Least Square Prewindowing

Homework 1 : Data Windowing

```
% Define some number
a=4;b=6;c=6;d=6;e=2;
```

Input signal

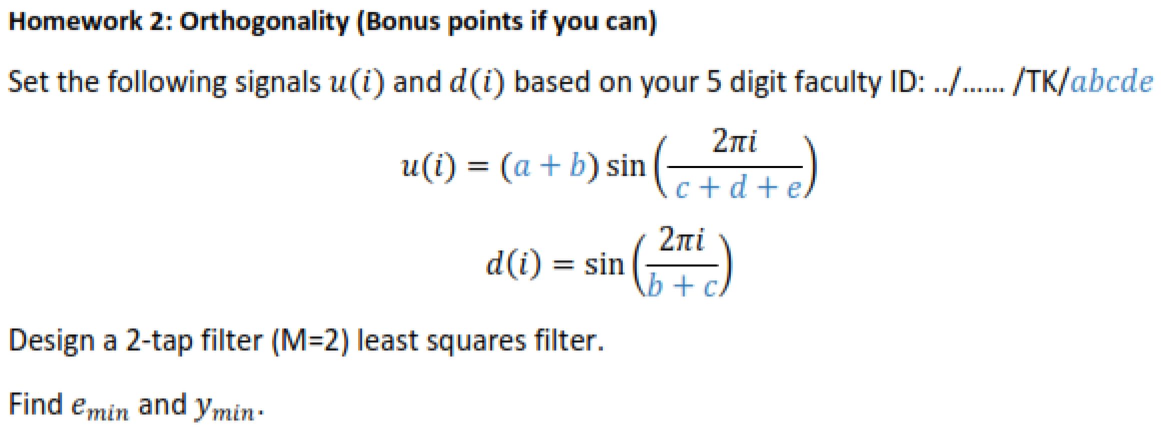
```
n = 1:10; % 10 data
u = a*sin((2*pi*n)/b);
m = 7; % number of tap
```

Prewindowing Method

```
X = [];
for i=1:m
    X = [X;[zeros(1,i-1) u(1:length(u)-i+1)]];
end
X
```

```
X = 7×10
    3.4641    3.4641    0.0000   -3.4641   -3.4641   -0.0000    3.4641    3.4641 ···
         0    3.4641    3.4641    0.0000   -3.4641   -3.4641   -0.0000    3.4641
         0         0    3.4641    3.4641    0.0000   -3.4641   -3.4641   -0.0000
         0         0         0    3.4641    3.4641    0.0000   -3.4641   -3.4641
         0         0         0         0    3.4641    3.4641    0.0000   -3.4641
         0         0         0         0         0    3.4641    3.4641    0.0000
         0         0         0         0         0         0    3.4641    3.4641
```

Homework 2 : Design 2 Tap filter least square



Homework 2: Orthogonality (Bonus points if you can)

Set the following signals $u(i)$ and $d(i)$ based on your 5 digit faculty ID: ../...... /TK/$abcde$

$$u(i) = (a+b)\sin\left(\frac{2\pi i}{c+d+e}\right)$$

$$d(i) = \sin\left(\frac{2\pi i}{b+c}\right)$$

Design a 2-tap filter (M=2) least squares filter.

Find e_{min} and y_{min}.

Input Signal (u)

```
n = 1:10; % 10 data
u = (a+b)*sin((2*pi*n)/(c+d+e));
```

Desired output

```
d = sin(2*pi*n)/(b+c);
```

```
d = d';
```

2 Tap Filter

```
m = 2; % number of delay in filter
```

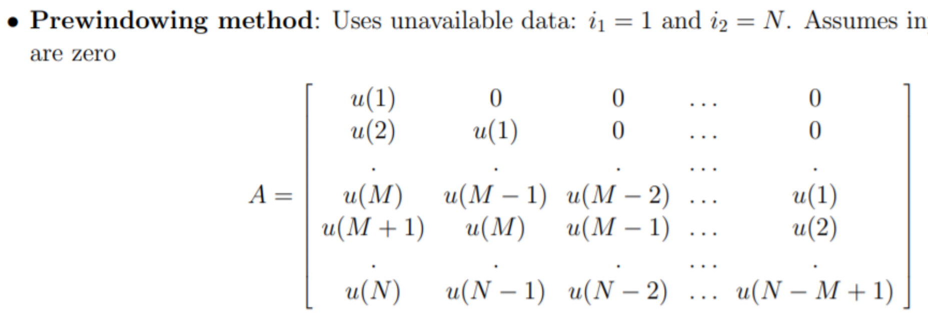
Prewindowing

```
X = [];
for i=1:m
    X = [X;[zeros(1,i-1) u(1:length(u)-i+1)]];
end
X
```

```
X = 2×10
    4.3388    7.8183    9.7493    9.7493    7.8183    4.3388    0.0000   -4.3388 ⋯
         0    4.3388    7.8183    9.7493    9.7493    7.8183    4.3388    0.0000
```

use this,

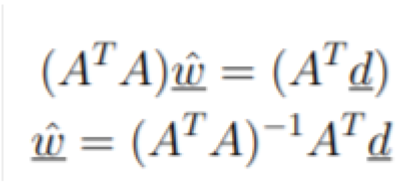
A = X'



- **Prewindowing method**: Uses unavailable data: $i_1 = 1$ and $i_2 = N$. Assumes input data prior to $u(1)$ are zero

$$A = \begin{bmatrix} u(1) & 0 & 0 & \dots & 0 \\ u(2) & u(1) & 0 & \dots & 0 \\ . & . & . & \dots & . \\ u(M) & u(M-1) & u(M-2) & \dots & u(1) \\ u(M+1) & u(M) & u(M-1) & \dots & u(2) \\ . & . & . & \dots & . \\ u(N) & u(N-1) & u(N-2) & \dots & u(N-M+1) \end{bmatrix}$$

and



$$(A^T A)\underline{\hat{w}} = (A^T \underline{d})$$
$$\underline{\hat{w}} = (A^T A)^{-1} A^T \underline{d}$$

```
A = X';
At = A';
w = inv(At*A)*At*d
```

```
w = 2×1
10^-16 ×
    0.2520
   -0.2873
```

Y prediction (Y min)

$$\underline{\hat{y}} = A\underline{\hat{w}} = A(A^T A)^{-1} A^T \underline{d}$$

```
y_pred = A*w
```

```
y_pred = 10×1
10^-15 ×
    0.1093
    0.0724
    0.0210
   -0.0344
   -0.0831
   -0.1153
   -0.1247
   -0.1093
   -0.0724
   -0.0210
```

e min

```
e_min = d-y_pred
```

```
e_min = 10×1
10^-15 ×
   -0.1297
   -0.1132
   -0.0823
   -0.0472
   -0.0190
   -0.0072
   -0.0182
   -0.0540
   -0.1113
   -0.1831
```