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Measuring the systemic importance of interconnected banks

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ABSTRACT

We propose a method for measuring the systemic importance of interconnected banks. In order to capture contributions to system-wide risk, our measure accounts fully for the extent to which a bank (i) propagates shocks across the system and (ii) is vulnerable to propagated shocks. An empirical implementation of this measure and a popular alternative reveals that interconnectedness is a key driver of systemic importance. However, since the two measures reflect the impact of interbank borrowing and lending on system-wide risk differently, they can disagree substantially about the systemic importance of individual banks.

1. Introduction

It is commonly thought that interconnectedness is a key driver of systemic importance. Yet, the literature has *produced* few concrete insights to support this view. Who is more systemically important – the lender or the borrower in an interbank market transaction? How should the two counterparties share the rise in system-wide risk that the link between them creates? These fundamental questions remain unanswered despite recent policy initiatives to strengthen the financial system by tightening the regulatory requirements for interconnected banks.[1] To assess whether interconnectedness is indeed a key driver of systemic importance and how it affects different participants in the interbank market, we propose a measurement methodology and implement it empirically.

We quantify each bank's systemic importance as its share in the overall level of system-wide risk. Our measure of system-wide risk is the expected loss that the banking system as a whole imposes on non-banks in systemic events, i.e. those events in which aggregate losses exceed a critical level. And

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[1] See Basel Committee on Banking Supervision (2011).

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we explore two approaches to quantifying systemic importance that decompose the same measure of system-wide risk but allocate it differently across individual institutions.

One of the approaches focuses on the expected losses that a bank imposes on its non-bank investors in systemic events. It equates systemic importance with the expected participation of individual banks in systemic events. Thus, we label it the participation approach (PA).

A bank's participation in systemic events is conceptually different from its contribution to system-wide risk. Consider, for example, a bank that is small in the sense that it can impose only small losses on its non-bank investors. As this bank participates little in systemic events, PA assigns only limited systemic importance to it. The same bank, however, might be highly interconnected in the interbank market and contribute materially to system-wide risk by transmitting distress from one bank to another. PA is not designed to capture such transmission mechanisms, which prompts us to consider an alternative: a contribution approach.

The key methodological innovation of our paper is to generalise an existing contribution approach – based on Shapley values – to the case of interconnected institutions. We call this the generalised contribution approach (GCA). It considers different groups of banks (or subsystems) in isolation and measures how much each bank contributes to the hypothetical risk of each subsystem. The systemic importance of a bank is then simply the average of its risk contributions to all subsystems. When considering a subsystem within a system of interconnected banks, it is important to: (i) keep the interbank network structure in that subsystem intact and (ii) remove all the risk associated – directly or indirectly – with banks outside the subsystem. Only then do we fully capture the risk contribution of an interconnected bank to a particular subsystem. With GCA, we achieve this by (i) taking the positions among banks in each particular subsystem as given and (ii) modelling banks outside the subsystem as entities that impose no risk either on other banks or on non-banks.

We argue that PA and GCA are both valid approaches but are suited for different objectives. Measures obtained under PA can be used to form the basis of insurance schemes that protect banks' creditors against losses in systemic events and that banks have to fund by paying actuarially fair premia. By contrast, GCA can be used to determine how much to charge a bank in order to penalise it for its contribution to system-wide risk. This contribution stems not only from the risk that this bank imposes on its own investors but also from the risk that it imposes, through interbank linkages, on the investors in other banks.

The first main conclusion of our empirical analysis is to confirm the intuition that interconnectedness is a key driver of banks' systemic importance. Analysing stylised banking systems and a system of 20 large globally active banks, we find that interbank linkages raise materially the measured level of system-wide risk and, by extension, the portions of this risk allocated to individual banks. Thus, systemic importance rises in the presence of an interbank market, with the rise being greater for banks with greater interbank market activity.

Our second key finding is that the choice of a particular approach to measuring systemic importance matters not only from a conceptual but also from an empirical point of view. Quantitative differences between the two approaches are particularly pronounced in the presence of interbank linkages, as PA consistently assigns a higher (lower) degree of systemic importance to an interbank lender (borrower) than GCA.

To offer some intuition why PA and GCA differ, the following deliberately stylised setup zooms on the risk created by interbank lending, keeping everything else constant. There are two banks, A and B, and a policymaker concerned only with the losses to the non-bank creditors in her jurisdiction. Initially, bank A funds a risk-free investment with debt purchased by domestic non-banks and bank B funds a risky investment of the same amount with debt purchased by foreign entities. Since there is no risk to the non-bank creditors of bank A, the policymaker perceives zero system-wide risk. The picture changes, however, if bank A replaces its risk-free investment with an interbank loan to bank B, which uses this loan to replace its foreign funding. Assume that this leads to contagion from B to A, exposing A's domestic non-bank creditors to losses in systemic events. Then, from the perspective of the policymaker, the interbank loan raises system-wide risk.

PA and GCA allocate differently the increase in system-wide risk to the two banks. The introduction of an interbank link in the thought experiment raises the expected participation of the lending bank A in systemic events (the risk faced by its domestic non-bank creditors increases) but leaves the participation of the borrowing bank B unchanged (it still imposes no direct risk on domestic non-bank creditors). Since participation in systemic events is all that matters to PA, this approach attributes the entire increase in system-wide risk to the interbank lender A. By contrast, GCA splits this increase equally between the two counterparties. In this way, GCA captures the idea that an interconnected bank can contribute to system-wide risk not only by imposing losses on its own non-bank creditors but also by imposing losses on the creditors of banks from which it has borrowed.

The rest of the paper is organised as follows: In Section 2, we review the related literature. We outline the analytic setup in Section 3, where we first present our definition of system-wide risk and then define PA and GCA. In Section 4, we describe our empirical setup. We analyse stylised banking systems in Section 5 and measure systemic importance in a system of 20 real-world large banks in Section 6. We conclude with Section 7.

2. Literature review

The literature has recently proposed and used a number of alternative measures of systemic importance. PA, for example, has been implemented, albeit under different names, by Tasche (2008), Huang et al. (2010), Acharya et al. (2009) and Brownlees and Engle (2010). These papers compute the system-wide loss distribution, define a set of systemic events, and equate the systemic importance of a particular bank with the expected losses it generates in these events.[2] Thus, the papers measure systemic importance as the expected participation of individual institutions in systemic events.[3]

Tarashev et al. (2010) propose an alternative measure that captures the contribution of institutions to system-wide risk. This measure is based on the Shapley value, developed by Shapley (1953) for allocating the value created in cooperative games across individual players. When the methodology is applied in the context of system-wide risk, a bank's Shapley value is a measure of its systemic importance.

Applications of the Shapley value methodology lead to important insights. For example, Tarashev et al. (2010) highlight that bank size, institution-specific probabilities of default and exposures to common risk factors interact in a non-linear fashion to determine the contribution of financial institutions to system-wide risk. In a related paper, Staum (2012) uses Shapley values to design a deposit insurance scheme in the presence of fire-sale externalities and mergers. The insights of these papers are however incomplete because, just like articles adopting PA, they do not consider explicitly the interbank network structure as a driver of systemic importance.

There is a large literature – surveyed recently by Upper (2011) and Allen and Babus (2009) – on how linkages in an interbank network influence system-wide risk. A number of the theoretical and empirical results of this literature, as well as some of the methodological challenges faced by it, reemerge in our analysis below. An example is the finding, first reported by Allen and Gale (2000), that the structure of the interbank network determines the extent of contagion in the system and, thus, has a first-order impact on system-wide risk.

The large size of the interbank network literature notwithstanding, few papers have measured the systemic importance of interconnected institutions. In one of these papers, Liu and Staum (2010) do tackle key challenges related to the measurement of systemic importance in the presence of an interbank network. The approach they propose is different from ours, however: it requires complex linear programming techniques and is used only in extremely stylized settings. Another example is Gauthier et al. (2012), who apply several measures of systemic importance to a system comprised of five large Canadian banks. In particular, they consider a specific application of PA and, following an earlier version of the present paper (Drehmann and Tarashev, 2011b), have also included GCA in their analysis.

[2] As a starting point, both PA and GCA require a measure of system-wide (or systemic) risk. The growing literature on system-wide risk includes e.g. Elsinger et al. (2006), Giesecke and Baeho (2011) and Suh (2012).

[3] Another strand of the literature gauges systemic importance by the impact that distress at one bank has on the rest of the system. An often cited measure from this literature is CoVaR, which has been popularised by Adrian and Brunnermeier (2011). We abstract from CoVaR and related measures as they do not attempt to allocate system-wide risk to individual institutions and, thus, are not additive across institutions. That said, as illustrated by Drehmann and Tarashev (2011a), such measures can be easily implemented in the empirical framework we develop below.

While they do show that different approaches lead to different measures of the systemic importance of the five Canadian banks in their sample, we cast and explain the differences between PA and GCA in a general context. This helps us to argue why the two approaches are suited to address different policy objectives.

Our paper is also related to articles in game theory analysing the allocation of value created in networks. For instance, Navarro (2007) proves that the Shapley value is an efficient and fair allocation rule under general externalities across the network's components. In turn, Ni and Wang (2007) consider the negative value created by river pollution and study the resulting cost game in order to derive the fair cleaning charges that each polluter should pay. In their context, the chemical reaction among polluting substances can lead to one polluter increasing the damage imposed by another; just as one bank in our context can increase the riskiness of another bank in the same network. Ni and Wang (2007) find that the Shapley value solution to their cost game satisfies the principles of two key doctrines in international disputes.

