The missing links: A global study on uncovering financial network structures from partial data[*]

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Abstract

Capturing financial network linkages and contagion in stress test models are important goals for banking supervisors and central banks responsible for micro- and macroprudential policy. However, granular data on financial networks is often lacking, and instead the networks must be reconstructed from partial data. In this paper, we conduct a horse race of network reconstruction methods using network data obtained from 25 different markets spanning 13 jurisdictions. Our contribution is two-fold: first, we collate and analyze data on a wide range of financial networks. And second, we rank the methods in terms of their ability to reconstruct the structures of links and exposures in networks.

Keywords: Network reconstruction, market structure, intermediation

JEL classifications: G20, L14, D85, C63

1 Introduction

In the era prior to the global financial crisis, banking supervisors followed a microprudential approach to assessing the resilience of banks. As such, the first generation of bank stress-test models tended to focus on individual banks' solvency risks, while remaining silent on the issue of liquidity risk, and ignored interlinkages within banking systems. However, as the crisis attests, the failure to capture these features led to an underestimation of the risks to financial systems in many advanced economies.

The crisis spurred much reflection over the reasons behind the explosive growth of interlinkages in the financial system in years prior to the crisis (e.g., Billio et al., 2012; Merton et al., 2013). At the same time, bank supervision authorities and central banks have been busy developing new stress-testing models and tools that more rigorously account for the interconnections between banks and the interactions between banks' liquidity and solvency risks.[1] In particular, models of financial network contagion have become popular as they can shed light on the risk transmission mechanisms within financial systems. For example, banks that have suffered solvency shocks may cut lending to their counterparties. These counterparties, in turn, anticipating the cut in funding, will also cut lending to their counterparties, and so on. Depending on the structure of the network and distribution of initial shocks, such hoarding may lead to a freeze in interbank markets.[2]

Employing network contagion models in stress tests requires granular data on both the credit exposures and funding structures of financial institutions. Yet the collection of granular micro-level bank data is patchy across countries. Indeed, a crisis is often needed to spur data-reporting efforts. For example, after the 1994 Mexican peso currency crisis, the Banco de México started collecting detailed information on daily exposures both among domestic banks and from domestic to foreign banks. Similarly, in response to the global

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[1] Examples include the European Central Bank's "Stress-Test Analytics for Macroprudential Purposes in the euro area" (STAMP€) and Bank of Canada's "Macro-Financial Risk Assessment Framework" (MFRAF).

[2] See, for example, Gai et al. (2011) and Lee (2013). As an alternate mechanism, Zawadowski (2011) consider how information asymmetries make it costly for a bank's creditors to monitor the quality of the bank's assets following a solvency shock, which can lead to an endogenous withdrawal of liquidity.

financial crisis, the G-20 set up the Data Gaps Initiative in 2009 to strengthen the reporting and collection of financial data by member countries. However, while data-gathering capabilities have improved markedly, frequent data on *bilateral* linkages, which are needed to operationalize network contagion models, is often lacking.

To overcome the data limitations, several methods have been developed to reconstruct financial networks from available aggregate data. The methods vary in terms of the emphasis placed on the network features to reproduce. Some methods, for example, seek to minimize the exposures of individual links, while others seek to minimize the number of links required. The methods also vary in their outputs: some produce unique reconstructed networks, while others generate a distribution of possible networks. To date, the different methods have been tested and validated using partial data from very different periods and financial markets, which renders any comparison between them difficult.

In this paper, we present the results of a joint exercise, spanning 13 jurisdictions and 25 different financial markets, to comprehensively analyze the performance of different network reconstruction methods.[3] The reconstruction process is as follows. For the markets analyzed, we have access to actual data and thus known the true bilateral links in the financial networks. However, for each market, we postulate that the only information available is the aggregate asset and liability positions for all banks. This information is fed into the different methods, which reconstruct the network of bilateral exposures. Summary statistics for the properties of the reconstructions are recorded, as well as estimates for how similar they are to the true financial networks. In the end, a ranking of the different methods is produced by comparing their performance across the different financial markets.

A key challenge in running the horse race stems from the confidentiality of the data, which prevents their sharing, especially across jurisdictions. To overcome this issue, we devise a de-centralized approach to running the horse race, wherein a suite of code is run on data provided by a participating jurisdiction, and only a summary of the results is commu-

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[3]We thus do not describe the use of these results as in van Lelyveld and Liedorp (2006), for example, or survey them as in Upper (2011) and Hüser (2015).

nicated publicly. This mechanism ensures that there is no violation of data confidentiality while facilitating a meaningful comparison of results across jurisdictions.

We find that the winner of the horse race depends on the feature of the network that we seek to preserve during the reconstruction. This, in turn, is captured by our choice of similarity measure. In particular, those measures that emphasize reproducing the structure of links between financial institutions favor methods that produce more sparse networks. In contrast, similarity measures that emphasize reproducing bilateral exposure sizes favor methods that allocate exposures as evenly as possible.

To the best of our knowledge, this is the first paper that documents a cross-country and cross-market comparison of financial network data, together with a comparison of network reconstruction methods. In two related and recent papers, Mistrulli (2011) and Anand et al. (2015), the authors compare stress-test outcomes on Italian and German interbank markets, respectively, where the networks are reconstructed using two different methods. We consider the methods and markets analyzed by these authors, as well.

Our paper is organized as follows. Section 2 provides an overview of the different financial markets we consider, as well as our panel analysis (detailed market summaries are provided in Appendix A). Section 3 provides brief summaries of the different network reconstruction methods and similarity measures we consider (further details in Appendices B and C). The results of our horse race are provided in Section 4, and a final section concludes.

2 Summary of the financial network data

Our horse race stems from the liquidity stress testing work stream of the Basel Committee on Banking Supervision's (BCBS's) Research Task Force (RTF) which has published a series of reports BCBS (2013a,b, 2015). A new area of focus for the work stream is macro prudential stress testing, with particular emphasis on integrating liquidity, solvency and systemic risks into stress-test models. With particular reference to network analysis, the work stream has written that (emphasis ours) "*network analysis and agent based models [prove] useful for broadening stress tests, as these models consider contagion through common exposure, interbank funding relationships and the endogenous behaviour of banks*" (BCBS, 2015).

Operationalizing these models requires granular data, which is inferred using network reconstruction methods. To evaluate the performance of these methods, the RTF authorized a collaborative effort across several jurisdictions with two key objectives.[4] The first, to collect and analyze data on financial networks across the jurisdictions, and the second, to test the performance of network reconstruction methods using the collected data.

Table 1 provides an overview of the financial networks we covered. The majority of these are interbank networks.[5] This is followed by payments networks and several other networks for different financial contracts: repurchase agreements, i.e., repos, foreign exchange derivatives (FX), credit default swaps (CDS) and equities. For many networks, we also analyze several consecutive time-shots. Further details on the time frames and other institutional information are provided in Appendix A.

Table 2 summarizes the properties of the interbank networks.[6] As can be seen, the networks vary greatly in their size and other properties. The German interbank network (DE01), for example, has on average 592 banks (with over 11 thousand links), while the

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[4]See BCBS (2015) for further details.

[5]For Mexico, we disaggregated the interbank network into three different networks, each for a different financial instrument. A similar disaggregation is carried out for the Mexican payments networks.

[6]See Appendix C for a description of the different network properties we compute. We group the metrics in the tables into those that are binary-based and those that are weight-based.

