The missing links: A global study on uncovering financial network structure from partial data[1]

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Abstract

We conduct a horse race of methods to reconstruct financial networks from partial data. The various methods are back-tested using financial network data obtained from 25 different markets, across 13 different jurisdictions. Our contribution is two-fold. We conduct what is arguably the first comprehensive and consistent cross-country analysis of financial networks. The analysis sheds new and robust insights on features of financial networks that transcend jurisdictional boundaries. And, second, the horse race ranks the performance of the different methods relative to the type of financial networks considered. This can guide supervisors in choosing a network reconstruction method for their stress-tests.

Keywords: Network reconstruction, market structure, intermediation
JEL classifications: G20, L14, D85, C63

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1. Introduction

Network analysis can help shed light on the transmission mechanisms for solvency risk and liquidity risk, as well as their interactions, in financial systems. For example, financial intermediaries that have suffered solvency shocks or deposit run-offs may decide to cut lending to their counterparties. These counterparties, in turn, anticipating the cut in funding, will also cut lending to their counterparties, and so on. Depending on the structure of the network and distribution of initial shocks, such hoarding may ultimately lead to a freeze in aggregate interbank markets (see Gai *et al.,* 2011 and Lee, 2013).[2]

Employing network analysis in stress-test models requires granular data on credit exposures and funding structures of financial intermediaries, hereafter simply referred to as banks.[3] In the run up to the Global Financial Crisis, the collection of such high-frequency and granular micro-level data was, at best, patchy, with notable exceptions.[4] The G20 Data Gaps Initiative was set up in the aftermath of the crisis to strengthen the reporting and collection of financial data by member countries. While data gathering capabilities have surged in response to the initiative, data are generally not available beyond regulatory perimeters. For example, domestic exposures between banks and non-bank financial institutions are not tracked systematically, and cross-border data are only exchanged sporadically (e.g., Alves *et al.*, 2013). Thus, most empirical research focuses on the interbank market in a single country, ignoring international links or those to the non-bank financial sector.

To overcome these data limitations, several methods to reconstruct networks from partial data have been developed. The leading method assumes that all banks interact with all other banks to an equal extent. This so-called *maximum-entropy* method has, at its core, a simple risk-sharing mechanism, whereby banks seek to spread risks as evenly as possible with all other banks. Other methods, such as the *minimum-density* method, *probability map* approach, and the *enhanced configuration model*, amongst others, have been developed.

[2] Other mechanisms may also be considered. For example, Zawadoski (2011) and Anand *et al*. (2012) consider how information asymmetries make it costly for a bank's creditors to monitor the quality of the bank's assets following a solvency shock, which can lead to an endogenous withdrawal of liquidity. This, in turn, will leads to a reorganization of lending relationships, which in the limit can lead to a wholesale freeze of lending.
[3] The BIS International Banking Statistics' Funding template, which is currently being rolled out, aims to fill this gap (Cerutti *et al.* 2012).
[4] After the 1994 Tequila crises in México, the Banco de México started collecting detailed information on daily exposures between both domestic banks, and also from domestic to foreign banks.

These different methods have been tested and validated using very different financial markets and data, which renders any comparison between the methods meaningless.

In this paper we present a comprehensive analysis of network reconstruction methods and their applicability to different network data and markets. Specifically, we conduct a horse race of seven methods using data on 25 different financial markets, contributed from 13 separate jurisdictions. For the markets analysed, comprehensive data are available, and the *true* financial networks are known. These networks serve as natural benchmarks against with the different reconstructions are back-tested.

The reconstruction process is as follows. For each market, we postulate that the only information available is aggregate data on the total exposures (assets and liabilities) of banks. This data is fed to the different methods, which produce reconstructions for the network of bilateral exposures. Summary statistics for the properties of the reconstructions are recorded, as well as estimates for how similar they are to the true financial networks. For some methods, a single reconstruction is produced, while others produce a distribution of possible reconstructions. In the end, a ranking of the different methods is produced by comparing their performance across the different financial markets.

Our contribution is two-fold. First, we collate and present detailed summary statics for the 25 real financial networks. This is the first time that such diverse financial network data has been brought together and presented in a way that allows for cross-country and cross-market comparison. And, second, the results of our horse race constitute a road-map that supervisors can use to guide their stress-test model development. The horse race ranks the performance of the different methods relative to the type of financial networks considered. By identifying how the financial market to be stressed compares with those in the horse race, supervisors can make an informed choice on the most appropriate network reconstruction method to use.

Our paper is organized as follows. Section 2 provides a summary of the different methods we consider as part of our analysis. Section 3 provides descriptive statics on the 25 different financial markets considered. In Section 4, we present the results of our horse race between the different methods. A final section concludes.

2. Network reconstruction methods

Table 1 provides a summary of the seven network reconstruction methods used in the horse race. The methods can be grouped as either being based on the maximum entropy method, or not. Methods that are based on maximum entropy are: *Bara*, *Dreh, Maxe* and *Musm*. While, the *Anan*, *Hala* and *Mast* methods are not based on the maximum-entropy method. In what follows, we provide a brief description of each method. Further details are relegated to Appendix A.

Authors	Code	Short Description
Anand et al. (2014)	*Anan*	A "Minimum Density"-method which minimises the number of links necessary for distributing a given volume of loans
Baral and Fique (2012)	*Bara*	Uses a copula to allocate the marginals
Battiston et al. (2012)	*Batt*	A "fitness model" determines the likelihood of linkage, fitness being determined by capital
Drehmann and Tarashev (2013)	*Dreh*	Perturbed maximum entropy matrices (with the RAS-algorithm)
Halaj and Kok (2013)	*Hala*	Assumes a probability map driving link probabilities. Set to be uniform for comparability
Mastrandrea et al. (2014)	*Mast*	Reconstructs the network with information on the degree distribution of the node

Table 1: Overview of methods included in the horse race

The *Anan* method draws on techniques from information theory and the economic rationale that establishing links is costly. The authors provide a heuristic to find a network that has the fewest number of links. Additionally, the method shapes the resulting output to be *disassortative*, i.e., banks that have large interbank assets lend to other institutions with small interbank liabilities and visa-versa. The output is a single network.

Bara uses a copula to estimate adjacency matrices of links between banks. A copula is a multivariate distribution where the complex interdependencies between banks can be easily summarized using marginal distributions. The exposures are applied to the adjacency matrices using the maximum entropy method. The output of the method is a series of reconstructed networks.

Dreh creates networks with a core-periphery structure. The authors postulate that the network should have a dense core with large exposures and a sparse periphery with smaller exposures. This prior information is applied to the standard maximum entropy method to produce a series of reconstructed networks.

Hala is based on a so-called probability map, which specifies the probabilities with which any two financial institutions are linked. To determine the exposure between pairs of banks, the authors propose a simple iterative algorithm, wherein the fraction of one bank's liabilities that are satisfied from the second bank is drawn at random from the unit interval. The output of the *Hala* method is an ensemble of weighted networks.

