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Systemic liquidity shortages and interbank network structures☆

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ABSTRACT

This paper aims to shed light on the systemic nature of liquidity risk and to propose a method for calculating systemic liquidity shortages. Our method incorporates not only direct liquidity shortages but also indirect liquidity shortages due to the knock-on effects through interbank linkages. We perform a simulation with a simple banking system model and find that a deficit bank can mitigate a liquidity shortage by holding more claims on a surplus bank. Meanwhile, a greater imbalance in liquidity positions across banks tends to aggravate the liquidity shortage of a deficit bank. According to comparative analysis between different types of network structures, a core-periphery network with a deficit money center bank gives rise to the highest level of systemic liquidity shortage, and a banking system becomes more vulnerable to liquidity shocks as its interbank network becomes more ill-matched.

1. Introduction

The 2007–09 global financial crisis has highlighted the fragility of interbank funding markets. In particular, following the collapse of Lehman Brothers in September 2008, short-term wholesale funding markets suddenly dried up, causing system-wide liquidity shortages (IMF, 2010). Banks have traditionally specialized in maturity transformation of short-term deposits to long-term loans, but in recent years have relied increasingly on wholesale funding sources – such as repos, interbank loans, and wholesale deposits – to expand their balance sheets. The scale of long-term illiquid assets financed by short-term wholesale funding has made financial institutions vulnerable to liquidity shocks.

The interbank market is an important means through which banks cover short-falls in liquidity. Typically, interbank linkages are based on short-term borrowing and lending. This kind of linkage can help to make the banking system more efficient by re-allocation of resources from banks that have idle liquid reserves to those that have pressing need for these reserves (Allen and Gale, 2000). However, since the interbank markets also play a role as channels propagating liquidity shocks, the entire banking system becomes exposed to the knock-on effects of liquidity shocks. The knock-on effects of liquidity crunch have been key processes in the recent deleveraging of the global financial system that is taking place as most financial institutions contract their balance sheets at the same time.

Because of balance sheet interconnections among banks, liquidity risk becomes systemic. If a bank faces a mass withdrawal of liquidity, it tries to call in its interbank claims before starting to liquidate its illiquid assets, thus spreading its liquidity problems throughout the financial system. For this reason, we need to assess liquidity risk not at the level of an individual bank, but at the level of the financial system as a whole. Liquidity and solvency problems interact and can cause each other through the banking system (Diamond and Rajan, 2005). If a bank faces a liquidity shortage, it is forced to sell its illiquid long-term assets. Therefore, liquidity shortages may lead to fire sales which, in turn, result in a sharp reduction in asset prices, causing solvency concerns. Conversely, insolvency can exacerbate liquidity shortages. If the insolvency of a bank is anticipated, depositors will immediately withdraw their money from that bank, causing a liquidity shortage.

Most previous studies have focused on solvency concerns, and we thus try to depart from them to focus on systemic liquidity concerns. In this paper, we propose a framework that allows us to capture the systemic nature of liquidity shortages. We also develop a method for calculating the systemic liquidity shortages that a

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banking system faces when it is hit by liquidity shocks. This method incorporates not only direct liquidity shortages but also indirect liquidity shortages due to the knock-on effects through interbank linkages.

We set up a simple banking system model that is described by several exogenous parameters – including a reserve ratio, deposit shares, surplus funds, and cross holdings. We assume that each bank meets its external liquidity withdrawals by calling in its claims on other banks and selling its external liquid assets before starting to liquidate illiquid assets. In order to investigate the determinants of systemic liquidity shortages, we perform a simulation with this simple system. The results of simulation suggest that a deficit bank can prevent a liquidity shortage by holding more reserves or interbank claims, a finding consistent with the risk-sharing argument in Allen and Gale (2000).[1] We also find that a greater imbalance in liquidity positions across banks or a greater deposit share of a surplus bank tend to aggravate the liquidity shortage of a deficit bank. A deficit bank may suffer from liquidity shortages even without direct deposit withdrawals, whereas a surplus bank may avoid liquidity shortages even when there are large deposit withdrawals, since it can transfer its liquidity needs to a deficit bank by calling in its interbank claims.

The network structure of the interbank markets is also a critical determinant of systemic liquidity shortages. To study the effects of network structures on liquidity shortages, we construct six types of interbank network structures – (1) a complete network; (2) a disconnected network; (3) a circular well-matched network; (4) a circular ill-matched network; (5) a core-periphery network with a surplus money center bank; and (6) a core-periphery network with a deficit money center bank. These network structures can be characterized by four properties – completeness, interconnectedness, matchedness and centeredness. The results of simulation show that a core-periphery network with a deficit money center bank gives rise to the highest level of systemic liquidity shortage, and that the banking system becomes more vulnerable to liquidity shocks as its interbank network becomes more ill-matched.

Our paper is related to a number of important earlier papers. Bhattacharya and Gale (1987) first developed the theory of systemic risk in the interbank market. They examined the effects of customers' preference shocks on a multi-bank market and showed that banks tend to hold less liquid assets when they can borrow in the interbank market. Freixas et al. (2000) studied financial contagion in the payment system, through which banks are exposed to the positive probability of contagion if the failure of one bank triggers a domino effect. Allen and Gale (2000) modeled financial contagion via interbank exposures and investigated how the banking system responds to liquidity preference shocks when banks are connected under different interbank market structures. They suggested that if a bank has more links with other banks the initial impact of a financial crisis on that bank may be attenuated, since each of its neighbors takes a small hit when the bank has a run. Diamond and Rajan (2005) examined how liquidity shortages and solvency problems at banks interact, and how each can cause the other. They argued that bank failures can shrink the common pool of liquidity, exacerbating an aggregate liquidity shortage, which could lead to a contagion of failures and total meltdown of the system. Cifuentes et al. (2005) developed a model of contagion in which fire-sale effects play an important role. They showed that for appropriate parameter values the fire-sale effects greatly amplify the extent of contagion. Nier et al. (2007) analyzed how the banking system structure affects systemic risk. They showed that the size of interbank liabilities tends to increase knock-on defaults, and that the more concentrated the banking system is the larger the systemic risk will be. May et al. (2008) and Schweitzer et al. (2009) highlighted the importance of developing models of financial system resilience using more general techniques and insights from the literature on networks and complex systems. Gai and Kapadia (2010) applied statistical techniques from network theory to develop a general model of contagion in complex financial systems. They argued that financial systems have a "robust-yet-fragile" tendency. Although it is true that a greater degree of financial linkages can lower the likelihood of contagion, given a severe contagious default high connectivity will lead to widespread contagion. Recent analyses in this vein include May and Arinaminpathy (2010) and Gai et al. (2011). Much of this literature finds that greater connectivity reduces the likelihood of widespread default, as in Allen and Gale (2000).

In addition to the theoretical studies, there is a growing empirical literature that analyzes the contagion risk through interbank connections for particular countries, see Upper (2011) for a detailed survey. Most of these papers use balance sheet-based network models in which the interbank linkages are modeled in the forms of interbank exposure matrices. The contagion effects arising from the interbank markets are assessed by simulating the failure of single banks. Among others, such balance sheet-based models include those of Boss et al. (2004) for Austria, Upper and Worms (2004) for Germany, Elsinger et al. (2006) for Austria, Degryse and Nguyen (2007) for Belgium, Mistrulli (2007) for Italia, Becher et al. (2008) for the UK large-value payment system, and Cocco et al. (2009) for Portugal. More recently, Chan-Lau et al. (2009) and Aikman et al. (2009) have measured systemic risk using balance sheet-based models. These studies have found that the banking systems demonstrate high resilience even to large shocks.

However, most previous studies have focused on interbank credit losses and default contagion among banks, but have not taken into account liquidity shortages and liquidity contagion such as through the sudden drying up of liquidity in the interbank markets. The knock-on process of a liquidity shock differs from a default shock in two ways. Firstly, the default losses spread from debtors to lenders, whereas liquidity needs are transferred from lenders to debtors. Secondly, default contagion may occur only if there is a bank whose loss from the initial shock exceeds its capital, with the result that default contagion stops if no additional banks fail. On the other hand, a liquidity shock could continue to spread throughout the interbank market until the initial liquidity needs are met by liquidation of external assets since the interbank market cannot create liquidity (Allen and Gale, 2000).