3. System-wide risk and systemic importance

In this section, we present the Shapley value methodology as a general tool for attributing system-wide risk to individual institutions. Even though the methodology can be applied in a wide range of different settings, we frame the discussion within a parsimonious specification of the banking system and system-wide losses and focus on a popular measure of system-wide risk: expected shortfall. At the end of the section, we introduce and discuss two concrete approaches to the Shapley value methodology: the participation and generalised contribution approaches.

3.1. Measuring risk in the entire system and any subsystem

Let the system be a set N comprised of n banks, indexed by $i \in \{1, 2, ..., n\}$. On the asset side of each bank's balance sheet, there are claims on non-banks and on other banks in the system, i.e. non-bank assets (*NBA*) and interbank-assets (*IBA*), respectively. Likewise, the liability side consists of non-bank liabilities (*NBL*), interbank-liabilities (*IBL*) and equity held by non-banks (*EQ*).

The only fundamental source of distress in the system is exogenous random shocks to the value of each bank's *NBA*. These can push banks into default in two different ways. First, an adverse realisation of *NBA* shocks can drive the value of a bank's total assets below its non-equity liabilities. In this case the bank experiences a *fundamental* default. Second, there can be *contagion* defaults. These materialise if the default by one or several banks pushes an otherwise solvent interbank creditor into default by depressing the value of its *IBA*.

Even though a defaulting bank imposes (stochastic) losses on all its creditors – irrespective of the type of default – we consider only the losses of non-bank creditors for our measure of system-wide risk. We do take into account the losses to interbank creditors and banks' equity holders, but only indirectly, to the extent that they lead to losses to non-bank creditors. There are two reasons why we do this.

First, losses on interbank exposures should not enter directly any measure of risk that takes a system-wide perspective. Since the interbank liabilities of one bank are the interbank assets of another, losses to the interbank creditors of one bank are ultimately incurred by the equity holders or non-bank creditors of one or more other banks in the system. As a result, including interbank losses in our measure of risk would involve double counting at a system-wide level.

Second, we emulate the spirit of financial stability mandates, which, although refraining from formal definitions of system-wide risk, call on authorities to ensure that the financial system runs smoothly and promotes real economic growth (e.g. ECB (2005), Bank of England (2013)). Thus, by excluding losses to equity holders from our measure of system-wide risk, we implicitly assume that such losses do not entail systemic repercussions: i.e. equity performs successfully its shock-absorbing role. This admittedly extreme assumption helps us to streamline the exposition in the main text. Reassuringly, relaxing the assumption in order to adopt a more general, social-welfare perspective is largely inconsequential. In Appendix A, we show that none of the insights of the paper is affected when losses to both equity investors and non-bank creditors enter directly the measure of system-wide risk.

In concrete terms, we measure risk by non-bank creditors' expected shortfall (ES). At an intuitive level, ES is the expected value of non-bank creditors' aggregate losses, given that these losses exceed a critical level. We think of such extreme losses as corresponding to systemic events, in which severe disruptions in the financial sector affect detrimentally the real economy.

We gauge the ES of the whole system N as well as of all subsystems $N^{sub} \subseteq N$. As it will become clear in Section 3.2, ES measures at the level of subsystems are the key building blocks of Shapley values.

When the probability distribution of losses is continuous, the ES of any subsystem, $N^{sub} \subseteq N$, takes a simple form[4]:

$$ES(N^{sub}) = E\left(\sum_{i \in N^{sub}} L_i^{N^{sub}} \middle| \sum_{i \in N^{sub}} L_i^{N^{sub}} \geqslant q^{N^{sub}}\right) \equiv E\left(\sum_{i \in N^{sub}} L_i^{N^{sub}} \middle| e(N^{sub})\right) \tag{1}$$

where $L_i^{N^{sub}}$ stands for the losses to the non-bank creditors of bank i, and $q^{N^{sub}}$ is some, typically high, quantile of the probability distribution of the total losses in N^{sub}. In turn, $e(N^{sub})$ is the set of loss configurations that deliver aggregate losses equal to or greater than $q^{N^{sub}}$. At the level of the whole system, $e(N)$ is the set of *systemic events*.

We define the losses to the non-bank creditors of bank i as

$$L_i^{N^{sub}} \equiv NBL_i \cdot LGD_i^{N^{sub}} \cdot I_i^{N^{sub}} \quad \text{for all } i \in \{1, 2, \cdots, n\} \tag{2}$$

where $LGD_i^{N^{sub}}$ is the loss given default, i.e. the share of NBL_i that is lost if bank i defaults. In turn, $I_i^{N^{sub}} = 1$ if bank i defaults and $I_i^{N^{sub}} = 0$ otherwise.

The notation in Eqs. (1) and (2) flags that measured risk changes with the subsystem. For one, each bank's losses on interbank exposures and, thus, the likelihood and severity of a contagion default (affecting respectively $I^{N^{sub}}$ and $LGD^{N^{sub}}$) depend on the identity of the bank's counterparties in a subsystem. Thus, the statistical properties of $L_i^{N^{sub}}$ depend on the subsystem. This implies that the set of conditioning events, $e(N^{sub})$, depends on the subsystem as well. We concretise these points in Section 3.2.3 below.

3.2. Shapley values as measures of systemic importance

Intuitively, a method for the attribution of systemic risk to individual institutions should possess several basic features. First, it should deliver institution-specific values of systemic importance that add up *exactly* to system-wide risk. Second, it should assign zero systemic importance to any institution that does not carry any risk. Third, if two institutions are ex ante identical, then the attribution method should assign to them equal levels of systemic importance. Fourth, the attribution method should be linear in the risk metric as this would allow a practitioner to account for model uncertainty by combining alternative risk metrics in a straightforward way.

The only attribution method that possesses these four features is the Shapley value methodology, which we present in Section 3.2.1.

The objective at hand determines the specific application of the Shapley value methodology. Suppose that the plan is to introduce a scheme for insuring non-bank creditors against losses in systemic events and to make each bank pay an actuarially fair insurance premium. The application of the Shapley value methodology that delivers such bank-specific premia is the participation approach (PA), which we present in Section 3.2.2. Alternatively, policymakers may try to avoid the build-up of systemic vulnerabilities by charging each bank according to its contribution to system-wide risk. Such a contribution should reflect not only the risk that the bank imposes on its own non-bank creditors but also the risk that it imposes, through interbank linkages, on the non-bank creditors of other banks.

[4] A more cumbersome definition of ES, which is general enough to accommodate any loss distribution, is provided in Appendix B. We use that definition for our numerical analysis.

We argue in Section 3.2.3 that, in this case, it is the generalised contribution approach (GCA) that provides the appropriate measure of systemic importance.

3.2.1. General specification[5]

At the heart of Shapley values is a so-called characteristic function ϑ. In the context of a banking system, ϑ maps any subsystem $N^{sub} \subseteq N$ into a measure of risk and needs to satisfy two weak conditions. First, it should be defined on each of the 2^n possible subsystems of banks.[6] Second, when it is applied to the entire system, it should coincide with the chosen measure of system-wide risk. Thus, in the light of Eq. (1), it should be the case that $\vartheta(N) = ES(N)$.

The Shapley value of bank i is a weighted average of the increments of risk, as measured by ϑ, that this bank generates when it joins any possible subsystem comprised of other banks. Denoting the risk of subsystem $N^{sub} \subseteq N$ by $\vartheta(N^{sub})$ and the risk in that subsystem without bank i by $\vartheta(N^{sub} - i)$, the Shapley value of bank i in system N is:

$$ShV_i(N;\vartheta) = \frac{1}{n}\sum_{n_s=1}^{n}\frac{1}{c(n_s)}\sum_{\substack{N^{sub} \supset i \\ |N^{sub}| = n_s}}(\vartheta(N^{sub}) - \vartheta(N^{sub} - i)) \quad \text{for all } i \in N \tag{3}$$

In this expression, $N^{sub} \supset i$ are all the subsystems $N^{sub} \subseteq N$ that contain bank i, $|N^{sub}|$ stands for the number of banks in subsystem N^{sub}, and $c(n_s) = (n-1)!/(n-n_s)!(n_s-1)!$ is the number of subsystems that contain bank i and are comprised of n_s banks.

For a given characteristic function ϑ, the four desirable features we mentioned at the beginning of Section 3.2 define *uniquely* the Shapley value.[7] In anticipation of our analysis below, we underscore that these features lead to a *fairness property*. This property implies that the increment of the Shapley value of bank i that is due to the presence of bank k in the system equals the increment of the Shapley value of bank k that is due to the presence of bank i:

$$ShV_i(N;\vartheta) - ShV_i(N-k;\vartheta) = ShV_k(N;\vartheta) - ShV_k(N-i;\vartheta) \quad \text{for all } i,k \in N \tag{4}$$

While Eq. (4) holds for any well-defined characteristic function ϑ, the size of the increments on each side of the equality sign *does change* with ϑ. Thus, the choice of a characteristic function is crucial in determining the extent to which – because of interbank links, for example – the measured systemic importance of bank i depends on the risk generated by bank k, and vice versa. Since the fairness property plays a key role in explaining our empirical results, we will discuss it further once we have introduced two characteristic functions that define two specific applications of the Shapley value methodology: PA and GCA.

