THEOREM $\land\ Init \Rightarrow Inv$

$\quad\quad\land\ Inv \land Next \Rightarrow Inv'$

$\quad\quad\land\ Inv \Rightarrow Safe$

$\langle 1\rangle 1.$ $Init \Rightarrow Inv$

  BY $MNPosInt$ DEF $Init$, Inv, $TypeOK$, $GCDInv$

$\langle 1\rangle 2.$ $Inv \land Next \Rightarrow Inv'$

  $\langle 2\rangle 1.$ SUFFICES ASSUME Inv, $Next$

   PROVE Inv'

   OBVIOUS

  $\langle 2\rangle 2.$ CASE $y > x$

   $\langle 3\rangle 1.$ $(y - x \in Nat \setminus \{0\}) \ \land \ \neg(x > y)$

    BY $\langle 2\rangle 1$, $\langle 2\rangle 2$, $SimpleArithmetic$ DEF Inv, $TypeOK$

   $\langle 3\rangle 2.$ QED

    BY $\langle 2\rangle 1$, $\langle 3\rangle 1$, $GCD3$ DEF Inv, $TypeOK$, $GCDInv$, $Next$

  $\langle 2\rangle 3.$ CASE $x > y$

   $\langle 3\rangle 1.$ $(x - y \in Nat \setminus \{0\}) \ \land \ \neg(y > x)$

    BY $\langle 2\rangle 1$, $\langle 2\rangle 3$, $SimpleArithmetic$ DEF Inv, $TypeOK$

   $\langle 3\rangle 2.$ $GCD(y, x - y) = GCD(y, x)$

    BY $\langle 2\rangle 1$, $\langle 3\rangle 1$, $GCD3$ DEF Inv, $TypeOK$, $Next$

   $\langle 3\rangle 3.$ QED

    BY $\langle 2\rangle 1$, $\langle 3\rangle 1$, $\langle 3\rangle 2$, $GCD2$ DEF Inv, $TypeOK$, $GCDInv$, $Next$

  $\langle 2\rangle 4.$ QED

   BY $\langle 2\rangle 1$, $\langle 2\rangle 2$, $\langle 2\rangle 3$ DEF $Next$

$\langle 1\rangle 3.$ $Inv \Rightarrow Safe$

  BY $GCD1$ DEF Inv, $Safe$, $TypeOK$, $GCDInv$

$\langle 1\rangle 4.$ QED

  BY $\langle 1\rangle 1$, $\langle 1\rangle 2$, $\langle 1\rangle 3$