OPTIMAL SUPPLY CHAIN STRATEGY WITH PRICING (SCP)

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1. The Problem Description

Let's consider the problem of a company owning a set of factories $i \in \mathcal{F}$ that produce a set of different products $p \in \mathcal{P}$. Each one of these products $p \in \mathcal{P}$ can be sold to any of the markets $j \in \mathcal{M}$ at a price γ_j^p. The company would like to know the price at which these products should be sold so that the total profit earned by the company along a given set of time periods $t \in \mathcal{T}$ is maximized. In order to do so, a linearly constrained nonlinear optimization problem named (SCP) that includes the following assumption on the *manufacturing process*, the *inventory dynamics*, *transportation* between factories and markets, *demand elasticity,* and *pricing policy* will be formulated.

1.1. Manufacturing and inventory dynamics

Each factory $i \in \mathcal{F}$ at every time period $t \in \mathcal{T}$ can manufacture any of the products $p \in \mathcal{P}$. However, there is a limited available amount of each one of the manufacturing resources $r \in \mathcal{R}$ required to process each good. The amount of resource r spent in the production of a unit of product p (a_{ir}^p) is assumed to be known. There is also a linear production unit cost c_i^{pt} [€] that depends on the product p, factory i and time period t.

Therefore, the total amount of resource $r \in \mathcal{R}$ used to produce all the products $p \in \mathcal{P}$ at factory $i \in \mathcal{F}$ at any given period $t \in \mathcal{T}$ must be less or equal than the limited amount of that resource $r \in \mathcal{R}$ at factory $i \in \mathcal{F}$ at time period $t \in \mathcal{T}$.

$$\sum_{p \in P} a_{ir}^p x_i^{pt} \leq b_{ri}^t \quad \forall\, i \in F, t \in T, r \in R$$

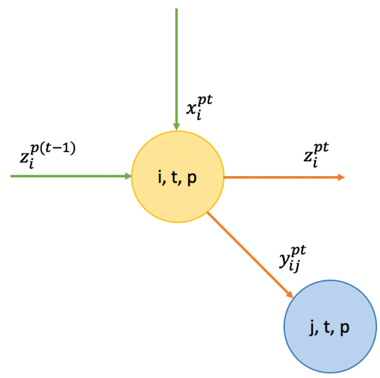
Where,

a_{ir}^p: resource $r \in R$ spent to produce one unit of product $p \in P$ at factory $i \in F$
x_i^{pt}: amount of product $p \in P$ made in factory $i \in F$ at time period $t \in T$
b_{ri}^t: availability of resource $r \in R$ at factory $i \in F$ at time period $t \in T$

The unsold production at the end of each time period remains available for the subsequent periods. It is assumed that there is enough warehouse space at each factory to store any amount of product but every unit held in the inventory incurs a known holding unit cost h_i^p [€] that does not depend on the time period. Each factory stores an initial inventory z_0^p at the beginning of the first time period.

For each factory $i \in \mathcal{F}$, product $p \in \mathcal{P}$ and time period $t \in \mathcal{T}$ the remaining inventory will be of the form:

$$z_i^{pt} = x_i^{pt} + z_i^{p(t-1)} - \sum_{j \in M} y_{ij}^{pt} \quad \forall\, i \in F, p \in P, t \in T$$



Where,

z_i^{pt}: amount of product $p \in P$ inventoried in factory $i \in F$ at time period $t \in T$
x_i^{pt}: amount of product $p \in P$ made in factory $i \in F$ at time period $t \in T$
$z_i^{p(t-1)}$: initial amount of product $p \in P$ inventoried in factory $i \in F$ at time period $t \in T$
y_{ij}^{pt}: amount of product $p \in P$ sold in market j ∈ M at time period t ∈ T

1.2. Goods Transportation

At any time period $t \in \mathcal{T}$, the company can sell any available quantity of product $p \in \mathcal{P}$ produced in any factory $i \in \mathcal{F}$ to any market $j \in \mathcal{M}$. There is a known linear transportation unit cost f_{ij}^p [€] for every product $p \in \mathcal{P}$ from factory i to market j that does not depend on the time period.

1.3. Demand elasticity and pricing policy

We assume that the demand of each market is a linear function of the price with known (negative) slope. Let $\tilde{\gamma}p$ be the current price for each product (the same for all markets and time periods), and $\tilde{d}_j^{pt}$ the current demand for each product, time period and market. Then, the actual demand in terms of the price γ_j^p can be expressed as:

$$d_j^{pt} = \tilde{d}_j^{pt} + \alpha_j^p\left(\gamma_j^p - \tilde{\gamma}^p\right) \quad \forall\, t \in T, p \in P, j \in M$$

Where,

the slope αjp < 0 is the *demand response* to the price γ_j^p.

The company considers that the price difference for the same product in different markets, γ_j^p and γ_k^p, $j \neq k$, should be bounded. In order to impose that condition, the optimal solution must satisfy that every price γ_j^p must be greater than a given fraction βmin and less that a given fraction βmax ($0 < \beta min < 1 < \beta max$) of the average price for product p among all the markets j such that:

$$\beta^{min} \frac{\sum_{k \in M} \gamma_k^p}{|M|} \leq \gamma_j^p \leq \beta^{max} \frac{\sum_{k \in M} \gamma_k^p}{|M|}$$

2. Mathematical Formulation

2.1. Sets

F: Set of origins (Factories)
M: Set of destinations (Markets)
P: Set of products
T: Set of time periods
R: Set of resources

2.2. Parameters

c_i^{pt}: unit production cost of product $p \in P$ at factory at time period $t \in T$ in [€/Tm]
h_i^p: inventory cost of product $p \in P$ at factory $i \in F$ in [€/Tm]
f_{ij}^p:unit transportation cost of product $p \in P$ through arc $(i, j) \in FxM$ in [€/Tm]
a_{ir}^p: resource $r \in R$ spent to produce one unit of product $p \in P$ at factory $i \in F$ in [Tm/r.u.*]
b_{ri}^t: availability of resource $r \in R$ at factory $i \in F$ at time period $t \in T$ in [r.u.*]
s_{i0}^p: stock of product $p \in P$ in factory $i \in F$ at time period $t = 0$ in [Tm]
α_j^p: demand response of product $p \in P$ in market $j \in M$ in [Tm²/€]
$\tilde{d}_j^{pt}$: current demand of product $p \in P$ in market $j \in M$ at time period $t \in T$ in [Tm]
$\tilde{\gamma}^p$: current price of product $p \in P$ in [€/Tm]
β^{min}: minimum fraction of the price spread policy
β^{max}: maximum fraction of the price spread policy