From a normative perspective, the fairness property is desirable in the context of risk allocation. Banks choose their own risk profiles: they can choose freely their size and exposures to risk factors, and are consenting counterparties in financial contracts. Thus, the symmetric role of the counterparties in an interbank transaction naturally calls for splitting the risk created by this transaction equally between the borrower and the lender.

3.2.2. Participation approach (PA)

As discussed in Section 2, several papers have proposed to measure a bank's systemic importance by the expected losses the bank generates in systemic events. We refer to this approach as PA. In the notation of Eq. (1), systemic events, $e(N)$, are states of the world in which aggregate losses generated

[5] Shapley values were first introduced in Shapley (1953).

[6] These subsystems are: *Ø*, *{1}*, *{2}*, *{3}*,. . .,*{n}*, *{1,2}*, *{1,3}*,. . .,*{n − 1,n}*,. . .,*{1,2,3,. . .,n}*.

[7] Mas-Colell et al. (1995) provides a formal definition of the four features of Shapley values: "additivity", "dummy axiom", "symmetry" and "linearity". These four features determine uniquely the Shapley values when the characteristic function is specified, as here, over *sets* of players, e.g. groups of banks (see Mas-Colell et al. (1995) and Denault (2001)). That said, if the characteristic function is defined over real numbers – representing, for example, player's *fractional* participation in a set – then Shapley values are uniquely determined by the four above features plus monotonicity and scale invariance (see Moulin and Sprumont (2007)).

by the banks in the whole system exceed a particular threshold: $\sum_{i\in N} L_i^N > q^N$. Thus, PA measures systemic importance of bank i as $E(L_i^N|\sum_{i\in N} L_i^N > q^N) = E(L_i^N|e(N))$, which reflects the losses that bank i imposes on *its own* non-bank creditors in systemic events. And the PA measure delivers exactly the actuarially fair premium that bank i would have to pay to a scheme insuring non-bank creditors against losses in systemic events.[8]

Interestingly, PA can also be viewed through the lens of the Shapley value methodology. For this, we only need to find a function ϑ^{PA} that: (i) can be applied to each subsystem $N^{sub} \subseteq N$; (ii) when evaluated at the level of the whole system N, coincides with the measure of system-wide risk: $\vartheta^{PA}(N) = ES(N)$; and (iii) when used as a characteristic function, leads to a Shapley value equal to the PA measure of systemic importance, i.e.

$$ShV_i(N;\vartheta^{PA}) = E\left(L_i^N|e(N)\right) \quad \text{for } \forall i \in N^{sub} \text{ and } \forall N^{sub} \subseteq N \tag{5}$$

Such a characteristic function indeed exists and we can define it as:

$$\vartheta^{PA}(N^{sub}) \equiv E\left(\sum_{i\in N^{sub}} L_i^N|e(N)\right) \quad \text{for each } N^{sub} \subseteq N. \tag{6}$$

Defined in this way, ϑ^{PA} holds two arguments constant across subsystems. First, irrespective of the subsystem, N^{sub}, ϑ^{PA} conditions on events determined at the level of the whole system: $e(N)$. Second, again irrespective of the subsystem in focus, ϑ^{PA} incorporates the losses L_i^N that pertain to bank i when the whole system N is in place. This implies that $\vartheta^{PA}(N^{sub}) - \vartheta^{PA}(N^{sub} - i) = E(L_i^N|e(N))$, for $\forall i \in N^{sub}$ and $\forall N^{sub} \subseteq N$.

Our choice of ϑ^{PA} in Eq. (6) has a purely expositional goal. The chosen function will allow us to draw sharp parallels with the characteristic function of an alternative approach to measuring systemic importance. However, it will lead to the *same* numerical findings as *any* other function consistent with the PA measure in Eq. (5).

3.2.3. Generalised contribution approach (GCA)

The alternative, GCA approach captures the risk that a bank generates on its own as well as the bank's contribution to the risk in each subsystem of other banks. To gauge such contributions, the approach equates the exclusion of a bank from a subsystem with the removal of the *entire* risk that the bank generates in that subsystem.

We define GCA with the following function:

$$\vartheta^{GCA}(N^{sub}) \equiv E\left(\sum_{i\in N^{sub}} L_i^{N^{sub}}|e(N^{sub})\right) = ES\left(N^{sub}\right) \quad \text{for each } N^{sub} \subseteq N. \tag{7}$$

As it satisfies the two criteria we stated in Section 3.2.1, ϑ^{GCA} qualifies as a characteristic function.

Even though the PA and GCA characteristic functions are consistent with the same measure of system-wide risk – i.e. $\vartheta^{GCA}(N) = \vartheta^{PA}(N) = ES(N)$ – the allocation of this risk across individual banks differs between the two approaches for two important reasons. First, in contrast to ϑ^{PA}, ϑ^{GCA} incorporates conditioning events, $e(N^{sub})$, that can change with the subsystem.

Second, unlike ϑ^{PA}, ϑ^{GCA} allows the stochastic losses $L_i^{N^{sub}}$ incurred by the non-bank creditors of bank i to depend on the subsystem considered. Of course, such dependence is redundant if there are no interbank links and failures occur only because of shocks coming from outside the banking system. In such a setting, Tarashev et al. (2010) propose their contribution approach, which we generalise here to account for an interbank network.

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[8] In this paper, we do not take a stand on the optimal insurance scheme. That said, an insurance scheme based on PA would penalise banks for being exposed to systematic factors and, thus, would mitigate the deficiencies in standard deposit insurance contracts identified by Pennacchi (2006). By taking account of bank size, the correlation of bank returns and bank interconnectedness, it would also fulfill the criteria of Acharya et al. (2010) for an efficient deposit insurance scheme. In a related paper, Chiang et al. (2007) argue that only a mandatory insurance scheme would deliver optimal social welfare.

When there is an interbank network, the absence of some banks from a particular subsystem can imply the absence of some interbank links and, thus, the absence of contagion risk for the banks remaining in the subsystem. This implies that the risks faced by the non-bank creditors of a particular bank do depend on which other banks are in the subsystem. To capture this, we implement the following two adjustments before evaluating the *ES* in a subsystem $N^{sub} \subseteq N$. First, we replace the non-equity liabilities of banks in N^{sub} to banks outside N^{sub} with liabilities to hypothetical entities outside the *entire* system.[9] Second, the exposures of banks inside N^{sub} to banks outside N^{sub} are replaced with a risk-free asset with an equal value. Specifically, if bank j is not in a subsystem containing bank i, an interbank loan from i to j, $IBA_{i,j}$, is replaced with a risk free asset of value $IBA_{i,j} * (1 - LGD_j * PD_j)$, where PD_j and LGD_j are the probability of default of bank j and the (expected) loss-given-default on the interbank exposure. Not implementing these adjustments leads to measures that do not equate the removal of a bank from a (sub)system with the removal of the entire risk that this bank generates.[10]

It is important to note that GCA gauges the consequences of the hypothetical removal of a bank from a (sub)system, keeping everything else constant. Thus, we preserve the links among the banks that remain in the (sub)system as well as their links to non-banks. Adjusting these links would bias the measure of systemic importance. It would effectively change the structure and level of system-wide risk, while the aim of GCA (as well as PA) is to allocate a *given* level of this risk to individual banks.

3.2.4. The fairness property under PA and GCA

The fairness property of Shapley values helps underscore key differences between PA and GCA. By Eq. (6), ϑ^{PA} does not allow the Shapley value of a bank to change with the subsystem considered: $ShV_i(N^{sub}; \vartheta^{PA}) = ShV_i(N; \vartheta^{PA})$ for each $N^{sub} \subseteq N$. It thus gives rise to an uninformative fairness property. For each $i, k \in N$:

$$ShV_i(N; \vartheta^{PA}) - ShV_i(N - k; \vartheta^{PA}) = ShV_k(N; \vartheta^{PA}) - ShV_k(N - i; \vartheta^{PA}) = 0. \quad (8)$$

GCA Shapley values exhibit a different fairness property. This comes to the fore if the simultaneous presence of two banks $i, k \in N$ raises system-wide risk, which occurs as long as these banks have strictly positive sizes and PDs and positively correlated assets. In such a case, the Shapley value that ϑ^{GCA} assigns to either bank in the entire system is higher than the corresponding Shapley value in a system excluding the other bank:

$$ShV_i(N; \vartheta^{GCA}) - ShV_i(N - k; \vartheta^{GCA}) = ShV_k(N; \vartheta^{GCA}) - ShV_k(N - i; \vartheta^{GCA}) > 0. \quad (9)$$

To see what the fairness property of GCA implies concretely, recall the thought experiment at the end of the introduction, which illustrates how a link between an interbank lender (e.g. bank i) and an interbank borrower (bank k) raises system-wide risk. Eq. (9) implies that ϑ^{GCA} splits the rise in system-wide risk equally between the interbank lender and the interbank borrower. In this way, GCA captures the idea that an interconnected bank can contribute to system-wide risk through two channels. First, as illustrated by bank i in the example, by directly imposing losses on its own non-bank creditors. Second, as illustrated by bank k, by indirectly imposing losses on the non-bank creditors of other banks that it has borrowed from.

