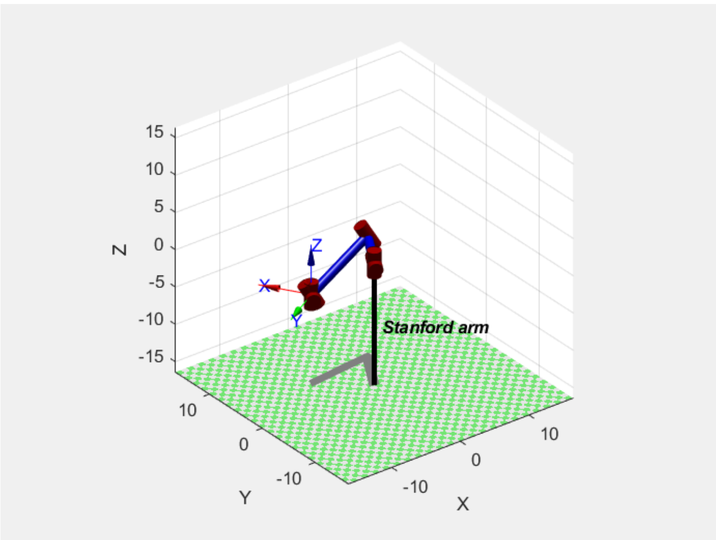
擴充功能:

Matlab Robotics ToolBox:

- 操作方式先需下載官方套件
- 參閱: http://petercorke.com/wordpress/toolboxes/robotics-toolbox
- 安裝完成後:
  - 於 RVCx/rvctools startup_rvc.m 執行
  - 在執行 Standford_model_sim.m
- 於 Commnad line 輸入 Kinematics: joint variables
  - Ex: [100 100 10 100 100 100]
- Plot



機器人學 Project 1

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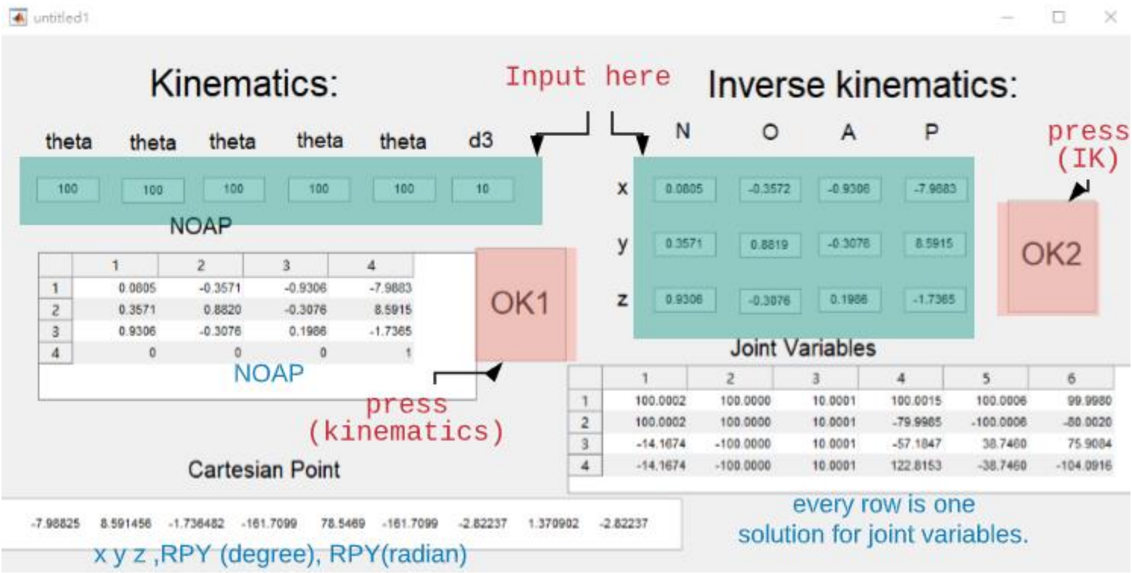
介面說明

- 使用 Maple 作矩陣代數數學運算
- 使用 Matlab R2019a 開發
  1. 打開 kinematics 或 inverse kinematics 儲存文件
  2. RUN
  3. 預設參數:
  - kinematics 文件檔:
    - joint_variables = [100,100,100,100,100]
    - d3 = 10
  - inverse_kinematics 文件檔:
    - N = [0.0805 0.3571 0.9306];
    - O= [-0.3572 0.8819 -0.3076];
    - A = [-0.9306 -0.3076 0.1986];
    - P = [-7.9883 8.5915 -1.7365];
- Matlab GUIDE 開發輸入介面