2.3. Variables

γ_j^p: purchase price of product $p \in P$ in market $j \in M$ in [€/Tm]
y_{ij}^{pt}: amount of product $p \in P$ sold in market j ∈ M at time period t ∈ T in [Tm]
x_i^{pt}: amount of product $p \in P$ made in factory $i \in F$ at time period $t \in T$ in [Tm]
z_i^{pt}: amount of product $p \in P$ inventoried in factory $i \in F$ at time period $t \in T$ in [Tm]
d_j^{pt}: demand of product $p \in P$ in market j ∈ M at time period $t \in T$ in [Tm]

2.4. Objective Function

$$max \sum_{p \in P} \sum_{t \in T} \sum_{i \in F} \left(\sum_{j \in M} \gamma_j^p y_{ij}^{pt} - \left(c_i^{pt} x_i^{pt} + h_i^p z_i^{pt} + \sum_{j \in M} f_{ij}^p y_{ij}^{pt} \right) \right)$$

2.5. Constraints

(1) Inventory Balance: $x_i^{pt} + z_i^{p(t-1)} - \sum_{j \in M} y_{ij}^{pt} - z_i^{pt} = 0 \quad \forall\, i \in F, p \in P, t \in T$

(2) Initial inventory: $z_i^{p0} = s_i^{p0} \quad \forall\, i \in F, p \in P$

(3) Resource Availability: $\sum_{p \in P} \frac{1}{a_{ir}^p} x_i^{pt} \leq b_{ri}^t \quad \forall\, i \in F, t \in T, r \in R$

(4) Demand and supplay equiliblium: $\sum_{i \in F} y_{ij}^{pt} = d_j^{pt} \quad \forall\, t \in T, p \in P, j \in M$

(5) Demand elasticity: $d_j^{pt} = \tilde{d}_j^{pt} + \alpha_j^p \left(\gamma_j^p - \tilde{\gamma}^p \right) \quad \forall\, t \in T, p \in P, j \in M$

*r.u: resource units

(6) Price lower bound: $\gamma_j^p \geq \beta^{min} \frac{\sum_{k \in M} \gamma_k^p}{|M|} \quad \forall\, p \in P, j \in M$

(7) Price upper bound: $\gamma_j^p \leq \beta^{max} \frac{\sum_{k \in M} \gamma_k^p}{|M|} \quad \forall\, p \in P, j \in M$

(8) Non negativity: $\gamma_j^p, y_{ij}^{pt}, x_i^{pt}, z_i^{pt}, d_j^{pt} \geq 0 \quad \forall\, t \in T, p \in P, j \in M, i \in F$

3. The Data and the Test Case

In our test case, we will consider the following four time periods, three products, three factories, seven markets, and one resource.

i. $\mathcal{T}$ = {1,2,3,4}
ii. $\mathcal{P}$ = {bands, coils, plate}
iii. $\mathcal{F}$ = {GARY, CLEV, PITT}
iv. $\mathcal{M}$ = {FRA, DET, LAN, WIN, STL, FRE, LAF}
v. $\mathcal{R} = \{\text{hours}\}$

The data used to solve the problem can be found in the dataset number 8 of the steelSCP.dat file. This dataset contains the following information in different several tables:

i. *Table avail:* This table contains the total number of hours available to produce one unit of product $p \in \mathcal{P}$ in every Factory $i \in F$ at time period $t \in T$ in [h].

ii. *Table inv0:* This table contains the inventory at time t=0 for every factory $i \in F$ and product $p \in \mathcal{P}$ in [Tm].

iii. *Table rate:* This table contains the number of hours necessary to produce one unit of product $p \in \mathcal{P}$ in factory $i \in F$ [Tm/h].

iv. *Tables demand_current [*,bands,*]:* There are three tables that contain the current demand in market $j \in M$ at time period $t \in T$ for every product $p \in \mathcal{P}$ in [Tm].

v. *Table demand_response:* This table contains the demand response for every product $p \in \mathcal{P}$ and market $j \in M$ in [Tm].

vi. *Table price_current:* This table contains the current price for each product $p \in \mathcal{P}$ in [€/Tm].

vii. *Tables make_cost[CLEV,*,*]:* There are three tables that contain the cost to produce every product $p \in \mathcal{P}$ at time period t ∈ T in factory i ∈ F in [€/Tm].

viii. *Table trans_cost [CLEV,*,*]:* There are three tables that contain the cost to ship one unit of product $p \in \mathcal{P}$ from factory $i \in F$ to market $j \in M$ in [€/Tm].

ix. *Table inv_cost:* This table contains he cost to hold one unit of product $p \in \mathcal{P}$ in factory $i \in F$ [€/Tm].