PA treats the same interbank link in a different way. To see why, note that this link raises the potential losses to the non-bank creditors of the interbank lender (bank i) but does not affect directly the non-bank creditors of the interbank borrower (bank k). Thus, the link raises the expected participation of the interbank lender in systemic events but leaves that of the interbank borrower unchanged. And since participation in systemic events is all that matters to PA (recall Section 3.2.2), ϑ^{PA} attributes the entire risk associated with the interbank link to the interbank lender.

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[9] We could equally assume that all banks remained in the (sub)system but some were fully funded with equity and, thus, could not impose credit losses on non-banks or other banks.

[10] In Annex 1 of Drehmann and Tarashev (2011b), we show that such measures would be roughly in line with PA but in substantial disagreement with GCA.

4. Empirical implementation

The Shapley value methodology can be implemented in any stochastic environment that gives rise to a well-defined loss distribution in each subsystem. In this section, we explain how we simulate one such stochastic environment.

Our calibration and simulation procedure has four general steps. First, we specify the exogenous stochastic shocks that affect banks' claims on non-banks and, thus, drive fundamental defaults. Second, taking fundamental defaults as given, we obtain contagion defaults, which depend on the network of interbank exposures. Third, we derive how fundamental and contagion defaults impose losses on banks' non-bank creditors. Fourth, we combine the first three steps in order to generate a probability distribution of bank-level losses. On the basis of this distribution, we measure system-wide risk and the systemic importance of individual banks.

We implement this procedure for two types of banking systems: one comprised of nine hypothetical banks and another comprised of 20 real-world banks. The following four subsections outline the four steps of the procedure, taking banks' balance sheets and, thus the interlinkages in each system, as given. Concrete descriptions of these balance sheets appear in Sections 5 and 6.

4.1. Shocks to non-bank assets and fundamental defaults

The banking system is subject to exogenous shocks that affect only the value of non-bank assets. We consider a one-period specification, in which the non-bank assets of bank i, NBA_{i0}, are hit by a common shock (M) and an idiosyncratic shock (Z_i). These shocks are mutually independent normal variables with zero means and equal variances:

$$NBA_{i1} = NBA_{i0} + \rho_i \cdot M + \sqrt{1-\rho_i^2}Z_i \quad \text{for all } i \in \{1, 2, \cdots, n\} \tag{10}$$

where $\rho_i \in [0, 1]$ is the common factor loading.[11] To simplify the notation, we henceforth suppress the time subscripts 0 and 1, as they will be obvious from the context.

If the exogenous shocks M and Z_i, depress a bank's total assets (i.e. $NBA_i + IBA_i$) below the value of its non-equity liabilities (i.e. $NBL_i + IBL_i$), this bank experiences a fundamental default. Being driven exclusively by exogenous shocks, fundamental defaults are independent of the (sub)system considered. We refer to the probability of a fundamental default as the bank's *fundamental PD*. In the light of Eq. (10), the probability of joint fundamental defaults increases in ρ_i.

In turn, a contagion default occurs if bank i has not experienced a fundamental default, but losses on interbank assets, IBA_i, add to those on NBA_i so that the combined losses wipe out EQ_i. We refer to the probability of a contagion default as *contagion* PD.

It remains to specify the variance of the two shocks and the common-factor loadings ρ_i. As we explain in detail in Section 4.4, we calibrate the shock variance such that each bank's *overall* PD, i.e. the sum of its fundamental and contagion PDs, matches Moody's KMV estimates of the bank's probability of default. In turn, we calibrate common factor loadings, ρ_i, on the basis of correlation matrices estimated by Moody's KMV for 2006–2009. We condense these matrices to 20 bank-specific common-factor loadings (see Appendix B), which we assign to the 20 real-world banks in our sample. These loadings average *0.67*, implying an average correlation of exogenous shocks of roughly $0.67^2 = 0.45$. When calibrating a hypothetical banking system, we use this average value for the common factor loading of each bank.

4.2. Interbank networks and contagion defaults

We study the implications of interbank networks in two different ways. To illustrate specific conceptual arguments, we first analyse four hypothetical systems with stylised interbank networks (see

[11] In a recent article, Suh (2012) analyses system-wide risk and the systemic importance of individual institutions on the basis of a similar specification of bank assets and PA. While he considers correlated defaults that are driven by an observable common factor, he abstracts from interbank linkages.

Section 5). Then, we analyse the interbank network linking 20 real-world banks (see Section 6). In the rest of this subsection, we explain how we derive this network on the basis of available data.

The lack of publicly available information on bilateral interbank exposures implies that we need to estimate the interbank network matrix for the system of 20 real-world banks. Most of the literature does so via a maximum entropy (ME) matrix of bilateral positions, which possesses the following properties. First, the sum of the entries in each row/column corresponding to a particular bank equals the aggregate level of this banks' liabilities/assets vis-à-vis the other banks in the system. Second, the interbank assets and liabilities of each bank are distributed as uniformly as possible across the other banks in the system. Since the second property is clearly ad hoc, however, focusing purely on the ME matrix could bias our results.[12]

This prompts us to consider probable alternatives, without attempting to conduct an exhaustive analysis of network structures.[13] To derive one alternative, we randomly generate 225 matrices as perturbations around the ME matrix. Each of these matrices possesses the first property of the ME matrix and is thus consistent with the observed data on each bank's aggregate interbank assets and liabilities. Then, we choose the randomly generated matrix that differs the most (in terms of the 2-norm distance) from the ME matrix. As 75% of the off-diagonal entries of the chosen matrix are zero – meaning that it concentrates all interbank positions in 25% of the possible links – we refer to it as a high-concentration (HC) matrix.[14] In addition, in order to parallel the analysis of hypothetical systems, we also consider the zero interbank matrix, which rules out interbank positions.

An interbank network propagates the exogenous shocks through the system, thereby generating contagion defaults. There could be several rounds of contagion. For instance, it can be the case that shocks to non-bank assets lead to the failure of one bank. Fundamental shocks plus this failure can lead, in turn, to the contagion default of another bank, which then may induce more contagion defaults of other banks. We account for contagion chains by implementing the "clearing algorithm" of Eisenberg and Noe (2001). They prove that their algorithm delivers the unique set of contagion defaults for any draw of exogenous shocks.

4.3. Losses to a bank's creditors

Irrespective of whether a bank has experienced a fundamental or a contagion default, it imposes losses on its creditors via two channels. First, any drop in a bank's assets below the value of its non-equity liabilities causes the bank to default and leads to a write-down of these liabilities. We do not net out interbank positions and assume that non-bank creditors are senior to bank creditors. The latter assumption means that the drop in assets translates into a write-down of non-banks liabilities, *NBL*, only when interbank liabilities, *IBL*, have been written down to zero.[15] Second, there are bankruptcy costs, which reduce further the value of a defaulting bank's liabilities vis-à-vis both non-banks and other banks by a fraction a. For concreteness, we set $a = 20\%$ in all simulations.[16]

Thus, the loss-given-default to non-banks, LGD_i, which appears in Eq. (2) and enters our measure of system-wide risk, is the product of two terms. The first is the faction of *NBL* that is written down because of the deterioration of the bank's assets. The second is the bankruptcy cost, a.

[12] Mistrulli (2011) and van Lelyveld and Liedorp (2006) show that the ME assumption tends to lead to an underestimation of contagion, and thus of system-wide risk, when applied to real-world banking systems.

[13] For example, we do not try to detect and analyse specific patterns in banks' interconnectedness, for which Squartini and Garlaschelli (2011) have developed a rigorous methodology. Likewise, in contrast to David and Lehar's (2011) theoretical investigation, we do not consider the impact of interlinkages on banks' behaviour in the event of a counterparty's default.

[14] Appendix B contains further detail on the construction of ME and HC matrices.

[15] As reported by Upper (2011), non-bank depositors are for example senior to interbank creditors in Germany. Relaxing the assumption about the seniority of non-bank depositors would weaken the impact of the interbank network on system-wide risk but would not alter qualitatively the results we obtain below.

[16] For a sample of failed US banks, James (1991) estimates that losses on bank assets amount to around 30% on average. This reflects direct losses as well as losses of charter value and at least 10 percentage points of administrative and legal expenses. Background calculations reveal that the insights of our paper are preserved for any a between 10% and 30%, even if a is allowed to differ between bank and non-bank creditors.

4.4. Probability distribution of losses, ES and measures of systemic importance

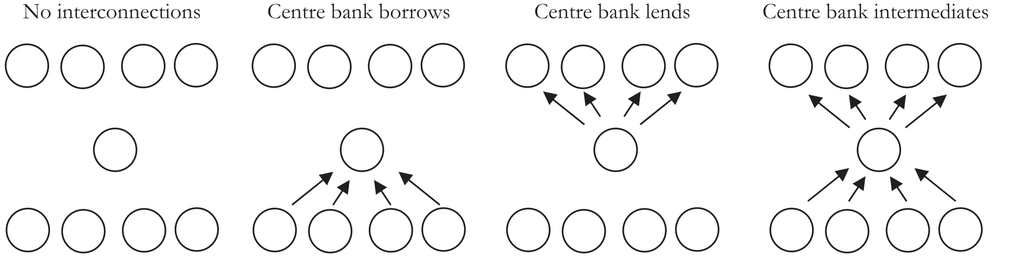
For any subsystem, N^{sub}, we proceed as follows. First, we draw one million sets of exogenous shocks.[17] In line with Eq. (10), each set contains a cross-section of correlated bank-specific shocks to non-bank assets. Such a set of shocks determines which banks in N^{sub} experience fundamental defaults (see Section 4.1). If some banks do experience fundamental defaults, we derive which other banks experience contagion defaults (Section 4.2). Then, on the basis of the fundamental and contagion defaults, we calculate the losses to non-bank creditors, $\left\{L_i^{N^{sub}}\right\}_{i\in N^{sub}}$ (Section 4.3). Repeating this for all sets of simulated shocks, we arrive at the joint distribution of $\left\{L_i^{N^{sub}}\right\}_{i\in N^{sub}}$.