程式架構說明

- Kinematics:
  - 輸入Joint variables, degree to radian
  - 參考kinematics table 求出各軸的旋轉矩陣A1-A6
  - 求出總旋轉矩陣 T6=A1*A2*A3*A4*A5*A6相對應之NOAP
  - 由NOAP直接看出px,py,pz. RPY代入以下公式解

i. $\phi = \tan^{-1}\left[\frac{a_y}{a_x}\right]$ or $\tan^{-1}\left[\frac{a_y}{a_x}\right] + 180^o$

ii. $\theta = \tan^{-1}\left(\frac{s\theta}{c\theta}\right) = \tan^{-1}\left[\frac{c\phi a_x + s\phi a_y}{a_z}\right]$

iii. $\psi = \tan^{-1}\frac{s\psi}{c\psi} = \tan^{-1}\left\{\frac{-s\phi n_x + c\phi n_y}{-s\phi o_x + c\phi o_y}\right\}$

- Inverse Kinematics:
  - 輸入NOAP
  - 參考kinematics table 求出各軸的旋轉矩陣A1-A6
  - 設定總共四組解
  - 使用代數法求出theta 1共有兩組解
  - 依序計算其分別對應之theta2-6,以及d3

數學運算說明

[Kinematics運算部分]

- 目的為求 **Noap**, 以及 **RPY angles**

1. Noap=T6
2. RPY angles:

$$if\ \ \theta \neq 0, \qquad \phi = \tan^{-1}\frac{ay}{ax} \ \text{ or } \ \tan^{-1}\frac{ay}{ax} + 180$$

$$if\ \ c\theta = az, \qquad \theta = \tan^{-1}\frac{s\theta}{c\theta} = \tan^{-1}\left\{\frac{c\phi ax + s\phi ay}{az}\right\}$$

$$\psi = \tan^{-1}\frac{s\psi}{c\psi} = \tan^{-1}\left\{\frac{-s\phi nx + c\phi ny}{-s\phi ox + c\phi oy}\right\}$$

3. 使用 Maple 作代數運算,此部分可用於接下來的 Inverse Kinematics 手寫部分的矩陣運算並當檢查用.

restart :

with(*linalg*) :

$A1 := Matrix([[c1, 0, -s1, 0], [s1, 0, c1, 0,], [0, -1, 0, 0], [0, 0, 0, 1]])$

$$\begin{bmatrix} c1 & 0 & -s1 & 0 \\ s1 & 0 & c1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A2 := Matrix([[c2, 0, s2, 0], [s2, 0, -c2, 0,], [0, 1, 0, 6.375], [0, 0, 0, 1]])$

$$\begin{bmatrix} c2 & 0 & s2 & 0 \\ s2 & 0 & -c2 & 0 \\ 0 & 1 & 0 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A3 := Matrix([[c3, -s3, 0, 0], [s3, c3, 0, 0,], [0, 0, 1, d3], [0, 0, 0, 1]])$

$$\begin{bmatrix} c3 & -s3 & 0 & 0 \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A4 := Matrix([[c4, 0, -s4, 0], [s4, 0, c4, 0], [0, -1, 0, 0], [0, 0, 0, 1]])$

$$\begin{bmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A5 := Matrix([[c5, 0, s5, 0], [s5, 0, -c5, 0,], [0, 1, 0, 0], [0, 0, 0, 1]])$

$$\begin{bmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A6 := Matrix([[c6, -s6, 0, 0], [s6, c6, 0, 0,], [0, 0, 1, 0], [0, 0, 0, 1]])$

$$\begin{bmatrix} c6 & -s6 & 0 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T56 := multiply(A5, A6)$

$$\begin{bmatrix} c5\,c6 & -c5\,s6 & s5 & 0 \\ s5\,c6 & -s5\,s6 & -c5 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T46 := multiply(A4, A5, A6)$

$$\begin{bmatrix} c4\,c5\,c6 - s4\,s6 & -c4\,c5\,s6 - s4\,c6 & c4\,s5 & 0 \\ s4\,c5\,c6 + c4\,s6 & -s4\,c5\,s6 + c4\,c6 & s4\,s5 & 0 \\ -s5\,c6 & s5\,s6 & c5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T36 := multiply(A3, A4, A5, A6)$