Mast is based on the fundamental principle of maximum entropy. The method generates a probability distribution that is free from biases – maximises the information entropy – and satisfies, on average, the aggregate asset and liability positions of all banks. The output produces a series of reconstructed networks.

Finally, *Musm* is based on a "fitness model". Each financial institution has a fitness-score that determine the probabilities with which two financial institutions are linked. If both has a high fitness-score, then the probability of a link is high. The *Musm* method produces a series of adjacency matrices, for which the exposures are determined using the maximum entropy method.

3. Summary of the financial market data

Table 2 provides a summary of the data sets we use in this exercise. Detailed descriptions of the individual markets are provided in Appendix B.

Country	Type	Code	Short Description
Brazil	Interbank	BR02	Interbank market exposures between financial institutions, both banking and non-banking, related to unsecured operations
	Payment	BR04	Payments between banks, on their own account, taken from the Brazilian LVPS and aggregated for one day.
BIS	Interbank	BIS03	Bilateral financial system exposures on banks as given in the BIS International Banking Statistics.
Canada	Interbank	CA01	Aggregate bilateral exposures between Canadian D-SIBs consisting of: banker's acceptances, debt securities holdings, unsecured lending (drawn and undrawn), OTC derivatives (potential future credit exposures), repurchase agreements (before collateral) and deposits.
Denmark	Interbank	DK01	Interbank loans derived from the KRONOS large value payment system
	Repo	DK02	Net bilateral repo exposure between Danish banks
France	Interbank	FR01	Bilateral exposures between French bank holding groups in December 2011 with values higher than 10% of banks' capital or above 300 millions of euro. It is obtained from the quarterly Credit Register report.
Germany	Interbank	DE01	Bilateral exposures between German banks with total assets above 1 billion euro. Data is derived from the national credit register which includes bilateral exposures (loans, bonds, derivatives, guarantees) above 1.5 million euro.
Hungary	Interbank	HU06	HUF interbank unsecured deposit transactions between Hungarian banks. All institutions and all transactions are included
	FX swap	HU07	All the transactions on the HUF/FX currency swap market where at least one participant is Hungarian
Italy	Interbank	IT01	Interbank unsecured market exposures (short term loans - up to one year)
Korea	Interbank	KR01	Interbank exposures with a remaining maturity of less than 3 months. These bilateral exposures, which include all on-balance sheet items such as deposits, loans and debt securities, are estimated using flow of funds data and a survey of bilateral interbank holdings.
Mexico	Interbank	MX0103 MX0303 MX0603	Total bilateral exposures (MX01..), Outstanding deposits and loans (MX03..), and Transacted deposit and loans (MX06..)
	Repo	MX0203	Repo amounts lent and borrowed between banks without considering the risk mitigation associated with the collateral
	Equity	MX0403	Cross Holding of securities
	Derivatives	MX0503	Outstanding derivatives
	Payment	MX0703 MX0803 MX0903	Total flow of payments (MX07..), Participant to participant payments (MX08..), and Third party to third party payments (MX09..).
Netherlands	Interbank	NL03	Using data for the payment system TARGET2, i1nterbank loans are inferred including all loans involving a Dutch bank.
UK	CDS	UK02	DTCC's Trade Information Warehouse data. Including all exposures on single names CDS contracts where the reference entity or at least one of two counterparties is UK domiciled. UK02 is the 30% largest in volume.
US	Payments	US02	Fedwire large value payment system for two time periods.
	CDS	OFR03	See UK. In addition index with a majority formed by UK firms

Table 2: Description of the data sets

Table 3 provides summary statistics for the different networks along several dimensions. We readily note the following features. Interbank networks tend to be denser than other types of networks. Interbank networks also have more banks in their cores. In fact, preliminary analysis suggests that the relative core size has some predictive power on the lender and borrower dependencies. As the core size increases, the networks become more concentrated with larger lender and borrower dependencies. The robustness of these results is to be verified once additional data is made available.

	Interbank												
	BIS03	BR02	CA01	DE01	DK01	FR01	HU06	IT01	KR01	MX0103	MX0303	MX0602	NL01
Num of links	812	418	29	10675	77	46	131	3084	263	420	129	50	576
Density	87.3	4.1	96.7	3.3	42.3	41.8	14.1	1	85.9	23.3	7.1	2.8	2.4
Avg Degree	26.2	4.1	4.8	18.9	5.5	4.2	4.2	5.6	14.6	9.8	3	1.2	3.7
Med Degree	28	1.5	5	14	5	4	3	3	15	10	2	1	1
Assortativity	-.12	-.36		-.63	-.3	-.42	-.44	-.43	-.17	-.26	-.36	-.2	-.48
Clustering	22.4	3.1	17.8	41.3	21.5	10.8	23	19.7	21.2	16.2	6.5	3.4	7.5
Lender Dep	28.8	63.9	54.4	42.9	37.5	38.7	51.1	74.9	31.6	56.6	76.3	84.4	79.4
Borrower Dep	31.6	60.2	46.3	71	39.6	39	44.9	88.2	24.9	53.4	63.6	78.8	73.1
Mean HHI Assets	.17	.41	.41	.29	.27	.18	.41	.67	.19	.41	.51	.44	.62
Median HHI Assets	.15	.31	.42	.22	.25	.22	.29	.71	.17	.33	.47	.46	.73
Mean HHI Liabilities	.19	.42	.35	.6	.25	.29	.2	.82	.15	.38	.33	.32	.42
Median HHI Liabilities	.15	.35	.28	.6	.23	.26	.08	1	.14	.28	.28	0	.35
Core Size (% banks)	77.4	9.8	66.7	6.5	42.9	36.4	22.6	3.2	77.8	32.6	16.3	7	7
Error score (% links)	4.4	46.7	3.4	11.2	14.3	10.9	17.6	19.9	3.4	23.6	38	76	26.9

	Payments					CDS			Repo		Other	
	BR04	MX0703	MX0803	MX0903	US02	MX0503	OFR02	UK01	DK02	MX0203	HU07	MX0403
Num of links	1396	800	149	302	169027	70	3267	2004	18	74	221	99
Density	13.3	43.3	8.3	16.3	.5	3.9	.6	1.8	13.6	4.1	2.3	5.5
Avg Degree	13.6	18.6	3.5	7	29.4	1.6	4.4	6	1.5	1.7	2.2	2.3
Med Degree	6	18	0	0	10	1	2	1	.5	1	1	1
Assortativity	-.52	-.45	-.37	-.26	-.27	-.15	-.79	-.72	-.73	-.16	-.57	-.31
Clustering	12.2	17.9	5.8	9.5	14.7	2.8	18.2	13.4	3.5	3.2	1.3	5.9
Lender Dep	62.9	52.2	63.9	39.2	60.8	74.4	69.4	59.7	71.4	75.2	77	67.1
Borrower Dep	59.3	53.5	63.7	49.1	61.4	71	69.1	63.4	95.1	69.7	74.1	60
Mean HHI Assets	.5	.38	.27	.12	.46	.44	.51	.36	.32	.45	.39	.31
Median HHI Assets	.41	.3	0	0	.38	.38	.44	.2	.11	.45	.12	.12
Mean HHI Liabilities	.47	.41	.24	.17	.49	.3	.31	.51	.84	.3	.47	.21
Median HHI Liabilities	.39	.3	0	0	.42	0	0	.46	1	0	.35	0
Core Size (% banks)	20.4	44.2	18.6	32.6	2.6	9.3	2	5.4	16.7	9.3	7.1	11.6
Error score (% links)	10.7	4	12.1	2.6	27.4	61.4	5.2	.9	22.2	55.4	33.5	56.6