Table 1: Overview of data set

Type of Network	Number of Networks	Countries
Interbank	13	Brazil, BIS, Canada, Denmark, Eurozone, Germany, Hungary, Italy, Mexico (x3), Netherlands
Payments	5	Brazil, Mexico (x3), US
Repo	2	Denmark, Mexico
FX	1	Hungary
CDS	2	UK, US
Equity	1	Mexico
Derivatives	1	Mexico

Canadian network (CA01) only includes the 6 major banks (with a link count below 30). The density of the networks ranges from almost fully connected, with 96.7% (CA01) to very sparse, with only 1% of the links for the Italian network (IT01). Another interesting feature is the core size, which is a proxy for tiering in the market. The fewer the banks in the core, the more tiered is the interbank market. Across our sample of interbank markets, the core size varies from 3.5% of banks (IT01) to 77.8% for the Korean network (KR01). Table 3 summarizes the properties for the remaining networks.

Table 2: Description of interbank network data

	BIS1	BR01	CA01	DE01	DK01	EU01	HU01	IT01	KR01	MX01	MX03	MX06	NL01
Number of nodes	31	111.9	6	592.4	14	26	35.8	535.4	18	43	43	43	159
Number of links	742.7	512.7	29.5	11623.5	77	197.7	274.8	3158.9	263	408.3	127.3	53.3	546.1
Density	79.9	4.1	98.3	3.3	42.3	29.2	22.2	1.1	85.9	22.6	7.1	3	2.2
Average degree	24	4.6	4.9	19.6	5.5	7.6	7.7	5.9	14.6	9.5	3	1.2	3.4
Median degree	25.3	2.2	5	14.7	5	8.4	7.8	3.2	15	9.3	2	1	1
Assortativity	-.19	-.37		-.6	-.3	-.33	-.31	-.43	-.17	-.22	-.23	-.39	-.49
Clustering	28.5	4.4	67.3	40.3	21.5	15.5	22.2	19.1	21.2	12.9	5.9	4.3	6.6
Lender dependency	28.7	65.2	35.8	43.6	37.5	71.2	32.4	71.6	31.6	54.6	71.7	84.6	78.6
Borrower dependency	30.6	59.8	40.2	69.4	39.6	71.1	39.4	87.8	24.9	51.8	61.6	74.8	76.2
Mean HHI assets	.16	.5	.26	.3	.27	.46	.24	.64	.19	.39	.47	.54	.54
Median HHI assets	.15	.44	.25	.22	.25	.35	.14	.61	.17	.33	.42	.6	.57
Mean HHI liabilities	.16	.38	.29	.59	.25	.56	.25	.84	.15	.36	.33	.24	.48
Median HHI liabilities	.13	.31	.26	.59	.23	.6	.11	1	.14	.26	.25	0	.45
Core size (% banks)	73.1	9.8	76.7	6.6	42.9	36.3	31.5	3.5	77.8	31	16.3	7	6.5
Error score (% links)	3.5	41.3	1.4	12.5	14.3	12.2	22.1	22.8	3.4	24.7	39.4	55.2	25.2
Number of slices	3	12	10	12	1	9	12	10	1	3	3	3	10

Note: The table shows the average over all network slices available (bottom row). The definitions of the metrics are given in Appendix C .

Table 3: Network statistics for remaining networks

	Payments					CDS			Repo		Other	
	BR02	MX07	MX08	MX09	US01	MX05	UK01	US02	DK02	MX02	HU02	MX04
Number of nodes	100.9	43.3	43	43.3	5733	43	336.3	985	12	43	148.6	43
Number of links	1604	734.3	229.7	476.7	180917.5	135	1856.8	4298.3	18	62.3	533.5	84.7
Density	15.8	40.3	12.7	26.3	.6	7.5	1.6	.4	13.6	3.5	2.4	4.7
Average degree	15.9	17.1	5.3	11.1	31.6	3.1	5.5	4.4	1.5	1.4	3.6	2
Median degree	10.2	15.7	3.3	8.3	10.5	1.3	1	2	.5	.7	1	.7
Assortativity	-.5	-.44	-.3	-.42	-.27	-.18	-.71	-.81	-.73	-.21	-.61	-.17
Clustering	18.7	17.3	6.8	11.9	14.3	6.1	7.4	17.6	3.5	4	3.7	4.6
Lender dependency	59.9	50.8	61.2	47	59.6	71.1	64.2	72.1	71.4	75.6	76.4	67.1
Borrower dependency	65.8	53.5	61	58.5	59.9	71.8	66.4	74.5	95.1	68.9	72.9	66.7
Mean HHI assets	.48	.36	.35	.22	.44	.39	.37	.52	.32	.37	.56	.28
Median HHI assets	.4	.29	.23	.16	.37	.35	.21	.5	.11	.28	.7	.07
Mean HHI liabilities	.53	.4	.37	.38	.47	.39	.48	.36	.84	.26	.57	.24
Median HHI liabilities	.46	.28	.23	.26	.41	.24	.41	.18	1	0	.6	0
Core size (% banks)	22.2	42.6	22.5	34.9	2.7	15.5	5.3	1.6	16.7	9.3	7.5	10.9
Error score (% links)	11.8	4	22	2.9	27.5	34	1	4.2	22.2	48.8	13.4	52.9
Number of slices	10	3	3	3	2	3	4	3	1	3	10	3

Note: The table shows the average over all network slices available (bottom row). The definitions of the metrics are given in Appendix C.

Figure 1 provides scatter plots showing the relationships between the size of interbank networks and their density, market diversity, and core-size. We measure a network's size as the logarithm of the number of nodes it has. We measure market diversity as the inverse of the Herfindahl-Hirschman Index (HHI) for interbank assets and liabilities (Baumgärtner, 2004). We readily note the following: large interbank networks tend to have a low density, a low diversity, and only a small number of financial institutions in the core. One interpretation for these results is that interbank networks are tiered networks. Additionally, we observe that in general, the larger the network, the more pronounced is the tiering: only a few – core – financial institutions are tightly interlinked, and intermediate on behalf of all other financial institutions.[7]

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[7]Formally exploring the tiering structure of interbank and, indeed, other types of networks is beyond the scope of this paper, and we leave it as possible future work.

