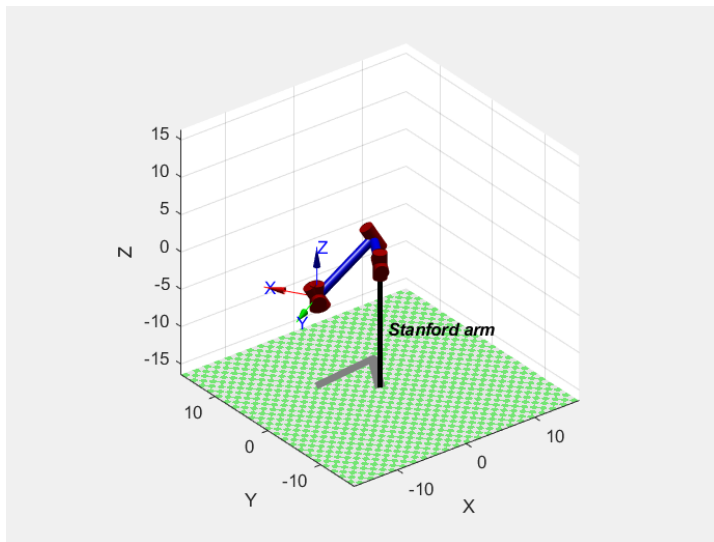


擴充功能：

## Matlab Robotics ToolBox:

- 操作方式先需下載官方套件
- 參閱：<http://petercorke.com/wordpress/toolboxes/robotics-toolbox>
- 安裝完成後：
  - 於 RVCx/rvctools startup\_rvc.m 執行
  - 在執行 Stanford\_model\_sim.m
- 於 Command line 輸入 Kinematics: joint variables
  - Ex: [100 100 10 100 100 100]
- Plot



# 機器人學 Project 1

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## 介面說明

使用 Maple 作矩陣代數數學運算

使用 Matlab R2019a 開發

1. 打開 kinematics 或 inverse kinematics 儲存文件

2. RUN

3. 預設參數:

➤ kinematics 文件檔:

◆ joint\_variables = [100,100,100,100,100]

◆ d3 = 10

➤ inverse\_kinematics 文件檔:

◆ N = [0.0805 0.3571 0.9306];

◆ O = [-0.3572 0.8819 -0.3076];

◆ A = [-0.9306 -0.3076 0.1986];

◆ P = [-7.9883 8.5915 -1.7365];

Matlab GUIDE 開發輸入介面

The screenshot shows a Matlab GUIDE window titled 'untitled1' with two main sections: 'Kinematics:' and 'Inverse kinematics:'.  
**Kinematics Section:**  
- Inputs: Five 'theta' buttons (all set to 100) and one 'd3' button (set to 10).  
- A red arrow points to the 'd3' input with the text 'Input here'.  
- Below the inputs is a table labeled 'NOAP' with 4 rows and 4 columns of numerical values.  
- A red button labeled 'OK1' is next to the table.  
- Below the table is a red button labeled 'press (kinematics)'.  
- Below that is a label 'Cartesian Point'.  
- At the bottom, a row of numerical values is displayed: -7.98825, 8.591456, -1.736482, -161.7099, 78.5469, -161.7099, -2.82237, 1.370902, -2.82237.  
- Below the values is a label 'x y z, RPY (degree), RPY(radian)'.  
**Inverse kinematics Section:**  
- Inputs: Four buttons labeled N, O, A, P with numerical values.  
- A red arrow points to the 'N' input with the text 'Input here'.  
- A red button labeled 'press (IK)' is next to the inputs.  
- Below the inputs is a red button labeled 'OK2'.  
- Below that is a table labeled 'Joint Variables' with 4 rows and 6 columns of numerical values.  
- At the bottom, a label says 'every row is one solution for joint variables.'

|   | 1      | 2       | 3       | 4       |
|---|--------|---------|---------|---------|
| 1 | 0.0805 | -0.3571 | -0.9306 | -7.9883 |
| 2 | 0.3571 | 0.8819  | -0.3076 | 8.5915  |
| 3 | 0.9306 | -0.3076 | 0.1986  | -1.7365 |
| 4 | 0      | 0       | 0       | 1       |

|   | 1        | 2         | 3       | 4        | 5         | 6         |
|---|----------|-----------|---------|----------|-----------|-----------|
| 1 | 100.0002 | 100.0000  | 10.0001 | 100.0015 | 100.0006  | 99.9980   |
| 2 | 100.0002 | 100.0000  | 10.0001 | -79.9985 | -100.0006 | -80.0020  |
| 3 | -14.1674 | -100.0000 | 10.0001 | -57.1847 | 38.7460   | 75.9084   |
| 4 | -14.1674 | -100.0000 | 10.0001 | 122.8153 | -38.7460  | -104.0916 |

