$= \sqrt{5} \cdot \angle\left(\frac{\tan^{-1}(4/3)}{2}\right)$

$$x = \frac{(4+5j)(3+6j)}{(8+4j)(9+3j)}$$

$$= \frac{\sqrt{41}\angle\tan^{-1}(5/4) \cdot \sqrt{45}\angle\tan^{-1}(2)}{\sqrt{80}\angle\tan^{-1}(4/8) \cdot \sqrt{90}\angle\tan^{-1}(1/3)}$$

$$= \frac{\sqrt{41}\cdot\sqrt{45}}{\sqrt{80}\cdot\sqrt{90}}\left(\frac{\angle\tan^{-1}(5/4) + \angle\tan^{-1}(2)}{\angle\tan^{-1}(1/2) + \angle\tan^{-1}(1/3)}\right) \text{degree}.$$

$$= \frac{42.953}{84.852}.$$



* It must be bounded
* If a signal is periodic then it is power signal.

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$\boxed{y = mx + c}$

m = slope
c = ~~rise~~ y - intercept.

$y_{ab} = -\frac{2A}{T}\cdot t + 0$ for $-T/2$ to 0

$y_{bc} = \frac{2A}{T}\cdot t + 0$ for $0 < t < T/2$

$$\boxed{\begin{aligned} y_{ab} &= \frac{2A}{-T}t + 0 \\ y_{bc} &= \frac{2At}{T} + 0 \end{aligned}}$$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \frac{1}{T}\left[\int_0^{T/2} |x(t)|^2 dt + \int_{-T/2}^{0} |x(t)|^2 dt\right]$$

$$= \frac{1}{T}\left[\int_0^{T/2} \left(\frac{2A}{T}\cdot t\right)^2 dt + \int_0^{T/2} \left(\frac{2A}{T}\cdot t\right)^2 dt\right]$$

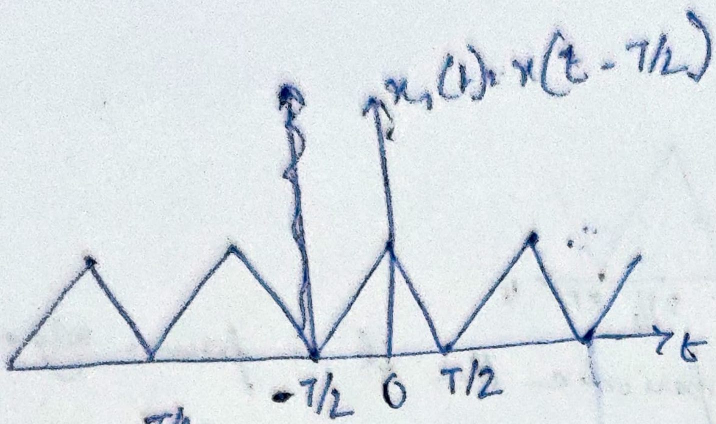
$$= \frac{1}{T}\cdot\left[\frac{4A^2}{T^2}\left[\frac{t^3}{3}\right]_0^{T/2} + \cancel{\frac{4A^2}{}}\frac{4A^2}{T^2}\cdot\left[\frac{t^3}{3}\right]_0^{T/2}\right]$$

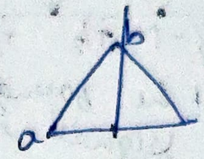
$$= \frac{1}{T}\cdot\frac{4A^2}{T^2}\left[\frac{T^3}{24} + \frac{T^3}{24}\right]$$

$$= \frac{4A^2}{\cancel{T^3}} \times \frac{2\cancel{T^3}}{24} = \frac{A^2}{3} \approx \text{power of signal.}$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= P_x \times \text{time} = \infty$$





$m = \frac{2A}{T}$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} |x(t - T/2)|^2\, dt$$

$$= \frac{1}{T}\left[\int_{0}^{T/2} |x(t - T/2)|^2\, dt + \int_{-T/2}^{0} |x(t - T/2)|^2\, dt\right]$$