Figure 1: Relationships between network size and selected network characteristics
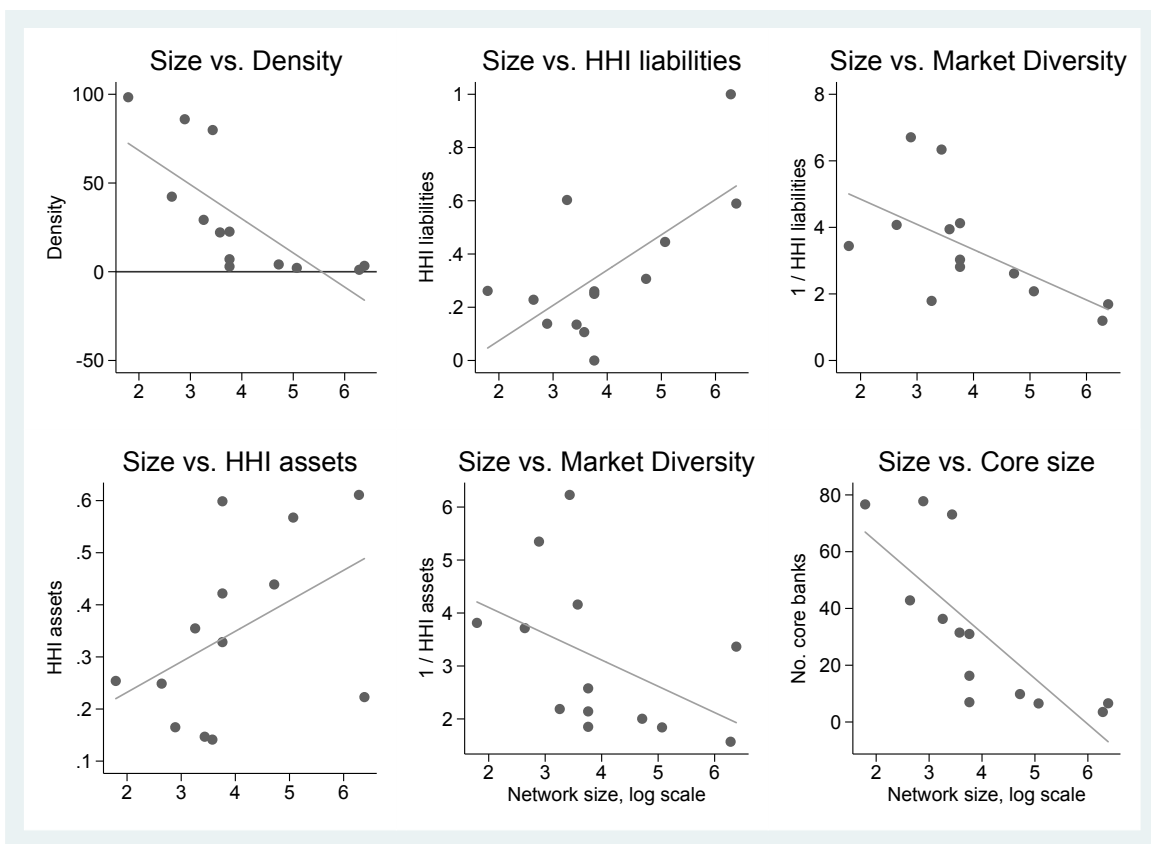
3 Network reconstruction methods

The number of new financial network reconstruction methods is growing rapidly. We concentrate on seven, which we selected early on in the process after a comprehensive search of published methods. The most important selection criteria are, first, that the method should be able to reconstruct the network based only on aggregate positions and, second, that the code can be fitted in our modular suite of Matlab codes.[8]

The methods can be broadly classified into two categories. The first one, labeled 'Iterative,' starts with an initial guess for the network. The entries in the network are then repeatedly re-scaled until the aggregate positions satisfy their targets (henceforth, referred to as the 'marginal constraints'). The methods in this category differ in their initial assumptions regarding the structure of the network. The three methods in this category have the following mnemonics: *Bara*, *Dreh* and *Maxe*. The second category, labeled 'Sampling,' consists of methods that use Monte Carlo sampling and other heuristics to generate financial networks. There are four methods in this category: *Anan*, *Cimi*, *Hala* and *Musm*. Table 4 provides an overview of the seven methods included followed by short descriptions. Full technical details are provided in Appendix B. Note that in our implementation *Anan*, *Bara*, *Hala* and *Maxe* generate only single reconstructed networks while *Cimi*, *Dreh* and *Musm* produce a series of reconstructed financial networks (i.e., ensembles).

Anan reconstructs networks with the smallest number of links, while still satisfying the marginal constraints. Additionally, the method shapes the network to be disassortative, i.e., banks with large aggregate positions will be linked to banks with small aggregate positions.

Bara consists of three steps. First, the aggregate positions for the financial institutions are fitted to a multivariate copula distribution. Second, financial networks are sampled from the copula. Finally, the rows and columns of the sampled networks are re-scaled until the marginal constraints are satisfied. The re-scaling is achieved using the *Maxe* method.

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[8]The full set of codes, the results and the networks descriptives are available at `https://github.com/imanvl/RTF_NTW_Horse.git`.

Table 4: Overview of network reconstruction methods

Authors	Code	Category	Description
Anand et al. (2015)	*Anan*	Sampling	Minimises the number of links necessary for distributing a given volume of loans
Baral and Fique (2012)	*Bara*	Iterative	Exposure sizes drawn from a copula fitted to the aggregate exposures of all banks
Cimini et al. (2015)	*Cimi*	Sampling	A fitness model determines the likelihood of directed linkages and exposures
Drehmann and Tarashev (2013)	*Dreh*	Iterative	Postulates that the network should have a core of banks with large exposures between themselves, and a periphery of other banks with smaller exposures
Halaj and Kok (2013)	*Hala*	Sampling	Links are drawn at random, where all links have an equal probability, and exposures are assigned according to an iterative procedure
Upper and Worms (2004)	*Maxe*	Iterative	The standard maximum-entropy method
Musmeci et al. (2013)	*Musm*	Sampling	A fitness model determines the likelihood of undirected linkages, and exposures are allocated via *Maxe*

In *Cimi*, the probability for a link between any two banks increases in their pre-specified 'fitness scores'. The method proceeds in two steps. First, a directed (binary) network of links between institutions is generated based on their fitness scores. The weights are then assigned to the links, also according to the fitness scores. Importantly, the marginal constraints are only binding *on average.* Thus, while marginal constraints may be violated for individual network realizations, the constraints will be binding when we average over a large number of network realizations.

Dreh generates networks with 'core-periphery' structures.[9] As an initial guess for the network, the method assumes there are a few institutions (the core) with large exposures between themselves. Other institutions (the periphery) have smaller exposures, and tend to link to core institutions. These initial networks are re-scaled using the *Maxe* method.

Hala samples links between institutions, where all links are ex-ante assumed to be equally likely. The exposure that is allocated to the link between institutions i and j is

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[9]The core-periphery model was first proposed by Craig and von Peter (2014). See in 't Veld and van Lelyveld (2014) for a cross-country comparison.

equal to institution j's aggregate position scaled by a term drawn at random from the unit interval. The method iterates until all aggregate position constraints are satisfied.

Maxe is the basis for all other iterative methods. In the initial guess network, institution i's exposure to institution j is the product of i's aggregate interbank asset position and institution j's aggregate interbank liability position. This network is subsequently re-scaled by the aggregate positions, first along the rows and then the columns, until the aggregate position constraints are satisfied. Bacharach (1965) proves that, as long as the initial network is 'connected,' the re-scaling always yields a unique network that satisfies the marginal constraints. A connected network is one where each financial institution has at least one link with another institution.

Finally, the *Musm* method is similar to *Cimi* in that the probabilities of observing links are based on fitness scores in both methods. However, in *Cimi*, the underlying adjacency matrix is directed, while for *Musm* it is undirected. Further, the assignment of exposures under *Musm* follows the *Maxe* method, while for *Cimi* it does not.

4 The horse race

The financial network data we consider are confidential, and their sharing is restricted. To overcome these restriction, we devise a de-centralized approach to run the horse race of network reconstruction methods. In particular, a suite of codes is applied within a jurisdiction by participating researchers, who report the results to the wider group. The suite consists of two parts: network reconstruction and similarity estimation. We treat each one in turn.

4.1 Procedure

We focus on reconstructing networks when the only information available is on the aggregate positions. These reconstructed networks are then compared with the true networks, also available to us. While restrictive, using only the aggregate positions ensures that we treat all the methods equally. For each jurisdiction, we first select the market and a particular date, and subsequently perform the following:

- Compute aggregate positions for financial institutions

- Reconstruct the network based on the aggregate positions using the seven methods

- Compare the reconstructed networks with the true one and compute similarity scores

Members shared the similarity scores and the descriptives for the true networks across jurisdictions. As described below, the similarity measures are aggregate statistics from which the true financial networks cannot be inferred.