## 程式架構說明

### Kinematics:

- 輸入Joint variables, degree to radian
- 參考kinematics table 求出各軸的旋轉矩陣A1-A6
- 求出總旋轉矩陣  $T6=A1*A2*A3*A4*A5*A6$  相對應之NOAP
- 由NOAP直接看出px, py, pz. RPY代入以下公式解

i.  $\phi = \tan^{-1} \left[ \frac{a_y}{a_x} \right] \text{ or } \tan^{-1} \left[ \frac{a_y}{a_x} \right] + 180^\circ$

ii.  $\theta = \tan^{-1} \left( \frac{s\theta}{c\theta} \right) = \tan^{-1} \left[ \frac{c\phi a_x + s\phi a_y}{a_z} \right]$

iii.  $\psi = \tan^{-1} \frac{s\psi}{c\psi} = \tan^{-1} \left\{ \frac{-s\phi n_x + c\phi n_y}{-s\phi o_x + c\phi o_y} \right\}$

### Inverse Kinematics:

- 輸入NOAP
- 參考kinematics table 求出各軸的旋轉矩陣A1-A6
- 設定總共四組解
- 使用代數法求出theta 1共有兩組解
- 依序計算其分別對應之theta2-6, 以及d3

## 數學運算說明

[Kinematics運算部分]

### 目的為求 Noap, 以及 RPY angles

1. Noap=T6
2. RPY angles:

$$\text{if } \theta \neq 0, \quad \phi = \tan^{-1} \frac{ay}{ax} \text{ or } \tan^{-1} \frac{ay}{ax} + 180$$

$$\text{if } c\theta = az, \quad \theta = \tan^{-1} \frac{s\theta}{c\theta} = \tan^{-1} \left\{ \frac{c\phi ax + s\phi ay}{az} \right\}$$

$$\psi = \tan^{-1} \frac{s\psi}{c\psi} = \tan^{-1} \left\{ \frac{-s\phi nx + c\phi ny}{-s\phi ox + c\phi oy} \right\}$$

3. 使用 Maple 作代數運算, 此部分可用於接下來的 Inverse Kinematics 手寫部分的矩陣運算並當檢查用.

*restart :*

*with(linalg) :*

$A1 := \text{Matrix}([ [c1, 0, -s1, 0], [s1, 0, c1, 0], [0, -1, 0, 0], [0, 0, 0, 1] ])$

$$\begin{bmatrix} c1 & 0 & -s1 & 0 \\ s1 & 0 & c1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A2 := \text{Matrix}([ [c2, 0, s2, 0], [s2, 0, -c2, 0], [0, 1, 0, 6.375], [0, 0, 0, 1] ])$

$$\begin{bmatrix} c2 & 0 & s2 & 0 \\ s2 & 0 & -c2 & 0 \\ 0 & 1 & 0 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A3 := \text{Matrix}([ [c3, -s3, 0, 0], [s3, c3, 0, 0], [0, 0, 1, d3], [0, 0, 0, 1] ])$

$$\begin{bmatrix} c3 & -s3 & 0 & 0 \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A4 := \text{Matrix}([ [c4, 0, -s4, 0], [s4, 0, c4, 0], [0, -1, 0, 0], [0, 0, 0, 1] ])$

$$\begin{bmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A5 := \text{Matrix}([ [c5, 0, s5, 0], [s5, 0, -c5, 0], [0, 1, 0, 0], [0, 0, 0, 1] ])$

$$\begin{bmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A6 := \text{Matrix}([ [c6, -s6, 0, 0], [s6, c6, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1] ])$

$$\begin{bmatrix} c6 & -s6 & 0 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T56 := \text{multiply}(A5, A6)$$

$$\begin{bmatrix} c5 c6 & -c5 s6 & s5 & 0 \\ s5 c6 & -s5 s6 & -c5 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T46 := \text{multiply}(A4, A5, A6)$$

$$\begin{bmatrix} c4 c5 c6 - s4 s6 & -c4 c5 s6 - s4 c6 & c4 s5 & 0 \\ s4 c5 c6 + c4 s6 & -s4 c5 s6 + c4 c6 & s4 s5 & 0 \\ -s5 c6 & s5 s6 & c5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T36 := \text{multiply}(A3, A4, A5, A6)$$

