

THEOREM $\text{Induction} \Rightarrow \text{Inv} / \text{Next} \Rightarrow \text{Inv}$

<1> 1. $\text{SUFFICES} \text{ ASSUME } \text{Inv}, \text{Next}$

PROVE Inv

OBVIOUS

<1> 2. $\text{CASE} / \text{Case } x > y$

$x = x - y$

$y = y$

<2> 1. TypeOK

BY <1> 1, <1> 2, $\text{SimpleArithmetic_DEF_Inv}, \text{TypeOK}$

<2> 2. GCDInv

<3> 1. $\text{GCD}(y, x) = \text{GCD}(y', x')$

BY <1> 1, <1> 2, $\text{GCD3_DEF_Inv}, \text{TypeOK}$

<3> 2. QED

BY <1> 1, <3> 1, <2> 1, $\text{GCD2_DEF_Inv}, \text{TypeOK}, \text{GCDInv}$

<2> 3. QED

BY <2> 1, <2> 2, DEF_Inv

<1> 3. $\text{CASE} / \text{Case } y > x$

$y = y - x$

$x = x$

<2> 1. TypeOK

BY <1> 1, <1> 3, $\text{SimpleArithmetic_DEF_Inv}, \text{TypeOK}$

<2> 2. GCDInv

BY <1> 1, <1> 3, $\text{GCD3_DEF_Inv}, \text{TypeOK}, \text{GCDInv}$

<2> 3. QED

BY <2> 1, <2> 2, DEF_Inv

<1> 4. QED

BY <1> 1, <1> 2, <1> 3, DEF_Next