Finally, we measure system-wide risk and individual banks' systemic importance. To calculate system-wide risk, we use the joint distribution of losses for the whole system, i.e. $\left\{L_i^N\right\}_{i\in N}$, to obtain the distribution of $\sum_{i=1}^{n} L_i^N$ and then derive ES in Eq. (1). Given Eq. (5), we only need the joint distribution of $\left\{L_i^N\right\}_{i\in N}$ to measure systemic importance under PA. By contrast, GCA requires that we derive the joint distribution of $\left\{L_i^{N^{sub}}\right\}_{i\in N^{sub}}$ for each $N^{sub} \subseteq N$. This amounts to 2^n joint probability distributions, one for each subsystem in a system of n banks. In all cases, we set the threshold $q^{N^{sub}}$ to the 99th percentile of the relevant distribution.

An intermediate output of our simulation procedure is the overall PD of each bank, which is equal to its fundamental PD plus its contagion PD. For the system of 20 real-world banks, we calibrate the variance of exogenous shocks in Eq. (10) so that, when the ME matrix of interbank exposures is in place, each bank's overall PD matches exactly the average value of the corresponding 1-year expected default frequency (EDF), as estimated by Moody's KMV for 2006–2009. These EDFs average 0.6% in the cross section, have a median of 0.25% and a standard deviation of 0.8%. When we work with hypothetical systems, we calibrate exogenous shocks so that the fundamental PD of each bank is 0.42%, halfway between the median and mean EDFs of the 20 real-world banks.

5. Hypothetical networks

In this section, we build intuition for the way in which the PA and GCA measures of systemic importance reflect interbank linkages. We do so by analysing four stylized 9-bank systems that differ only with respect to the interbank network (Graph 1). One of the four systems is without interbank connections. In each of the other three systems, there is an interbank network and one bank plays a central role, henceforth the centre bank. In the first two of these networks, four periphery banks either borrow from or lend to the centre bank, while the remaining four banks do not participate in the



Graph 1. Hypothetical interbank networks.

[17] Given that we need to work with a finite number of shock sets, our numerical results are inevitably affected by simulation noise. To quantify the effect of this noise, we evaluate how our findings would change if we were to start with different, random draws of one million sets of exogenous shocks. We establish that the changes are quantitatively negligible and do not alter any of our conceptual messages (presented below). This leads us to henceforth abstract from simulation noise in our discussion.

Table 1
Balance sheets in hypothetical banking systems.

Bank type	No interbank connections			Centre bank borrows			Centre bank lends			Centre bank intermediates		
	NBL	IBL	IBA	NBL	IBL	IBA	NBL	IBL	IBA	NBL	IBL	IBA
Unconnected	87	0	0	87	0	0	87	0	0			
PB lender				87	0	8	87	0	0	87	0	8
PB borrower				87	0	0	87	8	0	87	8	0
Centre bank				87	32	0	87	0	32	87	32	32

PB = periphery bank; NBL = non-bank liabilities (=size), IBL = interbank liabilities; IBA = interbank assets. Each bank has 5 units of equity. Non-bank asset holdings ensure the balance sheet identity.

interbank market. The last network captures in a stylised fashion the real-life phenomenon where the centre bank intermediates between periphery banks, four of which borrow and four of which lend to it.[18]

Balance sheets in the different systems are reported in Table 1. In all cases, banks have 5 units of equity and borrow 87 units from non-banks. This means that they are of the same size (recall Section 3.1). Periphery banks have 8 units of interbank liabilities (if they borrow from the centre bank) or 8 units of interbank assets (if they lend), which fully determines the interbank positions of the centre bank. The resulting share of interbank positions in a periphery (centre) bank's balance sheet is close to the mean (maximum) of the corresponding shares in our sample of 20 large banks (see Section 6 below). When simulating the probability distribution of losses, we follow Section 4.

5.1. Systemic importance under different network structures

A priori, the network structure should affect both the absolute and relative levels of systemic importance of individual banks. If all banks are ex ante the same, the network that is more vulnerable to contagion defaults should lead to higher levels of system-wide ES and thus to higher uniform Shapley values. In a system of heterogeneous banks, however, it seems intuitive that the most interconnected bank – e.g. the centre bank – should have the highest Shapley value as it propagates shocks through the system and is subject to propagated shocks.

As expected, system-wide risk increases with the potential for contagion (Table 2, first column). By design, the system without an interbank network does not experience contagion defaults and, as a result, has the lowest ES. In comparison, the level of ES is higher in the two systems where the centre bank either borrows from or lends to the periphery, thus making contagion defaults possible. Finally, ES is highest when the centre bank intermediates between periphery banks. In this case, there is an additional channel of shock propagation, as the default of one (or several) borrowers in the periphery can be transmitted, via the default of the centre bank, to lenders in the periphery.

More interestingly, Table 2 also shows how the network structure and banks' position in it affect systemic importance.[19] For concreteness, we focus only on GCA Shapley values in this subsection (second to last column in the table) and compare GCA and PA Shapley values in the next. Not surprisingly, all banks feature their lowest Shapley values in the system without an interbank network, when system-wide ES is at its lowest level.

Across the three setups with interbank linkages, the Shapley value of the centre bank is always higher than that of any periphery bank. While this result is intuitive, it is not obvious when looking simply at banks' PDs. The system in which the centre bank only borrows from the periphery is a case in point. Here, the centre bank is a source of risk for periphery banks but is never pushed into default by another bank, i.e. the centre bank has a contagion PD of zero. This leads to an overall PD of the cen-

[18] For theoretical and empirical investigation of such "tiered" network structures, see e.g. Bech and Atalay (2010) or Craig and von Peter (2010).

[19] All the insights we draw from Table 2 are robust to including losses to equity holders in our measures (see Table A.1 in Appendix A).

Table 2
System-wide risk and systemic importance in hypothetical banking systems.[a]

	ES	f.PD[b]	c.PD[c]	Shapley values	
				GCA	PA
No interconnections	4.01				
All (9 banks)		0.42	0	0.45	0.45
Centre bank borrows	4.95				
Centre bank		0.42	0	0.90	0.64
PB lender (4 banks)		0.42	0.10	0.56	0.66
Unconnected (4 banks)		0.42	0	0.45	0.42
Centre bank lends	5.16				
Centre bank		0.42	0.51	1.06	1.63
PB borrower (4 banks)		0.42	0	0.57	0.48
Unconnected (4 banks)		0.42	0	0.45	0.40
Centre bank intermediates	7.73				
Centre bank		0.42	0.51	2.06	1.78
PB lender (4 banks)		0.42	0.29	0.71	1.00
PB borrower (4 banks)		0.42	0	0.71	0.49

[a] All values are in per cent. ES and Shapley values are expressed per unit of system size. ES pertains to the system as a whole. All other values pertain to a bank in the particular group.
[b] Fundamental PD.
[c] Contagion PD.

tre bank that is lower than the PD of a lender in the periphery (0.42% vs. 0.52%). Nonetheless, the GCA Shapley value of the former bank is roughly 60% larger than that of the latter.

This result is driven by the fairness property of Shapley values. As discussed in Section 3.2.4, this property implies that, when an interbank link raises system-wide risk, GCA splits the rise equally between the borrower and the lender. Our stylised systems underscore the fairness property, as they keep constant across banks the drivers of system-wide risk that are unrelated to the interbank network – i.e. fundamental PDs, correlations of exogenous shocks and size.

Consider, for instance, the top two panels in Table 2. Switching from the system without a network to the system in which the centre bank only borrows from the periphery raises the system-wide ES by 0.94 percentage points (from 4.01% to 4.95%). Attributing this rise equally to interbank borrowers and lenders implies that $0.5 * 0.94\% = 0.47\%$ should be allocated to the centre bank and $(0.5 * 0.94\%)/4 = 0.1175\%$ to each of the four lending periphery banks. This matches almost perfectly the actual increases in Shapley values relative to the system without an interbank network. These are 0.45% (=0.9% − 0.45%) for the centre bank and 0.11% (=0.56% − 0.45%) for the periphery banks.[20] The same intuition holds if the centre bank acts only as an interbank lender or if it intermediates between periphery banks.

When the centre bank intermediates between periphery banks (Table 2, bottom panel), its Shapley value (2.06%) is larger than the sum of its Shapley values when it only borrows or lends (0.90% + 1.06%). The reason is that an intermediating centre bank creates indirect links between periphery banks. Because of these links, an adverse shock to an interbank borrower in one part of the system can cause the default of an interbank lender in another part. This transmission of distress is captured by our measure of contributions to system-wide risk, as it adopts a holistic perspective and takes into account both direct and indirect linkages in the system.