4. AMPL Implementation

4.1. Sets

F: Set of origins (Factories)	**set** FACT;
M: Set of destinations (Markets)	**set** MARK;
P: Set of products	**set** PROD;
T: Set of time periods	**set** TIME;
R: Set of Resources	**Set** RESO;

4.2. Parameters

b_{ri}^{t}: availability of resource $r \in R$ at factory $i \in F$ at time period $t \in T$	**param** avail {RESO,FACT,TIME} >= 0;
s_{i0}^{p}: stock of product $p \in P$ in factory $i \in F$ at time period $t = 0$	**param** inv0 {FACT,PROD} >= 0;
a_{ir}^{p}: resource $r \in R$ spent to produce one unit of product $p \in P$ at factory $i \in F$	**param** rate {RESO,FACT,PROD} >= 0;
$\tilde{d}_{j}^{pt}$: current demand of product $p \in P$ in market $j \in M$ at time period $t \in T$	**param** demand_current {MARK,PROD,TIME} >= 0;
$\tilde{\gamma}^{p}$: current price of product $p \in P$	**param** price_current {PROD} >= 0;
α_{j}^{p}: demand response of product $p \in P$ in market $j \in M$	**param** demand_response {MARK,PROD};
β^{min}: minimum fraction of the price spread policy	**param** price_min = 0.75;
β^{max}: maximum fraction of the price spread policy	**param** price_max = 1.25;
c_{i}^{pt}: unit production cost of product $p \in P$ at factory at time period $t \in T$	**param** make_cost {FACT,PROD,TIME} >= 0;
h_{i}^{p}: inventory cost of product $p \in P$ at factory $i \in F$	**param** inv_cost {FACT,PROD} >= 0;
f_{ij}^{p}:unit transportation cost of product $p \in P$ through arc $(i,j) \in FxM$	**param** trans_cost {FACT,MARK,PROD} >= 0;

4.3. Variables

γ_{j}^{p}: purchase price of product $p \in P$ in market $j \in M$	**var** pricet {MARK,PROD} >= 0;
y_{ij}^{pt}: amount of product $p \in P$ sold in market j ∈ M at time period t ∈ T	**var** sellt {FACT,MARK,PROD,TIME} >= 0;

x_i^{pt}: amount of product $p \in P$ made in factory $i \in F$ at time period $t \in T$	**var** prodt {FACT,PROD,TIME} >= 0;
z_i^{pt}: amount of product $p \in P$ inventoried in factory $i \in F$ at time period $t \in T$	**var** invent {FACT,PROD,{0} **union** {TIME}} >= 0;
d_j^{pt}: demand of product $p \in P$ in market j ∈ M at time period $t \in T$	**var** demant {MARK,PROD,TIME} >= 0;

4.4. Objective function and constraints

$\sum_{p \in P} \sum_{t \in T} \sum_{i \in F} \sum_{j \in M} \gamma_j^p y_{ij}^{pt}$	**var** Revenue = **sum** {p **in** PROD, t **in** TIME,i **in** FACT,j **in** MARK} pricet[j,p] * sellt[i,j,p,t];
$\sum_{p \in P} \sum_{t \in T} \sum_{i \in F} c_i^{pt} x_i^{pt}$	**var** Prod_cost = **sum** {p **in** PROD, t **in** TIME,i **in** FACT} make_cost[i,p,t]*prodt [i,p,t];
$\sum_{p \in P} \sum_{t \in T} \sum_{i \in F} h_i^p z_i^{pt}$	**var** Inv_cost = **sum** {p **in** PROD, t **in** TIME,i **in** FACT} inv_cost[i,p]*invent[i,p,t];
$\sum_{p \in P} \sum_{t \in T} \sum_{i \in F} \sum_{j \in M} f_{ij}^p y_{ij}^{pt}$	**var** Trans_cost = **sum** {p **in** PROD, t **in** TIME,i **in** FACT,j **in** MARK} trans_cost[i,j,p]*sellt[i,j,p,t];
$\sum_{p \in P} \sum_{t \in T} \sum_{i \in F} \left(\sum_{j \in M} \gamma_j^p y_{ij}^{pt} - \left(c_i^{pt} x_i^{pt} + h_i^p z_i^{pt} + \sum_{j \in M} f_{ij}^p y_{ij}^{pt} \right) \right)$	**maximize** Profit: Revenue-Prod_cost-Inv_cost-Trans_cost;
$x_i^{pt} + z_i^{p(t-1)} - \sum_{j \in M} y_{ij}^{pt} - z_i^{pt} = 0 \quad \forall\, i \in F, p \in P, t \in T$	**subject to** Balance {i **in** FACT, p **in** PROD, t **in** TIME}: prodt[i,p,t]+invent[i,p,t-1]-(**sum** {j **in** MARK} sellt[i,j,p,t]) = invent[i,p,t]; **subject to** Ini {i **in** FACT, p **in** PROD}: invent[i,p,0]=inv0[i,p];
$z_i^{p0} = s_i^{p0} \quad \forall\, i \in F, p \in P$	**subject to** Ini {i **in** FACT, p **in** PROD}: invent[i,p,0]=inv0[i,p];
$\sum_{p \in P} \frac{1}{a_{ir}^p} x_i^{pt} \leq b_{ri}^t \quad \forall\, i \in F, t \in T, r \in R$	**subject to** Resource {r **in** RESO, i **in** FACT,t **in** TIME}: (**sum** {p **in** PROD} (1/rate[r,i,p])*prodt[i,p,t])<=avail[r,i,t];
$\sum_{i \in F} y_{ij}^{pt} = d_j^{pt} \quad \forall\, t \in T, p \in P, j \in M$	**subject to** Satdemand {p **in** PROD,t **in** TIME, j **in** MARK}: (**sum** {i **in** FACT} sellt[i,j,p,t])=demant[j,p,t];

$d_j^{pt} = \tilde{d}_j^{pt} + \alpha_j^p\left(\gamma_j^p - \tilde{\gamma}^p\right) \quad \forall\, t \in T, p \in P, j \in M$	**subject to** Demand {p **in** PROD,t **in** TIME, j **in** MARK}: demant[j,p,t]=demand_current[j,p,t]+(demand_response[j,p]*(pricet[j,p]-price_current[p]));
$\gamma_j^p \geq \beta^{min} \frac{\sum_{k \in M} \gamma_k^p}{\lvert M \rvert} \quad \forall\, p \in P, j \in M$	**subject to** Pricelb {p **in** PROD, j **in** MARK}: pricet[j,p] >= price_min*((**sum** {k **in** MARK} pricet[k,p])/7);
$\gamma_j^p \leq \beta^{max} \frac{\sum_{k \in M} \gamma_k^p}{\lvert M \rvert} \quad \forall\, p \in P, j \in M$	**subject to** Priceub {p **in** PROD, j **in** MARK}: pricet[j,p] <= price_max*((**sum** {k **in** MARK} pricet[k,p])/7);
$\gamma_j^p, y_{ij}^{pt}, x_i^{pt}, z_i^{pt}, d_j^{pt} \geq 0 \quad \forall\, t \in T, p \in P, j \in M, i \in F$	**var** pricet {MARK,PROD} >= 0; **var** sellt {FACT,MARK,PROD,TIME} >= 0; **var** prodt {FACT,PROD,TIME} >= 0; **var** invent {FACT,PROD,{0} **union** {TIME}} >= 0; **var** demant {MARK,PROD,TIME} >= 0;

5. Results

5.1. Optimal solution

Using our dataset, the AMPL implementation and the MINOS (version 5.51) solver we obtain in 388 iterations that the value of the objective function for the solution that maximizes de profits of the company is ***20.080.293,48 €***.