Table 3: Descriptives of data

4. *Results of the horse race*

Postulating that the only data available for each of the different financial markets is information on the aggregate lending and borrowing of banks, we reconstruct networks using the seven different methods. For these reconstructed networks we a comprehensive set of statistics, covering local and global network characteristics. For each reconstructed network the number of links, their density, the degrees, assortativity, clustering, the concentration of in- and outgoing links, the core size and core-periphery fit (all defined in Appendix C) are computed. Second, we compute several metrics for how close the reconstructed networks were to their respective true networks. These include the Hamming, Jaccard, and Cosine distances and the Jensen-Shannon divergence measure. We also evaluate “confusion” matrices of the true/false positives/negatives in the reconstructed networks, relative to the true ones. Again, definitions of all of these measures can be found in the Appendix C. To conserve space we show only the Jensen-Shannon divergence in Table 4 since the other measures are broadly similar.

	Types of Networks					
Approaches	Interbank (Complete)	Interbank	Payment	Repo	CDS	Other
Maxe	.09 (.11) .14	.23 (.57) 1.22	.13 (.49) .91	.11 (.3) .49	.54 (.55) .55	.44 (.61) .77
Bara	.09 (.11) .14	.24 (.58) 1.24	.14 (.5) .91	.11 (.3) .5	.55 (.55) .55	.44 (.61) .78
Dreh	.27 (.28) .29	.39 (.75) 1.38	.29 (.67) 1.09	.13 (.4) .67	.71 (.72) .73	.59 (.75) .91
Mast	.42 (.45) .49	.33 (.57) .85	.55 (.73) .9	.2 (.23) .27	.22 (.31) .4	.26 (.36) .47
Anan	.31 (.33) .35	.46 (1.01) 1.52	.44 (.95) 1.52	.03 (.11) .2	1.13 (1.5) 1.87	.3 (.35) .4
Batt	.32 (.4) .47	.14 (.38) .67	.45 (.73) 1.16	.13 (.16) .18	.2 (.2) .2	.26 (.28) .3
Hala	.4 (.44) .48	.4 (.76) 1.51	0 (.67) .91	.08 (.19) .29	.28 (.28) .28	.45 (.45) .45

Table 4: Jensen-Shannon for different network types and algorithms

Figure 1 (left pane) provides a scatter plot of the density of the true networks (top 'row') and the density of the reconstructed matrices provided by the various methods. The sample includes a spread with some jurisdictions providing dense networks (CA, KR and BIS), while many others providing sparse networks. From the estimated matrices, we readily observe that *Anan*, *Hala* and *Mast* tend to estimate sparse networks for most jurisdictions, while *Dreh*, *Maxe* and *Musm* estimate denser networks – in many cases they estimate complete networks with a density of 100%.

A similar broad classification of results can be seen for borrower dependency (Figure 1, right pane), which measures the reliance of individual banks on their largest creditor. The larger is the borrower dependency, the more concentrated are nodes' connections. Once again, the *Anan*, *Hala* and *Mast* methods produce more concentrated networks than the *Dreh, Maxe* and *Musm* methods. For the original networks, borrower dependencies ranged from 20%-100%. The Hamming distance, which is supposed to be low if matrices are very similar, is lowest for maximum entropy (*Maxe*) and maximum entropy based measures which are applied to relatively dense networks. The mean for *Maxe* Hamming values for interbank networks is 33,000, much lower than the corresponding values for Payment and CDS networks. At the same time, the average density for interbank and networks is 44%. The other distance measures, which focus on aspects of similarity, show a qualitatively similar picture.

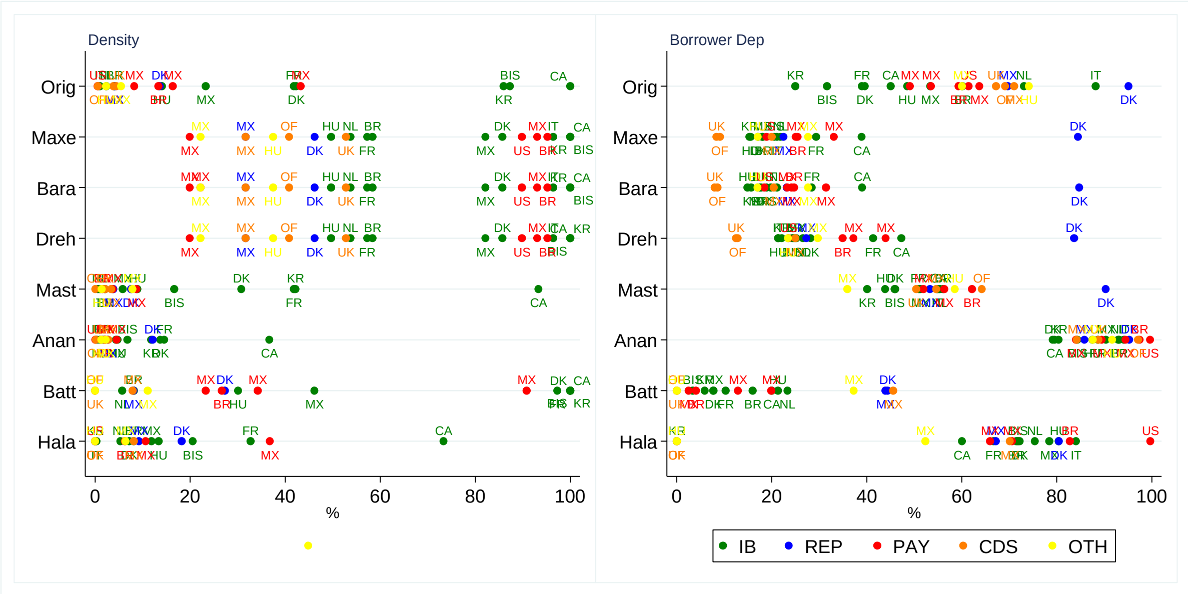

Figure 1: Density and Borrower dependence

The true-positives and true-negatives estimates from the confusion matrices shed further light on the structures of the estimated networks. As Figure 2 (left pane) demonstrates, the *Dreh*, *Maxe* and *Musm* methods are successful in identifying links among banks that are present in the original networks. This is a consequence of the maximal risk-sharing approach that underlies all three methods, resulting in these models estimating complete networks. On the other hand, the *Anan*, *Hala* and *Mast*2 methods correctly identify which links are absent in the original networks (high values in Figure 2, right pane). This stems from the fact that these three methods tend to produce sparse networks. However, in sum, the accuracy of the various partial network methods is mixed due to the identification of false positives and false negatives.