$$\begin{bmatrix} (c3\,c4 - s3\,s4)\,c5\,c6 + (-c3\,s4 - s3\,c4)\,s6 & -(c3\,c4 - s3\,s4)\,c5\,s6 + (-c3\,s4 - s3\,c4)\,c6 & (c3\,c4 - s3\,s4)\,s5 & 0 \\ (s3\,c4 + c3\,s4)\,c5\,c6 + (c3\,c4 - s3\,s4)\,s6 & -(s3\,c4 + c3\,s4)\,c5\,s6 + (c3\,c4 - s3\,s4)\,c6 & (s3\,c4 + c3\,s4)\,s5 & 0 \\ -s5\,c6 & s5\,s6 & c5 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T26 := multiply(A2, A3, A4, A5, A6)$

$$\begin{bmatrix} ((c2\,c3\,c4 - c2\,s3\,s4)\,c5 - s2\,s5)\,c6 + (-c2\,c3\,s4 - c2\,s3\,c4)\,s6 & -((c2\,c3\,c4 - c2\,s3\,s4)\,c5 - s2\,s5)\,s6 + (-c2\,c3\,s4 - c2\,s3\,c4)\,c6 & (c2\,c3\,c4 - c2\,s3\,s4)\,s5 + s2\,c5 & s2\,d3 \\ ((s2\,c3\,c4 - s2\,s3\,s4)\,c5 + c2\,s5)\,c6 + (-s2\,c3\,s4 - s2\,s3\,c4)\,s6 & -((s2\,c3\,c4 - s2\,s3\,s4)\,c5 + c2\,s5)\,s6 + (-s2\,c3\,s4 - s2\,s3\,c4)\,c6 & (s2\,c3\,c4 - s2\,s3\,s4)\,s5 - c2\,c5 & -c2\,d3 \\ (s3\,c4 + c3\,s4)\,c5\,c6 + (c3\,c4 - s3\,s4)\,s6 & -(s3\,c4 + c3\,s4)\,c5\,s6 + (c3\,c4 - s3\,s4)\,c6 & (s3\,c4 + c3\,s4)\,s5 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T6 := multiply(A1, A2, A3, A4, A5, A6)$

N:

$$\left[\begin{matrix} (((c1\,c2\,c3 - s1\,s3)\,c4 + (-c1\,c2\,s3 - s1\,c3)\,s4)\,c5 - c1\,s2\,s5)\,c6 + (-(c1\,c2\,c3 - s1\,s3)\,s4 + (-c1\,c2\,s3 - s1\,c3)\,c4)\,s6 \\ (((s1\,c2\,c3 + c1\,s3)\,c4 + (-s1\,c2\,s3 + c1\,c3)\,s4)\,c5 - s1\,s2\,s5)\,c6 + (-(s1\,c2\,c3 + c1\,s3)\,s4 + (-s1\,c2\,s3 + c1\,c3)\,c4)\,s6 \\ ((-s2\,c3\,c4 + s2\,s3\,s4)\,c5 - c2\,s5)\,c6 + (s2\,c3\,s4 + s2\,s3\,c4)\,s6 \\ 0. \end{matrix}\right.$$

O:

$$
\begin{matrix}
-(((c1\,c2\,c3 - s1\,s3)\,c4 + (-c1\,c2\,s3 - s1\,c3)\,s4)\,c5 - c1\,s2\,s5)\,s6 + (-(c1\,c2\,c3 - s1\,s3)\,s4 + (-c1\,c2\,s3 - s1\,c3)\,c4)\,c6 \\
-(((s1\,c2\,c3 + c1\,s3)\,c4 + (-s1\,c2\,s3 + c1\,c3)\,s4)\,c5 - s1\,s2\,s5)\,s6 + (-(s1\,c2\,c3 + c1\,s3)\,s4 + (-s1\,c2\,s3 + c1\,c3)\,c4)\,c6 \\
-((-s2\,c3\,c4 + s2\,s3\,s4)\,c5 - c2\,s5)\,s6 + (s2\,c3\,s4 + s2\,s3\,c4)\,c6 \\
0.
\end{matrix}
$$

A,P:

$$
\left.\begin{matrix}
((c1\,c2\,c3 - s1\,s3)\,c4 + (-c1\,c2\,s3 - s1\,c3)\,s4)\,s5 + c1\,s2\,c5 & c1\,s2\,d3 - 6.375\,s1 \\
((s1\,c2\,c3 + c1\,s3)\,c4 + (-s1\,c2\,s3 + c1\,c3)\,s4)\,s5 + s1\,s2\,c5 & s1\,s2\,d3 + 6.375\,c1 \\
(-s2\,c3\,c4 + s2\,s3\,s4)\,s5 + c2\,c5 & c2\,d3 \\
0. & 1.
\end{matrix}\right]
$$

