# Ejercicio Markov, Métodos y técnicas de AI

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# 1

## 1.1

La inferencia estaría basada en las probabilidades del estado estacionario. Sea Q la matriz de cambio de estado y $X_s$ el estado estacionario se cumple que $Q{*}\mathrm{X}_s = X_s$

$$Q = \begin{bmatrix} 0.85 & 0.5 \\ 0.15 & 0.5 \end{bmatrix}$$

$X_s$ =[ps pl]$^T$

Aplicando la ecuacion se obtiene el siguiente sistema de ecuaciones.

$0.85 \times ps + 0.5 = ps$
$0.15 \times ps + 0.5 \times pl = pl$
El cual tiene como solución la única ecuación $ps = 0.5/0.15 \times pl$ A mayores se añade la restricción $ps + pl = 1$, con lo que se obtiene ps = 0.77 y pl = 0.23

## 1.2

Los valores de $X_i$ son s para sol y l para lluvia

$\mathrm{P}(\mathrm{X}_{t+2} = s) = \sum_{x_i} P(X_{t+2} = s, X_{t+1} = x_i) = \sum_{x_i} P(X_{t+2} = s|X_{t+1} = x_i) \times P(X_{t+1} = x_i) =$

$\sum_{x_i}[P(X_{t+2} = s|X_{t+1} = x_i) \times \sum_{x_j} P(X_{t+1} = x_i|X_t = x_j) \times P(X_t = x_j)] =$

$\mathrm{P}(\mathrm{X}_{t+2} = s|X_{t+1} = s) \times [P(X_{t+1} = s|X_t = s) \times P(X_t = s) + P(X_{t+1} = s|X_t = l) \times P(X_t = l)] +$

$\mathrm{P}(\mathrm{X}_{t+2} = s|X_{t+1} = l) \times [P(X_{t+1} = l|X_t = s) \times P(X_t = s) + P(X_{t+1} = l|X_t = l) \times P(X_t = l)] =$

0.85 * ( 0.85 * 0.77 + 0.5 * 0.23) + 0.5 * (0.15 * 0.77 + 0.5 * 0.23 ) = 0.77

## 1.3

$P(X_{t+3} = x) = P(X_{t+3} = x|X_{t+2} = s) \times P(X_{t+2} = s) + P(X_{t+3} = x|X_{t+2} = l) \times P(X_{t+2} = l)(1)$

Where $P(X_{t+2} = s)$ and $P(X_{t+2} = l)$ are obtained from the point 1.2

$\mathrm{P}(\mathrm{X}_{t+3} = s) = 0.85 * 0.77 + 0.5 * 0.23 = 0.77$
$\mathrm{P}(\mathrm{X}_{t+3} = l) = 1 - P(X_{t+3} = s) = 0.23$

# 2

## 2.1

El estado estacionario, calculado en 1.1

## 2.2

P($X_{t+1} = l|Y_t = T, Y_{t+1} = T) = \alpha \times P(Y_{t+1} = T|X_{t+1} = l) \times \sum_{x_i} P(X_t = x_i|Y_t = T) \times P(X_{t+1} = l|X_t = x_i) = \mu \times 0.95 \times (P(Y_t = T|X_t = s) \times P(X_t = s) \times P(X_{t+1} = l|X_t = s) + P(Y_t = T|X_t = l) \times P(X_t = l) \times P(X_{t+1} = l|x = l) = \mu * 0.95(0.1 * 0.77 * 0.15 + 0.95 * 0.23 * 0.5) = \mu * 0.12$

P($X_{t+1} = s, Y_{t+1} = T, Y_t = T) = \alpha \times P(Y_{t+1} = T|X_{t+1} = s) \times [P(Y_t = T|X_t = s) \times P(X_t = s) \times P(X_{t+1} = s|x = s) + P(Y_t = T|X_t = l) \times P(X_t = l) \times P(X_{t+1} = s|X_t = l)] = \alpha * 0.1 * (0.1 * 0.77 * 0.85 + 0.95 * 0.23 * 0.5) = \alpha * 0.018$

P($X_{t+1} = l|Y_t = T, Y_{t+1} = T) = 0.12/(0.12 + 0.018) = 0.87$
P($X_{t+1} = s, Y_{t+1} = T, Y_t = T) = 0.018/(0.12 + 0.018) = 0.13$

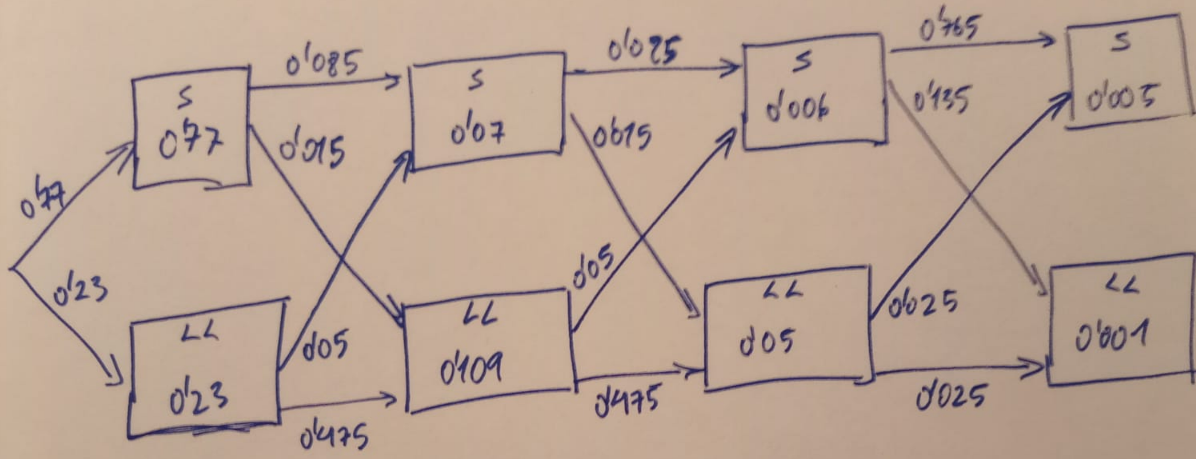
## 2.3

P($X_{t+2} = x|Y_t = T, Y_{t+1} = T, Y_{t+2} = F) = \alpha \times P(Y_{t+2} = F|X_{t+2} = x) \times [P(X_{t+1} = s|Y_t = T, Y_{t+1} = T) \times P(X_{t+2} = s|X_{t+1} = s) + P(X_{t+1} = l|Y_t = T, Y_{t+1} = T) \times P(X_{t+2} = s|X_{t+1} = l)$

P($X_{t+2} = s|Y_t = T, Y_{t+1} = T, Y_{t+2} = F) = \alpha * 0.9 * (0.13 * 0.85 + 0.87 * 0.5) = \alpha * 0.49$
P($X_{t+2} = l|Y_t = T, Y_{t+1} = T, Y_{t+2} = F) = \alpha * 0.05 * (0.13 * 0.85 + 0.87 * 0.5) = \alpha * 0.027$

P($X_{t+2} = s|Y_t = T, Y_{t+1} = T, Y_{t+2} = F) = 0.49/(0.49 + 0.027) = 0.95$
P($X_{t+2} = l|Y_t = T, Y_{t+1} = T, Y_{t+2} = F) = 1 - 0.95 = 0.05$

**2.4**




Arc weight: $P(X_t | X_{t-1}) \cdot P(e_t | X_t)$

Viterbi path: S → LL → LL → S