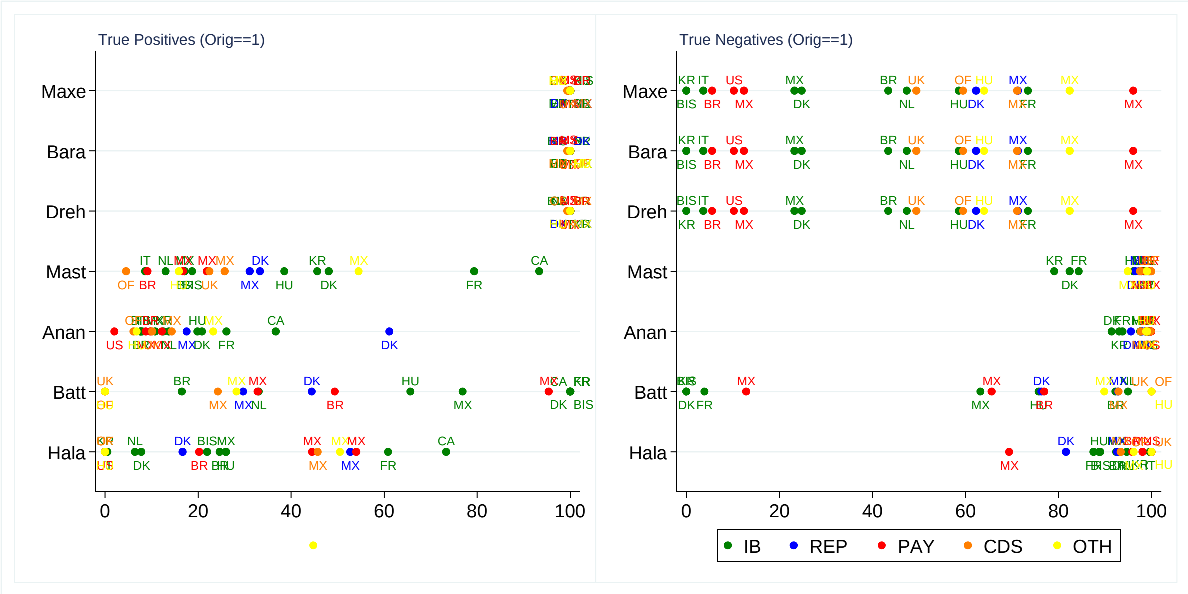

Figure 2: Correctly identified links (True Positives) and absence of links (True Negatives)

5. *Implications for supervisors*

The networks we consider vary greatly, both in size and composition. This heterogeneity is reflected in the quality of the different estimates obtained. As a general rule of thumb, dense networks are well estimated by the maximum-entropy based methods. However, significant differences may arise in estimates of the lending / borrowing dependencies and the Herfindahl indices (defined as sum of the squared "market" shares).

The maximum entropy method has, at its root, a simple risk-sharing mechanism, which implicitly assumes perfect competition, ie, all banks are equally willing to accept an equal share of risk. Consequently, the Herfindahl indices and dependency measures are all typically small. However, for those networks where these statistics were reported, we find that the *Dreh* and *Musm* estimate performs better than *Mast* and *Maxe*. The reason for the *Bara* estimate's superior performance is that it attempts to fit a core-periphery structure into the estimate, which forces some banks to have a larger market share.

For sparse networks, such as CDS networks, the *Anan*, *Hala* and *Mast* estimates are better fits. The *Anan* code, in particular, is designed to produce minimally dense networks. Both *Anan* and *Hala* rely on a "probability map": in *Anan* the probability that one bank is linked to another depends on the relative sizes of the two banks, while in *Hala* a uniform probability was selected. However, as both sets of authors point out, alternative specifications incorporating institution specific details may be used. The *Mast* method does not require any such additional information, but instead derives a maximum entropy (information theoretic) probability distribution for the interbank network, where constraints for the aggregate interbank positions of individual banks bind, on average.

Thus, between the three methods, the *Anan* estimates is always the least dense and most concentrated. The *Hala* and *Mast* estimates are denser and less concentrated. In particular, the *Mast* estimate for the average borrower/lender dependency and Herfindahl indices are closer to those of the original matrix. A similar picture emerges for the core size and error score. Thus, while the *Mast* estimates only satisfy the interbank assets and liabilities constraints, on average, the ensemble average matches the true statistics best for sparse networks.

In sum, both the *Anan* and *Hala* estimates serve as useful benchmarks. As an extreme, the *Anan* estimate provides useful information for how sparse a network can be made, while still ensuring a certain volume of interbank activity. Such information is useful when thinking about the ramifications of a liquidity crisis and the worst cases.

References

Alves, I., S. Ferrari, P. Franchini, J.-C. Heam, P. Jurca, S. Langfield, S. Laviola, F. Liedorp, A. Sánchez, S. Tavolaro, and G. Vuillemey (2013): "The Structure and Resilience of the European Interbank Market," *ESRB Occasional Paper Series*, 3.

Amundsen, E., and H. Arnt (2005): "Contagion risk in the Danish interbank market," *Danmarks Nationalbank Working Paper*, 25.

Anand, K., B. Craig, and G. von Peter (2014): "Filling in the Blanks: Interbank Linkages and Systemic Risk," *BIS Working Paper*, 455.

Anand, K., P. Gai, and M. Marsili (2012): "Rollover risk, network structure and systemic financial crises," *Journal of Economic Dynamics and Control*, 36, 1088-1100.

Arciero, L., R. Heijmans, R. Heuver, M. Massarenti, C. Picillo, and F. Vacirca (2013): "How to measure the unsecured money market? The Eurosystem's implementation and validation using TARGET2 data," *DNB Working Paper*, 369.

Baral, P. and J. Fique (2012): "Estimation of Bilateral Connections in a Network: Copula vs. Maximum Entropy," *Mimeo*.

Blasques, F., F. Bräuning, and I. van Lelyveld (2015): "A Dynamic Stochastic Network Model of the Unsecured Interbank Lending Market," *DNB Working Paper*, 460.

Blien, U. and F. Graef (1997): *Entropy Optimizing Methods for the Estimation of Tables*, Springer Verlag.

Cerutti, E., S. Claessens, and P. McGuire (2012): "Systemic Risks in Global Banking: What Available Data can tell us and What More Data are Needed?" *in Risk Topography: Systemic Risk and Macro Modeling*, NBER, 235-260.

Craig, B. and G. von Peter (2014): "Interbank Tiering and Money Center Banks," *Journal of Financial Intermediation*, 23, 322-347.

