Assignment 2: RREF and TSS
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Linear Algebra
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RREF

C10, C11, C30, C31, C32

C10.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

C11.

$$\begin{pmatrix} 1 & 0 & 1 & \frac{4}{5} & 0 \\ 0 & 1 & -1 & -\frac{1}{10} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

C30.

2 1 5 10
1 -3 -1 -2
4 -2 6 12

$R_1 = R_1/2$

1 0.5 2.5 5
1 -3 -1 -2
4 -2 6 12

$R_2 = R_2 - R_1$

1 0.5 2.5 5
0 -3.5 -3.5 -7
4 -2 6 12

$R_3 = R_3 - 4*R_1$

1 0.5 2.5 5
0 -3.5 -3.5 -7
0 -4 -4 -8

$R_3 = R_3*1.75$

1 0.5 2.5 5
0 -3.5 -3.5 -7
0 -7 -7 -14

$R_3 = R_3 - 2 * R_2$

1 0.5 2.5 5
0 -3.5 -3.5 -7
0 0 0 0

$R_2 = R_2 /-3.5$

1 0.5 2.5 5
0 1 1 2
0 0 0 0

$R_1 = R_1 - R_2/2$

1 0 2 4
0 1 1 2
0 0 0 0

C31.
1 2 -4
-3 -1 -3
-2 1 -7

$R_3 = R_3 + 2*R_1$

1 2 -4
-3 -1 -3
0 5 -15

$R_3 = R_3 - R_1 * 3$

1 2 -4
-3 -1 -3
-3 -1 -3

$R_3 = R_3 - R_1$

1 2 -4
-3 -1 -3
0 0 0

$R_2 = R_2 + R_1*3$

1 2 -4
0 -5 -15
0 0 0

$R_2 = R_2/-5$

1 2 -4
0 1 3
0 0 0

C32.
1 1 1
-4 -3 -2
3 2 1

$R_2 = R_2 + R_1$

1 1 1
-3 -2 -1
3 2 1

$R_3 = R_3 + R_2$

1 1 1
-3 -2 -1
0 0 0

$R_2 = R_2 + R_1*3$

1 1 1
0 1 2
0 0 0

$R_1 = R_1 - R_2$

1 0 -1
0 1 2
0 0 0

TSS

1,2,3. Exercises (for the C problems just solve and find r) C21, C22, C23 and M51, M52, M57.

1.
A system of equations is easily recognizable as inconsistent if the solution to an equation where all the coefficients are zeroes is a non-zero constant: e.g. $0x_1 + 0x_2 + 0x_3 = 8$

2.
The dependent variables are all those that have leftmost leading ones as their coefficients, and the independent or free variables are those that do not

3.
The Empty Set if the equations are parallel but do not intersect
An infinite number of solutions if the equations in the set are equivalent

