$$|K_0\rangle = |\psi_0\rangle$$

$$H|K_n\rangle = a_n |K_n\rangle + b_n |K_{n-1}\rangle + b_{n+1} |K_{n+1}\rangle$$

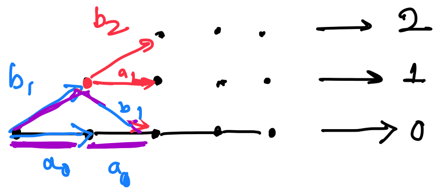
$$H|K_0\rangle = a_0|K_0\rangle + b_1|K_1\rangle$$

$$H|K_1\rangle = a_1|K_1\rangle + b_1|K_0\rangle + b_2|K_2\rangle$$

$$\mu_n = \langle\psi_0|(iH)^n|\psi_0\rangle$$

$$= \langle K_0|(iH)^n|K_0\rangle$$

$$= i^n \langle K_0| H^n |K_0\rangle$$



$$\mu_1 = i\langle K_0| H |K_0\rangle = i a_0$$

$$\mu_2 = i^2 \langle K_0| H^2 |K_0\rangle = i^2 (a_0^2 + b_1^2)$$

$$\boxed{a_0 = -i\mu_1}$$

$$a_0^2 + b_1^2 = -\mu_2$$

$$\Rightarrow -\mu_1^2 + b_1^2 = -\mu_2$$

$$\Rightarrow \boxed{b_1^2 = \mu_1^2 - \mu_2} = \langle K_0| H^2 |K_0\rangle - (\langle K_0|H|K_0\rangle)^2$$

- $\boxed{H = -\log\rho}$, $\lambda_\alpha = e^{-\varepsilon_\alpha}$

$$b_1^2 = \sum_\alpha e^{-\varepsilon_\alpha}(\varepsilon_\alpha)^2 - \left(\sum_\alpha e^{-\varepsilon_\alpha}\varepsilon_\alpha\right)^2$$

$$= \sum_\alpha \lambda_\alpha \log^2\lambda_\alpha - \left(\sum_\alpha \lambda_\alpha \log\lambda_\alpha\right)^2$$

for $\boxed{|\psi_0\rangle = \sum_\alpha e^{-\varepsilon_\alpha/2}|\alpha\rangle = |K_0\rangle}$

- $|\psi_0\rangle = \sum_\alpha e^{-\epsilon_\alpha/2} |\alpha\rangle$

$$\boxed{\rho_A = \sum_\alpha \lambda_\alpha |\alpha\rangle\langle\alpha|}$$

$$\boxed{H_A = \sum_\alpha \epsilon_\alpha |\alpha\rangle\langle\alpha|}$$

$$|\psi(s)\rangle = e^{-iH_A s} |\psi_0\rangle$$

$$= \sum_\alpha e^{-\epsilon_\alpha/2} e^{-i\epsilon_\alpha s} |\alpha\rangle$$

$$S(s) = \langle\psi(s)|\psi_0\rangle$$

$$= \sum_\alpha e^{-\epsilon_\alpha} e^{+i\epsilon_\alpha s} = \sum_\alpha \left(e^{-\epsilon_\alpha}\right)^{1-is}$$

$$= \sum_\alpha \lambda_\alpha^{1-is}$$

$$= \mathrm{Tr}\left(\rho_A^{1-is}\right)$$

$$\boxed{S(s) = \mathrm{Tr}\left(\rho_A^n\right)\Big|_{n\to 1-is}}$$

$$S_R^{(n)} = \frac{\log \mathrm{Tr}\left(\rho_A^n\right)}{1-n}$$

$$\Rightarrow \mathrm{Tr}\left(\rho_A^n\right) = \exp\left[(1-n)\, S^{(n)}\right]$$

$$\Rightarrow \boxed{\mathrm{Tr}\left(\rho_A^n\right)\Big|_{n=1-is} = \exp\left(is\, S_R^{(n)}\Big|_{n=1-is}\right)}$$

$$S(s) = \exp\left(is\, S_R^{(1-is)}\right)$$

Eg.