The remainder of this paper is organized as follows. Section 2 introduces a framework for modeling the interbank markets. Section 3 provides the methodologies for measuring systemic liquidity shortages in the banking system. In Section 4, we set up a simple banking system in order to identify the determinants of liquidity shortages. In Section 5, we construct six interbank network structures and compare the liquidity shortages arising in different types of network structures through simulations. Section 6 concludes.

2. Framework

Consider a set $\mathcal{N} = \{1, \quad \ldots, \quad n\}$ of interlinked banks. Table 1 illustrates a stylized balance sheet of a bank that participates in the interbank markets. The total liabilities of each bank consist of interbank liabilities b_{ij}, $\; j \in \mathcal{N}$, external non-bank liabilities d_i, and capital e_i. Here, b_{ij} denotes bank i's interbank borrowing from bank

[1] A surplus bank is defined as a bank that has a net lending position in the interbank markets. Similarly, a deficit bank is a bank that is in a net borrowing position in the interbank markets.

Table 1
Stylized bank balance sheet.

Assets	Liabilities
$\sum_{j\in\mathcal{N}} b_{ji}$	$\sum_{j\in\mathcal{N}} b_{ij}$
q_i	d_i
z_i	
	e_i

j, and d_i indicates the obligations of bank i to the external sector such as customer deposits. The asset side of the bank balance sheet consists of interbank assets b_{ji}, $j\in\mathcal{N}$, liquid external assets q_i, and illiquid external assets z_i. Since all interbank liabilities of a bank are other banks' assets, b_{ji} also indicates bank i's interbank loans to bank j. For simplicity, all interbank exposures are assumed to be liquid. The balance sheet identity implies that total assets must be equal to total liabilities for each bank $i\in\mathcal{N}$ in the system as

$$\sum_{j\in\mathcal{N}} b_{ji} + q_i + z_i = \sum_{j\in\mathcal{N}} b_{ij} + d_i + e_i. \tag{1}$$

In order to model a network structure of the banking system, it is necessary to combine the balance sheet information of individual banks. The structure of assets and liabilities of the banking system can be represented in a matrix form as in Table 2, which shows the uses and sources of funds for each individual bank:[2]

Let l_i be the total liquid assets of bank i, which include interbank loans and external liquid assets:

$$l_i = \sum_{j\in\mathcal{N}} b_{ji} + q_i.$$

The proportions of interbank liquid assets and external liquid assets can be written as

$$\phi_{ij} = \frac{b_{ij}}{l_j}, \quad i, \quad j\in\mathcal{N} \quad \text{and}$$
$$\phi_{n+1,j} = \frac{q_j}{l_j}, \quad j\in\mathcal{N}, \quad \text{respectively.}$$

By construction,

$$\sum_{i=1}^{n+1} \phi_{ij} = 1, \quad \forall j\in\mathcal{N}.$$

Note that Eisenberg and Noe (2001) introduce a relative interbank liabilities matrix to model default contagion, whereas we use here a relative liquid assets matrix to model systemic liquidity shortages. This is because while default losses spread from debtors to lenders, liquidity needs are transferred from lenders to debtors.

We make several assumptions concerning the liquidation of assets. First, that each bank i copes with its external liquidity withdrawals Δd_i by calling in its claims on other banks and selling its external liquid assets, proportionally to their fractions ϕ_{ji}, $j\in\mathcal{N}$ and $\phi_{n+1,i}$. Second, that a bank cannot short-sell liquid assets, i.e., $\Delta l_i \in [0, l_i]$. Third, that a bank sells all of its liquid assets before it starts liquidating its illiquid assets. Hence, bank i sells its illiquid assets only if it faces a liquidity shortage, i.e., $\Delta l_i = l_i$. This assumption is realistic since the premature liquidation of illiquid assets is costly. Lastly, that a bank can neither issue new stocks nor borrow new funds from outside investors to cope with liquidity withdrawals. This assumption is relevant because we aim to measure the systemic funding liquidity shortage of the banking system confronted by a liquidity squeeze.

For each bank $i\in\mathcal{N}$, liquidity withdrawals are paid initially by use of its liquid assets $\Delta l_i^{(0)}$ if $\Delta d_i < l_i$, and by liquidation of its illiquid assets $\Delta z_i^{(0)}$ if $\Delta d_i > l_i$:

$$\Delta d_i = \Delta l_i^{(0)} + \Delta z_i^{(0)} = \sum_{j\in\mathcal{N}} \Delta b_{ji}^{(0)} + \Delta q_i^{(0)} + \Delta z_i^{(0)}$$

where

$$\Delta b_{ji}^{(0)} = \phi_{ji} \min(\Delta d_i, \quad l_i)$$
$$\Delta q_i^{(0)} = \phi_{n+1,i} \min(\Delta d_i, \quad l_i)$$
$$\Delta z_i^{(0)} = \max(0, \quad \Delta d_i - l_i).$$

Calling in the interbank loans, $\sum_{j\in\mathcal{N}} \Delta b_{ji}^{(0)}$, will increase other banks' liquidity needs, and this knock-on process continues system-wide until the external assets of the banking system decrease by the same amount as the initial external liquidity withdrawals. This is because the interbank market can only redistribute but not create liquidity (Allen and Gale, 2000). Once external depositors withdraw their money from the banking system, the external assets of the system must be eventually liquidated in the same amount as the initial withdrawals as follows:

$$\sum_{i\in\mathcal{N}} \Delta d_i = \sum_{i\in\mathcal{N}} \Delta q_i + \sum_{i\in\mathcal{N}} \Delta z_i. \tag{2}$$

We define the liquidity needs of bank i, l_i^n, as the total liquidity withdrawn by other banks and depositors:

$$l_i^n = \sum_{j\in\mathcal{N}} \Delta b_{ij} + \Delta d_i. \tag{3}$$

If bank equity is constant, because of the balance sheet identity (1) the liquidity needs can also be computed by adding up the liquidations of interbank assets and external assets:

$$l_i^n = \sum_{j\in\mathcal{N}} \Delta b_{ji} + \Delta q_i + \Delta z_i. \tag{4}$$

By aggregating Eq. (4) and comparing with Eq. (2), we have

$$\sum_{i\in\mathcal{N}} l_i^n = \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} \Delta b_{ji} + \sum_{i\in\mathcal{N}} \Delta q_i + \sum_{i\in\mathcal{N}} \Delta z_i > \sum_{i\in\mathcal{N}} \Delta d_i. \tag{5}$$

The inequality (5) implies that the liquidity needs of the banking system can be much greater than the initial liquidity withdrawals, due to the knock-on effects in the interbank market. Consequently, even a small liquidity withdrawal can have a huge impact on the banking system as a whole.

3. Systemic liquidity shortages

In this section, we propose a method for measuring systemic liquidity shortages incorporating knock-on effects in the interbank markets.

A bank is said to face a liquidity shortage if its total liquidity needs l_i^n exceed its total liquid assets l_i. We define bank i's liquidity shortage l_i^s as

$$l_i^s \equiv \max(0, \quad l_i^n - l_i).$$

[2] Since banks do not lend to themselves, all diagonal elements of the interbank matrix are zero.

Table 2
Matrix representation of banking system.