$$\begin{bmatrix} (c3 c4 - s3 s4) c5 c6 + (-c3 s4 - s3 c4) s6 & -(c3 c4 - s3 s4) c5 s6 + (-c3 s4 - s3 c4) c6 & (c3 c4 - s3 s4) s5 & 0 \\ (s3 c4 + c3 s4) c5 c6 + (c3 c4 - s3 s4) s6 & -(s3 c4 + c3 s4) c5 s6 + (c3 c4 - s3 s4) c6 & (s3 c4 + c3 s4) s5 & 0 \\ -s5 c6 & s5 s6 & c5 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T26 := \text{multiply}(A2, A3, A4, A5, A6)$$

$$\begin{bmatrix} ((c2 c3 c4 - c2 s3 s4) c5 - s2 s5) c6 + (-c2 c3 s4 - c2 s3 c4) s6 & -((c2 c3 c4 - c2 s3 s4) c5 - s2 s5) s6 + (-c2 c3 s4 - c2 s3 c4) c6 & (c2 c3 c4 - c2 s3 s4) s5 + s2 c5 & s2 d3 \\ ((s2 c3 c4 - s2 s3 s4) c5 + c2 s5) c6 + (-s2 c3 s4 - s2 s3 c4) s6 & -((s2 c3 c4 - s2 s3 s4) c5 + c2 s5) s6 + (-s2 c3 s4 - s2 s3 c4) c6 & (s2 c3 c4 - s2 s3 s4) s5 - c2 c5 & -c2 d3 \\ (s3 c4 + c3 s4) c5 c6 + (c3 c4 - s3 s4) s6 & -(s3 c4 + c3 s4) c5 s6 + (c3 c4 - s3 s4) c6 & (s3 c4 + c3 s4) s5 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T6 := \text{multiply}(A1, A2, A3, A4, A5, A6)$$

N:

$$\begin{bmatrix} (((c1 c2 c3 - s1 s3) c4 + (-c1 c2 s3 - s1 c3) s4) c5 - c1 s2 s5) c6 + (-c1 c2 c3 - s1 s3) s4 + (-c1 c2 s3 - s1 c3) c4) s6 \\ (((s1 c2 c3 + c1 s3) c4 + (-s1 c2 s3 + c1 c3) s4) c5 - s1 s2 s5) c6 + (-s1 c2 c3 + c1 s3) s4 + (-s1 c2 s3 + c1 c3) c4) s6 \\ ((-s2 c3 c4 + s2 s3 s4) c5 - c2 s5) c6 + (s2 c3 s4 + s2 s3 c4) s6 \\ 0. \end{bmatrix}$$

O:

$$\begin{aligned}
& -(((c1c2c3 - s1s3)c4 + (-c1c2s3 - s1c3)s4)c5 - c1s2s5)s6 + (-c1c2c3 - s1s3)s4 + (-c1c2s3 - s1c3)c4)c6 \\
& -(((s1c2c3 + c1s3)c4 + (-s1c2s3 + c1c3)s4)c5 - s1s2s5)s6 + (-s1c2c3 + c1s3)s4 + (-s1c2s3 + c1c3)c4)c6 \\
& \quad -((-s2c3c4 + s2s3s4)c5 - c2s5)s6 + (s2c3s4 + s2s3c4)c6 \\
& \quad 0.
\end{aligned}$$

A,P:

$$\begin{aligned}
& ((c1c2c3 - s1s3)c4 + (-c1c2s3 - s1c3)s4)s5 + c1s2c5 \quad c1s2d3 - 6.375s1 \\
& ((s1c2c3 + c1s3)c4 + (-s1c2s3 + c1c3)s4)s5 + s1s2c5 \quad s1s2d3 + 6.375c1 \\
& \quad (-s2c3c4 + s2s3s4)s5 + c2c5 \quad c2d3 \\
& \quad 0. \quad 1.
\end{aligned}$$

[Inverse Kinematics運算部分]