Cont, R., E. Santos (2010): "The Brazilian Interbank Network Structure and Systemic Risk," *Banco Central do Brasil Working Paper Series,* 219.

Drehmann, M. and N. Tarashev (2013): "Measuring the Systemic Importance of Interconnected Banks," *Journal of Financial Intermediation*, 22, 586-607.

Elsinger, H., A. Lehar, and M. Summer (2013): "Network Models and Systemic Risk Assessment," in *Handbook on Systemic Risk*, ed. by Pierre Fouque and J. Langsam, Cambridge University Press, chap. 11.

Fang, S. C., J. R. Rajasekere, and H. S. J. Tsao (1997): *Entropy optimization and Mathematical Programming*, Kluwer Academic Publishers.

Fender, I. and P. McGuire (2010): "European Banks' US dollar Funding Pressures," *BIS Quarterly Review*, 57-64.

Furfine, C. H. (1999): "The Microstructure of the Federal Funds Market," *Financial Markets, Institutions and Instruments*, 8, 24-44.

Gai, P., A. Haldane, and S. Kapadia (2011): "Complexity, Concentration and Contagion," *Journal of Monetary Economics*, 58, 453-470.

Garratt, R. J., L. Mahadeva, and K. Svirydzenka (2011): "Mapping Systemic Risk in the International Banking Network," *Bank of England Working Paper*, 413.

Halaj, G. and C. Kok (2013): "Assessing Interbank Contagion using Simulated Networks," *ECB Working Paper*, 1506.

Jaynes, E. (1957): "Information Theory and Statistical Mechanics," *Physical Review*, 106.

Lee, S. H. (2013): "Systemic Liquidity shortages and Interbank Network Structures," *Journal of Financial Stability*, 9, 1-12.

MacKay, D. (2003): *Information Theory, Inference, and Learning Algorithms*, Cambridge University Press.

Martínez-Jaramillo, S., B. Alexandrova-Kabadjova, B. Bravo-Benítez, and J. P. Solórzano-Margain (2014): "An Empirical Study of the Mexican Banking System's Network and its Implications for Systemic Risk," *Journal of Economic Dynamics and Control*, 40, 242-265.

Mastrandrea, R., T. Squartini, G. Fagiolo, and D. Garlaschelli (2014): "Enhanced Reconstruction of Weighted Networks from Strengths and Degrees," *New Journal of Physics*, 16, 043022.

Minoiu, C. and J. A. Reyes (2011): "A Network Analysis of Global Banking: 1978-2009," *IMF Working Paper*, 1174.

Miranda, R. C., S. R. Souza, T. C. Silva, and B. M. Tabak (2014): "Connectivity and Systemic Risk in the Brazilian National Payments System," *Journal of Complex Networks*, 2, 585-613.

Musmeci, N., S. Battiston, G. Caldarelli, M. Puliga, and A. Gabrielli (2013): "Bootstrapping Topological Properties and Systemic Risk of Complex Networks Using the Fitness Model," *Journal of Statistical Physics*, 151, 720-734.

Poledna, S., M. Borboa, M. V. D. Leij, and S. Thurner (2015): "Multiplex structure of systemic risk in financial networks," *Journal of Network Theory in Finance*, 1, 99-138.

Rordam, K. B., and M. L. Bech (2008): "The Topology of Danish Interbank Money Flows," *Danmarks Nationalbank Working Paper,* 59.

Upper, C. (2011): "Simulation Methods to Assess the Danger of Contagion in Interbank Markets," *Journal of Financial Stability*, 7, 111-125.

Upper, C. and A. Worms (2004): "Estimating Bilateral Exposures in the German Interbank Market: Is there a Danger of Contagion?" *European Economic Review*, 48, 827-849.

Zawadowski, A. (2011): "Interwoven Lending , Uncertainty , and Liquidity Hoarding," *Boston University Mimeo*.

Appendix A: Network reconstruction methods

The standard approach in the literature is to estimate the matrix of bilateral links (denoted by X) by maximum entropy methods (Upper 2011, Elsinger *et al.* 2013). This entails maximizing the entropy function $-\sum_{i,j} X_{ij} \log\left(X_{ij}/Q_{ij}\right)$ subject to constraints (typically a firm's total assets, A_i, and liabilities, L_i, to all other participants), relative to prior information (Q_{ij}) on bilateral exposures, if available. As entropy is a measure of probabilistic uncertainty, this approach is optimal when selecting a probability distribution in the sense of using least information (Jaynes 1957, MacKay 2003). Entropy optimization is widely used across disciplines (Fang *et al.,* 1997), and can be implemented by a standard iterative algorithm (eg RAS), which can be generalized to handle additional constraints (Blien and Graef 1997, Elsinger *et al*. 2013). In the simple case where only the marginals are known, the maximum entropy (ME) solution takes the form of a gravity equation where the estimate of X_{ij} is approximately proportional to the product of marginals $A_i L_i$. To the extent that these marginals are positive, ME produces a complete network where each bank lends to all other active banks.

Anan

In Anan *et al.* (2014), the authors propose an approach which combines information-theoretic arguments with economic incentives to produce networks preserving realistic characteristic of interbank networks. The authors argue that interbank networks are sparse given that interbank activity is based on relationships.

The MD approach is formulated as a constrained optimization problem. Let c represent the fixed cost of establishing a link; let N be the number of banks, X the matrix of bilateral gross exposures, X_{ij} represents the exposure of bank i to bank j, the aggregated interbank assets of bank i are $\sum_{j=1}^{N} X_{ij}$ and its aggregated liabilities are $\sum_{i=1}^{N} X_{ij}$. Then the MD approach is formulated as:

$$\min_{\mathbb{X}} c \sum_{i=1}^{N} \sum_{j=1}^{N} 1_{[X_{ij}>0]} \qquad s.t.$$

$$\sum_{j=1}^{N} X_{ij} = A_i \qquad \forall i = 1,2,\dots N$$

$$\sum_{i=1}^{N} X_{ij} = L_j \qquad \forall j = 1,2,\dots N$$

$$X_{ij} \geq 0 \qquad \forall i,j$$

Where integer function 1 equals one only if bank i lends to bank j and zero otherwise. This problem, however, is computationally expensive to solve. The authors propose a heuristic to solve this problem, which involves the smooth value function, $V(X)$, which is high whenever the network X has a few links and satisfies the asset and liability constraints. The second input is the set of prior beliefs, Q_{ij}, which assumes that each small bank prefers to match its lending and funding needs for a large bank (dissasortative mixing).

Bara

In Baral and Fique (2012), the authors use a bivariate copula to estimate adjacency matrices. A copula is a multivariate distribution where the complex interdependencies between banks can be easily summarized using marginal distributions.