$m_{ab} = \frac{2A}{T}$

$y_{ab} = \frac{2A}{T} \cdot t + A$

$y_{bc} = -\frac{2A}{T} \cdot t + A$

$$= \int_{0}^{T/2} \left(\frac{2A}{T}\cdot t + A\right)^2 dt$$

$$= \left[\frac{4A^2 \cdot t^2}{T^2} + A^2 + 2\cdot\frac{2At}{T}\cdot A\right]_0^{T/2}$$

$$= \frac{4A^2 \cdot t^2}{T^2} + A^2 + \frac{4A^2 t}{T}$$

$$= \frac{1}{T}\cdot A^2 \left\{\int_{0}^{T/2}\left[\frac{4t^2}{T^2} + 1 + \frac{4t}{T}\right] dt + \int_{-T/2}^{0}\left[\frac{4t^2}{T^2} + 1 - \frac{4t}{T}\right] dt\right.$$

$$= \frac{1}{T}\cdot A^2 \left\{\left[\frac{4t^3}{3T^2}\right]_0^{T/2} + [t]_0^{T/2} + \frac{4}{T}\left[\frac{t^2}{2}\right]_0^{T/2} + \left[\frac{4t^3}{T^2\cdot 3}\right]_{-T/2}^{0} + [t]_{-T/2}^{0} - \frac{4}{T}\left[\frac{t^2}{2}\right]_{-T/2}^{0}\right\}$$

$$= \frac{A^2}{T}\left\{\frac{4 \cdot T^3}{8 \cdot 3T^2} + \frac{T}{2} + \frac{4T^2}{2 \cdot 4T} + \frac{4T^3}{3T^2 \cdot 8_2} + \frac{T}{2} - \frac{4T^2}{8T}\right\}$$

$$= \frac{A^2}{T}\left[\frac{T}{6} + \frac{T}{2} + \frac{T}{6} + \frac{T}{2}\right]$$

$$= \frac{A^2}{T}\left[\frac{T + 3T + T + 3T}{6}\right] = \frac{\cancel{8}^{4}T}{\cancel{6}_{3}} = \frac{2A^2}{3}$$

Q. $e^{2t} \cdot u(t)$

$$P_x = \cancel{\frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\, dt}$$

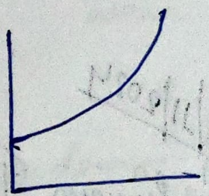
$$= |e^{At} \cdot u(t)|^2\, dt$$

$$= e^{2At} \cdot u(t)\, dt$$

m =

find

$$\boxed{\begin{matrix} E = \infty \\ P = 0 \end{matrix}}$$



$x(t) = 4 + 4\sin 500t + 8\cos 300\pi t$

rms value of $4 = 4$

" " $4\sin 500t = \cancel{4/\sqrt{2}}\ 4^2/2$

$8\cos 300\pi t = 8^2/2$

$P_x = 4^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{8}{\sqrt{2}}\right)^2$

$= 16 + \frac{256}{4} + \frac{4096}{\cancel{8}4}$

$= \frac{64 + 256 + 4096}{4}$

$$\begin{array}{r} 64 \\ \times 64 \\ \hline 256 \\ 384\times \\ \hline 4096 \end{array}$$

28/11/2024

Continuous Time Fourier Transform

Technique to convert a signal to frequency domain.

Limitation :

- A periodic signal can be represented in its frequency domain

Linear Combination of Harmonically related complex exponential

Linear combination

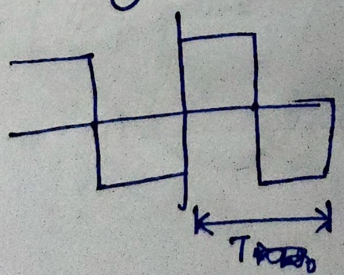
$F[a_1x_1(t) + a_2x_2(t)] = a_1F[x_1(t)] + a_2F[x_2(t)]$

$= a_1y_1(t) + a_2y_2(t)$

$\Rightarrow \boxed{ay(t)}$

Harmonically related

Considering periodic but not sinusoidal.