5.2. Table i: Revenues and Costs

This table summarizes the value of the objective function for each time period. The first column represents the total revenue in [€] generated from the company sales, the second column shows the total production costs, the third column shows the total inventory costs incurred in time period t, the fourth column shows the transportation costs, and the fifth column is the net profit, which is the result of subtract the total costs to the total revenue (Col. 1 - Col. 2 - Col. 3 - Col. 4).

Time period	Revenue	Production Costs	Inventory Costs	Transportation Costs	Profit
1	18011800	12519300	151734	3335420	2005360
2	17435200	8654200	108266	3221270	5451480
3	17480700	7482050	50040,8	3362810	6585760
4	17040500	7910240	0	3092570	6037710

Table 1: Summary of Revenues and Costs for each time period

The code used to create this tables and the original AMPL output can be found in the annex of the document (point 6.1.2).

5.3. Table ii: Optimal pricing policy and its effects on demand.

The following tables show information about the pricing policy and its effects on demand for all the products sold in each market. The first column represents the market in which the product is sold, the second is the product original price [€] at every market, the third is the optimum price [€] at which the products should be sold to maximize the total profits, the fourth column is the difference between column 3 and column 4, the fifth column is the product original demand [Tm], the sixth column is the product demand [Tm] resulted from changing the original price, and the seventh column is the difference between column 6 and column 5.

As expected, the demand and the price in each market are governed by the expression presented in point 1.3 of the document; the difference between the demand and the current demand is proportional to the difference between de price and the current price for every product and market. The constant of proportionality is, as we already know, the parameter alpha.

The code used to create these three tables and the original AMPL output can be found in the annex of the document (point 6.1.3)

5.3.1. Bands optimal pricing policy

Market	Current price	Price	Price Difference	Current demand	Demand	Demand Difference
DET	259,15	271,07	11,9199	14350	10488	-3862,03
FRA	259,15	274,614	15,4643	11748	6552	-5196
FRE	259,15	286,092	26,9419	25016	14454,8	-10561,2
LAF	259,15	294,014	34,8644	20642	12414	-8228
LAN	259,15	285,232	26,082	20685	13277,7	-7407,3
STL	259,15	264,411	5,26089	6076	5002,78	-1073,22
WIN	259,15	272,352	13,2021	26147	21183	-4964

Table 2: Optimal pricing policy for bands

5.3.2. Coils optimal pricing policy

Market	Current price	Price	Price Difference	Current demand	Demand	Demand Difference
DET	290,72	264,175	-26,545	10857	19457,6	8600,59
FRA	290,72	269,594	-21,1265	8878	15723	6845,98
FRE	290,72	287,594	-3,12634	18916	19741,4	825,353
LAF	290,72	285,825	-4,89472	15606	16761,2	1155,15
LAN	290,72	279,544	-11,1762	15639	18366	2727,98
STL	290,72	257,678	-33,0423	4592	12654,3	8062,31
WIN	290,72	291,59	0,869639	19780	19546,9	-233,063

Table 3: Optimal pricing policy for coils

5.3.3. Plate optimal pricing policy

Market	Current price	Price	Price Difference	Current demand	Demand	Demand Difference
DET	216,64	195,25	-21,3902	2632	10931,4	8299,39
FRA	216,64	200,639	-16,0006	2155	8491,26	6336,26
FRE	216,64	202,094	-14,5461	4592	9595,85	5003,85
LAF	216,64	200,813	-15,8272	3791	9552,1	5761,1
LAN	216,64	198,261	-18,379	3798	9458,74	5660,74
STL	216,64	194,895	-21,7453	1115	7203,68	6088,68
WIN	216,64	204,693	-11,9466	4797	8524,34	3727,34

Table 4: Optimal pricing policy for plate

5.4. Table iii: Average variation in % of the price and the demand for each product.

Since the current price for all the products is the same, we define the average price variation per product in [%] as:

$$\overline{\Delta\gamma^p} = \frac{\sum_{j\in M} \frac{\gamma_j^p - \tilde{\gamma}^p}{\tilde{\gamma}^p}}{|M|} \cdot 100. \quad \forall\, p \in P$$

Since the current demand for all the products is not the same, we define the average demand variation per product in [%] as:

$$\overline{\Delta d^p} = \frac{\sum_{j\in M} \left(\frac{\sum_{t\in T}\left(d_j^{pt} - \tilde{d}_j^{pt}\right)}{\sum_{t\in T} \tilde{d}_j^{pt}} \right)}{|M|} \cdot 100 \quad \forall\, p \in P$$

Appling the above expressions to our results we obtain the following table:

Product	Price variation [%]	Demand variation [%]
bands	7,37221	-32,2398
coils	-4,8668	51,4164
plate	-7,90218	234,729

Table 5: Average variation in % of price and demand for every product

The code used to create these three tables and the original AMPL output can be found in the annex of the document (point 6.1.4)

5.5. Table iv: Optimal inventory policy for each product and factory.

The following tables show information about the inventory policy and the production dynamics for each product in every factory. More precisely, the information presented illustrates the inventory balance equation introduced in point 1.1:

$$z_i^{pt} = x_i^{pt} + z_i^{p(t-1)} - \sum_{j\in M} y_{ij}^{pt} \qquad \forall\, i \in F, p \in P, t \in T$$

In each time period and for every combination of factories and products, the tables show the inventory [Tm] at time t-1 in the fourth column, the total amount [Tm] produced in time period t in the fifth column, the total amount sold [Tm] at time period t in the fifth column, and the final inventory [Tm] in the sixth column.