The copula is constructed as follows. First, the authors assume the copula to be of the Gumbel type, which are often used in extreme value theory. The authors construct the empirical distribution for the aggregate lending and borrowing of banks using the available data. This distribution is transformed into a copula using a maximum-likelihood method. The copula density function is

$$c_\theta\left(A_i, A_j\right) = \exp\left(-\left[(-\ln A_i)^\theta + \left(-\ln A_j\right)^\theta\right]\right)^{\frac{1}{\theta}}$$

where θ is the estimated dependency parameter. The copula matrix is the prior that is fed into the maximum entropy method. The exposures are re-scaled to ensure that the aggregate lending and borrowing constraints for each bank are satisfied.

paper.

Dreh

Drehmann & Tarashev (2013) generate a series of *high concentration* networks by perturbing the network produced by the maximum entropy method. The authors begin with the standard prior assumption that the exposure between banks i and j is equal to $A_i L_j$. The authors subsequently treat each element of the prior matrix Q_{ij} as a uniformly distributed random variable over the interval $\left[0,2 \times A_i L_j\right]$. After generating a series of prior matrices, the authors use the standard maximum entropy to rescale and determine the exposures.

Hala

Halaj and Kok (2013) introduce an iterative algorithm to generate a series of networks. At the initial step 0, the matrix X^0 has all entries equal to 0 and the unmatched interbank assets and liabilities are initiated as $A^0 \coloneqq A$ and $L^0 \coloneqq L$. At a step $k+1$ a pair of banks (i, j) is drawn at random, where all pairs have an equal probability of being selected. Next, a random number f is drawn from the unit interval and indicates the percentage of bank i's liabilities that are serviced by bank j. The exposure $X^{k+1}(i, j)$ is updated as follows:

$$X^{k+1}(i, j) = X^k(i, j) + \min\left\{f * L_i^k, A_j^k\right\}$$

and the unmatched assets and liabilities goes as:

$$L_i^{k+1} = L_i - \sum_{j=1}^{n} X^{k+1}(i,j) \text{ and } A_j^{k+1} = A_j - \sum_{i=1}^{n} X^{k+1}(i,j)$$

The stock of interbank liabilities and assets reduces as the volume of the assigned (matched) placements increases. The procedure is repeated until no more interbank liabilities are left to be assigned as placements from one bank to another.

Mast

In Mastrandrea *et al*. (2014) the authors develop a method for the reconstruction of weighted networks from local properties alone. They introduced an unbiased analytical maximum-entropy method to reconstruct ensembles of weighted networks using information only on the strengths and degrees. They only considered undirected networks.

The method determines the optimal probability distribution, $P(X)$, for the reconstructed networks that ensures the constraints on the aggregate lending and borrowing of banks is satisfied, and that there is no bias. This can be achieved by requiring that $P(X)$ maximizes Shannon's entropy with a constraint on the expected degree and strength sequences. As a result we can get the probability of the existence of a link with weight X between banks i and *j* as

$$Q_{ij}\ (X|A,L\ ) = \frac{(A_i\ A_j\ )^{\theta(w)}\ \left(L_i L_j\right)^{w}\ (1-\ L_i L_j)}{1-\ L_i L_j + A_i A_j L_i L_j}$$

In the expression $\theta(x) = 1\ if\ x > 0\ and\ \theta(x) = 0$. The reconstructed matrices are produced by sampling from the probabilities.

Musm

Musmeci *et al.* (2013) develop a bootstrap method to reconstruct financial networks. At the core of their method is a `fitness' model, which postulates that the probability of a bank to acquire links is proportional to its fitness. Formally, if banks i and j have fitness f_i and f_j, then the probability for a link between the two banks is

$$Q_{ij} = \frac{z f_i f_j}{1 + f_i f_j}.$$

Where the endogenous parameter z captures how binding the aggregate exposure constraints will be.

The method proceeds as follows. First, from the aggregate lending and borrowing constraints of banks, the parameter z is estimated. Second, using the probabilities p_{ij} a series of adjacency matrices are sampled. Finally, the exposures are determined using the standard maximum entropy method.

Appendix B: Summary of the data

In this appendix we provide a summary of all the financial markets analysed. The data are categorized according to their jurisdiction.

Brazil

The Brazilian contribution included: two snapshots of the interbank exposures (June 2011 and June 2012) and two snapshots of the national payments system (31 January 2013 and 30 December 2013).

Interbank exposures: This network is formed by exposures between banking or non-banking financial institutions in the Brazilian interbank market and has been analysed by Cont and Santos (2010). These institutions are either financial conglomerates or isolated institutions that do not belong to a conglomerate. We perform the network estimation task for two different dates: June 2011 and June. We form these networks by aggregating the end-of-month interbank market exposures for pairs of financial institutions, regardless of time to maturity and without netting. These exposures are comprised of the following unsecured assets: interbank deposit operations (59% in volume), debentures (23%) and repos collateralised with securities issued by the borrower (18%).

For these networks, the number of market participants are 107 (June 2011) and 102 (June 2012). They are sparse (maximum density is 4.1%) and present disassortative mixing, which means that they present a few highly-connected banks along with a comparatively high number of banks linked to one or two counterparties. This is confirmed by the average degree being around three times the corresponding median for these networks. We also observe, on average, the same distribution for assets and liabilities for a given institution: assets or liabilities of higher amounts are allocated to fewer counterparties than those of lower amounts. This effect is consistent with the ratios of average HHI to their medians for both assets and liabilities. However, this finding is distorted in cases in which an institution has a single counterparty. We also see high borrower and lender dependencies, which may be a result from sparse networks with low clustering coefficients.

Payments systems: The Brazilian Payments System (BPS) provides services for the settlement of obligations involving transfers of funds, securities, and foreign exchanges and has amongst others been analyzed by Miranda *et al.* (2014). The system is structured as a set of settlement systems interconnected by the Brazilian National Financial System Network. These settlement systems are segmented according to the target market and the type of assets traded.

We analyse the performance of the network estimation methods on transfers performed by the Reserves Transfer System (STR), which is the backbone of the BPS. The STR is a RTGS system that performs transfers between accounts held by the system's participants at the Central Bank of Brazil. Banks hold bank reserves accounts, while non-banking institutions hold, when authorised, settlement accounts.

We perform the network estimation task for data from two different dates: January 31, 2013 (104 participants), and December 30, 2013 (103 participants). In this analysis, we only consider financial conglomerates of the banking sector, comprising commercial and investment banks. Moreover, intra-conglomerate transactions are ignored.

In relation to the previous interbank exposures network, the payments network is denser (minimum density of the period is 13.3%) and presents more disassortative behaviour, meaning that banks with different degrees tend to attach to each other. Also, the network core is significantly higher than the previous case, suggesting that there is an expressive fraction of banks acting as intermediates in the chain of payments. Considering also that the degree distribution shows an apparent right-skewness property (median is smaller than mean degree), we can see that the majority of the network is composed of low-degree vertices that are connected to a few high-degree counterparties. Even though being denser, the average clustering coefficient is still small, due to the sparse structural topology (no triangular structures) of the low-degree vertices. This scenario also leads to high borrower and lender dependencies, by virtue of the low substitutability of the counterparties. Employing the same reasoning, we expect high assets and liabilities concentration values, as we can confirm from the computed HH indices. As the median is smaller than the mean HHI both for assets and liabilities, there are a majority of low-valued payments and few high-valued ones.