$\Rightarrow \omega_0 = \frac{2\pi}{T}$

If a sinusoidal signal which is represented with same frequency (w_0) that signal will be called first harmonic of the given signal.

When it is T/2 it is second harmonic of the given signal i.e. $2w_0$.

Similarly, third harmonic is $3w_0$.

harmonically related

$sq(x) = \sin(w_0 t) + \sin(2w_0 t) + \sin(3w_0 t) \ldots$

Complex exponential

$e^{jw_0 t}, e^{j2w_0 t}, e^{j3w_0 t}, \ldots$

This are all harmonically related

$e^{jw_0 t}$ = sinusoidal wave

Synthesis

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}$$

K => 1 = 1^{st} harmonic

2 = 2^{nd} harmonic

$\vdots$

a_k => fourier series coefficient

frequency domain then can be represented in form of phase & magnitude

Analysis

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jkw_0 t}\, dt$$

a_k we need to calculate magnitude and phase

Synthesis

When periodic sinusoidal signal is given

$x(t) = A \cos w_0 t$

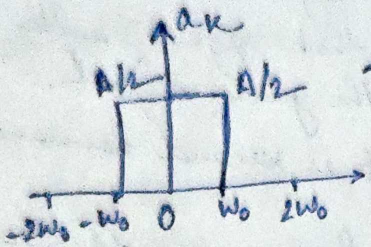
$x(t) = A \sin w_0 t$

Step 1: Conversion to complex exponential

$$= \frac{A.(e^{jw_0 t} + e^{-jw_0 t})}{2}$$

$$= \frac{A}{2}e^{jw_0 t} + \frac{A}{2}e^{-jw_0 t}$$

$a_1 = \frac{A}{2}$, $a_{-1} = \frac{A}{2}$ ⇐ coefficient of wave



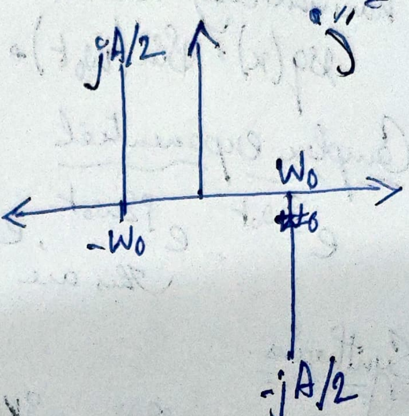
This is not the representation of magnitude & phase

So we need to represent it in magnitude and phase

$$\sin w_0 t = A\left(\frac{e^{jw_0t} - e^{-jw_0t}}{2j}\right)$$

$j = \sqrt{-1}$

$a_1 = \frac{A}{2j} = -j\frac{A}{2}$ $\quad a_2 = \frac{A}{2j} = j\frac{A}{2}$



$$x(t) = 4 + 2\cos\frac{2\pi}{3}t + 4\sin\frac{5\pi}{3}t$$

Calculating fundamental periodicity

$w_0 = \frac{2\pi}{3}$, $\quad w_{0_2} = \frac{5\pi}{3}$

$T_1 = 3 \quad T_2 = 6/5$

$T = LCM \cdot [\frac{3}{1}, \frac{6}{5}]$

$= \frac{6}{1} = 6$ sec.

↑ fundamental time

$w_0 = \frac{2\pi}{6} = \frac{\pi}{3}$ → fundamental period.

This means $x(t)$ will repeat after $\pi/3$

↑ 1st harmonic frequency

Conversion to exponential term.