		Interbank liabilities					Nonbank liabilities	Capital
		1	...	i	...	n		
Interbank assets	1	0	...	b_{1i}	...	b_{1n}	d_1	e_1
	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
	i	b_{i1}	...	0	...	b_{in}	d_i	e_i
	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
	n	b_{n1}	...	b_{ni}	...	0	d_n	e_n
Nonbank assets	Liquid	q_1	...	q_i	...	q_n		
	Illiquid	z_1	...	z_i	...	z_n		

Let $l_i^{(k)}$ denote the remaining liquid assets of bank i in the kth step of the knock-on process. Similarly, $l_i^{n(k)}$, $l_i^{s(k)}$ and $\Delta l_i^{(k)}$ denote the liquidity needs, liquidity shortages and changes in liquid assets of bank i, respectively, in the kth step of the knock-on process. Suppose that external depositors withdraw Δd_i from bank $i \in \mathcal{N}$. Then, bank i's initial liquidity shortage $l_i^{s(0)}$ can be computed as follows:

$$\begin{aligned} l_i^{(0)} &= l_i, \\ l_i^{n(0)} &= \Delta d_i, \\ \Delta l_i^{(0)} &= \min(l_i^{n(0)}, \quad l_i^{(0)}), \text{ and} \\ l_i^{s(0)} &= \max(0, \quad l_i^{n(0)} - l_i^{(0)}), \quad \forall i \in \mathcal{N}. \end{aligned}$$

The calling in of interbank loans will induce additional liquidity needs in the banking system. The kth step in the knock-on process can be written as

$$\begin{aligned} l_i^{(k)} &= l_i^{(k-1)} - \Delta l_i^{(k-1)}, \\ l_i^{n(k)} &= \sum_{j \in \mathcal{N}} \phi_{ij} \Delta l_j^{(k-1)}, \\ \Delta l_i^{(k)} &= \min(l_i^{n(k)}, \quad l_i^{(k)}), \text{ and} \\ l_i^{s(k)} &= \max(0, \quad l_i^{n(k)} - l_i^{(k)}), \quad \forall i \in \mathcal{N}. \end{aligned}$$

For each bank $i \in \mathcal{N}$, this knock-on process will continue to repeat as long as $l_i^{n(k)} > 0$.

The total liquidity need of bank i is the sum of the liquidity needs in all steps of the knock-on process:

$$l_i^n = \sum_{n=0}^{\infty} l_i^{n(k)}, \quad \forall i \in \mathcal{N}.$$

Similarly, the total changes in liquid assets Δl_i can be expressed as

$$\Delta l_i = \sum_{k=0}^{\infty} \Delta l_i^{(k)}, \quad \forall i \in \mathcal{N}.$$

Finally, the systemic liquidity shortage l_i^s is given by

$$l_i^s = \sum_{n=0}^{\infty} l_i^{s(k)}, \quad \forall i \in \mathcal{N}.$$

We should iterate the knock-on process infinitely to compute the exact amount of liquidity shortage which arises in the entire knock-on process. This method may be computationally inefficient and inaccurate, however, and so we develop an alternative methodology for computing the exact solution without such a computational burden.

Table 3
Simple banking system, $n = 2$.

0	β	d_1	e_1
$\alpha + \beta$	0	d_2	e_2
q_1	q_2		
z_1	z_2		

The total change in liquid assets Δl_i can be calculated exactly and efficiently by solving the following fixed point equation:

$$\Delta l_i = \min\,[l_i, \quad \sum_{j \in \mathcal{N}} \phi_{ij} \Delta l_j + \Delta d_i], \quad \forall i \in \mathcal{N}. \tag{6}$$

In order to solve this fixed point equation, we develop an algorithm similar to the fictitious default algorithm (Eisenberg and Noe, 2001), which gives exact solutions in at most n iterations (see Appendix A for more details on the algorithm). Then the liquidity needs of bank i are derived from (3) as

$$l_i^n = \sum_{j \in \mathcal{N}} \phi_{ij} \Delta l_j + \Delta d_i, \quad \forall i \in \mathcal{N}. \tag{7}$$

Therefore, the systemic liquidity shortage l_i^s is obtained by

$$l_i^s = max(0, \quad l_i^n - l_i), \quad \forall i \in \mathcal{N}. \tag{8}$$

4. Determinants of systemic liquidity shortages

4.1. Simple banking system

In order to investigate the possible determinants of systemic liquidity shortages, we set up a simple banking system consisting of two banks, i.e., $n = 2$.[3] We model an interbank market using exogenous parameters that describe the banking system. This allows us to study how systemic liquidity shortages depend upon the interbank market structure.

The simple banking system is illustrated in Table 3. For simplicity and without loss of generality, we may assume that $e_1 = e_2 = 0$, since we take into account liquidity concerns only. The total external liabilities of the banking system are normalized to one, i.e., $d_1 + d_2 = 1$. A bank is said to be a surplus bank if its external liabilities are greater than its external assets, and a deficit bank if the opposite is the case. Note that external liabilities and assets exclude only interbank transactions. Therefore, the surplus bank holds excess funds and lend them to the deficit bank in the interbank markets.

Supposing that Bank 1 specializes in deposit-taking and Bank 2 specializes in lending, Bank 1 is thus a surplus bank and Bank 2 a deficit bank. Let α and β denote the excess funds of the surplus bank and the deficit bank's claims on the surplus bank respectively. The

[3] For simplicity, we set $n = 2$. The results of this section also hold for $n > 2$.

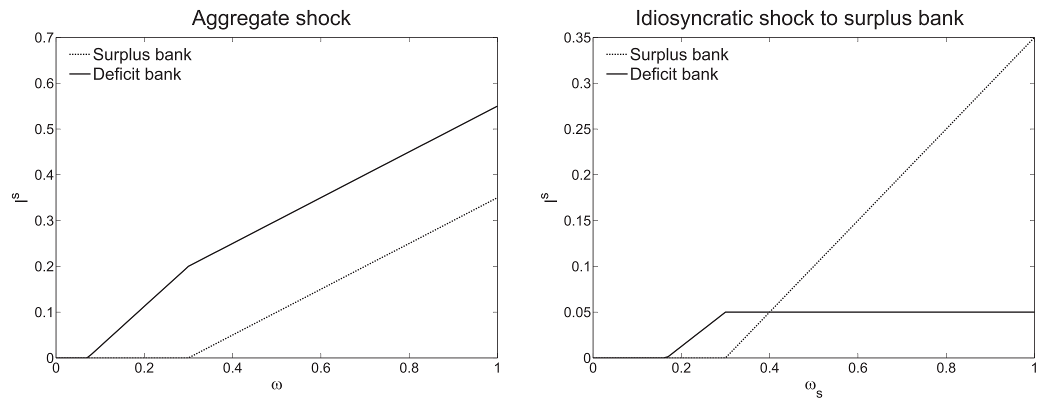

Fig. 1. Liquidity shock and liquidity shortage.

deposits of Bank 1 exceed its external assets by α, while the external assets of Bank 2 exceed its deposits by α. The size of interbank cross-holdings is parameterized by β. Note that banks are allowed to hold each other's assets and liabilities. Thus, Bank 1 will lend $\alpha+\beta$ to Bank 2, and Bank 2 will lend β to Bank 1 in the interbank market. Let δ denote the deposit share of the surplus bank. Since the total external liabilities of the banking system are normalized to one, the deposit structure of the banking system can be expressed as

$$d_1 = \delta, \quad \text{and}$$
$$d_2 = 1 - \delta.$$

We assume that each bank holds liquid assets in an amount satisfying the minimum reserve requirement. Thus, each bank must hold reserves against its deposits in the form of liquid assets as follows:

$$q_1 = \gamma\delta \quad \text{and}$$
$$q_2 = \gamma(1-\delta),$$

where γ is the reserve requirement ratio. By applying the balance sheet identity, each bank's illiquid assets are given by

$$z_1 = (1-\gamma)\delta - \alpha \quad \text{and}$$
$$z_2 = (1-\gamma)(1-\delta) + \alpha.$$

Since liquid assets comprise interbank loans and external liquid assets, each bank's total liquid asset holdings are written as

$$l_1 = \gamma\delta + \alpha + \beta, \quad \text{and}$$
$$l_2 = \gamma(1-\delta) + \beta.$$

Now consider aggregate liquidity shock hitting both banks in the system. The aggregate liquidity shock is assumed to reduce a fraction ω of the external deposits of both banks at the same time. Let Δd_i be the size of the initial liquidity shock, which is expressed as

$$\Delta d_1 = \omega\delta, \quad \text{and}$$
$$\Delta d_2 = \omega(1-\delta),$$

where ω is the deposit withdrawal rate. Using Eq. (6), the changes in total liquid assets are obtained by

$$\Delta l_1 = \min[\gamma\delta + \alpha + \beta, \quad \phi_{12}\Delta l_2 + \omega\delta], \quad \text{and}$$
$$\Delta l_2 = \min[\gamma(1-\delta) + \beta, \quad \phi_{21}\Delta l_1 + \omega(1-\delta)],$$

where

$$\phi_{12} = \frac{\beta}{\gamma(1-\delta)+\beta}, \quad \text{and}$$
$$\phi_{21} = \frac{\alpha+\beta}{\gamma\delta+\alpha+\beta}.$$

Using Eq. (7), the liquidity needs are given by

$$l_1^n = \phi_{12}\Delta l_2 + \omega\delta, \quad \text{and}$$
$$l_2^n = \phi_{21}\Delta l_1 + \omega(1-\delta).$$

Finally, using Eq. (8), the liquidity shortages are written as

$$l_1^s = \max[0, \quad (\omega-\gamma)\delta - \alpha - \beta + \phi_{12}\Delta l_2], \quad \text{and}$$
$$l_2^s = \max[0, \quad (\omega-\gamma)(1-\delta) - \beta + \phi_{21}\Delta l_1].$$

The explicit solutions for the changes in liquid assets, liquidity needs, and liquidity shortages are provided in Appendix B.