4.2 Similarity measures

We consider six similarity measures, where the first five can be classified into two groups: link-based and exposure-based. Link-based measures capture whether the presence or absence of a link in the true network is reproduced in the reconstructed network. We consider three measures in this category: (i) Hamming distance, (ii) Jaccard score, and (iii) Accuracy score. Exposure-based measures, on the other hand, take into account the size of links and check whether these have been faithfully reproduced. We consider two measures in this category: (i) Cosine measure and (ii) Jensen score.

Our sixth measure is based on DebtRank, which is a model for interbank contagion (Battiston et al., 2012). The DebtRank for a particular financial institution is a measure of the aggregate interbank assets of all institutions that are at risk from the failure of the

single institution. We order all financial institutions based on their DebtRank scores and compute the rank correlation between the ordering of institutions in the true network versus the reconstructed networks.

Table 5 provides an overview of the measures, along with a brief description. Note that for consistency, we have re-based the Hamming and Jensen measures so that, for all metrics, higher values correspond to greater similarity. In what follows, we report on the average values for the similarity measures over the time horizon of true network observations for each jurisdiction.

Table 5: Similarity measures

Metric	Category	Description	Range
Hamming	Link	Sum over all links of the difference between the original and reconstructed networks	$[0, \infty)$
Jaccard	Link	Inverse of the number of links belonging to the original and reconstructed networks divided by the number of links that belong to at least one network	$[0,1]$
Accuracy	Link	Percentage of true-positive and true-negatives links in the reconstructed network relative to the original network	$[0, 1]$
Cosine	Exposure	Cosine of the angle between the original and reconstructed networks	$[0, 1]$
Jensen	Exposure	Jensen-Shannon divergence between original and reconstructed networks, normalizing all entires in the networks to sum up to one	$[0, \infty)$

4.3 Summary of results

We begin by comparing the outputs of the different network reconstruction methods using standard network-based measures (a full summary of these measures is provided in Appendix C). Figure 2 provides a heat-map that compares the density of the true networks with the reconstructed networks for all jurisdictions and markets. The performance of each method is measured by how close its color matches that of the actual network (first row). Not surprisingly, there is a large variation among jurisdictions from dense to sparse networks.

From the estimated matrices, we observe that *Anan*, *Cimi*, *Hala* and *Musm* tend to estimate sparse networks for most jurisdictions, while *Bara*, *Dreh* and *Maxe* estimate denser networks.

Figure 2: Density

Note: Density is defined as the number of realized links over the total number of possible links (excluding self loops). The cells are shaded according to the relative density ranging from green to red.

A similar broad classification of results can be seen for borrower dependency (Figure 3), which measures the reliance of individual banks on their largest creditor. The larger the borrower dependency, the more concentrated the network is. Once again, the *Anan*, *Cimi*,

Hala and *Musm* methods produce more concentrated networks than the *Bara, Dreh* and *Maxe* methods.

Figure 3: Borrower dependency

Note: Borrower dependency is defined as the average of the market share of the largest borrower over total borrowing and lending. The cells are shaded according to the relative borrower dependency ranging from green to red.

Further insights into the performance of the different methods can be gained from the estimates for the true links, i.e., links that are present in the true and reconstructed

