Stochastic Optimization lab assignment
Benders decomposition of a stochastic problem

A bank branch wants to determine the amount of money to be deposited in an ATM on Fridays, before the weekend. The branch estimates that money has a cost (associated to the loss of benefits by interest rates) of c € for each € in the ATM. The demand of money during the weekend is a discrete random variable ξ, taking s values ξ_i with probabilities p_i, $i = 1, \ldots, s$. The ATM has a capacity of u €, with a technical minimum of l €. If the demand is greater than x then the ATM has to be refilled, with a cost of q € for each € the demand exceeds x. The bank branch formulates the following stochastic optimization problem in extensive form:

$$\begin{aligned} \min \quad & cx + \sum_{i=1}^{s} p_i q y_i \\ \text{s. to} \quad & l \leq x \leq u \\ & x + y_i \geq \xi_i \quad i = 1, \ldots, s \\ & y_i \geq 0 \quad i = 1, \ldots, s \end{aligned}$$

Write and AMPL code to solve the above problem using Benders decomposition. Check the solution coincides with the one obtained by directly solving the extensive form as a linear optimization problem.

You have to deliver:

- A report describing the Benders master and subproblem formulations; the AMPL code, including a short description of it; the computational results obtained. In particular, detail and justify the formulation of the Benders subproblem.
- A .zip file including the AMPL .mod, .dat and .run files.

Use the following parameters for the problem:

$$\begin{aligned} c &= 0.00025 \\ q &= 0.0011 \\ l &= 21 \\ u &= 147 \\ s &= 7 \end{aligned}$$

i	p_i	ξ_i
1	0.04	150
2	0.09	120
3	0.10	110
4	0.21	100
5	0.27	80
6	0.23	60
7	0.06	50

The units of l, u and ξ_i are thousands of €.