The code used to create these three tables and the original AMPL output can be found in the annex of the document (point 6.1.5)

5.5.1. Bands-GARY

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	bands	GARY	4123	22935,1	8808,39	18249,7
2	bands	GARY	18249,7	5122,79	12926,4	10446,1
3	bands	GARY	10446,1	52,178	7806,39	2691,92
4	bands	GARY	2691,92	6095,47	8787,39	0

Table 6: Optimal inventory policy for bands in GARY

5.5.2. Bands-CLEV

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	bands	CLEV	4473	16768	12053,1	9187,88
2	bands	CLEV	9187,88	8251,19	7568,17	9870,9
3	bands	CLEV	9870,9	1323,45	1326,17	9868,17
4	bands	CLEV	9868,17	0	9868,17	0

Table 7: Optimal inventory policy for bands in CLEV

5.5.3. Bands-PITT

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	bands	PITT	8864	5364,04	1032,56	13195,5
2	bands	PITT	13195,5	0	91,4918	13104
3	bands	PITT	13104	0	11754,5	1349,49
4	bands	PITT	1349,49	0	1349,49	0

Table 8: Optimal inventory policy for bands in PITT

5.5.4. Coils-GARY

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	coils	GARY	6090	3196,84	9286,84	0
2	coils	GARY	0	17772,9	17772,9	0
3	coils	GARY	0	20602,9	20602,9	0
4	coils	GARY	0	14612,9	14612,9	0

Table 9: Optimal inventory policy for coils in GARY

5.5.5. Coils-CLEV

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	coils	CLEV	3247	5934,53	9181,53	0
2	coils	CLEV	0	9652,53	9652,53	0
3	coils	CLEV	0	11652,6	9995,68	1656,88
4	coils	CLEV	1656,88	13602,8	15259,7	0

Table 10: Optimal inventory policy for coils in CLEV

5.5.6. Coils-PITT

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	coils	PITT	4072	11716,3	12936,2	2852,07
2	coils	PITT	2852,07	97,0765	2949,15	1,06E-10
3	coils	PITT	1,06E-10	0	0	5,13E-11
4	coils	PITT	5,13E-11	0	0	0

Table 11: Optimal inventory policy for coils in PITT

5.5.7. Plate-GARY

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	plate	GARY	746	1546,96	2292,96	0
2	plate	GARY	0	4061,13	4061,13	0
3	plate	GARY	0	7762,86	7762,86	0
4	plate	GARY	0	5842,86	5842,86	0

Table 12: Optimal inventory policy for plate in GARY

5.5.8. Plate-CLEV

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	plate	CLEV	1167	3679,46	4846,46	0
2	plate	CLEV	0	4961,46	4961,46	0
3	plate	CLEV	0	8172,48	8172,48	0
4	plate	CLEV	0	9954,48	9954,48	0

Table 13:Optimal inventory policy for plate in CLEV

5.5.9. Plate-PITT

Time period	Product	Factory	Inventory (t-1)	Production	Sell	Inventory (t)
1	plate	PITT	1197	7798,92	8995,92	0
2	plate	PITT	0	6866,75	6866,75	0
3	plate	PITT	0	0	5,68E-13	0
4	plate	PITT	0	0	1,59E-12	0

Table 14: Optimal inventory policy for plate in PITT

5.6. Table v: Optimal transportation policy

The following tables show for every combination of products and factories the optimal quantities shipped or sold [Tm] to each of the corresponding markets at each time period.

The code used to create these three tables and the original AMPL output can be found in the annex of the document (point 6.1.6)

5.6.1. CLEV-Bands

	t=1	t=2	t=3	t=4
DET	5123,93	0	0	0
FRA	0	0	0	0
FRE	0	0	0	0
LAF	4801	1934	0	5679
LAN	2128,17	5634,17	1326,17	4189,17
STL	0	0	0	0
WIN	0	0	0	0

Table 15: Optimal transportation policy for bands from CLEV

5.6.2. CLEV-Coils

	t=1	t=2	t=3	t=4
DET	0	0	5066,15	3896,15
FRA	0	0	0	0
FRE	0	0	0	0
LAF	5484,79	3307,79	1843,79	6124,79
LAN	3696,75	6344,75	3085,75	5238,75
STL	0	0	0	0
WIN	0	0	0	0

Table 16: Optimal transportation policy for coils from CLEV

5.6.3. CLEV-Plate

	t=1	t=2	t=3	t=4
DET	0	0	2780,85	2499,85
FRA	0	0	0	0
FRE	0	0	0	0
LAF	2700,28	2172,28	1816,28	2863,28
LAN	2146,19	2789,19	1997,19	2526,19
STL	0	0	1578,17	2065,17
WIN	0	0	0	0

Table 17: Optimal transportation policy for plate from CLEV

5.6.4. GARY-Bands

	t=1	t=2	t=3	t=4
DET	0	0	0	0
FRA	0	693	3158	2701
FRE	3034,7	3409,7	4606,7	3403,7
LAF	0	0	0	0
LAN	0	0	0	0
STL	1575,69	702,695	41,6947	2682,69
WIN	4198	8121	0	0

Table 18: Optimal transportation policy for bands from GARY

5.6.5. GARY-Coils

	t=1	t=2	t=3	t=4
DET	0	0	0	0
FRA	1367,92	3217,24	5082,24	4728,24
FRE	4506,34	4782,34	5687,34	4765,34
LAF	0	0	0	0
LAN	0	0	0	0
STL	3412,58	2750,58	2249,58	4241,58
WIN	0	7022,73	7583,73	877,734

Table 19: Optimal transportation policy for coils from GARY

5.6.6. GARY-Plate

	t=1	t=2	t=3	t=4
DET	0	0	0	0
FRA	0	0	2400,06	2320,06
FRE	2292,96	2360,96	2578,96	2362,96
LAF	0	0	0	0
LAN	0	0	0	0
STL	0	1700,17	0	0
WIN	0	0	2783,84	1159,84