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networks, and true non-links, i.e., links that are absent in both the true and reconstructed networks. As Figure 4 demonstrates, the *Bara*, *Dreh* and *Maxe* methods are successful in identifying links among banks that are present in the original networks. This is a consequence of the methods estimating complete networks.
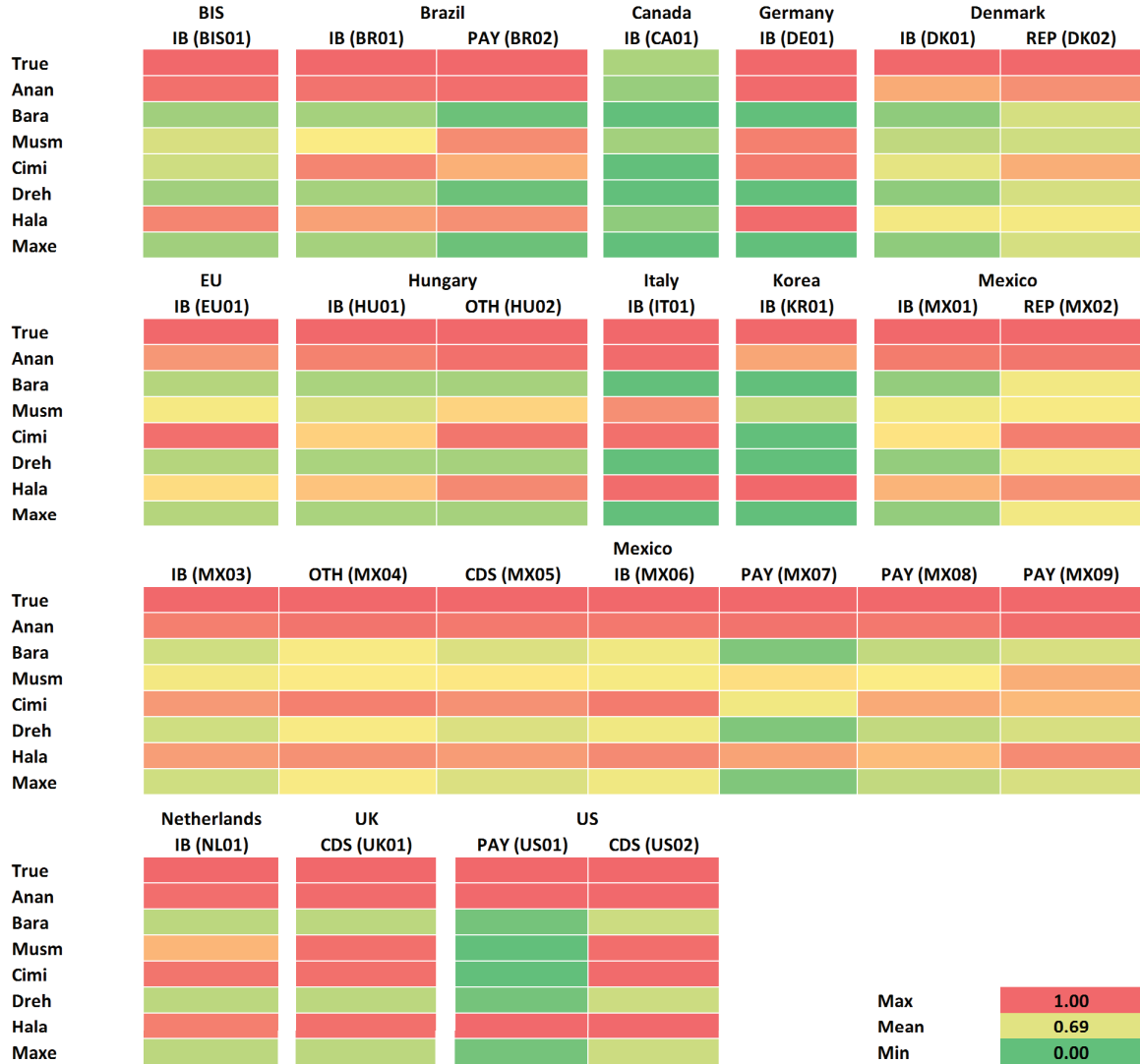
Figure 4: Percentage of true-links
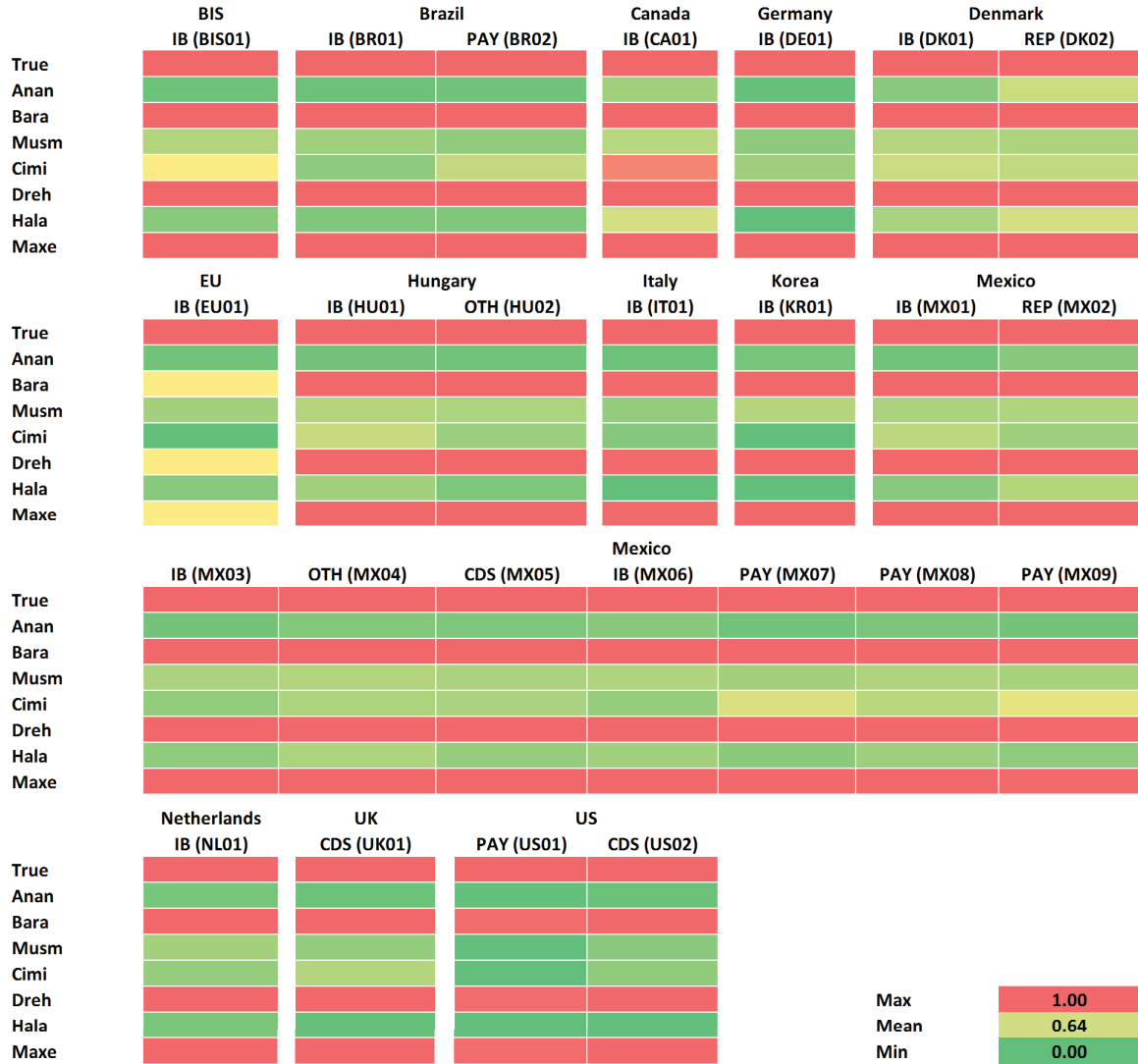
Note: true-links are defined as the percentage of realized links also found in the reconstructed network. The cells are shaded according to the relative true-links ranging from green to red.

On the other hand, the *Anan*, *Cimi*, *Hala* and *Musm* methods correctly identify which links are absent in the original networks (high values in Figure 5). This stems from the fact that these three methods tend to produce sparse networks. However, in sum, the accuracy of the various partial network methods is ambiguous, as it weighs both true links and true non-links equally.

Figure 5: Percentage of true non-links

Note: true-non links are defined as the percentage of absent links also not present in the reconstructed network. The cells are shaded according to the relative true-links ranging from green to red.

4.4 Horse race league table

In presenting our results we face the challenge that some algorithms produce an ensemble of networks while others produce a single matrix. This makes a straightforward comparison difficult. Fortunately the *Cimi* method is the clear winner between the ensemble methods.[10]

The horse race results for methods that produce unique networks are summarized in Table 6.[11] Each row corresponds to a different network. The last six columns indicate the 'winning' methods for each of the different similarity measures. If, however, there was no clear winner, i.e several methods performed equally well, then the cell has been left blank.

The winner crucially depends on whether we aim for a deterministic (single matrix) or probabilistic (ensemble) outcome and – in case of deterministic methods – the feature of the network that we are most keen to preserve. The latter is reflected by the choice of the similarity measure. For the probabilistic methods, the *Cimi* method is the clear winner across the board while for the deterministic methods it depends on the network feature. If, for example, we are focused on reproducing the structure of links, then from the link-based similarity measures – Hamming distance and Accuracy score – we note that *Anan* is the clear winner across all financial networks with *Hala* as the runner up. The *Anan* method seeks to minimise the number of links required to allocate the aggregate positions of all financial institutions. The method, thus, focuses on reducing the incidence of false-positives, i.e., reconstructing a link that is not present in the true network. This, however, may be countered by a higher incidence of false-negatives, i.e., not reconstructing a link when it is present in the true network. However, as the results suggest, the *Anan* method's ability to reduce the number of false-positives gives it a clear advantage over the other methods under link-based similarity measures.

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[10] It is worth noting that both the *Anan* and *Hala* methods can also be extended to produce ensembles of networks. However, for our analysis, we focus on the cases of single realizations.

[11] For each network, we first compute the average over all available time slices and then we run the horse race. We also tabulate the results if we first run the race for each available slice and then find the mode across slices. The latter results are not materially different.