4.2. Systemic liquidity shortage in simple banking system

In order to identify the relationships between liquidity shortage and the parameters that describe the banking system, we measure liquidity shortages in the simple banking system. The excess funds of the surplus bank are set at $\alpha = 0.1$, and the claims of the deficit bank on the surplus bank at $\beta = 0.05$. The deposit share of the surplus bank is in addition set at $\delta = 0.5$ and the reserve ratio at $\gamma = 0.1$. Finally, we set the size of the liquidity shock at $\omega = 0.1$.[4]

Fig. 1 illustrates the liquidity shortages arising from different sizes of liquidity shocks – aggregate shocks ω to both banks (left panel) and idiosyncratic shocks ω_s to the surplus bank (right panel). A simulation with benchmark parameter values suggests that the deficit bank is vulnerable to both aggregate shocks and idiosyncratic shocks to the surplus bank. The deficit bank may suffer from liquidity shortage even without a direct deposit withdrawal, whereas the surplus bank may avoid liquidity shortage even in the case of a large deposit withdrawal as it can transfer its liquidity needs to the deficit bank by calling in interbank claims.

In reality, it is more common that depositors transfer their deposits from problem banks to safer banks. Let τ denote the portion of deposit withdrawal transferred to safer banks. Then, only $1-\tau$ of the deposit withdrawal will be turned into cash or foreign

[4] The simulation results are robust with respect to the parameter values.

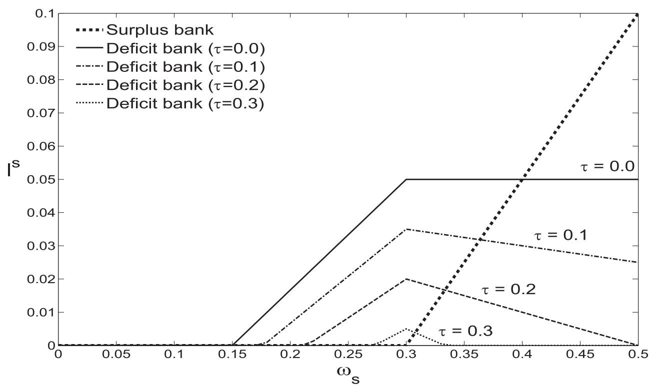

Fig. 2. Deposit transfer and liquidity shortage.

currency. In the case of an idiosyncratic shock to the surplus bank, $\tau\omega_S\delta$ units of deposits will be redeposited in the deficit bank. The right panel of Fig. 1 is equivalent to liquidity shortage with τ of 0. Fig. 2 shows the effects of deposit transfer on liquidity shortages. As τ increases, the liquidity shortage of the deficit bank decreases. Thus, a deposit transfer can reduce a systemic liquidity shortage.

Fig. 3 summarizes the relationships between liquidity shortages and the parameter values. The deficit bank is able to effectively prevent a liquidity shortage by holding more reserves. The crossholdings between the different bank types also mitigate the liquidity shortage of the deficit bank, consistent with the risk-sharing argument in Allen and Gale (2000). On the contrary, a greater deposit share of the surplus bank tends to aggravate the liquidity shortage of the deficit bank. Furthermore, a greater imbalance in liquidity positions across the banks is associated with an increase in the deficit bank's liquidity shortage.

5. Interbank network structures

5.1. Network types

An interbank network consists of a set of banks that are connected by interbank claims. Following Allen and Gale (2000) and Freixas et al. (2000), we construct several stylized network structures of interbank markets based on completeness and interconnectedness. These network structures are also characterized by the matchedness between two types of banks – surplus and deficit. An interbank network is said to be well-matched if all banks are linked to banks of opposite types, and ill-matched otherwise. Further, we introduce a core-periphery structure in which a money center bank is the hub or core that has complete connection with all other banks, whereas other banks connect only to the money center bank.

Consider a banking system $\mathcal{N} = \{1, \ldots, n\}$ that consists of a set of surplus banks $\mathcal{S} = \{i : d_i + e_i \geq q_i + z_i\}$ and a set of deficit banks $\mathcal{D} = \{i : d_i + e_i < q_i + z_i\}$. Let n_S and n_D denote the numbers of surplus banks and of deficit banks, respectively, so that $n_S + n_D = n$. For simplicity and without loss of generality, let $e_i = 0$, $\forall i$ and $\sum_{i\in\mathcal{N}} d_i = 1$. It is also assumed that their external assets and liabilities are homogeneous for all banks within the sub-groups – $\mathcal{S}$ and $\mathcal{D}$. Note that the interbank asset and liability structures of the banks within the subgroups may differ depending upon the interbank network structure. The distribution of external deposits

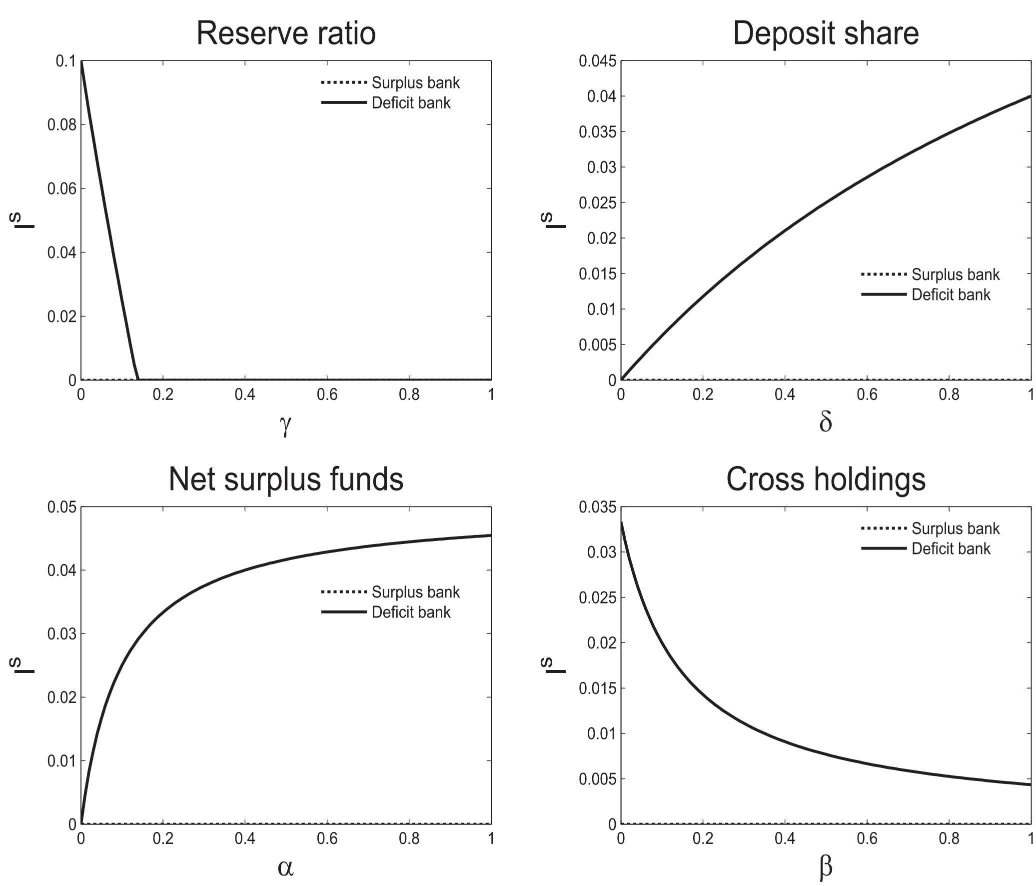

Fig. 3. Associations between liquidity shortage and parameter value.