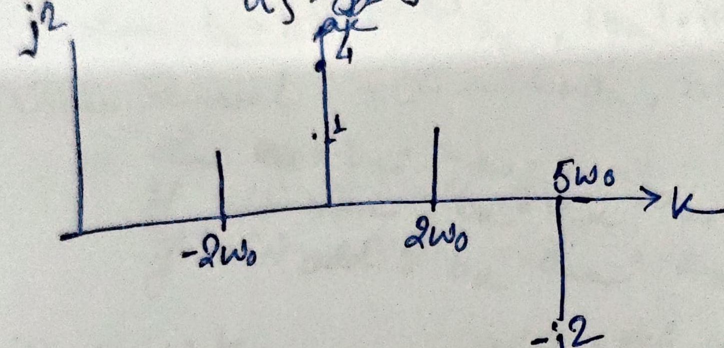
$$= 4e^{j0w_0t} + 2\left(\frac{e^{j2\pi/3\,t} + e^{-j2\pi/3\,t}}{2}\right) + 4\left(\frac{e^{j5\pi/3\,t} - e^{-j5\pi/3\,t}}{2}\right)$$

$= 4e^{j0\omega_0 t} + e^{j(2\pi/3)t} + e^{-j(2\pi/3)t} + \frac{2}{j}e^{j(5\pi/3)t} - \frac{2}{j}e^{-j(5\pi/3)t}$

$= 4e^{j0\omega_0 t} + e^{j(2\pi/3)t} + e^{-j(2\pi/3)t} + (-j)2e^{j(5\pi/3)t} + j2e^{-j(5\pi/3)t}$

$a_0 = 4 \quad a_2 = 1 \quad a_{-2} = 1$

$a_5 = -j2 \quad a_{-5} = j2$



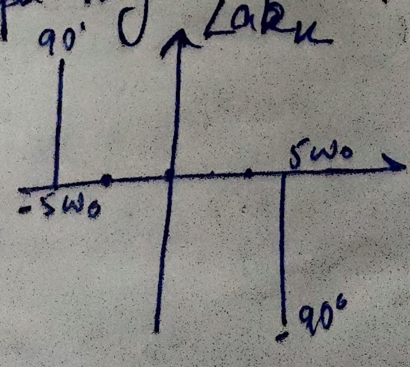
This is fourier series

Calculating magnitude & phase response

a_k	Values	$\lvert a_k \rvert$ (magnitude)	$\angle a_k$
a_0	4	4	0°
a_2	1	1	0°
a_{-2}	1	1	0°
a_5	$-j2$	2	$-90°$
a_{-5}	$+j2$	2	90°

No complex term so phase angle is 0

Draw the graph magnitude & phase angle vs values.



H/W

$$x(t) = 2 + \sin \omega_0 t + 2\cos \omega_0 t + \cos(2\omega_0 t + \pi/4)$$

Properties of Fourier Series Coefficient

1) Linearity : $x(t) \xleftrightarrow{FS} a_k$, $y(t) \xleftrightarrow{FS} b_k$

$z(t) = [\alpha x(t) + \beta y(t)] \xleftrightarrow{FS} c_k$

then, $c_k = \alpha a_k + \beta b_k$

2) Time ~~shifting~~ shifting : $x(t) \xleftrightarrow{FS} a_k$, $x(t \pm t_0) \xleftrightarrow{FS} b_k$ $(k\omega_0(\pm t_0))$

then $b_k = e^{jk\omega_0(\pm t_0)} \cdot a_k$, $|b_k| = (a+jb) = \cos(k\omega_0(\pm t_0))] + j\sin$

3) Time Reversal : $x(t) \xleftrightarrow{FS} a_k$, $x(-t) \xleftrightarrow{FS} b_k$

then $b_k = a_{-k}$

if $x(t)$ even : $b_k = a_{-k} = a_k$

if $x(t)$ odd : $b_k = a_{-k} = -a_k$

Suppose

Q1 $x(t) = \cos \omega_0 t$ $\qquad$ $y(t) = \sin \omega_0 t$

$= a_k$ $\qquad$ $= b_k$ $\qquad$ $\alpha = 1$, $\beta = 1$

$= \dfrac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$ $\qquad$ $= \dfrac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$