Table 6: Horse race results for deterministic methods

Country	Type	Code	Hamming	Accuracy	Jaccard	Cosine	Jensen	DebtRank
BIS	IB	BIS1				maxe	maxe	bara
Brazil	IB	BR01	anan	anan	hala	maxe	hala	maxe
	PAY	BR02	anan	anan		bara	bara	bara
Canada	IB	CA01				maxe	maxe	maxe
Germany	IB	DE01	anan	anan	anan	maxe	hala	maxe
Denmark	IB	DK01	hala	hala		maxe	maxe	
	REP	DK02	anan	anan	anan	hala	anan	
Eurozone	IB	EU01	anan	anan		maxe	bara	hala
Hungary	IB	HU01	anan	anan		maxe	maxe	bara
	OTH	HU02	anan	anan	anan	maxe	maxe	anan
Italy	IB	IT01	anan	anan	anan	maxe	hala	bara
Korea	IB	KR01				maxe	bara	maxe
Mexico	IB	MX01	anan	anan		bara	bara	bara
	REP	MX02	anan	anan	hala	bara	anan	bara
	IB	MX03	anan	anan		maxe	anan	maxe
	OTH	MX04	anan	anan		bara	bara	bara
	CDS	MX05	anan	anan		maxe	maxe	maxe
	IB	MX06	anan	anan	hala	bara	anan	maxe
	PAY	MX07	hala	hala		maxe	bara	bara
	PAY	MX08	anan	anan		maxe	maxe	bara
	PAY	MX09	hala	hala		maxe	maxe	bara
Nethlerlands	IB	NL01	anan	anan	anan	maxe	anan	bara
UK	CDS	UK01	anan	anan	anan	maxe	maxe	*
US	PAY	US01	hala	hala	anan	bara	hala	bara
	CDS	US02	hala	hala	anan	maxe	hala	*

Note: for each network we first compute the average over all available time slices and then we run the horse race. If two or more methods win jointly then we leave the cell blank. For the CDS networks we have no results for the DebtRank metric (marked with an *) because capital buffer data is not available. We do not include a similar table for the probabilistic methods, since *Cimi* is the clear winner.

If we focus on reproducing the structure of bilateral exposures, then from the exposure-based similarity measures – Cosine and Jensen – an altogether different picture emerges. These metrics compare the allocated exposure sizes in the reconstructed networks with the true networks. For the Cosine measure, we find that the *Bara* and *Maxe* methods perform best across all networks. There are two possible explanations for this result. First, for many of the financial institutions in each data market, a large fraction of the links are of an equal size. Second, the average link size is roughly similar to the aggregate exposure divided by the number of financial institutions. For the Jensen score, we find that the *Maxe* method is the clear winner.

Finally, for the DebtRank correlation measure, we find that the *Bara* method is the winner, with *Maxe* coming in as a close second. Insofar as both these methods are also the top performers for the exposure-based similarity measures, this suggests that the DebtRank contagion mechanism does not depend so much on the pattern of linkages, but rather on exposure sizes.

5 Conclusions

Capturing financial network linkages and interbank contagion in stress-test models are important goals for central banks tasked with oversight of macro-prudential policy. The operationalization of these models, however, requires granular financial network data, which is often unavailable. In this paper, we conduct a horse race of methods to reconstruct financial networks from partial data. The winner of the horse race depends on the network feature we are most keen on reproducing.

As such, we derive the following rules of thumb: If we seek to preserve the structure of links and expect the network to be sparse, then *Anan* and *Cimi* are the best performing methods for the deterministic and probabilistic methods, respectively. If, however, we are more interested in reproducing the structure of exposures, then *Bara* or *Maxe* tend to be

the best performers among the deterministic methods and *Cimi* among the probabilistic. Finally, if our emphasis is more on financial stability, in that we seek to maximize the rank correlation of DebtRank scores, we find that *Bara* is the winner for the deterministic method while *Cimi* is the winning probabilistic method.

A byproduct of our horse race is the collation of summary statistics – both point estimates and distributions – for a wide range of financial networks.[12] Consistent definitions for the network statistics are used and computed for the different networks. This, in turn, facilitates a meaningful comparison of the different networks, which was previously not possible. For many networks, we also collect up to 12 consecutive snapshots. Such data may be of use to the wider financial network research community, who could use our statistics to generate realistic networks for their own research.

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[12]As mentioned, the full set of codes, the results and the networks descriptives are available at `https://github.com/imanvl/RTF_NTW_Horse.git`

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A   Summary of the data

In this appendix we provide a summary of all the financial markets analyzed. The data are categorized according to their jurisdiction.

Bank for International Settlements The network constitutes the exposures between different national financial systems in 2013Q4. We derive the network from the International Banking Statistics (IBS, locational by residency), which the Bank for International Settlements (BIS) has been collecting since the late seventies (see the BIS website for further details). The data has also been studied in a network context (e.g. Fender and McGuire, 2010; Minoiu and Reyes, 2013; Garratt et al., 2011).

Both domestically owned and foreign-owned banking offices with significant external claims in the reporting countries report their on-balance sheet positions on other countries split out by sector (residency concept). A wide range of claims is included (e.g., standard loans and deposits, repurchase agreements, i.e., repos, and reverse repos, certificate of deposits, financial leases, promissory notes, subordinated loans, debt securities, and equity holdings and participations). Out of the possible reporters, data availability leads us to include 21 countries.[13]

Brazil Two types of networks are analyzed: the interbank exposures and the national payments system network.

Interbank exposures: This network is formed by exposures between banking or non-banking financial institutions in the Brazilian interbank market and is analyzed by Cont et al. (2010). These institutions are either financial conglomerates or isolated institutions that do not belong to a conglomerate. Data are monthly, from January to December 2012. These networks are formed by aggregating, without netting, the end-of-month interbank

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[13] Austria, Australia, Belgium, Canada, the Cayman Islands, Switzerland, Germany, Greece, Denmark (excl. Faeroe Islands and Greenland), Spain, Finland, France (incl. Monaco), United Kingdom (excl. Guernsey, Isle of Man and Jersey), Ireland, Italy, Japan, Luxembourg, Netherlands, Portugal, Sweden, and the United States.

market exposures for pairs of financial institutions, regardless of instrument and time to maturity. The instruments included in these exposures are unsecured interbank deposit operations (59% in volume), debentures (23%) and repos collateralized with securities issued by the borrower (18%). The average number of market participants is 112 (maximum, 115). They are sparse (maximum density is 4.3%)

Payments system: The Brazilian Payments System (BPS) provides services for the settlement of obligations involving transfers of funds, securities and foreign currencies and has previously been analyzed by Miranda et al. (2014). The system is segmented according to the target market and the type of assets traded. The system component selected for this study is the network of the Reserves Transfer System, which is an real-time gross settlement system (RTGS) that provides the backbone of the BPS. The network participants are banks that hold bank reserves accounts at the Central Bank of Brazil and non-banking institutions, that hold, when authorized, settlement accounts. Daily snapshots are considered from January 16th to 27th in 2012. These networks have, on average, 101 participants (maximum, 102). The payments networks are denser than the interbank ones (minimum density of the period is 12.5%) and present more disassortative behavior.

Canada The networks considered are monthly observations of interbank exposures between the six Canadian domestic systemically important banks, from June 2014 to March 2015.[14] The bilateral exposures are constructed by aggregating over six different interbank instruments as reported by the banks: (1) bankers' acceptances, (2) debt securities holdings,(3) lending (drawn and undrawn), (4) over-the-counter derivatives (potential future credit exposure), (5) repos (before collateral), and (6) deposits.

The total interbank exposures as a fraction of their Common Equity Tier 1 capital ranged between 10% and 60% across bank-month pairs. The ratios of total interbank assets to total (liquid) assets ranged between 0.7% (7%) and 4% (59%).[15]

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[14]A description of the reporting standards can be found at http://www.osfi-bsif.gc.ca/eng/fi-if/rtn-rlv/fr-rf/dti-id/Pages/imer.aspx.

[15]Liquid assets are defined as: cash, cash equivalents, t-bills and other short-term paper issued or guaran-

Denmark Two types of networks are analyzed: overnight interbank loans and repo transactions. Snapshots for both markets come from December 2011.