among the two types of banks is given by δ, which denotes the share of total external deposits that the surplus banks take:

$$d_i = \begin{cases} \dfrac{\delta}{n_S}, & \text{if } i \in \mathcal{S} \\ \dfrac{(1-\delta)}{n_D}, & \text{if } i \in \mathcal{D} \end{cases}$$

As in the previous section we assume that each bank holds liquid external assets in the amount necessary to satisfy its minimum reserve requirements:

$$q_i = \begin{cases} \dfrac{\gamma\delta}{n_S}, & \text{if } i \in \mathcal{S} \\ \dfrac{\gamma(1-\delta)}{n_D}, & \text{if } i \in \mathcal{D} \end{cases}$$

Similarly, in the simple case $n = 2$, α denotes the aggregate excess funds of the surplus banks and β the aggregate interbank cross holdings between different types of banks. Hence, the amount of bank i's excess funds s_i is given by

$$s_i = \begin{cases} \dfrac{\alpha}{n_S}, & \text{if } i \in \mathcal{S} \\ -\dfrac{\alpha}{n_D}, & \text{if } i \in \mathcal{D} \end{cases}$$

Since $z_i = d_i - q_i - s_i$, the amount of bank i's illiquid assets is given by

$$z_i = \begin{cases} \dfrac{(1-\gamma)\delta - \alpha}{n_S}, & \text{if } i \in \mathcal{S} \\ \dfrac{(1-\gamma)(1-\delta)+\alpha}{n_D}, & \text{if } i \in \mathcal{D} \end{cases}$$

We construct six types of interbank network structures which differ in their completeness, interconnectedness, matchedness and centeredness, but satisfy the following three constraints: (i) the aggregate excess funds of the surplus banks are α; (ii) the aggregate excess funds of the deficit banks are $-\alpha$; and (iii) the aggregate claims of the deficit banks on the surplus banks are β:

$$\sum_{i\in\mathcal{N}, j\in\mathcal{S}} b_{ij} - \sum_{i\in\mathcal{S}, j\in\mathcal{N}} b_{ij} = \alpha, \tag{9}$$

$$\sum_{i\in\mathcal{N}, j\in\mathcal{D}} b_{ij} - \sum_{i\in\mathcal{D}, j\in\mathcal{N}} b_{ij} = -\alpha \quad \text{and} \tag{10}$$

$$\sum_{i\in\mathcal{S}, j\in\mathcal{D}} b_{ij} = \beta. \tag{11}$$

5.1.1. Type 1. Complete network

A complete network is defined as an interbank market in which every bank has exposures to all other banks in the system. The complete and perfectly interconnected network is rather unrealistic. It is equivalent to a random graph with an Erdös–Rényi probability of 100%. Fig. 4 illustrates a complete network structure

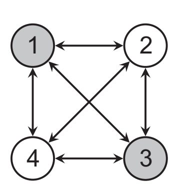

Fig. 4. Complete network, $\mathcal{S} = \{1, \quad 3\}$ and $\mathcal{D} = \{2, \quad 4\}$.

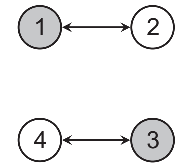

Fig. 5. Disconnected network, $\mathcal{S} = \{1, \quad 3\}$ and $\mathcal{D} = \{2, \quad 4\}$.

that consists of four banks. Under constraints (9)–(11), we may construct the interbank exposures of the complete network as[5]

$$b_{ij} = \begin{cases} \dfrac{\alpha+\beta}{n_S n_D}, & \text{if } j \in \mathcal{S} \text{ and } i \neq j \\ \dfrac{\beta}{n_S n_D}, & \text{if } j \in \mathcal{D} \text{ and } i \neq j \\ 0, & \text{if } i = j. \end{cases}$$

5.1.2. Type 2. Disconnected network

We define a disconnected network as an interbank market which is divided into two or more isolated submarkets. Allen and Gale (2000) argue that a network can be disconnected if banks specialize in particular areas of business or have connections with other banks operating in the same geographical or political unit. In order to isolate the effect of disconnectedness, we assume that there exist two submarkets which are complete and well-matched. Hence, every bank has exposures to all other banks in the submarket in which it participates. Fig. 5 illustrates such a disconnected structure consisting of four banks. Under constraints (9)–(11), we may construct the interbank exposures of the disconnected network as

$$b_{ij} = \begin{cases} \dfrac{\alpha+\beta}{n_S n_{D1}}, & \text{if } i \in \mathcal{S}_1 \cup \mathcal{D}_1, j \in \mathcal{S}_1 \text{ and } i \neq j \\ \dfrac{\beta}{n_S n_{D1}}, & \text{if } i \in \mathcal{S}_1 \cup \mathcal{D}_1, j \in \mathcal{D}_1 \text{ and } i \neq j \\ \dfrac{\alpha+\beta}{n_S n_{D2}}, & \text{if } i \in \mathcal{S}_2 \cup \mathcal{D}_2, j \in \mathcal{S}_2 \text{ and } i \neq j \\ \dfrac{\beta}{n_S n_{D2}}, & \text{if } i \in \mathcal{S}_2 \cup \mathcal{D}_2, j \in \mathcal{D}_2 \text{ and } i \neq j \\ 0, & \text{otherwise,} \end{cases}$$

where $\mathcal{S} = \mathcal{S}_1 \cup \mathcal{S}_2$, $\mathcal{D} = \mathcal{D}_1 \cup \mathcal{D}_2$, and n_{Dk} is the number of deficit banks in the kth submarket.

5.1.3. Type 3. Circular well-matched network

A circular network is defined as an interbank market in which each bank is allowed to make interbank loans to only one adjacent bank. Caballero and Simsek (2012) analyze the role of interbank exposure in generating flight-to-quality episodes using a simple circular network, which resembles some important features of information transmission in the actual financial markets.

A circular well-matched network is a circular network in which different types of banks are located alternately. In this structure every deficit bank borrows from a surplus bank. Fig. 6 illustrates a circular and well-matched network structure that consists of four banks. Under constraints (9)–(11), we may construct the interbank exposures of the circular well-matched network as follows:

[5] In the complete network, cross holdings between the same types of banks are also allowed.

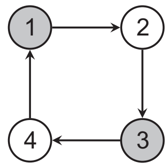

Fig. 6. Circular well-matched network, $\mathcal{S} = \{1, \quad 3\}$ and $\mathcal{D} = \{2, \quad 4\}$.

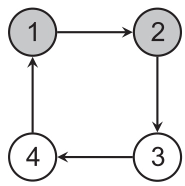

Fig. 7. Circular ill-matched network, $\mathcal{S} = \{1, \quad 2\}$ and $\mathcal{D} = \{3, \quad 4\}$.

$$b_{ij} = \begin{cases} \dfrac{\alpha+\beta}{n_S}, & \text{if } j \in \mathcal{S} \text{ and } i = c(j+1) \\ \dfrac{\beta}{n_D}, & \text{if } j \in \mathcal{D} \text{ and } i = c(j+1) \\ 0, & \text{otherwise,} \end{cases}$$

where

$$c(i) = n \cdot I_{[i<1]} + i \cdot I_{[1 \le i \le n]} + 1 \cdot I_{[i>n]},$$

$$n_S = n_D,$$

and $I_{[\cdot]}$ is an indicator function. Here, the function $c(i)$ transforms a consecutive serial number to a circular serial number.