C21.
1 4 3 -1 5
1 -1 1 2 6
4 1 6 5 9

$R_3 = R_3 - 4*R_2$

1 4 3 -1 5
1 -1 1 2 6
0 5 6 5 9

$R_1 = R_1 - R_3$

1 -1 -3 -6 -4
1 -1 1 2 6
0 5 6 5 9

$R_1 = R_1 - R_2$

0 0 -4 -8 -10
1 -1 1 2 6
0 5 6 5 9

$R_1 = R_1/-4$

0 0 1 2 2.5
1 -1 1 2 6
0 5 6 5 9

$R_3 = R_3/5$

0 0 1 2 2.5
1 -1 1 2 6
0 1 1.2 1 1.8

Swap R_3 and R_1

0 1 1.2 1 1.8
1 -1 1 2 6
0 0 1 2 2.5

Swap R_1 and R_2

1 -1 1 2 6 0
0 1 1.2 1 1.8 0
0 0 1 2 2.5 0

$R_1 = R_1 + R_2$

1 0 2.2 3 7.8
0 1 1.2 1 1.8
0 0 1 2 2.5

$R_2 = R_2 - 1.2*R_3$

1 0 2.2 3 7.8
0 1 0 -1.4 -1.2
0 0 1 2 2.5

$R_1 = R_1 - 2.2*R_3$

1 0 0 -1.4 2.3
0 1 0 -1.4 -1.2
0 0 1 2 2.5

r = 3

C22.
1 -2 1 -1 3
2 -4 1 1 2
1 -2 -2 3 1

$R_2 = R_2 - R_1$

1 -2 1 -1 3
1 -2 0 2 -1
1 -2 -2 3 1

$R_1 = R_1 - R_2$

0 0 1 -3 4
1 -2 0 2 -1
1 -2 -2 3 1

$R_3 = R_3 - R_2$

0 0 1 -3 4
1 -2 0 2 -1
0 0 -2 1 2

$R_3 = R_3 + 2*R_2$

0 0 1 -3 4
1 -2 0 2 -1
0 0 0 -5 10

$R_3 = R_3/-5$

0 0 1 -3 4
1 -2 0 2 -1
0 0 0 1 -2

Switch R_1 and R_2

1 -2 0 2 -1
0 0 1 -3 4
0 0 0 1 -2

$R_2 = R_2 + 3*R_3$

1 -2 0 2 -1
0 0 1 0 -2
0 0 0 1 -2

$r = 3$

C23.
1 -2 1 -1 3
1 1 1 -1 1
1 0 1 -1 2

$R_2 = R_2 - R_3$

1 -2 1 -1 3
0 1 0 -2 -1
1 0 1 -1 2

$R_1 = R_1 - R_3$

0 -2 0 0 3
0 1 0 -2 -1
1 0 1 -1 2

$R_1 = R_1 + 2*R_2$

0 0 0 -4 1
0 1 0 -2 -1
1 0 1 -1 2

$R_1 = R_1/-4$

0 0 0 1 -0.25
0 1 0 -2 -1
1 0 1 -1 2

$R_2 = R_2 + 2*R_1$

0 0 0 1 -0.25
0 1 0 0 -1.5
1 0 1 -1 2

$R_3 = R_3 + R_1$

0 0 0 1 -0.25
0 1 0 0 -1.5
1 0 1 0 1.75

Switch R_1 and R_3

1 0 1 0 1.75
0 1 0 0 -1.5
0 0 0 1 -0.25
$r = 3$

M51.

1. This system will have one solution
    a. **Theorem CSRN** Consistent Systems, r and n: “Suppose A is the augmented matrix of a consistent system of linear equations with n variables. Suppose also that B is a row-equivalent matrix in reduced row-echelon form with r pivot columns. Then r n. If r = n, then the system has a unique solution, and if r < n, then the system has infinitely many solutions.”
2. There will be 6 rows with leading ones (dependent variables)
3. There will be 2 rows that are all zeros

M52.

1. This system will have an infinite amount of solutions
    a. **Theorem CMVEI** “Consistent, More Variables than Equations, Infinite solutions Suppose a consistent system of linear equations has m equations in n variables. If n > m, then the system has infinitely many solutions.”
2. All 6 rows will have leading ones (dependent variables)
3. There will be two free/independent variables
    a. **Theorem FVCS** Free Variables for Consistent Systems: “Suppose A is the augmented matrix of a consistent system of linear equations with n variables. Suppose also that B is a row-equivalent matrix in reduced row-echelon form with r rows that are not completely zeros. Then the solution set can be described with n r free variables.”

M57.

1. This system is inconsistent

a. **Theorem RCLS** Recognizing Consistency of a Linear System: “Suppose A is the augmented matrix of a system of linear equations with n variables. Suppose also that B is a row-equivalent matrix in reduced row-echelon form with r nonzero rows. Then the system of equations is inconsistent if and only if column n+1 of B is a pivot column.”

2. If there are only 6 variables and seven pivot columns then one row must be: $0x_1 + 0x_2 + 0x_3 + 0x_4 + 0x_5 + 0x_6 = 1$
3. Therefore, this system of equations has no solution