Overnight interbank loans: The network is constructed using data from the Danish large-value payment system (Kronos) whose members include all Danish banks. This market has previously been analyzed by Amundsen and Arnt (2012). An algorithm similar to Furfine (1999) is used to in order to isolate transactions connected to the deliveries and returns of overnight money market loans.

Repo transactions: Major firms, including financial institutions, are required to report their end-of-month outstanding repo agreements vis-à-vis every other domestic institution. Repos with foreign institutions are reported on an aggregate basis. The network considered is thus the net bilateral repo exposure (excluding collateral) between Danish banks.

Eurozone The network constitutes bilateral exposures between the 26 largest banking groups that are domicile in the Eurozone. A banking group's size is measured in terms of its aggregate trading securities position, which includes long- and short-term debt and equity. The data are derived from the Securities Holding Statistics Group (SHS-G) database. The SHS-G data specifies the portfolio for each banking group at the level of individual securities. The quarterly slices span from September 2013 through December 2015.

Netherlands The network is constructed using data on bilateral transactions gleaned from the TARGET 2 large-value payment system on April 6th 2010. This was a typical day without any stress or extraordinary operational event. For this exercise we focus on the overnight market and thus leave out all longer maturity loans. Building on Furfine (1999), Arciero et al. (2016) have developed a methodology to identify loans with price and maturity information. The transaction-level data set thus has the time, volume and price of all transactions involving at least one Dutch bank. This data has been analyzed further in Blasques et al. (2015).

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teed by Canadian governments. Note that, even though the full network is complete at the aggregate level, there is some variation in the strength of the interlinkages between banks at an instrument level.

Germany Quarterly data from March 2013 to December 2015 is used on interbank loans in the German banking system. The network consists of German banks with total assets above 1 billion euros on a consolidated basis at the respective reporting date. Those banks capture approximately 95% of the total assets of the German banking system. The data is derived from the national credit register, which includes bilateral exposures covering loans, bonds, derivatives and guarantees. Until the end of 2014, only exposures above 1.5 million euros based on the group of borrowers were reported, while the respective reporting threshold was lowered to 1 million euros at the beginning of 2015. The data previously been analyzed by Anand et al. (2015).

Hungary Two types of networks are considered: interbank deposits and currency swaps.

Interbank deposits: The Hungarian interbank deposit market is the main market for Hungarian banks to manage their liquidity and the only market where they have direct credit risk against each other. The Central Bank of Hungary has been collecting data on this market since 2003. The dataset contains detailed information on every transaction (e.g., the name of both counterparties, the start and end dates of the transaction, the size and the interest rate of the transaction). Twelve consecutive monthly networks from July 2007 to June 2008 are considered.

Currency swaps: Ten monthly snapshots of FX swap transactions from June 2007 to April 2008 are considered. This market is one of the most important Hungarian financial markets. The Central Bank of Hungary obliges Hungarian credit institutions to report all of their foreign currency related transactions including FX swaps. The majority of the foreign currency contracts are US dollar denominated (roughly 82%) and a minor proportion in euros (roughly 15%) and Swiss Francs (less than 3%).

Korea The interbank exposures are constructed using banks' counterparty information collected from the flow of funds and surveys on interbank transactions in 2012Q4. These cover all the on-balance sheet items such as deposits, lending, repo transactions and debt

issuance of all 18 domestic banks. However, only exposures with a remaining maturity of less than three months are included. Thus these bilateral interbank exposures can be suitable for analyzing the structure of short-term interbank transactions.

Italy Monthly data from December 2013 to September 2014 is used to gather data on outstanding bilateral, unsecured, short-term interbank loans between banks domiciled in Italy. Short-term interbank loans include overnight deposits, certificates of deposit, other deposits and other borrowings.

Mexico Several networks are considered in the analysis: interbank exposures (unsecured loans, FX transactions, derivatives transactions), repo transactions, cross holding of securities and payment systems flows. The networks were drawn for three different dates: 31st October 2008, 28th June 2013 and the 30th June 2014. These networks have previously been studied by Martínez-Jaramillo et al. (2014) and Poledna et al. (2015).

Interbank exposures networks: These networks are generated by aggregating unsecured interbank loans, net positions from outstanding derivatives transactions and cross holding of securities between pairs of banks.

Payment systems networks: The payment system flow networks are divided in three different networks: the total flow, the large-value payments and the low-value payments networks. In Mexico, the large-value payment system accepts both low-value and large-value payments. The individual payment records include information on the purpose of the payment. This means that the payment may be done between two banks for transferring an unsecured interbank loan, this payment is classified as a large-value payment. On the other hand, the payment may be the result of a money transfer between two clients in two different banks, and this is classified as a small-value payment.

Outstanding derivatives exposures, outstanding interbank deposits, loans and credit lines, outstanding call money transactions and cross holding of securities: These networks can be seen as layers of the total exposures network.

Outstanding repo loans: This network only considers the total amount of an outstanding repo position between two banks. The collateral is not taken into account; this means that the weight of the links is the total amount lent by one bank to another, considering all the outstanding derivative transactions between them. This implies that many repo transactions are consolidated in one link regardless of the type of security used as collateral, the residual maturity or the premium.

United Kingdom The networks are created using Trade Information Warehouse data from the Depository Trust and Clearing Corporation (DTCC). The data used pertain to transactions among reporting counterparties on single-name CDS contracts where the reference entity is a UK firm. A detailed description of the UK CDS market is provided in Benos et al. (2013).

Two different types of networks are constructed from these data. First, for a selected date at the end of June 2010, two networks of gross notional exposures are generated by aggregating all outstanding trades across 1) 30% and 2) all reference entities, respectively. Second, following Ali et al. (2016), four monthly snapshots of networks of notional exposures, denominated in euro, from June 2010 to September 2010 are generated for CDS contracts referring to the largest 66 reference entities.

Over time the number of counterparties varies between 331 and 345, and properties such as network density, average degree, clustering coefficient and assortativity appear to be stable. All networks considered have low density, small average degree and negative assortativity.

United States Two networks are analyzed: the payments network and CDS network.

Payments network: Fedwire Funds is the RTGS system operated by the Federal Reserve System. In Fedwire, payments are identified by the ABA number (routing transit number) of the sender and receiver. Banks may maintain multiple ABA numbers for use when sending and receiving payments. However, each bank must designate one of its ABAs

as its "master" account. For this analysis, payments sent and received by sub-accounts are attributed to the master account.

The data used to measure the importance of and size of each participant is the net debit cap (i.e., maximum allowable uncollateralized daylight overdraft), which is only available at the master account level. The Federal Reserve calculates the net debit cap by multiplying a bank's qualifying capital by its appropriate cap multiple.[16]

Fedwire participants without net debit caps are excluded from the analysis. Participants with a net debit cap of 0 are included; this cap indicates that a bank should not incur any daylight overdrafts. During the week of the sample period, at least one payment was sent or received by 5722 unique master accounts.

CDS network: Positions on CDS exposures over three weekly snapshots covering September 5, 12, and 19, 2014 are used in this exercise. Our sample includes both centrally cleared and bilateral contracts. The data are obtained from DTCC, which makes its Trade Information Warehouse available to the Office of Financial Research under a written agreement. Positions used in this study include all exposures on single-name and index CDS contracts where the reference entity is US-domiciled (in the case of single names) or North American domiciled (in the case of indices), or where at least one of two counterparties is US-domiciled. On any given date, approximately 900-1000 counterparties trade positions on 3,500 to 4,000 underlying reference entities. From this data we have constructed a complete network of counterparty exposures.

