تقسیم و غلبه - بخش سوم

Divide and Conquer III

پارادایم تقسیم و غلبه

Divide the problem into a number of subproblems that are smaller instances of the same problem.

Conquer the subproblems by solving them recursively. If the subproblem sizes are small enough, however, just solve the subproblems in a straightforward manner.

Combine the solutions to the subproblems into the solution for the original problem.

- Tournament Scheduling
- Convex Hull

برنامه ریزی تورنمنت ←

پوشش محدب ←

زمان‌بندی تورنمنت

- In a *tournament* with *N* teams,
  - each team must play against other teams through (*N*-1) matches,
  - one team can play at most one match per day.
1. How many days a tournament of *N* teams can be fulfilled?
2. What is the minimum days an *N*-tournament can be fulfilled?
3. How an *N*-tournament is scheduled in a optimal way?

زمان‌بندی تورنمنت

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  - each team must play against other teams through (*N*-1) matches,
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- How many days a tournament of *N* teams can be fulfilled?
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- How an *N*-tournament is scheduled in a optimal way?
- *First of all, how to represent a scheduling timetable?*

اینکه شکل نمایش جواب را چگونه قرار دادیم

N=4

Day 1	
1	2
3	4

Day 2	
1	3
2	4

Day 3	
1	4
2	3

team	Day 1	Day 2	Day 3
1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

زمان‌بندی تورنمنت

- What is the minimum days an *N*-tournament can be fulfilled?
  - (*N*-1) days, if *N* is even
  - *N* days, if *N* is odd

team	Day 1	Day 2	Day 3
1	2	3	-
2	1	-	3
3	-	1	2

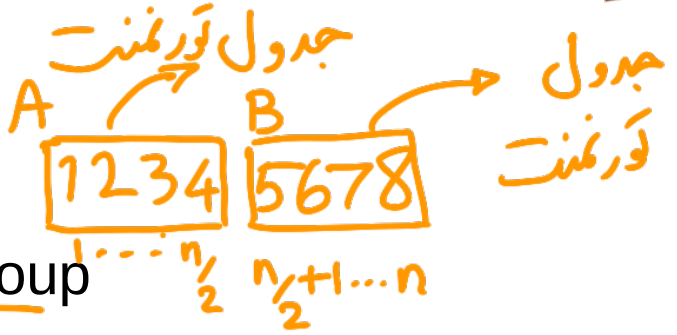
N=3

team	Day 1	Day 2	Day 3
1	2	3	4
2	1	4	3
3	4	1	2
4	3	2	1

N=4

یک الگوریتم تقسیم و غلبه برای زمان‌بندی تورنمنت

- Let $N=2^k$, (این فرض را بعدا کنار می‌گذاریم)
- Divide teams into two groups of $N/2$
- Schedule a $(N/2)$-tournament for each group
  - Only one recursive call for problem of size $(N/2)$
  - Duplicate the timetable for $(N/2)$ and relabled matching teams, i.e add $N/2$ to the team number
- Schedule $(N/2)$ matches between pairs of two groups
  - Use the ``*soccer hand-shaking*`` fashion



N=2

team	Day 1
1	2
2	1

N=4

team	Day 1	Day 2	Day 3
Group *A* 1	2		
Group *A* 2	1		
Group *B* 3	2		
Group *B* 4	1		

یک الگوریتم تقسیم و غلبه برای زمان‌بندی تورنمنت

- Let *N*=2^*k*,
- Divide teams into two groups of *N*/2
- Schedule a (*N*/2)-tournament for each group
  - Only one recursive call for problem of size (*N*/2)
  - Duplicate the timetable for (*N*/2) and relabled matching teams, i.e add *N*/2 to the team number
- Schedule (*N*/2) matches between pairs of two groups
  - Use the ``*soccer hand-shaking*`` fashion

