Gaussian Filtering

Gaussian filtering is used to blur images and remove noise and detail.

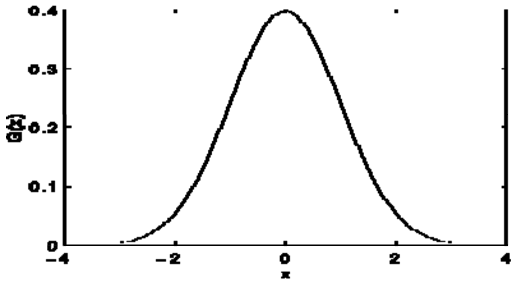
In one dimension, the Gaussian function is:

$$G(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$$

Where σ is the standard deviation of the distribution. The distribution is assumed to have a mean of 0.

Shown graphically, we see the familiar bell shaped Gaussian distribution.

Gaussian distribution with mean 0 and σ = 1



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Gaussian filtering

- Significant values

x	0	1	2	3	4
$\sigma * G(x) / 0.399$	1	$e^{-0.5/\sigma^2}$	e^{-2/σ^2}	$e^{-9/4\sigma^2}$	e^{-8/σ^2}
$G(x) / G(0)$	1	$e^{-0.5/\sigma^2}$	e^{-2/σ^2}	$e^{-9/4\sigma^2}$	e^{-8/σ^2}

For σ=1:

x	0	1	2
$G(x)$	0.399	0.242	0.05
$G(x) / G(0)$	1	0.6	0.125

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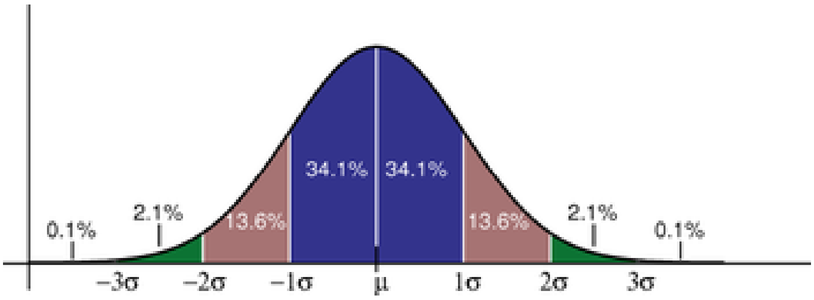
Gaussian Filtering

Standard Deviation

The Standard deviation of the Gaussian function plays an important role in its behaviour.

The values located between +/- σ account for 68% of the set, while two standard deviations from the mean (blue and brown) account for 95%, and three standard deviations (blue, brown and green) account for 99.7%.

This is very important when designing a Gaussian kernel of fixed length.



Distribution of the Gaussian function values (Wikipedia)

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Gaussian Filtering

The Gaussian function is used in numerous research areas:

- It defines a probability distribution for noise or data.
- It is a smoothing operator.
- It is used in mathematics.

The Gaussian function has important properties which are verified with respect to its integral:

$$I = \int_{-\infty}^{\infty} \exp\left(-x^2\right) dx = \sqrt{\pi}$$

In probabilistic terms, it describes 100% of the possible values of any given space when varying from negative to positive values

Gauss function is never equal to zero.

It is a symmetric function.

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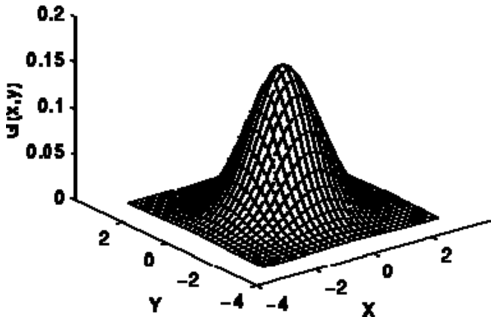
Gaussian Filtering

When working with images we need to use the two dimensional Gaussian function.

This is simply the product of two 1D Gaussian functions (one for each direction) and is given by:

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

A graphical representation of the 2D Gaussian distribution with mean(0,0) and σ = 1 is shown to the right.



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Gaussian Filtering

The Gaussian filter works by using the 2D distribution as a point-spread function.

This is achieved by convolving the 2D Gaussian distribution function with the image.

We need to produce a discrete approximation to the Gaussian function.

This theoretically requires an infinitely large convolution kernel, as the Gaussian distribution is non-zero everywhere.

Fortunately the distribution has approached very close to zero at about three standard deviations from the mean. 99% of the distribution falls within 3 standard deviations.

This means we can normally limit the kernel size to contain only values within three standard deviations of the mean.

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Gaussian Filtering

Gaussian kernel coefficients are sampled from the 2D Gaussian function.

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Where σ is the standard deviation of the distribution.

The distribution is assumed to have a mean of zero.

We need to discretize the continuous Gaussian functions to store it as discrete pixels.

An integer valued 5 by 5 convolution kernel approximating a Gaussian with a σ of 1 is shown to the right,

$$\frac{1}{273}$$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

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Gaussian Filtering

The Gaussian filter is a non-uniform low pass filter.

The kernel coefficients diminish with increasing distance from the kernel's centre.

