Introduction to quantum computing Session 1. Examples with wxMaxima

1 Using vectors and matrices

```
u:[1,2,sqrt(2)];
```
(%o1) $[1,2,\sqrt{2}]$

```
v:[%i,1+%i,2-%i];
```
(%o2) $[\%i,\%i+1,2-\%i]$

```
%e; %pi;
```
(%o4) $\%e$
(%o5) $\%pi$

```
transpose(u);
```
(%o6) $\begin{pmatrix} 1 \\ 2 \\ \sqrt{2} \end{pmatrix}$

```
3·4;
```
(%o7) 12

```
ratsimp(conjugate(u).transpose(v));
```
(%o12) $(3-\sqrt{2})\,\%i+2^{3/2}+2$

```
load("eigen");
```
(%o10) *C:/maxima-5.44.0/share/maxima/5.44.0/share/matrix/eigen.mac*

```
innerproduct(u,v);
```
(%o11) $(3-\sqrt{2})\,\%i+2^{3/2}+2$

```
inprod(u,v);
```
(%o13) $(3-\sqrt{2})\,\%i+2^{3/2}+2$

```
X:matrix([0,1],[1,0]);
```
(%o14) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

```
X.[1,2];
```
(%o15) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

```
rank(matrix(u,v));
```
(%o18) 2

norm of v

```
sqrt(inprod(v,v));
```
(%o19) $2^{3/2}$

2 Example in slide 8

```
e1:transpose(1/sqrt(2)·[1,1]);/*ket*/
```

(%o25) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

```
e2:transpose(1/sqrt(2)·[1,-1]);
```

(%o26) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

```
inprod(e1,e2);/*orthogonal*/
```

(%o27) 0

```
inprod(e1,e1);/*normalized*/
```

(%o28) 1

```
inprod(e2,e2);
```

(%o29) 1

```
P1:e1.conjugate(transpose(e1));/*ket . bra */
```

(%o30) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

```
P1.P1-P1;/*projection*/
```

(%o32) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

```
→ P2:e2.conjugate(transpose(e2));
```

(%o33) $\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$

```
→ P2.P2-P2;
```

(%o34) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

```
→ P1.P2;/*orthogonal*/
```

(%o36) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

```
→ P1+P2;/*completeness relation*/
```

(%o37) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

3 Example in slide 9

```
→ v1:transpose([1,%i]);
```

(%o38) $\begin{pmatrix} 1 \\ \%i \end{pmatrix}$

```
→ v2:transpose([2·%i,4]);
```

(%o39) $\begin{pmatrix} 2\,\%i \\ 4 \end{pmatrix}$

```
[f1,f2]:gramschmidt([v1,v2]);
```

(%o42) $\left[\begin{pmatrix} 1 \\ \%i \end{pmatrix},\begin{pmatrix} 3\,\%i \\ 3 \end{pmatrix}\right]$

```
e1:f1/sqrt(inprod(f1,f1));
```

(%o43) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{\%i}{\sqrt{2}} \end{pmatrix}$

```
e2:f2/sqrt(inprod(f2,f2));
```

(%o44) $\begin{pmatrix} \frac{\%i}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

4 Example in slide 15

```
Y:matrix([0,-%i],[%i,0]);
```

(%o45) $\begin{pmatrix} 0 & -\%i \\ \%i & 0 \end{pmatrix}$

```
transpose(conjugate(Y))-Y; /*Hermitian*/
```

(%o46) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

```
eigenvalues(Y);/* load("eigen") */
```

(%o47) $[[-1,1],[1,1]]$

-1 eigenvalue with multiplicity 1, 1 eigenvalue with multiplicity 1

```
→ ratsimp(determinant(Y-lambda·ident(2)));
```

(%o54) $lambda^2 - 1$

```
→ solve(%,lambda);
```

(%o55) $[lambda = -1, lambda = 1]$

```
→ eigenvectors(Y);
```

(%o56) $[[[-1,1],[1,1]],[[[1,-\%i]],[[1,\%i]]]]$

```
→ Y.[1,-%i];
```

(%o57) $\begin{pmatrix} -1 \\ \%i \end{pmatrix}$

```
→ Y.[1,%i];
```

(%o58) $\begin{pmatrix} 1 \\ \%i \end{pmatrix}$

```
→ l1:transpose([1,%i])/sqrt(inprod([1,%i],[1,%i]));
```

(%o59) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{\%i}{\sqrt{2}} \end{pmatrix}$

```
lm1:transpose([1,-%i])/sqrt(inprod([1,-%i],[1,-%i]));
```

(%o60) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{\%i}{\sqrt{2}} \end{pmatrix}$

```
inprod(l1,lm1);
```

(%o61) 0

```
l1.conjugate(transpose(l1))+lm1.conjugate(transpose(lm1));/*completeness relation*/
```

(%o63) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

```
U:transpose(matrix(transpose(l1)[1],transpose(lm1)[1]));
```

(%o65) $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\%i}{\sqrt{2}} & -\frac{\%i}{\sqrt{2}} \end{pmatrix}$

```
transpose(conjugate(U)).U;
```

(%o66) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

```
transpose(conjugate(U)).Y.U; /*diagonalization*/
```

(%o67) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

5 Example in page 16

```
eigenvectors(X);
```

(%o68) $[[[-1,1],[1,1]],[[[1,-1]],[[1,1]]]]$

```
A:matrix([1,0,0,0],[0,1,0,0],[0,0,0,1],[0,0,1,0]);
```

(%o69)
$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$$

```
eigenvectors(A);
```

(%o70) $[[[-1,1],[1,3]],[[[0,0,1,-1]],[[1,0,0,0],[0,1,0,0],[0,0,1,1]]]]$

```
gramschmidt([[1,0,0,0],[0,1,0,0],[0,0,1,1]]);
```

(%o72) $[[1,0,0,0],[0,1,0,0],[0,0,1,1]]$

```
U:transpose(matrix(1/sqrt(2)·[0,0,1,−1],[1,0,0,0],[0,1,0,0],(1/sqrt(2))·[0,0,1,1]));
```

(%o76)
$$\begin{pmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}}\end{pmatrix}$$

```
transpose(conjugate(U)).U;
```

(%o77)
$$\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

```
transpose(conjugate(U)).A.U;
```

(%o78)
$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$