Introduction to quantum computing
Session 2. Examples with wxMaxima

```
load("eigen");/* for eigenvalues, eigenvectors */
```

(%o183) C:/maxima-5.44.0/share/maxima/5.44.0/share/matrix/eigen.mac

```
load("linearalgebra"); /* for Kronecker product */
```

(%o82) C:/maxima-5.44.0/share/maxima/5.44.0/share/linearalgebra/linearalgebra.mac

1 Spectral decomposition. Example in slide 25

```
Y:matrix([0,-%i],[%i,0]);
```

(%o184)
$$\begin{pmatrix} 0 & -\%i \\ \%i & 0 \end{pmatrix}$$

```
eigenvalues(Y);
```

(%o185) [[-1,1],[1,1]]

```
P1:(Y-(-ident(2)))/(1-(-1));
```

(%o186)
$$\begin{pmatrix} \frac{1}{2} & -\frac{\%i}{2} \\ \frac{\%i}{2} & \frac{1}{2} \end{pmatrix}$$

```
P2:(Y-1·ident(2))/(-1-1);
```

(%o187)
$$\begin{pmatrix} \frac{1}{2} & \frac{\%i}{2} \\ -\frac{\%i}{2} & \frac{1}{2} \end{pmatrix}$$

```
P1.P1-P1;
```

(%o190)
$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
P2.P2-P2;
```

(%o191)
$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

```
P1+P2;/*completness relation*/
```

(%o192)
$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
P1-P2;/*spectral decomp*/
```

(%o193)
$$\begin{pmatrix} 0 & -\%i \\ \%i & 0 \end{pmatrix}$$

```
Y;
```

(%o194)
$$\begin{pmatrix} 0 & -\%i \\ \%i & 0 \end{pmatrix}$$

2 Example slide 9-10

```
(%i1) X:matrix([0,1],[1,0]) ;
```

$$(\%o1)\quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

```
(%i2) load("eigen");
```

$$(\%o2)\quad C:/maxima-5.44.0/share/maxima/5.44.0/share/matrix/eigen.mac$$

```
(%i3) eigenvalues(X);
```

$$(\%o3)\quad [[-1,1],[1,1]]$$

```
(%i4) ;
```

```
(%i5) P1:(X-(-ident(2)))/(1-(-1));
```

$$(\%o5)\quad \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

```
(%i6) P2:(X-1·ident(2))/(-1-1);
```

$$(\%o6)\quad \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

```
(%i7) P1-P2;X;
```

$$(\%o9)\quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(\%o10)\quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

```
U:exp(%i·omega·t/2)·P1+exp(-%i·omega·t/2)·P2;/*evolution operator (unitary)*/
```

(%o18)
$$\begin{pmatrix} \frac{\%e^{\frac{\%i\ omega\ t}{2}}}{2}+\frac{\%e^{-\frac{\%i\ omega\ t}{2}}}{2} & \frac{\%e^{\frac{\%i\ omega\ t}{2}}}{2}-\frac{\%e^{-\frac{\%i\ omega\ t}{2}}}{2} \\ \frac{\%e^{\frac{\%i\ omega\ t}{2}}}{2}-\frac{\%e^{-\frac{\%i\ omega\ t}{2}}}{2} & \frac{\%e^{\frac{\%i\ omega\ t}{2}}}{2}+\frac{\%e^{-\frac{\%i\ omega\ t}{2}}}{2} \end{pmatrix}$$

```
U.transpose([1,0]);
```

(%o19)
$$\begin{pmatrix} \frac{\%e^{\frac{\%i\ omega\ t}{2}}}{2}+\frac{\%e^{-\frac{\%i\ omega\ t}{2}}}{2} & \frac{\%e^{\frac{\%i\ omega\ t}{2}}}{2}-\frac{\%e^{-\frac{\%i\ omega\ t}{2}}}{2} \\ \frac{\%e^{\frac{\%i\ omega\ t}{2}}}{2}-\frac{\%e^{-\frac{\%i\ omega\ t}{2}}}{2} & \frac{\%e^{\frac{\%i\ omega\ t}{2}}}{2}+\frac{\%e^{-\frac{\%i\ omega\ t}{2}}}{2} \end{pmatrix}$$

```
rectform(%);
```

(%o20)
$$\begin{pmatrix} \cos\left(\frac{omega\ t}{2}\right) & \\ \%i\ \sin\left(\frac{omega\ t}{2}\right) & \end{pmatrix}$$

Probability of measuring spin up is (cos(omega*t/2))^2

```
U.(1/sqrt(2)·transpose([1,1]));
```

(%o21)
$$\begin{pmatrix} \frac{e^{\frac{i\,\omega\,t}{2}}}{\sqrt{2}}+\frac{e^{-\frac{i\,\omega\,t}{2}}}{\sqrt{2}} & \frac{e^{\frac{i\,\omega\,t}{2}}}{\sqrt{2}}-\frac{e^{-\frac{i\,\omega\,t}{2}}}{\sqrt{2}} \\ \frac{e^{\frac{i\,\omega\,t}{2}}}{\sqrt{2}}+\frac{e^{-\frac{i\,\omega\,t}{2}}}{\sqrt{2}} & \frac{e^{\frac{i\,\omega\,t}{2}}}{\sqrt{2}}-\frac{e^{-\frac{i\,\omega\,t}{2}}}{\sqrt{2}} \end{pmatrix}$$

```
rectform(%);/*parallel to (1,1) but with a phase factor */
```