$= a_1 = \frac{1}{2}$ $\qquad$ $b_1 = -\frac{j}{2}$

$a_{-1} = \frac{1}{2}$ $\qquad$ $b_{-1} = \frac{j}{2}$

$c_1(t) = \alpha\left(\frac{1}{2}\right) + \beta\left(-\frac{j}{2}\right)$ $\qquad$ $c_{-1} = \frac{1}{2} + \frac{j}{2}$

$c_{+1}(t) = \frac{1}{2} + -\frac{j}{2}$ $\qquad$ $c_{-1}(t) = \frac{1+j}{2}$

$= \frac{1-j}{2}$

Q2 $x(t+t_0) + x(t-t_0) \xleftrightarrow{F.S} b_k$

If $x(t+t_0) \longleftrightarrow e^{jk\omega_0 t_0} \cdot a_k$

$x(t-t_0) \longleftrightarrow e^{-jk\omega_0 t_0} \cdot a_k$

$e^{jk\omega_0 t_0} \cdot a_k + e^{-jk\omega_0 t_0} \cdot a_k$

$b_k = \dfrac{2 \cdot a_k \left(e^{jk\omega_0 t_0} + e^{-jk\omega_0 t_0}\right)}{2}$ $\qquad$ Multi & divide by 2

$b_k = 2a_k \cos(k\omega_0 t_0)$

$|b_k| = \cos(k\omega_0(\pm t_0)] + j\sin[k\omega_0(\pm t_0)]$

$= 1$

$\angle b_k = \angle e^{jk\omega_0(\pm t_0)} + \angle a_k$

$= \cancel{\cos(k\omega_0(\pm t_0)) + j\sin(}$

$= \tan^{-1}\dfrac{\sin k\omega_0(\pm t_0)}{\cos k\omega_0(\pm t_0)} + \angle a_k$

$= \angle a_k + k\omega_0(\pm t_0)$

Q $x(1-t) + x(t-1) \longleftrightarrow b_k$

$\cancel{x(1-t)}$ $x(t) = a_k$

$x(-t) = \cancel{b_k}\ \cancel{a_k}\ b_k = a_{-k}$

$x(-t+1) = e^{-jk\omega_0} \cdot a_{-k}$

$x(t) = a_k$

$x(t-1) = e^{-jk\omega_0} \cdot a_k$

$\left(e^{-jk\omega_0} \cdot a_{-k} + e^{-jk\omega_0} \cdot a_k\right)$

$\cancel{e^{-jk\omega_0}}$ $e^{-jk\omega_0}(a_{-k} + a_k)$

even : $b_k = e^{-jk\omega_0}[a_k + a_{-k}]$

$= 2a_k e^{-jk\omega_0}$

odd : $b_k = e^{-jk\omega_0}[a_k - a_k]$

$= 0$

Purely even signal.

Q Time scaling

$x(t) \longleftrightarrow a_k$

$x(2t) \longleftrightarrow b_k$

$b_k = a_k$

Coefficient of $\cos 2\omega_0 t$

$$\frac{e^{j2\omega_0 t} + e^{-j2\omega_0 t}}{2}$$

$$\underbrace{a_{-2} = \frac{1}{2} \quad a_2 = \frac{1}{2}}_{\text{for } 2\omega_0}$$

Now time scaling.

~~for cos~~ $a_1 = \frac{1}{2}$ & $a_{-1} = \frac{1}{2}$

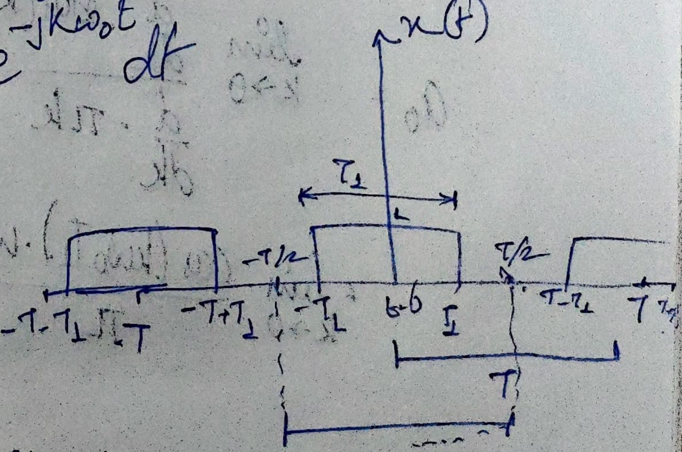
Coefficient same but value of k changes.