$$\boxed{|\psi_0\rangle = \sqrt{p}\,|0\,0\rangle + \sqrt{1-p}\,|1\,1\rangle}$$

(A, B)

$$\rho = |\psi_0\rangle\langle\psi_0|$$

$$= p\,|00\rangle\langle 00| + (1-p)\,|11\rangle\langle 11|$$

$$+ \sqrt{p(1-p)}\left(|00\rangle\langle 11| + |11\rangle\langle 00|\right)$$

$$\boxed{\rho_A = \mathrm{Tr}_B(\rho) = p\,|0\rangle\langle 0| + (1-p)\,|1\rangle\langle 1|}$$

$\lambda_1 = p$

$\lambda_2 = (1-p)$

$$S(s) = Tr\left(\rho_A^{1-is}\right)$$

$$= Tr\left(p^{1-is} |0\rangle\langle 0| + (1-p)^{1-is} |1\rangle\langle 1| \right)$$

$$S(s) = p^{1-is} + (1-p)^{1-is}$$

$$\mu_k = \frac{d^k}{ds^k} S(s) = (-i)^k \left[p \log^k p + (1-p)\log^k (1-p) \right]$$

$$|\psi(s)\rangle = e^{-iH_A s} |\psi_0\rangle$$

$$= \sum_{n=0}^{1} \psi_n(s) \, |k_n\rangle$$

$$C(s) = \sum_n n |\psi_n(s)|^2 = |\psi_1(s)|^2$$

---

pseudo

$$\rho_A = p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1| \qquad H_A = -\log \rho_A$$

$$p = \lambda_0 \Rightarrow \varepsilon_0 = -\log p$$

$$1-p = \lambda_1 \Rightarrow \varepsilon_1 = -\log(1-p)$$

$$|\psi_0\rangle = \sum_\alpha e^{-\varepsilon_\alpha/2} |\alpha\rangle$$

$$|\psi(s)\rangle = e^{-iH_s} |\psi_0\rangle = \ldots$$

$$\rho_A = \lambda_0 |0\rangle\langle 0| + \lambda_1 |1\rangle\langle 1| \quad ; \quad \boxed{\lambda_0 + \lambda_1 = 1}$$

$$= \lambda_0 |0\rangle\langle 0| + (1-\lambda_0)|1\rangle\langle 1|$$

$$\lambda_0 = p + iq \Rightarrow \boxed{\varepsilon_0 = -\log(p+iq)}$$

$$\lambda_1 = (1-p) - iq \Rightarrow \boxed{\varepsilon_1 = -\log(1-p-iq)}$$

- $$|\psi_R\rangle = e^{-\varepsilon_0/2}|0\rangle_R + e^{-\varepsilon_1/2}|1\rangle_R$$

$$|\psi_L\rangle = e^{-\varepsilon_0^*/2}|0\rangle_L + e^{-\varepsilon_1^*/2}|1\rangle_L$$

$$H_A = \varepsilon_0 |0\rangle_R {}_L\langle 0| + \varepsilon_1 |1\rangle_R {}_L\langle 1|$$

$$|P_0\rangle = |\psi_R\rangle, \quad \langle Q_0|P_0\rangle = 1$$

$$|Q_0\rangle = |\psi_L\rangle, \quad \boxed{a_0 = \langle Q_0|H_A|P_0\rangle = \varepsilon_0 e^{-\varepsilon_0} + \varepsilon_1 e^{-\varepsilon_1}}$$