(%o22)
$$\begin{pmatrix} \frac{i\,\sin\left(\frac{\omega\,t}{2}\right)}{\sqrt{2}}+\frac{\cos\left(\frac{\omega\,t}{2}\right)}{\sqrt{2}} \\ \frac{i\,\sin\left(\frac{\omega\,t}{2}\right)}{\sqrt{2}}+\frac{\cos\left(\frac{\omega\,t}{2}\right)}{\sqrt{2}} \end{pmatrix}$$

3 Example slide 12

```
N:n_x·matrix([0,1],[1,0])+n_y·matrix([0,-%i],[%i,0])+n_z·matrix([1,0],[0,-1]) ;
```

(%o29)
$$\begin{pmatrix} n_z & n_x-i\,n_y \\ i\,n_y+n_x & -n_z \end{pmatrix}$$

```
eigenvectors(N);
```

(%o30)
$$\left[\left[\left[-e^{t\sqrt{n_z^2+n_y^2+n_x^2}},\ e^{t\sqrt{n_z^2+n_y^2+n_x^2}}\right],[1,1]\right],\left[\left[1,-\frac{(i\,n_y+n_x)\,e^{t\sqrt{n_z^2+n_y^2+n_x^2}}+(i\,n_y+n_x)\,n_z}{n_y^2+n_x^2}\right]\right],\left[\left[1,\frac{(i\,n_y+n_x)\,e^{t\sqrt{n_z^2+n_y^2+n_x^2}}+(-i\,n_y-n_x)\,n_z}{n_y^2+n_x^2}\right]\right]\right]$$

```
P1:(N-(-ident(2)))/(1-(-1));
```

(%o34)
$$\begin{pmatrix} \frac{n_z+1}{2} & \frac{n_x-\%i\,n_y}{2} \\ \frac{\%i\,n_y+n_x}{2} & \frac{1-n_z}{2} \end{pmatrix}$$

```
P2:(N-1·ident(2))/(-1-1);
```

(%o35)
$$\begin{pmatrix} -\frac{n_z-1}{2} & -\frac{n_x-\%i\,n_y}{2} \\ -\frac{\%i\,n_y+n_x}{2} & -\frac{-n_z-1}{2} \end{pmatrix}$$

```
ratsimp(P1-P2);N;
```

(%o38)
$$\begin{pmatrix} n_z & n_x-\%i\,n_y \\ \%i\,n_y+n_x & -n_z \end{pmatrix}$$

(%o39)
$$\begin{pmatrix} n_z & n_x-\%i\,n_y \\ \%i\,n_y+n_x & -n_z \end{pmatrix}$$

```
U:(exp(%i·omega·t/2)·P1+exp(-%i·omega·t/2)·P2);/*evolution operator (unitary)*/
```

(%o45)
$$\begin{pmatrix} \frac{(n_z+1)\,\%e^{\frac{\%i\,omega\,t}{2}}}{2}-\frac{(n_z-1)\,\%e^{-\frac{\%i\,omega\,t}{2}}}{2} & \frac{(n_x-\%i\,n_y)\,\%e^{\frac{\%i\,omega\,t}{2}}}{2}-\frac{(n_x-\%i\,n_y)\,\%e^{-\frac{\%i\,omega\,t}{2}}}{2} \\ \frac{(\%i\,n_y+n_x)\,\%e^{\frac{\%i\,omega\,t}{2}}}{2}-\frac{(\%i\,n_y+n_x)\,\%e^{-\frac{\%i\,omega\,t}{2}}}{2} & \frac{(1-n_z)\,\%e^{\frac{\%i\,omega\,t}{2}}}{2}-\frac{(-n_z-1)\,\%e^{-\frac{\%i\,omega\,t}{2}}}{2} \end{pmatrix}$$

```
ratsimp(rectform(U.transpose([1,0])));
```

(%o47)
$$\begin{pmatrix} \%i\, n_z \sin\left(\frac{\mathrm{omega}\ t}{2}\right)+\cos\left(\frac{\mathrm{omega}\ t}{2}\right) \\ (\%i\, n_x - n_y)\sin\left(\frac{\mathrm{omega}\ t}{2}\right) \end{pmatrix}$$

4 Example slide 13

```
load("linearalgebra");
```

(%o48) C:/maxima-5.44.0/share/maxima/5.44.0/share/linearalgebra/linearalgebra.mac

```
sup:transpose([1,0]);sdown:transpose([0,1]);
```

(%o49)
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(%o50)
$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

```
u:1/sqrt(2)·(kronecker_product(sup,sup)+kronecker_product(sdown,sdown));
```

(%o52)
$$\begin{pmatrix} -\frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

```
(%i53) kronecker_product(c1·sup+c2·sdown,d1·sup+d2·sdown);
```

$$\begin{pmatrix} c1\ d1 \\ c1\ d2 \\ c2\ d1 \\ c2\ d2 \end{pmatrix}$$

```
(%i54) solve([c1·d1=1/sqrt(2),c1·d2=0,c2·d1=0,c2·d1=1/sqrt(2)],[c1,c2,d1,d2]);
```

$$[\,]$$