Gravitational N-body Problem

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February 26, 2023

Proposal

Abstract

The n-body problem aims to solve for the motion of n particles that influence each other "gravitationally". The mutual influence(s) is determined by the gravitational law(s) that the system obeys. The 2-body problem has a complete analytic solution for classical systems, systems that abide by the rules of classical mechanics. However, things get more complicated for n $\geq$ 3, these only have analytic solutions for special cases and have to be approached numerically otherwise.

Strategy

Since there is a range of laws and number of particles that we can pick from, we have candidates for **tests** and **approximations**. Assuming we have "pre-requisite" code to numerically solve a general n-body problem, we will approach the set of general n-body problems in the following order:

- **Classical 2-body problem**
  SANITY CHECK: The easiest way to verify that our pre-requisite code works is to test it on well known 2-body systems. The Earth-Sun and Earth-Moon systems are good candidates since they have been extensively tested in the past (for launching satelites, etc). We can solve this problem *analytically* and compare it to the results we get from our pre-requisite code (using integration methods). This mostly serves as a glorified sanity check.

- **Classical 3-body problem**
  LAGRANGE POINTS: As a continuation to the previous **test**, we will run our pre-requisite code for the analytically solveable, restricted Earth-Sun-Satellite 3-body problem. We will also explore the Lagrange Points of this system and the behaviour of the satellite at and around these points.
  SCATTERING: We will then look at a more general 3-body problem with a binary system and an incomming body in detail. The calculations for this will be done numerically.

- **Relativity and Quantum**
  Finally, we will increase the scope by considering other gravitational *laws*. These are harder to solve for even two bodies so we will look at special cases of the 2-body problem and other cases that our pre-requisite code supports.
  NUMERICAL RELATIVITY: We will look at a Kepler problem (special 2-body problem with an infinitesimal mass, $m \ll M$) involving a black hole (or a similarly massive body, with mass M) and a photon (or a similarly infinitesimal mass, with some mass m).
  FEW-BODY: We will briefly explore how a two-body system behaves in the quantum realm.

Program

The "pre-requisite" code will primarily be written in Python. With `C` and `C++` code as needed for performance. Libraries like NumPy and Matplotlib will be used for perfomant math operations and visuals respectively. There are also number of open-source Python/`C`/`C++` libraries aimed at solving n-body problems that we will compare our results against: mockingbirdnest/Principia, hannorein/rebound, Kushaalkumar-pothula/gravitylab, to name a few. We will also have a GUI for ease of use!