5.2. Comparing GCA and PA: borrowers vs. lenders

PA and GCA can generate materially different Shapley values in the presence of an interbank network. To a large extent, this is due to the two approaches treating the counterparties in interbank links differently. As explained in Section 3.2.2 above, PA attributes the risk generated by an interbank link to

[20] The match is not exact because we compare two different systems, underpinned by different interbank networks, whereas the fairness property in Eq. (9) holds exactly within a *given* system of banks.

Table 3
Centre bank as a central counterparty.

Bank type	Balance sheets[a]			Bank-specific risk measures[b]			
						Shapley values	
	NBL	IBL	IBA	f.PD	c.PD	GCA	PA
Centre bank	0	32	32	0	0.19	0.22	0
PB lender	87	0	8	0.42	0.07	0.58	0.70
PB borrower	87	8	0	0.42	0	0.58	0.52

[a] PB = periphery bank; NBL = non-bank liabilities (size); IBL = interbank liabilities; IBA = interbank assets. The centre bank has 3 units of equity and, to satisfy the balance sheet identity, holds 3 units in a risk-free asset. All other banks have 5 units of equity and their investments in risky non-bank assets ensure the balance sheet identity.

[b] In per cent. f.PD and c.PD = fundamental and contagion PD, respectively. Shapley values are per unit of system size.

the lender. By contrast, GCA treats the two counterparties of an interbank link symmetrically and, thus, splits the associated risk equally between them. This raises (lowers) the GCA Shapley value of an interbank borrower (lender) relative to the corresponding PA value.

Indeed, in all the stylised systems, an interbank borrower (lender) has a higher (lower) Shapley value under GCA than under PA (compare the last two columns in Table 2). Importantly, this leads the alternative approaches to rank-order differently individual banks' systemic importance. For example, if the centre bank only borrows from the periphery, PA Shapley values attribute lower systemic importance to it than to interbank lenders in the periphery (0.64% vs. 0.66%). By contrast, the centre bank has the highest GCA Shapley value in that system (0.9% vs. 0.56% for periphery lenders). This reflects the fact that, by being the only propagator of shocks, this bank is the main *contributor* to system-wide risk.

5.3. Comparing GCA and PA: intermediaries

It is also of interest to compare the implications of GCA and PA for a centre bank that intermediates on the interbank market, thus creating indirect exposures between periphery lenders and borrowers (Table 1, right-most panel). By the fairness property in Eq. (9), part of the risk stemming from these exposures raises the GCA Shapley value of the centre bank. By contrast, PA attributes this risk entirely to periphery lenders. In the end, the PA Shapley value of the intermediating bank is lower than the GCA value (1.78% vs. 2.06%, bottom panel of Table 2).

The difference between PA and GCA is particularly stark when we assume that the centre bank acts as a central counterparty (CCP), thus only intermediating on the interbank market but neither lending to nor borrowing from non-banks. As explained in Section 3.1, the centre bank is then of zero size. We report the complete balance sheet information for all banks and associated risk measures in Table 3.

Whereas the GCA Shapley values are in accordance with the intuition that the centre bank contributes to system-wide risk, the PA Shapley value of this bank is zero (Table 3, last two columns). The PA Shapley values reflect the fact that, since the centre bank does not borrow from non-banks, it does not contribute directly to their credit losses and, thus, does not participate in systemic events (in the sense introduced in Section 3.2.2). That said, this bank creates indirect links between lending and borrowing periphery banks, thereby contributing to system-wide risk. GCA captures this risk-transmission channel and assigns a Shapley value of 0.22% to the CCP.

6. Analysing the systemic importance of 20 large banks

In this section, we build on the insights of the stylized hypothetical networks in order to analyse the systemic importance of 20 large internationally active banks. The balance sheet data for these 20 banks – provided by Bankscope for end-2009 – are summarised in Table 4. As reported by the first row in this table, the largest bank in our sample (bank *P*) is 3.5 times larger than the smallest (bank *S*). The table also shows the shares of each bank's interbank assets (*IBA* = loans and advances to banks)

Table 4
Balance sheet statistics for 20 large internationally active banks (in per cent).

	Bank																			
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
Size	3.8	4.8	5	3.1	7.7	5.6	6.6	3.6	4.9	4.1	3.9	5.1	4.7	5.5	3.5	8.2	6.8	4.4	2.3	6.4
IBAs	5.8	3.8	21	12.6	5.4	9.6	3.1	3.2	3.7	6.6	10.3	18.6	3.4	13.4	8.4	4.3	7.9	7.2	20.8	21.7
IBLs	1.9	4	9.2	17.3	8.9	13	3.1	2.3	7.5	9.5	5	14.2	8.4	20.8	12.4	11.4	5.9	8.3	25.1	9

Size = liabilities to non-banks (NBL) divided by system-wide non-bank liabilities; IBAs = a bank's interbank assets divided by its total assets; IBLs = a bank's interbank liabilities divided by its total non-equity liabilities.

and interbank liabilities (*IBL* = deposits from banks) in the bank's total assets and non-equity liabilities, respectively. Both the *IBA* and *IBL* shares average approximately 10% across banks.[21]

We simulate the probability distribution of losses following the procedure we outlined in Section 4 above. As discussed in Section 4.2, we analyse three different interbank networks, two of which – the ME and HC matrix – are consistent with the available data. In line with the stylised system, we also consider the zero matrix, without interbank links. In order to study how the alternative network structures affect banks' PDs and systemic importance, we apply the same distribution of exogenous shocks to each interbank matrix (see Section 4.4 for the specification of these shocks' variance).

For each of the three interbank networks, Table 5 reports fundamental and contagion PDs. As we are not allowed to publish bank-specific PD estimates from Moody's KMV, we only report the rank of fundamental PDs (1 = highest and 20 = lowest). The values of these PDs remain the same across networks. By contrast, contagion PDs, for which we report actual values, do change with the structure of the network.

In the remainder of this section, we examine the dependence of system-wide risk and the systemic importance of individual banks on the interbank network. For the analysis of systemic importance, we first focus on results obtained under GCA and then compare these results to those obtained under PA. Of course, system-wide risk and systemic importance are also influenced by banks' size, PD and common factor loadings. We analyse these drivers in Appendix C.

6.1. Impact of the network on system-wide risk

In this section, we examine how the measured level of system-wide risk changes when we replace the zero matrix of interbank exposures with either the ME or HC matrix.

Not surprisingly, interbank linkages raise system-wide ES, as they give rise to positive contagion PDs. The contagion PDs generated by interbank linkages lead to a higher likelihood of joint defaults: for example, the probability of four or more defaults increases from 0.16% in the system without interbank links to 0.32% under the ME and 0.39% under the HC matrix. In turn, this underpins a roughly equal rise in system-wide ES: from 3.22% under the zero matrix to 4.34% under the ME matrix and to 4.50% under the HC matrix (Table 5).

The values of ES implied by the ME and HC matrices differ little from each other, even though these matrices differ materially with respect to the distribution of each bank's interbank lending and borrowing across counterparties. This is because switching from the ME to the HC matrix sets two counteracting forces at work. To see this, recall that the ME matrix spreads interbank positions as widely as possible, whereas these positions are highly concentrated in the HC matrix. Thus, if two banks are connected under both matrices, the shock caused by the default of one bank would tend to affect more strongly the other under the HC matrix, as the particular bilateral exposure would tend to be larger. On its own, this increases the probability of joint defaults under the HC matrix relative to that under the ME matrix, making the difference between the respective ESs positive. That said, each shock is

[21] When interpreting the *IBA* and *IBL* shares, it is useful to keep in mind that (i) total interbank positions of any of the 20 banks in our sample include positions vis-à-vis banks that we abstract from and (ii) part of the actual interbank links stem from off-balance sheet positions, which we cannot identify in our data. The former (latter) implies that we may overstate (understate) the importance of the network.

Table 5
System-wide risk and systemic importance in a system of 20 large banks (in per cent).

Bank	f.PD rank	No interbank network ES = 3.22 median PD = 0.14 Shapley values			ME matrix ES = 4.34 median PD = 0.25 Shapley values			HC matrix ES = 4.50 median PD = 0.31 Shapley values		
		c.PD	GCA	PA	c.PD	GCA	PA	c.PD	GCA	PA
A	1	0	0.39	0.37	0.07	0.37	0.31	0.04	0.37	0.32
B	2	0	0.54	0.58	0.05	0.53	0.52	0.04	0.54	0.44
C	3	0	0.43	0.45	0.12	0.53	0.48	0.17	0.54	0.46
D	4	0	0.17	0.11	0.19	0.28	0.21	0.15	0.28	0.21
E	5	0	0.50	0.49	0.07	0.59	0.56	0.08	0.55	0.56
F	6	0	0.27	0.28	0.05	0.35	0.28	0.06	0.36	0.32
G	7	0	0.32	0.34	0.08	0.34	0.40	0.12	0.37	0.46
H	8	0	0.09	0.07	0.02	0.09	0.07	0.02	0.10	0.08
I	9	0	0.10	0.10	0.07	0.13	0.15	0.12	0.15	0.18
J	10	0	0.08	0.08	0.10	0.14	0.14	0.04	0.11	0.10
K	11	0	0.04	0.03	0.14	0.09	0.13	0.29	0.13	0.21
L	12	0	0.07	0.07	0.10	0.16	0.16	0.24	0.28	0.28
M	13	0	0.06	0.06	0.06	0.08	0.10	0.04	0.08	0.08
N	14	0	0.06	0.07	0.14	0.24	0.22	0.09	0.21	0.19
O	15	0	0.03	0.02	0.06	0.05	0.07	0.04	0.06	0.05
P	16	0	0.04	0.04	0.07	0.09	0.15	0.05	0.12	0.12
Q	17	0	0.03	0.03	0.07	0.07	0.12	0.08	0.07	0.13
R	18	0	0.01	0.01	0.04	0.02	0.04	0.05	0.03	0.05
S	19	0	0.00	0.00	0.15	0.08	0.07	0.08	0.04	0.04
T	20	0	0.00	0.00	0.13	0.09	0.17	0.17	0.13	0.22

f.PD = fundamental PD; c.PD = contagion PD; GCA = generalised contribution approach; PA = participation approach; ME = maximum entropy; HC = high concentration. ES and Shapley values are per unit of system size.