5.1.4. Type 4. Circular ill-matched network

We define a circular ill-matched network as an interbank market in which each bank is allowed to make interbank loans only to one adjacent bank, and banks of the same type are located adjacently. In this structure some deficit banks are allowed to borrow only from other deficit banks. Fig. 7 illustrates a circular and ill-matched structure consisting of four banks. Under constraints (9)–(11), we may construct the interbank exposures of the circular ill-matched network as

$$b_{ij} = \begin{cases} b_{j,c(j-1)} + \dfrac{\alpha}{n_S}, & \text{if } j \in \mathcal{S} \text{ and } i = c(j+1) \\ b_{j,c(j-1)} - \dfrac{\alpha}{n_D}, & \text{if } j \in \mathcal{D} \text{ and } i = c(j+1) \\ 0, & \text{otherwise,} \end{cases}$$

where the circular interbank exposures b_{ij} can be pinned down by constraint (11).

5.1.5. Type 5. Core-periphery network with a surplus money center bank

A core-periphery network is defined as an interbank market in which the banks on the periphery have exposures to the banks at the center but not to each other.[6] This network structure reflects the fact that small banks tend to trade with large banks, but rarely among themselves. Fricke and Lux (2012) show that a core-periphery network provides a better fit for interbank overnight transaction data than do alternative network structures.

[6] Following Craig and von Peter (2010), we can combine a complete network and a core-periphery network into a multiple core-periphery network, in which a few large core banks form a complete network and a larger number of small banks in the periphery are only connected to core banks but not to other banks in the periphery.

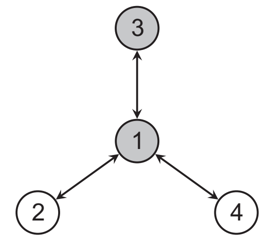

Fig. 8. Core-periphery network with a surplus money center bank, $\mathcal{S} = \{1, \quad 3\}$, $\mathcal{D} = \{2, \quad 4\}$, and $\mathcal{C} = \{1\}$.

The money center can be either a surplus bank or a deficit bank, depending upon the network formation. If the money center is a surplus bank, then surplus banks which are not linked to deficit banks exist, resulting in an ill-matched network. Fig. 8 illustrates a core-periphery network with a surplus money center bank. Under constraints (9)–(11), we may construct the interbank exposures of the core-periphery network with a surplus money center bank as

$$b_{ij} = \begin{cases} \dfrac{\alpha+\beta}{n_D}, & \text{if } i \in \mathcal{D} \text{ and } j \in \mathcal{C} \\ \dfrac{\beta}{n_S}, & \text{if } i \in \mathcal{S} - \mathcal{C} \text{ and } j \in \mathcal{C} \\ \dfrac{\beta}{n_D}, & \text{if } i \in \mathcal{C} \text{ and } j \in \mathcal{D} \\ \dfrac{\alpha+\beta}{n_S}, & \text{if } i \in \mathcal{C} \text{ and } j \in \mathcal{S} - \mathcal{C} \\ 0, & \text{otherwise,} \end{cases}$$

where $\mathcal{C}$ is the set of money center banks.

5.1.6. Type 6. Core-periphery network with a deficit money center bank

If the money center of the core-periphery network is a surplus bank, then deficit banks which are not linked to surplus banks exist, resulting again in an ill-matched network. Fig. 9 illustrates the core-periphery network with a deficit money center bank. Under constraints (9)–(11), we may construct the interbank exposures of the core-periphery network with a deficit money center bank as

$$b_{ij} = \begin{cases} \dfrac{\alpha+\beta}{n_D}, & \text{if } i \in \mathcal{D} - \mathcal{C} \text{ and } j \in \mathcal{C} \\ \dfrac{\beta}{n_S}, & \text{if } i \in \mathcal{S} \text{ and } j \in \mathcal{C} \\ \dfrac{\beta}{n_D}, & \text{if } i \in \mathcal{C} \text{ and } j \in \mathcal{D} - \mathcal{C} \\ \dfrac{\alpha+\beta}{n_S}, & \text{if } i \in \mathcal{C} \text{ and } j \in \mathcal{S} \\ 0, & \text{otherwise,} \end{cases}$$

where $\mathcal{C}$ is the set of money center banks.

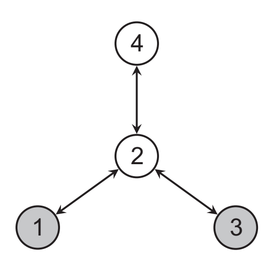

Fig. 9. Core-periphery network with a deficit money center bank, $\mathcal{S} = \{1, \quad 3\}$, $\mathcal{D} = \{2, \quad 4\}$, and $\mathcal{C} = \{2\}$.

Table 4
Different types of interbank network structures.

Network properties	Interbank networks					
	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
Completeness	Yes	No	No	No	No	No
Interconnectedness	Yes	No	Yes	Yes	Yes	Yes
Matchedness	Yes	Yes	Yes	No	No	No
Centeredness	No	No	No	No	Yes	Yes

Table 4 summarizes the characteristics of the six interbank network structures.

5.2. Simulation results

We assess the vulnerability to liquidity shocks of the six types of interbank network structures by simulations with the same parameter values as in the previous section. Fig. 10 shows the liquidity shortage of the different interbank network types. The core-periphery network with a deficit money center bank gives rise to the highest level of systemic liquidity shortage among the six network types. The next most vulnerable interbank network is the circular ill-matched network, followed by the core-periphery network with a surplus money center bank. The networks least vulnerable to liquidity shocks are the well-matched networks – the complete network, the circular well-matched network, and the disconnected network.

Fig. 11 depicts the relationships between the liquidity shortages and the numbers of banks in the systems. For well-matched networks, the liquidity shortages do not change as the numbers of banks n increase. For an ill-matched interbank network, the number of ill-matched links increases as n increases. Thus, the circular ill-matched network and the surplus bank-centered network become more vulnerable to liquidity shocks as their numbers of banks increase. However, the liquidity shortages of the deficit bank-centered network decreases as n increases, since the size of deficit

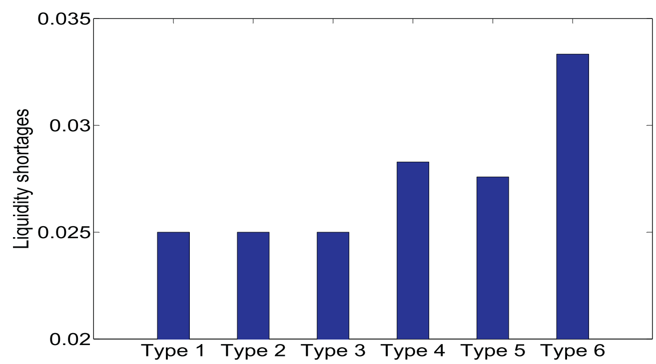

Fig. 10. Systemic liquidity shortages of different network types.