B Network reconstruction methods

The standard approach in the literature is to estimate the matrix of bilateral links (denoted by X) by the so-called maximum entropy method (Upper, 2011, Elsinger et al., 2013).

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[16]There are several levels of cap multiples, detailed at `www.clevelandfed.org/banking/ credit_risk_management/payment_system_risk/net_debit_cap.cfm`.

This entails maximizing the entropy function $-\sum_{i,j} X_{ij} \log\left(X_{ij}/Q_{ij}\right)$ subject to constraints (typically a firm's total assets, A_i, and liabilities, L_i, to all other participants), relative to prior information ($Q_i j$) on bilateral exposures, if available. As entropy is a measure of probabilistic uncertainty, this approach is optimal when selecting a probability distribution in the sense of using least information (MacKay, 2003). Entropy optimization is widely used across disciplines (Fang et al., 1997), and can be implemented by efficient iterative algorithms, which can be generalized to handle additional constraints (Blien and Graef, 1997, Elsinger et al., 2013).

B.1 *Anan*

In Anand et al. (2015), the authors propose an approach which combines information-theoretic arguments with economic incentives to produce networks preserving the realistic characteristic of interbank networks. The authors argue that interbank networks are sparse given that interbank activity is based on relationships. The Minimum Density (MD) approach is formulated as a constrained optimization problem. Let c represent the fixed cost of establishing a link, N be the number of banks, X the matrix of bilateral gross exposures, X_{ij} represents the exposure of bank i to bank j, the aggregated interbank assets of bank i are $\sum_{j=1}^{N} X_{ij}$ and its aggregated liabilities are $\sum_{i=1}^{N} X_{ij}$. Then the MD approach is formulated as:

$$\min_{x} c \sum_{i=1}^{N} \sum_{j=1}^{N} 1_{\{X_{ij}>0\}}, \quad \text{s.t.}$$

$$\sum_{j=1}^{N} X_{ij} = A_i \quad \forall i = 1, 2, \ldots, N$$

$$\sum_{i=1}^{N} X_{ij} = A_j \quad \forall j = 1, 2, \ldots, N$$

$X_{ij} > 0 \quad \forall i, j$ where integer function 1 equals one only if bank i lends to bank j, and zero otherwise. This problem, however, is computationally expensive to solve. The authors propose a heuristic to solve this problem, which involves the smooth value function, $V(X)$,

which is high whenever the network X has a few links and satisfies the asset and liability constraints. The second input is the set of prior beliefs, Q_{ij}, which assumes that each small bank prefers to match its lending and funding needs for a large bank (dissasortative mixing).

B.2 *Bara*

In Baral and Fique (2012), the authors use a bivariate copula to estimate adjacency matrices. A copula is a multivariate distribution where the complex interdependencies between banks can be easily summarized using marginal distributions.

The copula is constructed as follows. First, the authors assume the copula to be of the Gumbel type, which is often used in extreme value theory. The authors construct the empirical distribution for the aggregate lending and borrowing of banks using the available data. This distribution is transformed into a copula using a maximum-likelihood method. The copula density function is

$$c_\Theta(A_i, A_j) = \exp\left(-\left[(-\ln A_i)^\Theta + (-\ln A_j)^\Theta\right]\right)^\Theta$$

where Θ is the estimated dependency parameter. The copula matrix is the prior fed into the maximum entropy method. The exposures are then re-scaled to ensure that the aggregate lending and borrowing constraints for each bank are satisfied.

B.3 *Dreh*

Drehmann and Tarashev (2013) generate a series of high-concentration networks by perturbing the network produced by the maximum entropy method. The authors begin with the standard prior assumption that the exposure between banks i and j is equal to $A_i L_j$. They subsequently treat each element of the prior matrix Q_{ij} as a uniformly distributed random variable over the interval $[0, 2A_i L_j]$. After generating a series of prior matrices, the

authors use the standard maximum entropy to rescale and determine the exposures.

B.4 *Hala*

Halaj and Kok (2013) introduce an iterative algorithm to generate a series of networks. At the initial step 0, the matrix X^0 has all entries equal to 0 and the unmatched interbank assets and liabilities are initiated as $A^0 := A$ and $L^0 : L$. At a step $k+1$ a pair of banks (i, j) is drawn at random, where all pairs have an equal probability of being selected. Next, a random number f is drawn from the unit interval and indicates the percentage of bank i's liabilities that are serviced by bank j. The exposure X_{ij}^{k+1} is updated as follows:

$$X_{ij}^{k+1} = X_{ij}^k + f^{k+1} \min\{L_k^i, A_j^k\}$$

and the unmatched assets and liabilities is:

$$L_i^{k+1} = L_i^k - \sum_{j=1}^N X_{ij}^{k+1} \quad \text{and} \quad A_j^{k+1} = A_j^k - \sum_{i=1}^N X_{ij}^{k+1}$$

The stock of interbank liabilities and assets reduces as the volume of the assigned (matched) placements increases. The procedure is repeated until no more interbank liabilities are left to be assigned as placements from one bank to another.

B.5 *Cimi*

Cimini et al. (2015) present a model that is similar to the *Musm* method of Musmeci et al. (2013), but with some important differences. First, both methods generate adjacency matrices from so-called fitness models. However, in *Musm* the matrices are undirected, while for *Cimi*, they are directed. Second, for assigning the exposures, *Musm* utilizes the *Maxe* methods. While in *Cimi*, the exposure assignment also follows a fitness model. Importantly, the aggregate exposure constraints are not always satisfied for individual reconstructed

networks under *Cimi*. Instead, the constraints are binding only when we take an average over a large number of reconstructed networks.

B.6 *Musm*

Musmeci et al. (2013) develop a bootstrap method to reconstruct financial networks. At the core of their method is a 'fitness' model, which postulates that the probability of a bank acquiring links is proportional to its fitness. Formally, if banks i and j have fitness f_i and f_j, then the probability for a link between the two banks is

$$Q_{ij} = \frac{z f_i f_j}{1 + z f_i f_j} \,,$$

where the endogenous parameter z captures how binding the aggregate exposure constraints will be.

The method proceeds as follows. First, from the aggregate lending and borrowing constraints of banks, the parameter z is estimated. Second, using the probabilities p_{ij}, a series of adjacency matrices are sampled. Finally, the exposures are determined using the standard maximum entropy method.

C Description of metrics

Table 7 reports the network statistics we compute for the original networks.

Table 7: Summary of network statistics

Metric	Short Description
Number of links	Number of undirected links in the network
Density	Number of undirected links as a percentage of the total number of links (excluding self-loops)
Average degree	Average of undirected links of the nodes in the network
Median degree	Median of undirected links of the nodes in the network
Assortativity	Preference for a network's nodes to attach to others that are similar. Here, similarity is expressed in terms of a node's degree. A high assortativity implies that highly connected nodes tend to be connected with other high degree nodes
Clustering	The degree to which nodes in a graph tend to cluster together. In a undirected setting, this is defined as the number of closed triplets (any three nodes with links between all three) over the total number of triplets (also including triplets with one link missing)
Lender / borrower dependency	Average of the market share of the largest borrower or lender, respectively, over total borrowing and lending.
HHI	Herfindahl-Hirschman concentration index (mean and median) of both assets and liabilities. It is defined as the sum of the squared "market shares".
Core size	Percentage of banks classified as belonging to the core
Error score	The percentage of the actual links in violation of the perfect core-periphery structure