THEOREM $Induction \triangleq Inv \wedge Next \Rightarrow Inv'$

⟨1⟩1. SUFFICES ASSUME: 1. Inv
2. $Next$
PROVE: Inv'
PROOF: Obvious.

⟨1⟩2. CASE: $\wedge\ x > y$
$\wedge\ x' = x - y$
$\wedge\ y' = y$

⟨2⟩1. $TypeOK'$
PROOF: ⟨1⟩1.1 and the definitions of Inv and $TypeOK$ imply that x and y are in $Nat \setminus \{0\}$. By case assumption ⟨1⟩2, this implies that x' and y' are in $Nat \setminus \{0\}$, proving ⟨2⟩1.

⟨2⟩2. $GCDInv'$

⟨3⟩1. $GCD(y', x') = GCD(y, x)$
PROOF: ⟨1⟩1.1 and the definitions of Inv and $TypeOK$ imply that x and y are in $Nat \setminus \{0\}$, so ⟨3⟩1 follows from case assumption ⟨1⟩2 and GCD3 (substituting y for m and x for n).

⟨3⟩2. $GCD(x', y') = GCD(x, y)$
PROOF: ⟨1⟩1.1, the definitions of Inv and $TypeOK$, and ⟨2⟩1 imply that x, y, x', and y' are in $Nat \setminus \{0\}$, so ⟨3⟩2 follows from ⟨3⟩1 and GCD2.

⟨3⟩3. Q.E.D.
PROOF: ⟨3⟩2, ⟨1⟩1.1, and the definitions of Inv and $GCDInv$ imply ⟨2⟩2.

⟨2⟩3. Q.E.D.
PROOF: By ⟨2⟩1, ⟨2⟩2, and definition of Inv.

⟨1⟩3. CASE: $\wedge\ y > x$
$\wedge\ y' = y - x$
$\wedge\ x' = x$

⟨2⟩1. $TypeOK'$
PROOF: ⟨1⟩1.1 and the definitions of Inv and $TypeOK$ imply that x and y are in $Nat \setminus \{0\}$. By case assumption ⟨1⟩3, this implies that x' and y' are in $Nat \setminus \{0\}$, proving ⟨2⟩1.

⟨2⟩2. $GCDInv'$

⟨3⟩1. $GCD(x', y') = GCD(x, y)$
PROOF: ⟨1⟩1.1 and the definitions of Inv and $TypeOK$ imply that x and y are in $Nat \setminus \{0\}$, so ⟨3⟩1 follows from case assumption ⟨1⟩3 and GCD3 (substituting x for m and y for n).

⟨3⟩2. Q.E.D.

PROOF: ⟨3⟩1, ⟨1⟩1.1, and the definitions of *Inv* and *GCDInv* imply ⟨2⟩2.

⟨2⟩3. Q.E.D.

PROOF: By ⟨2⟩1, ⟨2⟩2, and definition of *Inv*.

⟨1⟩4. Q.E.D.

PROOF: By ⟨1⟩1.2 and the definition of *Next*, the cases ⟨1⟩2 and ⟨1⟩3 are exhaustive.

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