A Partially Expanded Proof of $\langle 4 \rangle 3$

$\langle 1 \rangle 1.$ ASSUME: *Inv*, *Producer*
PROVE: $C!Send$

$\langle 2 \rangle 1.$ $Len(chBar) \neq N$

$\langle 2 \rangle 2.$ $\exists v \in Msg : chBar' = Append(chBar, v)$

$\langle 3 \rangle 1.$ Pick $v \in Msg$ such that $buf' = [buf \text{ EXCEPT } ![p \% N] = v]$

$\langle 3 \rangle 2.$ $chBar' = Append(chBar, v)$

$\langle 4 \rangle 1.$ $p \ominus c \in 0 \,..\, (N-1)$

$\langle 4 \rangle 2.$ $p' \ominus c' = (p \ominus c) + 1$

$\langle 4 \rangle 3.$ Q.E.D.

$\langle 5 \rangle 1.$ $Append(chBar, v) =$
$$[\, i \in 1 \,..\, ((p \ominus c) + 1) \mapsto \text{IF } i \in 1 \,..\, (p \ominus c) \text{ THEN } buf[(c \oplus (i-1)) \% N] \text{ ELSE } v \,]$$

PROOF: By definition of *chBar* and *Append*, since *Inv* implies $p \ominus c$ is in *Nat*.

$\langle 5 \rangle 2.$ $chBar' = [\, i \in 1 \,..\, ((p \ominus c) + 1) \mapsto buf'[(c \oplus (i-1)) \% N] \,]$

PROOF: By definition of *chBar*, $\langle 4 \rangle 2$, and definition of *Producer*, which implies $c' = c$.

Remember that priming an expression means priming all the variables in it.

$\langle 5 \rangle 3.$ ASSUME: NEW $i \in 1 \,..\, ((p \ominus c) + 1)$
PROVE: $chBar'[i] = Append(chBar, v)[i]$

$\langle 6 \rangle 1.$ CASE: $i \in 1 \,..\, (p \ominus c)$

$\langle 6 \rangle 2.$ CASE: $i = (p \ominus c) + 1$

$\langle 6 \rangle 3.$ Q.E.D.
PROOF: By the $\langle 5 \rangle 3$ assumption, $\langle 6 \rangle 1$, $\langle 6 \rangle 2$, and *Inv*, which implies $p \ominus c$ is in *Nat*, so $1 \,..\, ((p \ominus c)+1)$ equals $(1 \,..\, (p \ominus c)) \cup \{(p \ominus c)+1\}$.

$\langle 5 \rangle 4.$ Q.E.D.
PROOF: By $\langle 5 \rangle 1$, $\langle 5 \rangle 2$, and $\langle 5 \rangle 3$.

$\langle 3 \rangle 3.$ Q.E.D.

$\langle 2 \rangle 3.$ Q.E.D.

$\langle 1 \rangle 2.$ ASSUME: *Inv*, *Consumer*
PROVE: $C!Rcv$

$\langle 1 \rangle 3.$ Q.E.D.

CLOSE