Table 20: Optimal transportation policy for plate from GARY

5.6.7. PITT-Bands

	t=1	t=2	t=3	t=4
DET	1032,56	91,4918	2890,49	1349,49
FRA	0	0	0	0
FRE	0	0	0	0
LAF	0	0	0	0
LAN	0	0	0	0
STL	0	0	0	0
WIN	0	0	8864	0

Table 21: Optimal transportation policy for bands from PITT

5.6.8. PITT-Coils

	t=1	t=2	t=3	t=4
DET	7546,15	2949,15	0	0
FRA	1327,32	0	0	0
FRE	0	0	0	0
LAF	0	0	0	0
LAN	0	0	0	0
STL	0	0	0	0
WIN	4062,73	0	0	0

Table 22: Optimal transportation policy for coils from PITT

5.6.9. PITT-Plate

	t=1	t=2	t=3	t=4
DET	3382,85	2267,85	0	0
FRA	1822,06	1949,06	0	0
FRE	0	0	0	0
LAF	0	0	0	0
LAN	0	0	0	0
STL	1860,17	0	0	0
WIN	1930,84	2649,84	5,68E-13	1,59E-12

Table 23: Optimal transportation policy for plate from PITT

6. ANNEX

6.1. AMPL Output Code

6.1.1. Optimal solution

```
reset;
model SCP.mod;
data SCP.dat;
option solver MINOS;
solve;

ampl: include SCP.run;
MINOS 5.51: optimal solution found.
388 iterations, objective 20080293.48
Nonlin evals: obj = 702, grad = 701.
```

6.1.2. Table i.

```
#########
#TABLE 1#
#########
#AUXILIAR VARIABLES (prefix O -output-)
var O_revenue {t in TIME} = sum {p in PROD,i in FACT,j in MARK}
        pricet[j,p] * sellt[i,j,p,t];
var O_prod_cost {t in TIME} = sum {p in PROD,i in FACT}
        make_cost[i,p,t]*prodt [i,p,t];
var O_inv_cost {t in TIME} = sum {p in PROD,i in FACT}
   inv_cost[i,p]*invent[i,p,t];
var O_trans_cost {t in TIME} = sum {p in PROD,i in FACT,j in MARK}
        trans_cost[i,j,p]*sellt[i,j,p,t];
var O_profit {t in TIME} = O_revenue[t]-O_prod_cost[t]-O_inv_cost[t]-O_trans_cost[t];

display O_revenue,O_prod_cost,O_inv_cost,O_trans_cost,O_profit;

:  O_revenue O_prod_cost O_inv_cost O_trans_cost  O_profit    :=
1   18011800   12519300    151734     3335420     2005360
2   17435200    8654200    108266     3221270     5451480
3   17480700    7482050    50040.8    3362810     6585760
4   17040500    7910240       0       3092570     6037710;
```

6.1.3. Tables ii.

```
#########
#TABLE 2#
#########
#AUXILIAR VARIABLES (prefix O -output-)
var O_difprice {p in PROD, j in MARK} = pricet[j,p]-price_current[p];
param O_demand_curr {p in PROD, j in MARK} = sum{t in TIME}
demand_current[j,p,t];
var O_demand {p in PROD, j in MARK} = sum{t in TIME} demant[j,p,t];
var O_difdemand {p in PROD, j in MARK} = (sum{t in TIME} demant[j,p,t])-(sum{t in
TIME} demand_current[j,p,t]);

display {p in PROD}: {j in MARK} (
```

```
            price_current[p],
            pricet[j,p],
            O_difprice[p,j],
            O_demand_curr[p,j],
O_demand[p,j],
O_difdemand[p,j]);

# $1 = price_current['bands']
# $3 = O_difprice['bands',j]
# $4 = O_demand_curr['bands',j]
# $5 = O_demand['bands',j]
# $6 = O_difdemand['bands',j]
:    $1   pricet[j,'bands']    $3      $4       $5        $6       :=
DET  259.15     271.07      11.9199   14350   10488      -3862.03
FRA  259.15     274.614     15.4643   11748   6552      -5196
FRE  259.15     286.092     26.9419   25016   14454.8   -10561.2
LAF  259.15     294.014     34.8644   20642   12414      -8228
LAN  259.15     285.232     26.082    20685   13277.7    -7407.3
STL  259.15     264.411     5.26089   6076    5002.78   -1073.22
WIN  259.15     272.352     13.2021   26147   21183      -4964;

# $1 = price_current['coils']
# $3 = O_difprice['coils',j]
# $4 = O_demand_curr['coils',j]
# $5 = O_demand['coils',j]
# $6 = O_difdemand['coils',j]
:    $1   pricet[j,'coils']    $3       $4      $5       $6      :=
DET  290.72     264.175     -26.545     10857   19457.6   8600.59
FRA  290.72     269.594     -21.1265     8878   15723    6844.98
FRE  290.72     287.594     -3.12634    18916   19741.4   825.353
LAF  290.72     285.825     -4.89472    15606   16761.2   1155.15
LAN  290.72     279.544     -11.1762    15639   18366     2726.98
STL  290.72     257.678     -33.0423     4592   12654.3   8062.31
WIN  290.72     291.59       0.869639   19780   19546.9   -233.063;

# $1 = price_current['plate']
# $3 = O_difprice['plate',j]
# $4 = O_demand_curr['plate',j]
# $5 = O_demand['plate',j]
# $6 = O_difdemand['plate',j]
:    $1   pricet[j,'plate']    $3      $4      $5       $6      :=
DET  216.64     195.25      -21.3902  2632   10931.4   8299.39
FRA  216.64     200.639     -16.0006  2155   8491.26   6336.26
FRE  216.64     202.094     -14.5461  4592   9595.85   5003.85
LAF  216.64     200.813     -15.8272  3791   9552.1    5761.1
LAN  216.64     198.261     -18.379   3798   9458.74   5660.74
STL  216.64     194.895     -21.7453  1115   7203.68   6088.68
WIN  216.64     204.693     -11.9466  4797   8524.34   3727.34;
```