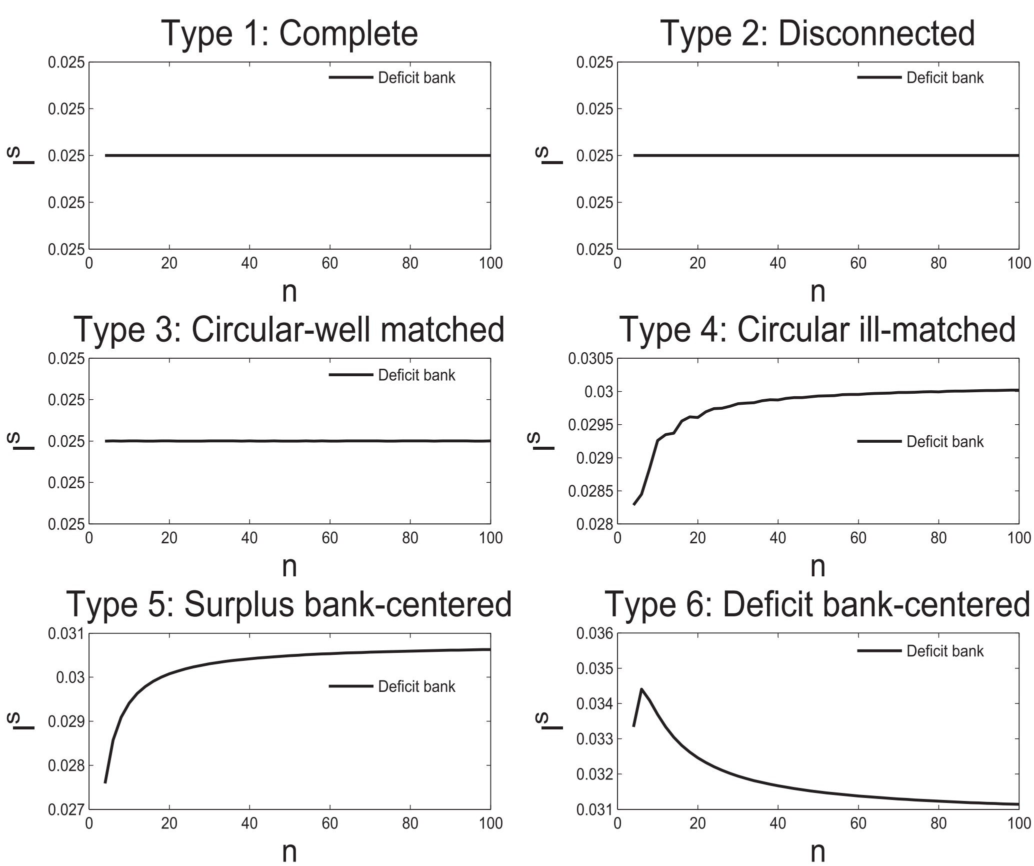

Fig. 11. Systemic liquidity shortages and numbers of banks.

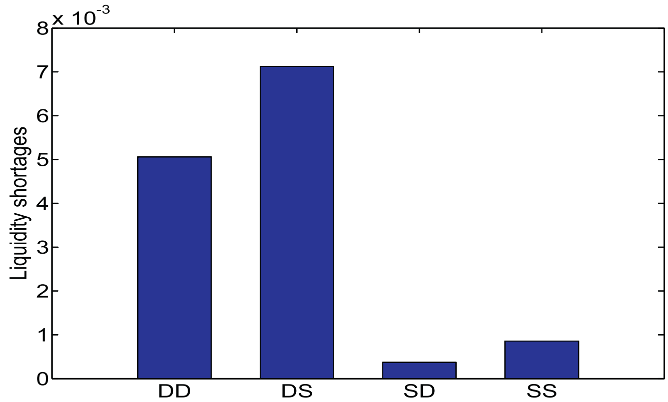

Fig. 12. Systemic liquidity shortages by deficit bank types.

of the money center bank also declines.[7] When n goes to infinity, the surplus bank-centered network and the deficit bank-centered network become closer to each other.

The fact that ill-matched interbank networks are vulnerable to liquidity shocks highlights the importance of counterparties. The liquidity shortages of a bank depend not only upon its liquidity position but also upon the counterparties to which it is linked. In order to investigate whether the type of counterparties of a bank is relevant to the liquidity shortages of that bank, let us consider a banking system consisting of ten deficit and ten surplus banks, i.e., $n_S = 10$, $n_D = 10$, and $n = 20$. Assume that every bank is linked to two banks: one for lending and the other for borrowing, resulting in a circular network. We classify the deficit banks into four types, in accordance with their counterparties. Let DD denote the type of deficit bank that borrows from a deficit bank and lends to a deficit bank; DS the type of deficit bank that borrows from a deficit bank and lends to a surplus bank; SD the type of deficit bank that borrows from a surplus bank and lends to a deficit bank; and SS the type of deficit bank that borrows from a surplus bank and lends to a surplus bank. The deficit banks of types DD and DS borrow from other deficit banks, making their networks more ill-matched, whereas the deficit banks of types SD and SS borrow from surplus banks, making their networks more well-matched. Fig. 12 depicts the average liquidity shortages for each type of deficit bank, calculated for all possible interbank networks.[8] The most vulnerable type of deficit bank is DS, while the least vulnerable type is SD. The deficit banks of DD type are also highly vulnerable to liquidity shocks. These results suggest that the more ill-matched an interbank network is, the more vulnerable to liquidity shocks the banking system becomes. This argument is also confirmed by Fig. 10, which shows that the systemic liquidity shortages of the ill-matched networks (Types 4, 5 and 6) are greater than those of the well-matched networks (Types 1, 2 and 3).

6. Conclusions

The global financial crisis reminds us of the significance of liquidity spillovers, and specifically that interconnectedness can lead to difficulties in rolling over liabilities which may spill over to the financial system as a whole. Liquidity risk might therefore be underestimated if it is assessed by individual institutions' liquidity positions alone. To address this problem, we propose a framework that allows us to capture the systemic nature of liquidity shortages. With this framework, we develop a method for computing the systemic liquidity shortages, including indirect liquidity shortages, due to the knock-on effects through interbank linkages.

We perform simulation with a simple banking system model and find that a deficit bank can prevent a liquidity shortage by holding more claims on a surplus bank. On the other hand, a greater imbalance in liquidity positions across banks tends to aggravate the liquidity shortage of a deficit bank. According to comparative analysis of different types of network structures, a core-periphery network with a deficit money center bank gives rise to the highest level of systemic liquidity shortage, and a banking system becomes more vulnerable to liquidity shocks as its interbank network becomes more ill-matched.

We have only examined the knock-on effects of liquidity shortages. Systemic liquidity shortages may lead to fire sales, which result in turn in sharp reductions in asset prices. This may cause a default contagion and total meltdown of the system. The role of systemic liquidity shortages in default contagion is left as a subject for future research.

Appendix A. Algorithm for calculating total changes in liquid assets

Let

$$\Delta\mathbf{l} = \begin{bmatrix} \Delta l_1 \\ \vdots \\ \Delta l_i \\ \vdots \\ \Delta l_n \end{bmatrix}, \quad \mathbf{l} = \begin{bmatrix} l_1 \\ \vdots \\ l_i \\ \vdots \\ l_n \end{bmatrix}, \quad \mathbf{l}^n = \begin{bmatrix} l_1^n \\ \vdots \\ l_i^n \\ \vdots \\ l_n^n \end{bmatrix}, \quad \Delta\mathbf{d} = \begin{bmatrix} \Delta d_1 \\ \vdots \\ \Delta d_i \\ \vdots \\ \Delta d_n \end{bmatrix} \quad \text{and}$$

$$\mathbf{\Phi} = \begin{bmatrix} \phi_{11} & \dots & \phi_{1j} & \dots & \phi_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \phi_{i1} & \dots & \phi_{ij} & \dots & \phi_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \phi_{n1} & \dots & \phi_{nj} & \dots & \phi_{nn} \end{bmatrix}.$$

(i) Set initial values of liquidity change vector

$$\Delta\mathbf{l}_{(0)} = (\mathbf{I} - \mathbf{\Phi})^{-1}\Delta\mathbf{d}.$$

(ii) Iterate $i = 1, \ldots, n$, where in each step we:

a. Calculate liquidity needs
$\mathbf{l}^n_{(i)} = \Delta\mathbf{d} + \mathbf{\Phi}\,\Delta\mathbf{l}_{(i-1)}$.

b. Evaluate liquidity shortages and obtain the $n \times n$ diagonal matrix defined as follows:
$\Lambda_{(i)} = diag(\mathbf{1}_{[\mathbf{l}^n_{(i)} > \mathbf{l}]})$.

c. Calculate the changes in liquidity

$$\Delta\mathbf{l}_{(i)} = [\mathbf{I} - (\mathbf{I} - \Lambda_{(i)})\Phi(\mathbf{I} - \Lambda_{(i)})]^{-1}[(\mathbf{I} - \Lambda_{(i)})\Delta\mathbf{d} + ((\mathbf{I} - \Lambda_{(i)})\Phi\Lambda_{(i)} + \Lambda_{(i)})\mathbf{l}].$$

d. Stop the iteration if
$\Delta\mathbf{l}_{(i)} = \Delta\mathbf{l}_{(i-1)}$.

[7] When n is small, the effect due to ill-matched links is very significant, dominating the effect due to the decrease in the size of deficit of the money center bank. Exceptionally, therefore, the liquidity shortages of the deficit bank-centered network rise when n increases from 4 to 6.

[8] Since all banks within each group are identical and located in a circular network, the number of all possible interbank networks is $(n-1)!/n_S!\,n_D! \approx 9238$, where $n_S = 10$, $n_D = 10$ and $n = 20$.