Bank for International Settlements

The network constitutes the2013Q4 exposures between different national financial systems. It is part of the International Banking Statistics (IBS, locational by residency) which the BIS has been collecting since the late seventies (See the BIS website for full details). The data has already been used widely, also in a network context (e.g. Fender and McGuire, 2010; Minoui and Reyes, 2011; and Garratt *et al*., 2011).

Both domestically owned and foreign-owned banking offices with significant external claims in the reporting countries report their on balance sheet positions on other countries split out by sector (residency concept). A wide range of claims is included (e.g. standard loans and deposits, repo and reverse repo, CDs, financial leases, promissory notes, subordinated loans, debt securities, and equity holdings and participations). Out of the possible reporters, data availability leads us to include 21 countries.[5]

Obviously the set-up of this data collection exercise leads to a very particular network structure. Not all countries are data reporters and thus bilateral claims between non-reporters and claims on reporting countries from non-reporting countries will not be included. Note that reporters' claims on non-reporters are included. However, the aim of the exercises is to achieve an adequate coverage of international claims and thus the most important countries are included. A notable exception is China which is becoming a more important player.

Canada

The network constitutes the averages of the total interbank exposures between the six Canadian Domestically Systemically Important Banks, from January 2014 to March 2014.[6] The total interbank exposures for banks' as a fraction of their core-tier 1 capital ranged between 17% to 85%. The ratios of total interbank assets to total assets and liquid assets were two-thousandth a percent and eight-thousandth a percent, respectively. The Figure

[5] I.e. Austria, Australia, Belgium, Canada, the Cayman Islands, Switzerland, Germany, Greece, Denmark (excl. Faeroe Islands and Greenland), Spain, Finland, France (incl. Monaco), United Kingdom (excl. Guernsey, Isle of Man and Jersey), Ireland, Italy, Japan, Luxembourg, Netherlands, Portugal, Sweden, and the United States.

[6] The 6 interbank instruments reported by banks are: (1) Banker's Acceptances, (2) Debt Securities holdings, (3) Unsecured Lending (drawn and undrawn), (4) OTC Derivatives (potential future credit exposure), (5) Repurchase Agreements (before collateral), and (6) Deposits.

below provides a snap-shot of the network, where all bilateral links are normalized over the lending bank's core-tier 1 capital.[7]

Denmark

Two separate financial markets were analysed: overnight interbank loans and repo transactions. Snapshots for both markets were taken from December 2011.

Interbank overnight loans: The network is based on information from the Danish large-value payment system (Kronos). This market has previously been analyzed by Amundsen and Arnt (2005) and Rordam and Bech (2008). An algorithm similar to Furfine (1999) is used to in order to isolate transactions connected to the deliveries and returns of overnight money market loans. Hence our data set consist of all unsecured overnight loans between Danish banks.

A couple of caveats are appropriate as the algorithm's selection criteria do not select overnight money market transactions perfectly. First, the algorithm can only capture overnight loans transferred via the payment system. Second, transactions can only observed at the time that transactions are settled, and not the actual point in time where banks enter into an agreement on a loan. An loan can be agreed upon earlier on the settlement day or even on a previous days.

Repo transactions: The repo dataset consist of monthly reports from major firms including financial institutes (although in the dataset used includes only banks). Each firm reports each outstanding repo agreement vis-à-vis every other institution at the end of each month. The network is comprised of the net bilateral repo exposure.

Since it is only major institutions who report their exposures, exposures between smaller institutions are not captured. Only Danish institutions file the report, and exposures against foreign institutions are collect in one single group. The data only covers "true" repo transactions and is used by the Statistical Department when calculating the holdings of each institution.

[7] The full network is complete on the aggregate level. However, for certain instruments – repurchase agreements and OTC derivatives – we have incomplete networks.

France

The network constitutes bilateral exposures between French bank holding groups in December 2011 with values higher than 10% of banks' capital or above 300 million euro. It is obtained from the Credit Register report collected quarterly. The French banking system consists of 11 major banking groups which, at the consolidated level, capture more than 80% of the total assets of the system.

Netherlands

The network is constructed using data on bilateral transactions gleaned from the TARGET 2 large value payment system on April 6th 2010. This was a typical day without any stress event or extraordinary operational event. For this exercise we have concentrated on the overnight market and thus leave out all longer maturity loans. Building on Furfine (1999), Arceiro (2013) have developed a methodology to identify loans with price and maturity information. The transaction-level data set thus has the time, volume and price of all transactions involving at least one Dutch bank. This data has been analysed further in Blasques *et al.* (2015).

Germany

The network consists of German banks with total assets above 1 billion Euros in December 2013. Altogether, more than 500 banks are included which capture more than 90% of the total assets of the German banking system. Data is derived from the national credit register which includes quarterly data on bilateral exposures (including loans, bonds, derivatives, guarantees) above 1.5 million Euros based on the group of borrowers. Bilateral exposures are captured on the consolidated level.

Hungary

Two markets were analysed: interbank deposits and currency swaps.

Interbank deposits: The Hungarian interbank deposit market is the main market for Hungarian banks to manage their Forint liquidity and this is the only market where they have direct credit risk against each other.

The Central Bank of Hungary is collecting data on this market since 2003. The collected dataset contains detailed information on every transaction. It means that we have the name of both counterparties, the date of the transaction (both the start and the end), the size of the transaction and the interest rate of the transaction.

Six separate monthly averaged snapshots of the market were considered. These include: January 2003, which is when data collected started; March 2005 and July 2007, which were periods when the Hungarian banking sector was characterized by strong lending activity; March 2009 was the most turbulent period for many central and eastern European countries, including Hungary. The size and activity within the interbank market fell dramatically; and finally May 2011 and March 2013, after financial markets normalized.

Currency swaps: FX Swap transactions averaged over the first week of 2005 are considered. This market is one of the most important Hungarian financial markets. The turnover of the Forint/foreign exchange swap market was 5-6 times of the Hungarian GDP in the last decade. Due to data constraints, only that part of the Hungarian currency swap market is known by us where at least one of the counterparties is a Hungarian bank.

The Central Bank of Hungary obliges Hungarian credit institutions to report all their foreign currency related transactions including FX swaps. We have information about the size, the currency, the implied yield, the maturity and the partners in the transaction which make it possible to use the data for network research. Based on the dataset mostly the foreign currency in the contracts is USD (more than 80% on average in the last about 10 years) and a minor proportion is EUR (cc. 15%) and CHF (less than 3 %). Regarding the maturity most of the transactions has 1-2 days long maturity (76 per cent on average) and another 14 per cent has less than 1 month maturity and only around 9 per cent is longer than one month.