Central pixels have a higher weighting than those on the periphery.

Larger values of σ produce a wider peak (greater blurring).

Kernel size must increase with increasing σ to maintain the Gaussian nature of the filter.

Gaussian kernel coefficients depend on the value of σ.

At the edge of the mask, coefficients must be close to 0.

The kernel is rotationally symmetric with no directional bias.

Gaussian kernel is separable, which allows fast computation.

Gaussian filters might not preserve image brightness.

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Gaussian Filtering examples

- Is the kernel

1	6	1

a 1D Gaussian kernel?
- Give a suitable integer-value 5 by 5 convolution mask that approximates a Gaussian function with a σ of 1.4.
- How many standard deviations from the mean are required for a Gaussian function to fall to 5%, or 1% of its peak value?
- What is the value of σ for which the value of the Gaussian function is halved at +/-1 x.
- Compute the horizontal Gaussian kernel with mean=0 and σ=1, σ=5.

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Gaussian Filtering examples

Apply the Gaussian filter to the image:
Borders: keep border values as they are

15	20	25	25	15	10
20	15	50	30	20	15
20	50	55	60	30	20
20	15	65	30	15	30
15	20	30	20	25	30
20	25	15	20	10	15

Original image

¼*

1	2	1

15	20	24	23	16	10
20	25	36	33	21	15
20	44	55	51	35	20
20	29	44	35	22	30
15	21	25	24	25	30
20	21	19	16	14	15

¼*

1
2
1

15	20	24	23	16	10
19	28	38	35	23	15
20	35	48	43	28	21
19	31	42	36	26	28
18	23	28	25	22	21
20	21	19	16	14	15

Or:

1	2	1
2	4	2
1	2	1

*1/16

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Gaussian Filtering examples

Apply the Gaussian filter ($\mu=0$, $\sigma=1$) to the image:

15	20	25	25	15	10
20	15	50	30	20	15
20	50	55	60	30	20
20	15	65	30	15	30
15	20	30	20	25	30
20	25	15	20	10	15

Original image

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Gaussian Filtering examples

Apply the Gaussian filter (μ=0, σ=0.2)
to the image:

15	20	25	25	15	10
20	15	50	30	20	15
20	50	55	60	30	20
20	15	65	30	15	30
15	20	30	20	25	30
20	25	15	20	10	15

Original image

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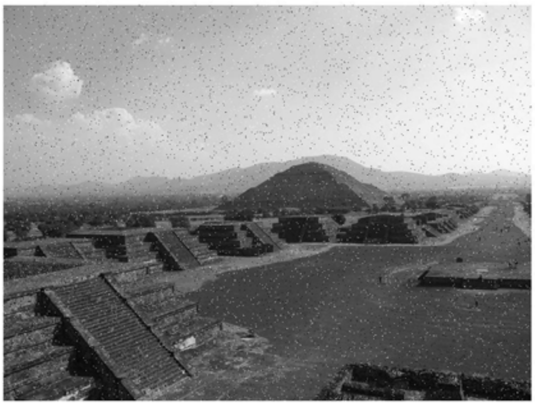
Gaussian Filtering

Gaussian filtering is used to remove noise and detail. It is not particularly effective at removing salt and pepper noise.

Compare the results below with those achieved by the median filter.



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Gaussian Filtering

Gaussian filtering is more effective at smoothing images. It has its basis in the human visual perception system. It has been found that neurons create a similar filter when processing visual images.

The halftone image at left has been smoothed with a Gaussian filter and is displayed to the right.

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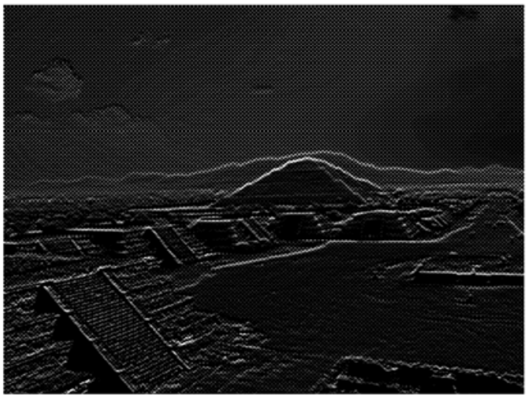
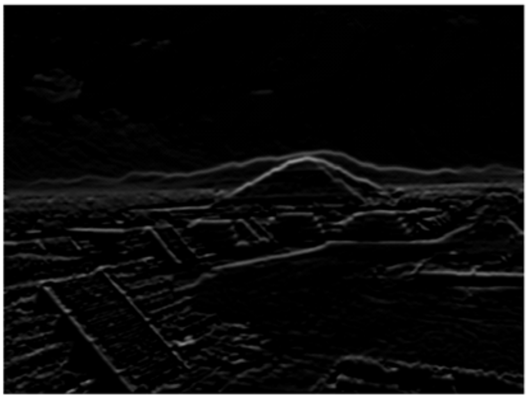
Gaussian Filtering

This is a common first step in edge detection.

The images below have been processed with a Sobel filter commonly used in edge detection applications. The image to the right has had a Gaussian filter applied prior to processing.





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