Appendix B. Solutions for simple banking system

The changes in liquid assets can be rewritten as[9]

$$\Delta l_1 = \begin{cases} \gamma\delta+\alpha+\beta, & \text{if}\, \omega \ge \gamma+\alpha/\delta, \\ \beta+\omega\delta, & \text{if}\, \dfrac{[1-\phi_{12}\phi_{21}][\gamma(1-\delta)+\beta]}{1-\delta+\phi_{21}\delta} \le \omega < r+\alpha/\delta, \\ [1-\phi_{12}\phi_{21}]^{-1}[\phi_{12}\omega(1-\delta)+\omega\delta], & \text{otherwise.} \end{cases}$$

$$\Delta l_2 = \begin{cases} \gamma(1-\delta)+\beta, & \text{if}\, \omega \ge \dfrac{[1-\phi_{12}\phi_{21}][\gamma(1-\delta)+\beta]}{1-\delta+\phi_{21}\delta}, \\ [1-\phi_{12}\phi_{21}]^{-1}[\phi_{21}\omega\delta+\omega(1-\delta)], & \text{otherwise.} \end{cases}$$

The liquidity needs can be rewritten as

$$l_1^n = \begin{cases} \beta+\omega\delta, & \text{if}\, \omega \ge \dfrac{[1-\phi_{12}\phi_{21}][\gamma(1-\delta)+\beta]}{1-\delta+\phi_{21}\delta}, \\ [1-\phi_{12}\phi_{21}]^{-1}[\phi_{12}\phi_{21}\omega\delta+\phi_{12}\omega(1-\delta)]+\omega\delta, & \text{otherwise.} \end{cases}$$

$$l_2^n = \begin{cases} \alpha+\beta+\omega(1-\delta), & \text{if}\, \omega \ge \gamma+\alpha/\delta, \\ \phi_{21}[\beta+\omega\delta]+\omega(1-\delta), & \text{if}\, \dfrac{[1-\phi_{12}\phi_{21}][\gamma(1-\delta)+\beta]}{1-\delta+\phi_{21}\delta} \le \omega < r+\alpha/\delta, \\ [1-\phi_{12}\phi_{21}]^{-1}[\phi_{12}\phi_{21}\omega(1-\delta)+\phi_{21}\omega\delta]+\omega(1-\delta), & \text{otherwise.} \end{cases}$$

Finally, the liquidity shortages can be rewritten as

$$l_1^s = \begin{cases} (\omega-\gamma)\delta-\alpha, & \text{if}\, \omega \ge r+\alpha/\delta, \\ 0, & \text{otherwise.} \end{cases}$$

$$l_2^s = \begin{cases} (\omega-\gamma)(1-\delta)-\alpha, & \text{if}\, \omega \ge \gamma+\alpha/\delta, \\ (\omega-\gamma)(1-\delta)+\phi_{21}\omega\delta-[1-\phi_{21}]\beta, & \text{if}\, \dfrac{[1-\phi_{12}\phi_{21}][\gamma(1-\delta)+\beta]}{1-\delta+\phi_{21}\delta} \le \omega < \gamma+\alpha/\delta, \\ 0, & \text{otherwise.} \end{cases}$$

[9] The case that the surplus bank's liquidity shortage>0 and the deficit bank's liquidity shortage= 0 is impossible. If that case were true, then

$$\Delta l_1 = \gamma\delta+\alpha+\beta < \frac{\beta}{\gamma(1-\delta)+\beta}[\alpha+\beta+\omega(1-\delta)]+\omega\delta \quad \text{and}$$
$$\Delta l_2 = \alpha+\beta+\omega(1-\delta) \le \gamma(1-\delta)+\beta.$$

Hence,

$$\gamma\delta+\alpha < \omega\delta \quad \text{and}$$
$$\alpha+\omega(1-\delta) \le \gamma(1-\delta),$$

resulting in a contradiction

$$\omega-\gamma > \frac{\alpha}{\delta} > 0 \quad \text{and}$$
$$\omega-\gamma \le -\frac{\alpha}{1-\delta} < 0.$$

References

Aikman, D., Alessandri, P., Eklund, B., Gai, P., Kapadia, S., Martin, E., Mora, N., Sterne, G., Willison, M., 2009. Funding Liquidity Risk in a Quantitative Model of Systemic Stability. Bank of England Working Paper No. 372.

Allen, F., Gale, D., 2000. Financial contagion. Journal of Political Economy 108 (1), 1–33.

Becher, C., Millard, S., Soramäki, K., 2008. The Network Topology of CHAPS Sterling. Bank of England Working Paper No. 355.

Bhattacharya, S., Gale, D., 1987. Preference shocks, liquidity, and central bank policy. In: Barnett, W., Singleton, K. (Eds.), New Approaches to Monetary Economics. Cambridge University Press, New York.

Boss, M., Elsinger, H., Thurner, S., Summer, M., 2004. Network topology of the interbank market. Quantitative Finance 4 (6), 677–684.

Caballero, R., Simsek, A., 2012. Fire Sales in a Model of Complexity. NBER Working Paper No. 15479.

Chan-Lau, J.A., Espinosa, M., Giesecke, K., Solé, J., 2009. Assessing the Systemic Implications of Financial Linkages. Chapter 2 in IMF Global Financial Stability Report, April.

Cifuentes, R., Ferrucci, G., Shin, H.S., 2005. Liquidity risk and contagion. Journal of European Economic Association 3 (2–3), 556–566.

Cocco, J., Gomes, F., Martins, N., 2009. Lending relationships in the interbank market. Journal of Financial Intermediation 18 (1), 24–48.

Craig, B., von Peter, G., 2010. Interbank Tiering and Money Center Banks. BIS Working Paper No. 322.

Degryse, H., Nguyen, G., 2007. Interbank exposures: an empirical examination of contagion risk in the Belgian banking system. International Journal of Central Banking 3 (2), 123–171.

Diamond, D.W., Rajan, R.G., 2005. Liquidity shortages and banking crises. Journal of Finance 60 (2), 615–647.

Eisenberg, L., Noe, T.H., 2001. Systemic risk in financial systems. Management Science 47 (2), 236–249.

Elsinger, H., Lehar, A., Summer, M., 2006. Risk assessment for banking systems. Management Science 52 (9), 1301–1314.

Freixas, X., Parigi, B., Rochet, J.C., 2000. Systemic risk, interbank relations and liquidity provision by the central bank. Journal of Money, Credit and Banking 32 (3), 611–638.

Fricke, D., Lux, T., 2012. Core-periphery structure in the overnight money market: evidence from the e-MID trading platform. Kiel Working Papers, No. 1759.

Gai, P., Kapadia, S., 2010. Contagion in financial networks. Bank of England Working Paper No. 383.

Gai, P., Haldane, A., Kapadia, S., 2011. Complexity, concentration and contagion. Journal of Monetary Economics 58 (5), 453–470.

IMF, 2010. Systemic liquidity risk: improving the resilience of financial institutions and markets. Global Financial Stability Report, October 2010.

May, R.M., Arinaminpathy, N., 2010. Systemic risk: the dynamics of model banking systems. Journal of the Royal Society Interface 7 (46), 823–838.

May, R.M., Levin, S.A., Sugihara, G., 2008. Complex systems: ecology for bankers. Nature 451, 893–895.

Mistrulli, P., 2007. Assessing financial contagion in the interbank market: maximum entropy versus observed interbank lending patterns. Bank of Italy Working Paper.

Nier, E., Yang, J., Yorulmazer, T., Alentorn, A., 2007. Network models and financial stability. Journal of Economic Dynamics and Control 31 (6), 2033–2060.
Schweitzer, F., Fagiolo, G., Sornette, D., Vega-Redondo, F., Vespignani, A., White, D.R., 2009. Economic networks: the new challenges. Science 325, 422–425.
Upper, C., 2011. Simulation methods to assess the danger of contagion in interbank markets. Journal of Financial Stability 7 (3), 111–125.
Upper, C., Worms, A., 2004. Estimating bilateral exposures in the German Interbank market: is there a danger of contagion? European Economic Review 48 (4), 827–849.