Korea

The interbank exposures are constructed using banks' counterparty information collected from the flow of funds and surveys on interbank transactions in 2012Q4. These cover all the on-balance sheet items such as deposits, lending, repo transactions and debt issuance of all 18 domestic banks. However, only exposures with a remaining maturity of less than three months are included since the data frequency of the SAMP is quarterly. Thus these bilateral interbank exposures can be suitable for analysing the structure of short-term interbank transactions.

Italy

Monthly data on outstanding short-term interbank loans from December 2013 between 550 Italian banks was used in this exercise. The data is broken down by counterparty (bilateral exchanges) between banks resident in Italy; interbank loans include deposits, overnight deposits, certificates of deposit, other deposits and other borrowings.

The network assortativity is equal to -0.43, which corresponds to a dissortative network. A negative value means that smaller nodes tend to link up with larger nodes and vice versa. The clustering coefficient, which measures whether nodes connect in groups, is equal to 19.74. The lender and borrower dependency measures are quite high.

Mexico

The data used for this report include data on various types of exposures (unsecured interbank loans, foreign exchange settlement, derivatives transactions), repo transactions, cross holding of securities and payment systems flows between Mexican banks in two different dates: the 31st October 2008 and the 28th June 2013. The dates were selected in order to appreciate difference in crises and post-crisis period. This data has been extensively described in previous works like Martinez-Jaramillo *et al*. (2014) and Poledna *et al.* (2015).

Interbank exposures networks: These networks are the most constantly used for contagion studies, stress testing and more recently for a study in link persistence and overlapping. Seen explicitly as networks, in Martinez-Jaramillo *et al.* (2014) there are a number of empirical facts reported, among them the authors report an average degree of 9, an average completeness index of 0.3, average reciprocity of 0.4 and average size of the largest strongly connected component of 27 for the period of time studied. Additionally, the authors report that the interbank exposures network exhibited a tiered structure, according to the Craig & von Peter (2014) core-periphery model, before the 2008 financial crisis but this changed after such an event and the network exhibited similitude with the scale free network with the same characteristics. In addition, the total exposures network is reported to exhibit the “dissassortative mixing” phenomenon, meaning that small degree nodes tend to have links with large degree nodes.

Payment systems networks: On the other hand, the payment system flow networks are divided in three different networks: the total flow, the large value payments and the low value payments networks. The total flow exhibited the following empirical properties: an average degree of 15, an average completeness index of 0.45, average reciprocity of 0.42 and average size of the largest strongly connected component of 34 for the period of time studied. The large value payments network exhibited properties similar to the total exposures network and the low value lies somewhere in between the large value and the total flows networks. Regarding the core-periphery model, this model presents a better fit than the scale free model, regardless of the period of time (pre-crisis, post crisis). Another relevant feature reported in Martinez-Jaramillo *et al*. (2014) is that the payments network exhibits the “dissasortative mixing” phenomenon to a larger or a smaller extend.

Outstanding repo transactions: The repo transactions networks are built on the basis of the total amount lent for a given day between two institutions, the weight of the link does not discount the value of the collateral used in such transactions. These networks are not built under any type of risk measure, such networks are meant to measure the intensity of repo trading between two banks. These networks do not exhibit high connectivity but are very good examples to see how the algorithms cope with low density matrices in contrast to highly connected networks like the total flow of payments network.

Outstanding derivatives and cross holding of securities: These networks can be seen as layers of the total exposures network as in both cases the weights in such networks are computed as the total exposure derived from either the net position in derivatives transactions or the total amount of debt instruments of a particular banks bought by another bank.

United Kingdom

Transactions data on credit default swaps from 30th June 2010 was used in this exercise. The data from obtained from DTCC's Trade Information Warehouse, which is available to the Bank of England under ODRF and CPSS-IOSCO agreements. These transactions include all trades on single name CDS contracts, in this time period, where the underlying assets were a UK firm. From this data we have reconstructed 89 complete networks of gross notional exposures, one for each of the available single name entities in the data set.

United States

Transactions data on CDS positions from 3rd October 2014 was used in this exercise. The data was obtained from DTCC's Trade Information Warehouse, which is available to the Office of Financial Research under a written agreement with DTCC. These positions include all exposures on single names CDS contracts where the reference entity is US domiciled, or at least one of two counterparties is US domiciled. Additionally, these positions include all exposures on credit indices where a majority of reference entities in an index are US-domiciled or at least one of two counterparties is US-domiciled. From this data we have reconstructed a complete network of counterparty exposures.

Appendix C: Description of metrics used

This section discusses the measures included in the standard output exchanged amongst the members of the group, grouped in two categories: 1) network statistics, and 2) matrix similarity. Table 3 summarizes the network statistics we employ.

Metric	Short Description
Number of links	The number of undirected links in the network
Density	The number of undirected links as a percentage of the total number of links (excluding self-loops)
Average Degree	The average number of undirected links of the nodes in the network
Median Degree	The median number of undirected links of the nodes in the network
Assortativity	The preference for a network's nodes to attach to others that are similar.

	Here similarity is expressed in terms of a node's degree. A high value means that highly connected notes tend to be connected with other high degree nodes. This tendency is referred to as assortative mixing, or assortativity.
Clustering	A measure of the degree to which nodes in a graph tend to cluster together. In a undirected setting this is defined as the number closed triplets (any three nodes with links between all three) over the total number of triplets (also including triplets with one link missing). This is generally defined as a Global Clustering coefficient.
Lender / borrower dependency	The average of the market share of the largest borrower or lender, respectively over total borrowing and lending.
HHI	The average Herfindahl-Hirschman concentration index (mean and median) of both assets and liabilities. It is defined as the sum of the squared "market shares".
Core size	The percentage of banks classified as belonging to the core
Error score	The percentage of the actual links in violation of the perfect Core-Periphery structure

Table 3: Network statistics

The similarity measures listed in Table 4 capture the closeness of the reconstructed networks to the true networks. For all measures lower values denote more similarity. In some cases this means that our definition is the inverse of the customary one.

Metric	Short Description
Hamming	Sum of the difference between the original and estimated *adjacency* matrices. This measure captures the number of instances where the original matrix had a link, but the estimated did not (false negative) and where the original matrix did not have a link, but the estimated matrix did (false positive). Range: $[0, \infty)$.
Jaccard	The inverse of the number of links belonging to both the original and estimated adjacency matrices divided by the number of links that belong to at least one of these matrices. Range: [0,1].
Cosine	The inverse of the transformation of the original and estimated $N \times N$ matricies into vectors of dimension N^2. The cosine similarity thus computes the angle between the two vectors. Range: [0,1]. .
Jensen	Jensen-Shannon divergence measure compares two probability distributions. We normalise the matrices such that the sum of entries is one. Range: [0,1].
True positives	Percentage of links in the estimated network that are also in the original network
True negatives	Percentage of links absent in the estimated network that are also absent in the original network
False positives	Percentage of links in the estimated network that are absent in the original network
False negatives	Percentage of links absent in the estimated network that are present in the original network
Accuracy	The sum of true positive and true negatives over all observations (percentage)