propagated to fewer institutions under the HC than under the ME matrix. On its own, this leads to a negative difference between the respective ESs. In our particular case, the two effects roughly balance each other.[22]

6.2. Impact of the network on systemic importance

As interbank linkages raise system-wide risk, they also tend to elevate the systemic importance of individual banks. And the impact is strongest on banks that are most active in the interbank market. Consider banks *L*, *N*, *S* and *T*, for example, which feature some of the largest interbank assets or liabilities as shares in the respective balance sheets (see Table 4). These are the banks that experience the greatest rises in GCA Shapley values when interbank linkages are introduced (compare the left to the middle and right-hand panels of Table 5).

The two network structures HC and ME can lead to materially different levels of individual banks' systemic importance. This is despite the similarity of the corresponding levels of system-wide risk, which we reported in the previous subsection. For example, the GCA Shapley value of bank *L* is 75% higher when interbank markets are characterised by the HC matrix (0.28%) rather than the ME matrix (0.16%) (Table 5, middle and right-hand panels). This increase is largely driven by the substantial rise in the bank's contagion PD (from 0.10% to 0.24%), which in turn is caused by the greater concentration of its interbank lending under the HC matrix. The story is reversed for bank *S*, as the HC matrix concentrates the exposures of this bank to low-PD counterparties. As we keep the distribution of

[22] The similarity between the measured levels of ES under the ME and HC matrices also stems from similarities of the underlying LGDs. Given our parameterisation of 20% bankruptcy costs and the assumption of non-bank creditors' seniority, we obtain roughly 20% LGD on non-bank debt, irrespective of the network structure. By contrast, LGD on interbank debt increases from zero under the zero matrix (by construction) to about 37% on average under both the ME and HC matrices.

Table 6
Systemic importance measures and two indicators of interbank activity.

	GCA_{ME}–GCA_0	GCA_{HC}–GCA_0	GCA_{ME}–GCA_{HC}	PA_{HC}–GCA_{HC}
LI_{nwk}	79	77		46
BI_{nwk}	64	59		−64
$LI_{ME} - LI_{HC}$			67	
$BI_{ME} - BI_{HC}$			83	

Correlations, in per cent. The subscript *nwk* stands for ME for the correlations of GCA_{ME}–GCA_0 with *LI* and *BI* and for HC for the other cases.

exogenous shocks fixed, this lowers the bank's contagion PD and, ultimately, its GCA Shapley value relative to that under the ME matrix.

The impact of the network structure goes beyond contagion PDs. Bank *P*, for example, experiences a 33% rise in its GCA Shapley value (from 0.09% to 0.12%), even though its contagion PD falls from 0.07% to 0.05% when the interbank network changes from the ME to the HC matrix. Having the second largest level of interbank liabilities, this bank is a key propagator of shocks through the system. And the impact of shock propagation is greater under the HC matrix where the interbank network is more concentrated. This is captured by GCA, thus underpinning a higher Shapley value for bank *P* under the HC matrix.

In order to study the impact of the network structure on systemic importance more formally, we construct two indicators. The first, lending indicator (*LI*) gauges how interbank lending affects the risk of a bank's own non-bank creditors. It is a product of the bank's contagion PD, which reflects the impact of interbank lending on the likelihood of default, and the bank's size (*NBL*), which is proportional to the losses that non-bank creditors incur if the bank defaults:

$$LI_{nwk,i} = c.PD_{nwk,i} * NBL_i \tag{11}$$

where *nwk* indicates that the contagion PD, and thus the indicator, changes with the network.

Our second indicator captures the notion that each interbank borrower is an indirect source of risk for the non-bank creditors of other banks. Paralleling *LI*, this indicator consists of two components. The first component reflects the impact that each bank *i* has on the risk of each other bank *j*. To measure this impact, we apply the Shapley value methodology to contagion PDs in order to allocate a portion of the contagion PD of bank *j* to bank i. We denote this component by $c.PD^i_{nwk,j}$. Note that this component captures the risk of direct contagion from bank *i* to bank *j*, as well as indirect channels of contagion, e.g. from bank *i*, via a default of bank *k*, to bank *j*.

The second component of the second indicator reflects the fact that the impact of bank *i* on the non-bank creditors of bank *j* is greater when bank *j* has borrowed more from non-banks, i.e. when NBL_j is higher. Putting the two components together and summing across all the counterparties of bank *i*, leads to the borrowing indicator (*BI*):

$$BI_{nwk,i} = \sum_{j \neq i} c.PD^i_{nwk,j} * NBL_j \tag{12}$$

We expect that, when we switch from the system without an interbank network to a system underpinned either by the ME or HC networks, the resulting change in GCA Shapley values would be positively related to both *LI* and *BI*. The reason is that these indicators gauge the impact of interbank activities on, respectively, a bank's own non-bank creditors and the non-bank creditors of other banks. By extension, we also expect that the difference between the Shapley values under the ME network and those under the HC network is related positively to the two indicator differences, $LI_{ME} - LI_{HC}$ and $BI_{ME} - BI_{HC}$.

This intuition is confirmed by the data. The bivariate correlations reported in the first two columns in Table 6 show that higher interbank borrowing or lending increases GCA Shapley values.[23] Likewise,

[23] The actual relationship between a simple indicator and a measure of systemic importance would generally be non-linear. For instance, Tarashev et al. (2010) derive formally a convex relationship between a bank's size and its systemic importance.

larger differences $LI_{ME} - LI_{HC}$ and $BI_{ME} - BI_{HC}$ are associated with greater increases in the Shapley values when we switch from the HC to the ME network (third column).

6.3. Comparing GCA and PA

GCA and PA can disagree substantially about the levels of systemic importance of individual banks (see Table 5). And even though the 20 banks we consider differ in a number of other aspects – such as size, probability of default and exposure to exogenous common factors – a banks' role in the interbank network is the main driver of the difference between its GCA and PA Shapley values. This can be seen more formally by calculating the sum of absolute differences between GCA and PA Shapley values under the zero network matrix (left-hand panel of Table 5) and dividing it by the corresponding sum under the ME or the HC matrix (centre and right-hand panels). The resulting ratios equal 0.35 and 0.31, respectively, indicating that roughly two-thirds of the disagreement between GCA and PA is rooted in their different approaches to the risk associated with interbank links. For some of the systemically most important institutions – i.e. banks *B*, *C* and *G* – this disagreement translates into 20% or larger differences between the measured levels of systemic importance under the two approaches. Ultimately, the message is that the choice of a particular approach to measuring systemic importance matters not only from a conceptual but also from an empirical point of view, and more so when interbank-market links are present.

In our analysis of hypothetical networks we showed that PA Shapley values are larger than GCA ones for an interbank lender, and smaller for an interbank borrower. We argued that this stems from PA and GCA incorporating different versions of Shapley values' fairness property. In this subsection, we show that this argument holds for the system of 20 real-world banks as well. For concreteness, we focus on results obtained under the HC matrix of interbank positions.[24]

We start the comparison between PA and GCA Shapley values by zooming onto two banks. The first bank, *D*, has an average lending indicator (*LI*), which means that the bank imposes an average level of risk on its direct non-bank creditors. However, the same bank has the third highest borrowing indicator (*BI*) and, thus, is a significant source of contagion in the interbank market, imposing significant risk on other banks. Since only GCA captures the latter characteristic of bank *D*, the GCA Shapley value of this bank is one-third higher than its PA Shapley value (0.28% vs. 0.21%). The relationship between GCA and PA Shapley values is reversed for the second bank, *K*, which has the third highest *LI* and the third lowest *BI*. Its high *LI* suggests that its non-bank creditors are quite vulnerable to risk from the interbank market. In other words, the expected participation of bank *K* in systemic events is high, which boosts the bank's PA Shapley value. GCA, however, attributes part of the risk borne by the non-bank creditors of bank *K* to banks borrowing from this bank. The upshot is that the measured level of systemic importance of bank *K* is roughly two-thirds larger under PA than under GCA (0.21% and 0.13%, respectively).

More generally, *BI* and *LI* are also helpful in explaining differences between PA and GCA (Table 6, right-most panel). The positive correlation between *LI* and the difference between PA and GCA Shapley values indicates that, as anticipated, PA tends to assign a higher Shapley value than GCA to institutions participating in the interbank market mainly as lenders. Likewise, the negative correlation between *BI* and the same difference confirms that GCA tends to assign a higher Shapley value to interbank borrowers than PA.