If changing frequency, changing coefficient position

10/12/2024

Analysis Equation

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) \cdot e^{-jk\omega_0 t}\, dt$$



For Synthesis of fourier series - period & sinusoidal

If not sinusoidal then use analysis equation.

$$\therefore a_k = \frac{1}{T}\int_{-T_1}^{T} x(t)\, e^{-jk\omega_0 t}\, dt$$

$\boxed{x(t) = 1}$

$$= \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\, dt = -\frac{1}{T}\left[\frac{e^{-jk\omega_0 t}}{jk\omega_0}\right]_{-T_1}^{T_1}$$

$$= -\frac{1}{T}\left[\frac{e^{-jk\omega_0 T_1}}{jk\omega_0} - \frac{e^{+jk\omega_0 T_1}}{jk\omega_0}\right]$$

$$= +\frac{1\times 2}{T}\left[\frac{e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}}{2jk\omega_0}\right]$$

$$= \frac{2}{Tk\omega_0}\left[\frac{e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}}{2j}\right]$$

$$= \frac{2}{Tk\omega_0} \cdot \sin(k\omega_0 T_1)$$

$$= \frac{2}{Tk \cdot \frac{2\pi}{T}} \cdot \sin\left(k \frac{2\pi}{T} \cdot T_1\right)$$

$$= \frac{1}{k\pi} \cdot \sin\left(\frac{2\pi k T_1}{T}\right) = \boxed{\frac{1}{k\pi} \cdot \sin(\omega_0 k T_1)}$$

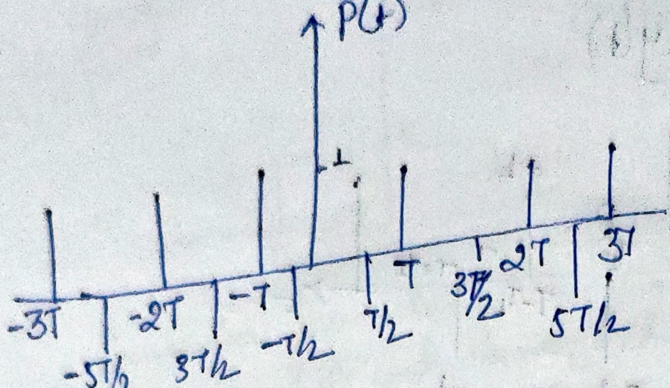
pulse width $= 2T_1$

T is time period.

if $k = 0$, use L'Hospital Rule w.r.t k

$$a_0 = \lim_{k \to 0} \frac{\frac{d}{dk} \cdot \sin(k\omega_0 T_1)}{\frac{d}{dk} \cdot \pi k}$$

$$= \lim_{k \to 0} \frac{\cos(k\omega_0 T_1) \cdot \omega_0 T_1}{\pi} = \frac{\omega_0 T_1}{\pi}$$



← impulse train.

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\cdot e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\cdot e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{T}\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-T/2}^{T/2}$$