- $$|A_1\rangle = H|P_0\rangle - a_0|P_0\rangle$$

$$= e^{-\varepsilon_0/2}\varepsilon_0|0\rangle_R + e^{-\varepsilon_1/2}\varepsilon_1|1\rangle_R - a_0\left(e^{-\varepsilon_0/2}|0\rangle_R + e^{-\varepsilon_1/2}|1\rangle_R\right)$$

$$\boxed{\begin{aligned} |A_1\rangle &= e^{-\varepsilon_0/2}(\varepsilon_0 - a_0)|0\rangle_R + e^{-\varepsilon_1/2}(\varepsilon_1 - a_0)|1\rangle_R \\ |B_1\rangle &= H^\dagger|Q_0\rangle - a_0^*|Q_0\rangle \\ &= e^{-\varepsilon_0^*/2}(\varepsilon_0^* - a_0^*)|0\rangle_L + e^{-\varepsilon_1^*/2}(\varepsilon_1^* - a_0^*)|1\rangle_L \end{aligned}}$$

$$|P_1\rangle = \frac{1}{c_1}|A_1\rangle \qquad \langle Q_1|P_1\rangle = 1$$

$$|Q_1\rangle = \frac{1}{b_1^*}|B_1\rangle$$

$$\Rightarrow \underbrace{\langle B_1|A_1\rangle}_{d_1^2 e^{2i\theta}} = \underline{\underline{b_1 c_1}}$$

$$d_1^2 e^{2i\theta} = b_1 c_1$$

$$\Rightarrow b_1 = d_1 e^{2i\theta/2} = d_1 e^{i\theta}$$

$$c_1 = d_1 e^{2i\theta/2} = d_1 e^{i\theta}$$

$$\langle B_1 | A_1 \rangle = e^{-\varepsilon_0} (\varepsilon_0 - a_0)^2 + e^{-\varepsilon_1} (\varepsilon_1 - a_0)^2 = b_1 c_1$$

$$= d_1^2 \, e^{2i\theta}$$

$$b_1 = d_1 e^{i\theta}$$

$$c_1 = d_1 e^{i\theta} = b_1$$

$$a_1 = \langle Q_1 | H_A | P_1 \rangle$$

$\boxed{a_0, b_1, a_1, c_1 \text{ known!}} \rightarrow$ Complex

- $$|\psi_R\rangle = e^{-\varepsilon_0/2} |0\rangle_R + e^{-\varepsilon_1/2} |1\rangle_R$$

$$|\psi_R(s)\rangle = e^{-iH_A s} |\psi_R\rangle$$

$$= e^{-\varepsilon_0/2} e^{-i\varepsilon_0 s} |0\rangle_R + e^{-\varepsilon_1/2} e^{-i\varepsilon_1 s} |1\rangle_R$$

$$|\psi_R(s)\rangle = \phi_0^R(s) |P_0\rangle + \phi_1^R(s) |P_1\rangle$$

$$\boxed{C_R(s) = \frac{|\phi_1^R(s)|^2}{|\phi_0^R(s)|^2 + |\phi_1^R(s)|^2}}$$

$$\phi_0^R(s) = \langle Q_0 | \psi_R(s) \rangle$$

$$\phi_1^R(s) = \langle Q_1 | \psi_R(s) \rangle$$

- $$|\psi_L\rangle = e^{-\varepsilon_0^*/2} |0\rangle_L + e^{-\varepsilon_1^*/2} |1\rangle_L$$

$$|\psi_L(s)\rangle = e^{-iH_A^\dagger s} |\psi_L\rangle = e^{-\varepsilon_0^*/2} e^{-i\varepsilon_0^* s} |0\rangle_L + e^{-\varepsilon_1^*/2} e^{-i\varepsilon_1^* s} |1\rangle_L$$

$$|\psi_L(s)\rangle = \phi_0^L(s) |Q_0\rangle + \phi_1^L(s) |Q_1\rangle$$

$$\phi_0^L(s) = \langle P_0 | \psi_L(s) \rangle$$

$$\phi_1^L(s) = \langle P_1 | \phi_L(s) \rangle$$

$$\boxed{C_L(s) = \frac{|\phi_1^L(s)|^2}{|\phi_0^L(s)|^2 + |\phi_1^L(s)|^2}}$$