انجام تورنمنتهای درون گروهی

N=2

team	Day 1
1	2
2	1

N=4

	team	Day 1	Day 2	Day 3
Group *A*	1	2		
	2	1		
Group *B*	3	4		
	4	3		

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  - Only one recursive call for problem of size (*N*/2)
  - Duplicate the timetable for (*N*/2) and relabled matching teams, i.e add *N*/2 to the team number
- Schedule (*N*/2) matches between pairs of two groups
  - Use the ``*soccer hand-shaking*`` fashion

Hand shaking

Group *A*	1	2
Group *B*	3	4

	team	Day 1	Day 2	Day 3
Group *A*	1	2	3	
	2	1	4	
Group *B*	3	4	1	
	4	3	2	

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- Schedule a $(N/2)$-tournament for each group
  - Only one recursive call for problem of size $(N/2)$,
  - Duplicate the timetable for $(N/2)$ and relabled matching teams, i.e add $N/2$ to the team number.
- Schedule $(N/2)$ matches between pairs of two groups
  - Use the ``*soccer hand-shaking*`` fashion.

Hand shaking

Group *A*	1	2
Group *B*	3	4

1	2
4	3

تورنمنت بین گروهی

	team	Day 1	Day 2	Day 3
Group *A*	1	2	3	4
	2	1	4	3
Group *B*	3	4	1	2
	4	3	2	1

یک الگوریتم تقسیم و غلبه برای زمان‌بندی تورنمنت

- If *N* is odd,
  - Solve the problem for (*N*+1)-tournament,
  - Remove the team with lable *N* and replace its matches by ``*rest*``.
- If *N* is even, but *N*/2 is odd,
  - Use the divide-and-conqure as for *N*=2^*k*,
  - But schedule a match for two ``rest`` teams on a day.

N=3

team	Day 1	Day 2	Day 3
1	2	3	--
2	1	--	3
3	--	1	2

+3

N=6

team	Day 1	Day 2	Day 3	Day 4	Day 5
1	2	3	--		
2	1	--	3		
3	--	1	2		
4	5	6	--		
5	4	--	6		
6	--	4	5		

Group *A*

Group *B*

یک الگوریتم تقسیم و غلبه برای زمان‌بندی تورنمنت

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  - But schedule a match for two ``rest`` teams on a day.

تورنمنت درون گروهی بعلاوه یک مسابقه بین گروهی برای هر تیم

N=3

team	Day 1	Day 2	Day 3
1	2	3	--
2	1	--	3
3	--	1	2

N=6

	team	Day 1	Day 2	Day 3	Day 4	Day 5
Group *A*	1	2	3	**4**		
	2	1	**5**	3		
	3	**6**	1	2		
Group *B*	4	5	6	**1**		
	5	4	**2**	6		
	6	**3**	4	5		

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- If *N* is even, but *N*/2 is odd,
  - Use the divide-and-conqure as for *N*=2^*k*,
  - But schedule a match for two ``rest`` teams on a day.

Hand shaking

Group *A*	1	2	3
Group *B*	4	5	6

ignore

Group *A*	1	2	3
Group *B*	5	6	4

Group *A*	1	2	3
Group *B*	6	4	5

N=6

	team	Day 1	Day 2	Day 3	Day 4	Day 5
Group *A*	1	2	3	4	5	6
	2	1	5	3	6	4
	3	6	1	2	4	5
Group *B*	4	5	6	1	3	2
	5	4	2	6	1	3
	6	3	4	5	2	1

آنالیز الگوریتم تقسیم و غلبه برای حل مسئله زمانبندی تورنمنت

فرض کنید $T(n)$ نشان دهنده هزینه زمان اجرای الگوریتم تقسیم و غلبه برای حل تورنمنت n تایی است.

$$T(n) = \begin{cases} T(n/2) + \Theta(n^2) & \text{if } n \text{ is even} \\ T(n+1) + \Theta(n) & \text{if } n \text{ is odd} \end{cases}$$

$$\Rightarrow \quad T(n) = T\left(\left\lfloor \frac{n+1}{2} \right\rfloor\right) + \Theta(n^2)$$

حالت سوم قضیه اساسی $\Rightarrow \quad T(n) = \Theta(n^2)$ ✓