[Inverse Kinematics運算部分]

- **目的為求各軸的 ：** θ1,θ2,θ4,θ5,θ6 and d3

此部分使用Handwriting並掃描

兩種算法比較

	Algebraic	**Geometric**
Pros	It’s easier to calculate by computer step by step.	It’s understandable in 3D space
Cons	Have to check all solutions to be right after calculate	1. The solution is not unique and is not continuous. 2. Complicated for high DOF

$$A_1=\begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2=\begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3=\begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$6.375 = d_2$

$$A_4=\begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5=\begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6=\begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6 = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $T_6 = \begin{bmatrix} n & o & a & \begin{matrix} P_x \\ P_y \\ P_z \end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad \begin{cases} P_x = d_3 C_1 S_2 - d_2 S_1 & \text{—①} \\ P_y = d_3 S_1 S_2 + d_2 C_1 & \text{—②} \\ P_z = d_3 C_2 \end{cases}$

$$\begin{cases} P = P_x^2 + P_y^2 \\ P_x = P \cdot \cos\phi \\ P_y = P \cdot \sin\phi \\ \phi = \text{atan2}(P_y, P_x) \end{cases} \xrightarrow{\text{代入}}$$

使用 ③ in $\boxed{A_1^{-1} T_6}$ section: $-S_1 P_x + C_1 P_y = d_2$

$-S_1 P C\phi + C_1 P S\phi = d_2$

$\rightarrow -S_1 C\phi + C_1 S\phi = \frac{d_2}{P}$

$\rightarrow S(\phi - \theta_1) = \frac{d_2}{P}$

by $C = \pm\sqrt{1-S^2}$

$\rightarrow C(\phi-\theta_1) = \sqrt{\frac{P^2-d_2^2}{P^2}}$

$\rightarrow \phi - \theta_1 = \text{atan2}(d_2, \pm\sqrt{P^2-d_2^2})$

$$\therefore \theta_1 = \text{atan2}(P_y, P_x) - \text{atan2}(d_2, \pm\sqrt{P^2-d_2^2})$$

2 solutions #

- $A_1^{-1} T_6 : \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2 A_3 A_4 A_5 A_6 = A_{26}$

$$= \begin{bmatrix} & & & d_3 S_2 \\ & & & -d_3 C_2 \\ & & & d_2 \\ & & & 1 \end{bmatrix}$$

$$\begin{cases} C_1 P_x + S_1 P_y = d_3 S_2 & \text{—①} \\ -S_1 P_x + C_1 P_y = d_2 & \text{—③} \\ -P_z = -d_3 C_2 & \text{—②} \end{cases}$$

$$\frac{①}{②} = -\tan\theta_2 = \frac{C_1 P_x + S_1 P_y}{-P_z}$$

if $d_3 \neq 0$,

$$\therefore \theta_2 = \text{atan2}(C_1 P_x + S_1 P_y, P_z)$$

#

$\cdot\ A_2^{-1} A_1^{-1} T_6 = A_3 A_4 A_5 A_6$

$$\begin{bmatrix} C_2 & S_2 & 0 & 0 \\ 0 & 0 & 1 & -6.375 \\ S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left[\quad \begin{matrix} 0 \\ 0 \\ d_3 \\ 1 \end{matrix} \right]$$

$$= \begin{bmatrix} C_1C_2 & C_2S_1 & -S_2 & 0 \\ -S_1 & C_1 & 0 & -d_2 \\ C_1S_2 & S_1S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & \begin{matrix} p_x \\ p_y \\ p_z \\ 1 \end{matrix} \end{bmatrix}$$

$\therefore$ $C_1 S_2 P_x + S_1 S_2 P_y + C_2 P_z = d_3$ #

$\theta_3 = 0^\circ$

$\cdot\ A_3^{-1} A_2^{-1} A_1^{-1} T_6 = A_4 A_5 A_6$. $\theta_3 = 0$.

