№2

XOR:

x	y	out
0	0	0
1	0	1
0	1	1
1	1	0

~~xor~~ $xor = \lambda xy.\ x\,(\text{not } y)\ y$

Proof:

1) x = false, y = false

~~xor x y =~~ xor false false = false

2) x = true, y = false

xor true false = true (not false) false = not false = true

3) x = false, y = true

xor false true = false (not true) true = true

4) x = true, y = true

xor true true = true (not true) true = not true = false

I proved, that xor function is valid, so

~~№3~~ because xor is originaly commutative this function ~~val~~ is also commutative.

№1

(1) $\forall M, N \quad \lambda x. MN =_\beta S(\lambda x.M)(\lambda x.N)$

$\triangle$

$S = \lambda fgx.\, fx(gx)$

$S(\lambda x.M)(\lambda x.N) =$

$= (\lambda fgx.\, fx(gx))(\lambda x.M)(\lambda x.N) \stackrel{\beta}{=}$

$= (\lambda gx.\, (\lambda x.M)x(gx))(\lambda x.N) \stackrel{\beta}{=}$

$= \lambda x.\, (\lambda x.M)x((\lambda x.N)x) = \lambda x.M((\lambda x.N)x) =$

$= \lambda x.MN$

∇