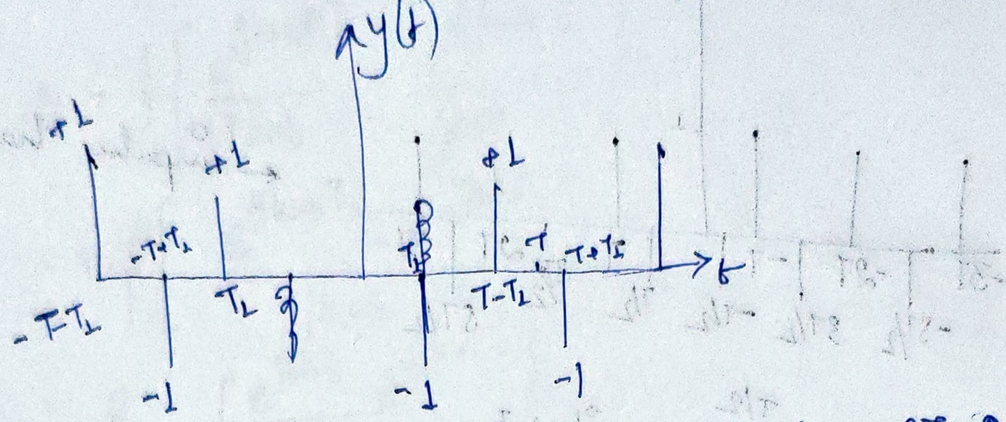
$$= \frac{1}{T}\left[\frac{e^{-jk\omega T/2}}{-jk\omega_0} - \frac{e^{+jk\omega T/2}}{-jk\omega_0}\right]$$

$$= \frac{1}{T} \quad \text{(independent of } k\text{)}$$

$$P(t) = \sum \delta(t-kT)$$

$= \delta t$ when $k=0$

$= \cancel{2}\ \cancel{\delta t}$



$$P(t-T_1) \longrightarrow -2T+T_1 \quad -T+T_1 \quad T_1 \quad T+T_1 \quad 2T+2T_1$$

$$P(t+T_1) \longrightarrow -2T-T_1 \quad -T-T_1 \quad -T_1 \quad T-T_1 \quad 2T-T_1$$

$$P(t+T_1)-P(t-T_1) \to y(t)$$

$$P(t) \xleftrightarrow{FSC} \frac{1}{T}$$

$$P(t+T_1) \xleftrightarrow{F.SC} e^{jk\omega_0 T_1}\frac{1}{T}$$

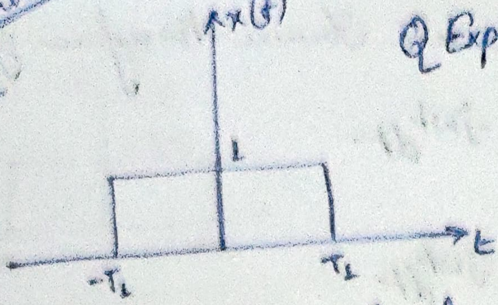
$$P(t-T_1) \xleftrightarrow{F.S.C} e^{-jk\omega_0 T_1}\frac{1}{T}$$

$$\Rightarrow \frac{2j}{T}\left(\frac{e^{jk\omega_0 T_1}-e^{-jk\omega_0 T_1}}{2j}\right)$$

$$\Rightarrow \frac{2j}{T}\sin(k\omega_0 T_1)$$

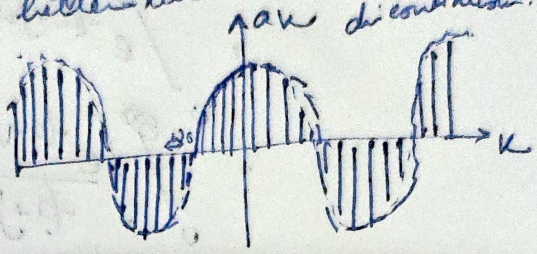
Fourier Transform
19/12/2024

Q Explain ~~by~~ FSC ~~from~~ to FT

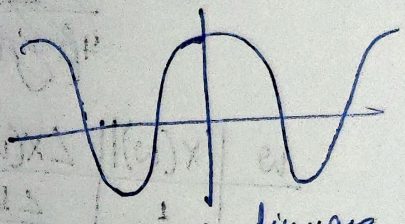


Conversion from time to frequency domain to understand it better and cancel out all the unnecessary noise