6.1.4. Tables iii.

```
#########
#TABLE 3#
#########
#AUXILIAR VARIABLES (prefix O -output-)
var O_pricevar {p in PROD} = (sum {j in MARK} ((pricet[j,p]-
price_current[p])/price_current[p]))*100/card(MARK);
var O_demandvar {p in PROD} = (sum {j in MARK} (sum{t in TIME} (demant[j,p,t])-
sum{t in TIME} (demand_current[j,p,t]))/sum{t in TIME}
(demand_current[j,p,t]))*100/card(MARK);

display {p in PROD} (
        O_pricevar[p],
        O_demandvar[p]);

:    O_pricevar[p] O_demandvar[p]    :=
bands    7.37221      -32.2398
coils   -4.8668       51.4164
plate   -7.90218      234.729;
```

6.1.5. Tables iv.

```
#########
#TABLE 4#
#########
display {p in PROD, i in FACT}: {t in TIME} (
        p,
        i,
        invent[i,p,t-1],
        prodt[i,p,t],
        sum{j in MARK} sellt[i,j,p,t],
        invent[i,p,t]);

# $3 = invent['GARY','bands',t - 1]
# $5 = sum{j in MARK} sellt['GARY',j,'bands',t]
# $6 = invent['GARY','bands',t]
: 'bands' 'GARY'     $3   prodt['GARY','bands',t]    $5       $6      :=
1  bands  GARY   4123         22935.1         8808.39  18249.7
2  bands  GARY  18249.7        5122.79        12926.4  10446.1
3  bands  GARY  10446.1         52.178        7806.39  2691.92
4  bands  GARY   2691.92       6095.47        8787.39     0;

# $3 = invent['CLEV','bands',t - 1]
# $5 = sum{j in MARK} sellt['CLEV',j,'bands',t]
# $6 = invent['CLEV','bands',t]
: 'bands' 'CLEV'     $3   prodt['CLEV','bands',t]    $5      $6      :=
1  bands  CLEV  4473         16768          12053.1   9187.88
2  bands  CLEV  9187.88       8251.19         7568.17  9870.9
3  bands  CLEV  9870.9        1323.45         1326.17  9868.17
4  bands  CLEV  9868.17         0           9868.17    0;
```

```
# $3 = invent['PITT','bands',t - 1]
# $5 = sum{j in MARK} sellt['PITT',j,'bands',t]
# $6 = invent['PITT','bands',t]
: 'bands' 'PITT'     $3    prodt['PITT','bands',t]      $5        $6
 :=
1  bands  PITT   8864          5364.04        1032.56    13195.5
2  bands  PITT  13195.5           0            91.4918   13104
3  bands  PITT  13104            0          11754.5      1349.49
4  bands  PITT   1349.49          0           1349.49       0;

# $3 = invent['GARY','coils',t - 1]
# $5 = sum{j in MARK} sellt['GARY',j,'coils',t]
# $6 = invent['GARY','coils',t]
: 'coils' 'GARY'   $3  prodt['GARY','coils',t]     $5    $6   :=
1  coils  GARY  6090        3196.84        9286.84  0
2  coils  GARY    0       17772.9        17772.9   0
3  coils  GARY    0       20602.9        20602.9   0
4  coils  GARY    0       14612.9        14612.9   0;

# $3 = invent['CLEV','coils',t - 1]
# $5 = sum{j in MARK} sellt['CLEV',j,'coils',t]
# $6 = invent['CLEV','coils',t]
: 'coils' 'CLEV'    $3    prodt['CLEV','coils',t]     $5       $6      :=
1  coils  CLEV  3247          5934.53        9181.53     0
2  coils  CLEV     0          9652.53        9652.53     0
3  coils  CLEV     0         11652.6         9995.68   1656.88
4  coils  CLEV  1656.88       13602.8        15259.7      0;

# $3 = invent['PITT','coils',t - 1]
# $4 = prodt['PITT','coils',t]
# $5 = sum{j in MARK} sellt['PITT',j,'coils',t]
# $6 = invent['PITT','coils',t]
: 'coils' 'PITT'       $3          $4        $5          $6
 :=
1  coils  PITT  4072           11716.3     12936.2   2852.07
2  coils  PITT  2852.07          97.0765    2949.15    1.05768e-10
3  coils  PITT    1.05768e-10     0          0       5.13061e-11
4  coils  PITT    5.13061e-11     0          0       0;

# $3 = invent['GARY','plate',t - 1]
# $5 = sum{j in MARK} sellt['GARY',j,'plate',t]
# $6 = invent['GARY','plate',t]
: 'plate' 'GARY'   $3  prodt['GARY','plate',t]    $5     $6    :=
1  plate  GARY  746        1546.96        2292.96   0
2  plate  GARY    0          4061.13        4061.13   0
3  plate  GARY    0          7762.86        7762.86   0
4  plate  GARY    0          5842.86        5842.86   0;

# $3 = invent['CLEV','plate',t - 1]
# $5 = sum{j in MARK} sellt['CLEV',j,'plate',t]
# $6 = invent['CLEV','plate',t]
: 'plate' 'CLEV'    $3  prodt['CLEV','plate',t]    $5     $6    :=
```

```
1  plate  CLEV  1167       3679.46       4846.46   0
2  plate  CLEV    0       4961.46       4961.46   0
3  plate  CLEV    0       8172.48       8172.48   0
4  plate  CLEV    0       9954.48       9954.48   0;

# $3 = invent['PITT','plate',t - 1]
# $5 = sum{j in MARK} sellt['PITT',j,'plate',t]
# $6 = invent['PITT','plate',t]
: 'plate' 'PITT'   $3  prodt['PITT','plate',t]       $5      $6   :=
1  plate  PITT  1197       7798.92       8995.92       0
2  plate  PITT    0       6866.75       6866.75       0
3  plate  PITT    0          0           5.68434e-13   0
4  plate  PITT    0          0           1.59162e-12   0;
```