7. Conclusion

In this paper we provide a framework for analysing the systemic importance of interconnected banks. We explore two approaches to measuring systemic importance that allocate the same quantum of system-wide risk differently across banks. The two approaches differ in the underlying concepts of systemic importance: (i) bank's participation in systemic events or (ii) banks' contributions to system-wide risk.

[24] The results are similar under the ME matrix.

In particular, the participation approach (PA) and the generalised contribution approach (GCA) disagree in the way in which they allocate the risk associated with an interbank transaction to its two counterparties. PA assigns this risk to the interbank lender, which is the direct source of potential losses to non-banks. In contrast, GCA splits the risk equally between the two counterparties. In this way, GCA captures the idea that the systemic importance of an interconnected bank depends not only on the risk it imposes directly on non-banks, but also on the risk it generates by both borrowing from and lending to other banks. Our empirical analysis shows that this is not only a theoretical consideration. The measured systemic importance of individual banks can differ materially across approaches, especially when banks are interconnected.

Our findings highlight two important messages for future research on systemic importance. First, the different implications of the ME and HC matrices, both of which are consistent with the same data on aggregate interbank lending, reveal how important it is to have information on bilateral positions. The aggregate data that are typically available at present do not allow for taking into account the actual interbank network structure and can thus lead to substantial errors in the measured levels of system-wide risk and the systemic importance of individual institutions.

Second, since alternative measures of systemic importance provide materially different results, researchers should have a clear understanding which measure is suited to address the question at hand. When exploring a scheme for insuring against losses in systemic events, the participation approach provides the natural measure. Alternatively, if the focus is on assessing the contribution of individual institutions to system-wide risk – which calls for analysing the propagation of shocks through the system – it is necessary to use the generalised contribution approach.

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Appendix A. Considering losses to both non-bank creditors and equity holders

In this appendix, we assess the robustness of our results to including losses to banks' equity holders alongside losses to non-bank creditors in our measures of system-wide risk and systemic importance. For this, we replace Eq. (2) with $L_i^{N^{sub}} \equiv NBL_i \cdot LGD_i^{N^{sub}} \cdot I_i^{N^{sub}} + \Delta E_i^{N^{sub}}$, where $\Delta E_i^{N^{sub}}$ stands for the change in the equity value of bank i in subsystem N^{sub}. We keep everything else the same and, for the sake of brevity, focus on the hypothetical banking systems.

The results reported in Table A.1 are qualitatively similar to those in Table 2 in the main text. Expected shortfall is now substantially larger for all interbank networks considered, which is unsurprising given the inclusion of losses to equity holders. But all other findings are confirmed. Namely, it is still the case that the centre bank has always the highest GCA Shapley value in a particular interbank network. Further, GCA still splits the increase in system-wide risk created by an interbank link equally between borrowers and lenders. For example, switching from the system without a network to the system in which the centre bank only borrows from the periphery raises the system-wide ES by 1.01 percentage points. Attributing this rise equally to borrowers and lenders implies that 0.51 should go to the centre bank and 0.125 to each of the four lending periphery banks. This matches almost exactly the actual increases in the corresponding GCA Shapley values. Likewise, PA still attributes the

Table A.1
System-wide risk and systemic importance in hypothetical banking systems: accounting for losses to non-bank creditors and equity holders.[a]

	ES	f.PD[b]	c.PD[c]	Shapley values	
				GCA	PA
No interconnections	7.38				
All (9 banks)		0.42	0	0.82	0.82
Centre bank borrows	8.39				
Centre bank		0.42	0	1.34	1.09
PB lender (4 banks)		0.42	0.10	0.95	1.05
Unconnected (4 banks)		0.42	0	0.82	0.78
Centre bank lends	8.47				
Centre bank		0.42	0.51	1.46	2.05
PB borrower (4 banks)		0.42	0	0.94	0.87
Unconnected (4 banks)		0.42	0	0.81	0.73
Centre bank intermediates	11.32				
Centre bank		0.42	0.51	2.58	2.25
PB lender (4 banks)		0.42	0.29	1.10	1.43
PB borrower (4 banks)		0.42	0	1.09	0.84

[a] All values are in per cent. ES and Shapley values are expressed per unit of system size. ES pertains to the system as a whole. All other values pertain to a bank in the particular group.
[b] Fundamental PD.
[c] Contagion PD.

risk associated with an interbank link fully to the interbank lender. Thus, interbank borrowers (lenders) continue to have a higher (lower) measure of systemic importance under GCA than under PA.

Appendix B. Technical details

In this appendix, we discuss three technical details of our empirical implementation: the formula for expected shortfall; the construction of interbank network matrices; and the derivation of common-factor loadings.

Expected. shortfall

All the numerical results are underpinned by Gordy's (2003) general formulation of expected shortfall, which can be applied to any loss distribution:

$$ES\left(N^{sub};q^{N^{sub}}\right) \equiv \frac{1}{1-q^{N^{sub}}}\left(E\left[\left(\sum_{i\in N^{sub}} L_i^{N^{sub}}\right)\cdot I_{\sum_{i\in N^{sub}} L_i^{N^{sub}} \geqslant VaR\left(N^{sub};q^{N^{sub}}\right)}\right] + VaR(N^{sub};q^{N^{sub}})\cdot\left(q^{N^{sub}} - \Pr\left(\sum_{i\in N^{sub}} L_i^{N^{sub}} \leqslant VaR\left(N^{sub};q^{N^{sub}}\right)\right)\right)\right), \tag{13}$$

where $I_\Omega = 1$ if the event Ω materialises and $I_\Omega = 0$ otherwise, and $VaR\left(N^{sub};q^{N^{sub}}\right) \equiv \inf\left\{x : \Pr\left(\sum_{i\in N^{sub}} L_i^{N^{sub}} \leqslant x\right) \geqslant q^{N^{sub}}\right\}$.

Interbank. matrices

In our data on 20 real-world banks, the sum of interbank assets is not equal to the sum of interbank liabilities. This is because these banks have interbank positions vis-à-vis banks outside our sample. In order to work with an internally consistent matrix of interbank positions, we create a "sink bank," which absorbs the excess amount of aggregate interbank assets or liabilities. We assume that this bank does not default and abstract from its potential losses on the interbank market. As a result, the sink bank does not create risk in the system.

In deriving the ME matrix and random perturbations around it, we proceed as follows. For the ME matrix, we use the so-called RAS algorithm and, thus, start with the relative entropy matrix as a prior (see Upper (2011)). This matrix assumes that the exposure of bank i to bank j is equal to $a_i * l_j$ if $i \neq j$ and 0 for $i = j$, where a_i (l_j) are bank-specific assets (liabilities) normalised by total interbank assets (liabilities) in the system. In turn, the HC matrix is one of the random interbank matrices that we generate by treating each entry of the prior matrix as a uniform variable distributed between zero and twice its initial value. In addition, we restrict a random set of off-diagonal entries to be equal to zero. We then apply the RAS algorithm to this modified prior and only consider matrices for which the algorithm converges.

Common-factor. loadings

When calculating common-factor loadings, we use Moody's KMV estimates of the correlations of banks' total asset returns from 2006 to 2009. First, we treat each of the yearly correlation matrices as corresponding to the returns on banks' non-bank assets. In principle, the correlation matrix should reflect the fact that the co-movement of banks' total asset values would be caused by (i) exogenous common factors affecting assets vis-à-vis non-banks but also by (ii) interbank linkages. That said, since we model interbank linkages as driven exclusively by buy-and-hold credit exposures and since banks' PDs are quite low, the impact of these linkages on asset-return correlations turns out to be negligible. Concretely, while asset return correlations estimated by Moody's KMV range between 0.30 and 0.60, interbank linkages increase correlations by roughly 0.01.

Second, we impose a single-common-factor structure on each correlation matrix of asset returns (see Tarashev and Zhu, 2008) and then average the common-factor loadings over time. This is not necessary for measuring system-wide risk and systemic importance but has the important expositional advantage of allowing us to describe the commonality of banks' exposures to non-banks with as many parameters as there are banks. Reassuringly, the single-common-factor structure captures roughly 75% of the cross-sectional variability of pair-wise correlations.

Appendix C. Additional drivers of systemic importance

Setting interbank linkages aside, the fundamental PD is the main driver of systemic importance in the system of 20 banks we consider. To illustrate this, we rank-order these banks according to the values of each of the three drivers – fundamental PD, size, and common factor loading – and according to their GCA Shapley values in the absence of an interbank network (Table C.1). The orderings of fundamental PDs and Shapley values are almost identical, having a rank correlation of 0.96. By contrast, the other two drivers provide virtually no information about systemic importance. Concretely, the rank correlation between Shapley values, and size and common-factor loadings is 0.13 and −0.42, respectively.

Table C.1
Shapley values and drivers of systemic importance: no network.

	Bank																			
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
GCA	4	1	3	7	2	6	5	8	9	10	13	12	15	11	16	14	17	18	20	19
f.PD	1	2	3	4	5	6	7	8	9	10	11	12	15	13	14	16	18	17	19	20
Size	16	11	9	19	2	6	4	17	10	14	15	8	12	7	18	1	3	13	20	5
CF	20	16	17	13	5	15	9	18	10	2	19	11	8	4	12	3	7	14	6	1

GCA Shapley values and drivers are ranked from 1 (=highest) to 20 (=lowest) within each category. Size = non-bank liabilities; f.PD = fundamental PD; CF = loading on the common risk factor.

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