$$\begin{bmatrix} \cancel{C_3} & \cancel{S_3} & 0 & 0 \\ \cancel{-S_3} & \cancel{C_3} & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1C_2 & C_2S_1 & -S_2 & 0 \\ -S_1 & C_1 & 0 & -d_2 \\ C_1S_2 & S_1S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \left[\quad \begin{matrix} C_4S_5 \\ S_4S_5 \\ C_5 \\ 0 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix} \right]$$

$$= \begin{bmatrix} C_1C_2 & C_2S_1 & -S_2 & 0 \\ -S_1 & C_1 & 0 & -d_2 \\ C_1S_2 & S_1S_2 & C_2 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & \boxed{\begin{matrix} a_x \\ a_y \\ a_z \\ 0 \end{matrix}} & \boxed{\begin{matrix} p_x \\ p_y \\ p_z \\ 1 \end{matrix}} \\ 0 & 0 & & \end{bmatrix}$$

$\xrightarrow{\times p} \begin{cases} C_1C_2P_x + C_2S_1P_y - S_2P_z = 0 \\ -S_1P_x + C_1P_y - d_2 = 0 \\ C_1S_2P_x + S_1S_2P_y + C_2P_z = d_3 \end{cases}$

$\times a$

$$\begin{cases} C_4S_5 = C_1C_2a_x + C_2S_1a_y - S_2a_z & \text{— ①} \\ S_4S_5 = -S_1a_x + C_1a_y & \text{— ②} \\ C_1S_2a_x + S_1S_2a_y + C_2a_z = C_5 & \text{— ③} \end{cases}$$

$\therefore$

$$\theta_4 = \frac{②}{①} = \mathrm{atan2}\left(\frac{\overbrace{-S_1a_x + C_1a_y}^{前項}}{\underbrace{C_1C_2a_x + S_1C_2a_y - S_2a_z}_{後項}}\right)$$ #

表示同 atan2(前, 後)

$$\begin{cases} \times o & C_1S_2O_x + S_1S_2O_y + C_2O_z = S_5S_6 & \text{— ④} \\ \times n & C_1S_2n_x + S_1S_2n_y + C_2n_z = -S_5C_6 & \text{— ⑤} \end{cases}$$

$$\theta_6 = \frac{④}{⑤} = \mathrm{atan2}\left(\frac{-C_1S_2O_x - S_1S_2O_y - C_2O_z}{+C_1S_2n_x + S_1S_2n_y + C_2n_z}\right)$$

- $A_4^{-1} A_3^{-1} A_2^{-1} A_1^{-1} T_6 = A_5 A_6$.

$$
\begin{bmatrix} C_4 & S_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_4 & C_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} C_1C_2 & C_2S_1 & -S_2 & 0 \\ -S_1 & C_1 & 0 & -d_2 \\ C_1S_2 & S_1S_2 & C_2 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\cdot
\begin{bmatrix} n & o & a & p_x \\ & & & p_y \\ & & & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
=
\begin{array}{c} \quad\quad\quad o \quad\quad a \quad\quad p \\ \begin{bmatrix} C_5C_6 & -C_5S_6 & S_5 & 0 \\ S_5C_6 & -S_5S_6 & -C_5 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}
$$

$$
= \begin{bmatrix} C_4C_1C_2 - S_4S_1 & C_4C_2S_1 + C_1S_4 & -C_4S_2 & -d_2S_4 \\ -C_1S_2 & -S_1S_2 & -C_2 & d_3 \\ -S_4C_1C_2 - C_4S_1 & -S_4C_2S_1 + C_1C_4 & S_2S_4 & -d_2C_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

if × p

$$
\rightarrow \begin{cases} (C_4C_1C_2 - S_4S_1)P_x + (C_4C_2S_1 + C_1S_4)P_y - (C_4S_2)P_z - d_2S_4 = 0 \\ (-C_1S_2)P_x - (S_1S_2)P_y - C_2P_z + d_3 = 0 \\ -(S_4C_1C_2 - C_4S_1)P_x - (S_4C_2S_1 + C_1C_4)P_y + (S_2S_4)P_z - d_2C_4 = 0 \end{cases}
$$

if × a

$$
\rightarrow \begin{cases} (C_4C_1C_2 - S_4S_1)a_x + (C_4C_2S_1 + C_1S_4)a_y - C_4S_2a_z = S_5 & — ① \\ -C_1S_2a_x - S_1S_2a_y - C_2a_z = -C_5 & — ② \end{cases}
$$

$$
\therefore \theta_5 = \frac{①}{②} = \operatorname{atan2}\left(\frac{(C_4C_1C_2 - S_4S_1)a_x + (C_4C_2S_1 + C_1S_4)a_y - C_4S_2a_z}{C_1S_2a_x + S_1S_2a_y + C_2a_z} \right)
$$

前

後項