6.1.6. Tables v.

```
#########
#TABLE 5#
#########
display sellt;

sellt [CLEV,*,bands,*]
:      1       2       3       4      :=
DET   5123.93     0       0       0
FRA     0       0       0       0
FRE     0       0       0       0
LAF   4801     1934       0     5679
LAN   2128.17   5634.17   1326.17   4189.17
STL     0       0       0       0
WIN     0       0       0       0

 [CLEV,*,coils,*]
:      1       2       3       4      :=
DET     0       0     5066.15   3896.15
FRA     0       0       0       0
FRE     0       0       0       0
LAF   5484.79   3307.79   1843.79   6124.79
LAN   3696.75   6344.75   3085.75   5238.75
STL     0       0       0       0
WIN     0       0       0       0

 [CLEV,*,plate,*]
:      1       2       3       4      :=
DET     0       0     2780.85   2499.85
FRA     0       0       0       0
FRE     0       0       0       0
LAF   2700.28   2172.28   1816.28   2863.28
LAN   2146.19   2789.19   1997.19   2526.19
STL     0       0     1578.17   2065.17
WIN     0       0       0       0

[GARY,*,bands,*]
:      1       2        3        4      :=
```

DET	0	0	0	0
FRA	0	693	3158	2701
FRE	3034.7	3409.7	4606.7	3403.7
LAF	0	0	0	0
LAN	0	0	0	0
STL	1575.69	702.695	41.6947	2682.69
WIN	4198	8121	0	0

[GARY,*,coils,*]

:	1	2	3	4	:=
DET	0	0	0	0	
FRA	1367.92	3217.24	5082.24	4728.24	
FRE	4506.34	4782.34	5687.34	4765.34	
LAF	0	0	0	0	
LAN	0	0	0	0	
STL	3412.58	2750.58	2249.58	4241.58	
WIN	0	7022.73	7583.73	877.734	

[GARY,*,plate,*]

:	1	2	3	4	:=
DET	0	0	0	0	
FRA	0	0	2400.06	2320.06	
FRE	2292.96	2360.96	2578.96	2362.96	
LAF	0	0	0	0	
LAN	0	0	0	0	
STL	0	1700.17	0	0	
WIN	0	0	2783.84	1159.84	

[PITT,*,bands,*]

:	1	2	3	4	:=
DET	1032.56	91.4918	2890.49	1349.49	
FRA	0	0	0	0	
FRE	0	0	0	0	
LAF	0	0	0	0	
LAN	0	0	0	0	
STL	0	0	0	0	
WIN	0	0	8864	0	

[PITT,*,coils,*]

:	1	2	3	4	:=
DET	7546.15	2949.15	0	0	
FRA	1327.32	0	0	0	
FRE	0	0	0	0	
LAF	0	0	0	0	
LAN	0	0	0	0	
STL	0	0	0	0	
WIN	4062.73	0	0	0	

[PITT,*,plate,*]

:	1	2	3	4	:=
DET	3382.85	2267.85	0	0	
FRA	1822.06	1949.06	0	0	
FRE	0	0	0	0	

```
LAF    0       0     0          0
LAN     0       0     0          0
STL   1860.17     0     0          0
WIN   1930.84   2649.84   5.68434e-13   1.59162e-12;
```

6.2. AMPL model

```
#SUPPLY CHAIN STRATEGY WITH PRICING

#SETS
set FACT; #Production plants
set MARK; #Markets
set PROD; #Products
set TIME; #Time period
set RESO; #Resource

#PARAMATERS
param avail {RESO,FACT,TIME} >= 0;
param inv0 {FACT,PROD} >= 0;
param rate {RESO,FACT,PROD} >= 0;
param demand_current {MARK,PROD,TIME} >= 0;
param price_current {PROD} >= 0;
param demand_response {MARK,PROD};
param price_min;
param price_max;
param make_cost {FACT,PROD,TIME} >= 0;
param inv_cost {FACT,PROD} >= 0;
param trans_cost {FACT,MARK,PROD} >= 0;

#VARIABLES
var pricet {MARK,PROD} >= 0;
var sellt {FACT,MARK,PROD,TIME} >= 0;
var prodt {FACT,PROD,TIME} >= 0;
var invent {FACT,PROD,{0} union {TIME}} >= 0;
var demant {MARK,PROD,TIME} >= 0;

#AUX VARIABLES TO DEFINE THE OBJECTIVE FUNCTION
var Revenue = sum {p in PROD, t in TIME,i in FACT,j in MARK}
    pricet[j,p] * sellt[i,j,p,t];
var Prod_cost = sum {p in PROD, t in TIME,i in FACT}
    make_cost[i,p,t]*prodt [i,p,t];
var Inv_cost = sum {p in PROD, t in TIME,i in FACT}
  inv_cost[i,p]*invent[i,p,t];
var Trans_cost = sum {p in PROD, t in TIME,i in FACT,j in MARK}
    trans_cost[i,j,p]*sellt[i,j,p,t];

#OBJECTIVE FUNCTION
maximize Profit:
    Revenue-Prod_cost-Inv_cost-Trans_cost;
```

```
#CONSTRAINTS
subject to Balance {i in FACT, p in PROD, t in TIME}:
  prodt[i,p,t]+invent[i,p,t-1]-(sum {j in MARK} sellt[i,j,p,t]) = invent[i,p,t];
subject to Ini {i in FACT, p in PROD}:
     invent[i,p,0]=inv0[i,p];
subject to Resource {r in RESO,i in FACT,t in TIME}:
  (sum {p in PROD} (1/rate[r,i,p])*prodt[i,p,t])<=avail[r,i,t];
subject to Satdemand {p in PROD,t in TIME, j in MARK}:
  (sum {i in FACT} sellt[i,j,p,t])=demant[j,p,t];
subject to Demand {p in PROD,t in TIME, j in MARK}:
  demant[j,p,t]=demand_current[j,p,t]+(demand_response[j,p]*(pricet[j,p]-
price_current[p]));
subject to Pricelb {p in PROD, j in MARK}:
  pricet[j,p] >= price_min*((sum {k in MARK} pricet[k,p])/card(MARK));
subject to Priceub {p in PROD, j in MARK}:
  pricet[j,p] <= price_max*((sum {k in MARK} pricet[k,p])/card(MARK));
```