Soln. $a_k = \frac{\sin k\omega_0 T_1}{\pi k}$ $\Rightarrow$



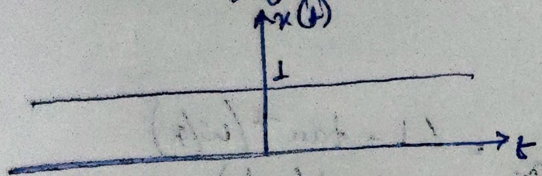
$\Downarrow$ Fourier transform



continuous

must $\omega_0 \to 0$ then gap = 0

$\hookrightarrow T = \frac{2\pi}{\omega_0} = \infty$



Consider $T \to \infty$ we calculate fourier transform.

analysis equation

$$a_k = \frac{1}{T} \int_{-T_1}^{T_1} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$a_k = \frac{1}{T} \int_{-\infty}^{\infty} x(t) \cdot e^{-jk\omega_0 t}\, dt$$

$k \ll \omega_0$, as $\omega \to \infty$ (discrete)

$k\omega_0 = \omega$ (continuous)

$$\lim_{T\to\infty} (Ta_k) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \to \text{Fourier transform.}$$

$$\boxed{X(\omega) = \lim_{T\to\infty} (T \cdot a_k) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt}$$

use this for any coefficient

Fourier Transform $\to X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} \cdot dt \to X(0) = \int_{-\infty}^{\infty} x(t)\, dt$ ← gives area in time domain

Inverse Fourier Transform $\to x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega \to x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, d\omega$ ← gives area in frequency domain

$x(t) = e^{-at} \cdot u(t)$. Calculate Fourier transform of the signal

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} \cdot dt$$

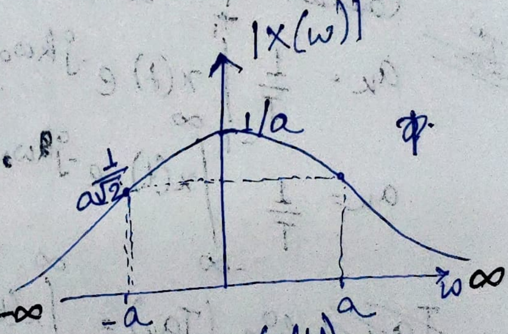
$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

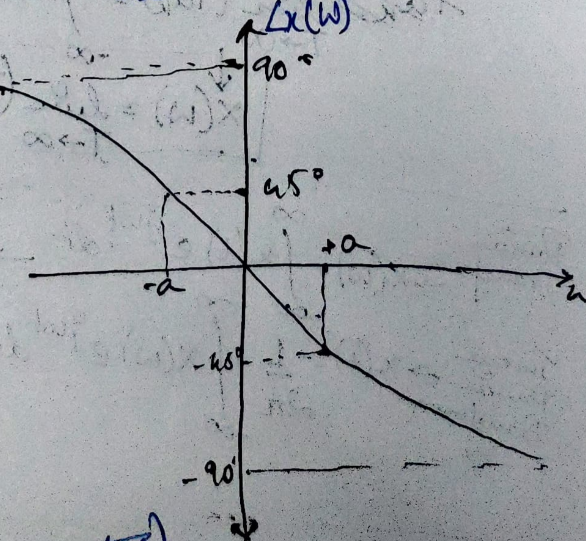
$$= \frac{1}{\cancel{+}(a+j\omega)} = \frac{1}{a+j\omega}$$

ω	$\lvert X(\omega) \rvert$	$\angle X(\omega)$
	$\frac{1}{\sqrt{a^2+\omega^2}}$	$\frac{\angle 1}{\tan^{-1} \omega/a}$
0	$\frac{1}{a}$	$0°$
$-a$	$\frac{1}{a\sqrt{2}}$	$45°$
$+a$	$\frac{1}{a\sqrt{2}}$	$-45°$
$-\infty$	0	$+90°$
$+\infty$	0	$-90°$

$= \angle 1 - \tan^{-1}(\omega/a)$

$= -\tan^{-1}(\omega/a)$





$\angle X(\omega)$

H/W Q $x(t) = e^{-a|t|}$

Q2