目的為求各軸的：θ1,θ2,θ4,θ5,θ6 and d3

此部分使用Handwriting並掃描

## 兩種算法比較

|             | Algebraic   | Geometric   |
|-------------|---|---|
| <b>Pros</b> | It's easier to calculate by computer step by step.      | It's understandable in 3D space   |
| <b>Cons</b> | Have to check all solutions to be right after calculate | <ol style="list-style-type: none"> <li>1. The solution is not unique and is not continuous.</li> <li>2. Complicated for high DOF</li> </ol> |

$$A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = d_3 \quad A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6 = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 = \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bullet T_6 = \begin{bmatrix} n & o & a \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \quad \begin{cases} p_x = d_3 C_1 S_2 - d_2 S_1 \quad \text{--- ①} \\ p_y = d_3 S_1 S_2 + d_2 C_1 \quad \text{--- ②} \\ p_z = d_3 C_2 \end{cases}$$

$$\begin{cases} P = p_x^2 + p_y^2 \\ p_x = P \cdot \cos \phi \\ p_y = P \cdot \sin \phi \\ \phi = \text{atan2}(p_y, p_x) \end{cases} \xrightarrow{\text{代换}} \text{使用 ③ in } [A_1^{-1} T_6] \text{ section: } -S_1 p_x + C_1 p_y = d_2$$

$$\begin{aligned} -S_1 p \cos \phi + C_1 p \sin \phi &= d_2 \rightarrow \text{by } C = \pm \sqrt{1 - S^2} \\ \rightarrow -S_1 C \phi + C_1 S \phi &= \frac{d_2}{P} \rightarrow C(\phi - \theta_1) = \sqrt{\frac{P^2 - d_2^2}{P^2}} \\ \rightarrow S(\phi - \theta_1) &= \frac{d_2}{P} \rightarrow \phi - \theta_1 = \text{atan2}(d_2, \pm \sqrt{P^2 - d_2^2}) \end{aligned}$$

$$\therefore \theta_1 = \text{atan2}(p_y, p_x) - \text{atan2}(d_2, \pm \sqrt{P^2 - d_2^2})$$

2 solutions #

$$\bullet A_1^{-1} T_6 = \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & o & a & p \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2 A_3 A_4 A_5 A_6 = A_{26}$$

$$= \begin{bmatrix} d_3 S_2 \\ -d_3 C_2 \\ d_2 \\ 1 \end{bmatrix}$$

$$\begin{cases} C_1 p_x + S_1 p_y = d_3 S_2 \quad \text{--- ①} \\ -S_1 p_x + C_1 p_y = d_2 \quad \text{--- ②} \\ -p_z = -d_3 C_2 \quad \text{--- ③} \end{cases} \quad \frac{①}{②} = -\tan \theta_2 = \frac{C_1 p_x + S_1 p_y}{-p_z}$$

if  $d_3 \neq 0$ ,

$$\therefore \theta_2 = \text{atan2}(C_1 p_x + S_1 p_y, p_z) \quad \#$$

$$A_2^{-1} A_1^{-1} T_6 = A_3 A_4 A_5 A_6$$

$$\begin{bmatrix} C_2 & S_2 & 0 & 0 \\ 0 & 0 & 1-b_3/5 & 0 \\ S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_o a_p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 C_2 & C_2 S_1 - S_2 & 0 \\ -S_1 & C_1 & 0 & -d_2 \\ C_1 S_2 & S_1 S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ n_o a_p \\ p_y \\ p_z \\ 1 \end{bmatrix}$$

$$\therefore \frac{C_1 S_2 p_x + S_1 S_2 p_y + C_2 p_z = d_3}{\theta_3 = 0^\circ} \quad \#$$

$$A_3^{-1} A_2^{-1} A_1^{-1} T_6 = A_4 A_5 A_6 \quad \theta_3 = 0$$

$$\begin{bmatrix} C_3 & S_3 & 0 & 0 \\ -S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1-d_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_1 C_2 & C_2 S_1 - S_2 & 0 \\ -S_1 & C_1 & 0 & -d_2 \\ C_1 S_2 & S_1 S_2 & C_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_o a_p \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C_4 S_5 & 0 \\ S_4 S_5 & 0 \\ C_5 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 C_2 & C_2 S_1 - S_2 & 0 \\ -S_1 & C_1 & 0 & -d_2 \\ C_1 S_2 & S_1 S_2 & C_2 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x p_x \\ n_o a_p \\ a_y p_y \\ a_z p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} x^p \\ \\ \\ \\ \end{matrix} \rightarrow \begin{cases} C_1 C_2 p_x + C_2 S_1 p_y - S_2 p_z = 0 \\ -S_1 p_x + C_1 p_y - d_2 = 0 \\ C_1 S_2 p_x + S_1 S_2 p_y + C_2 p_z = d_3 \end{cases}$$

$$\therefore \begin{matrix} \text{前項} \\ \text{後項} \end{matrix} \quad \theta_4 = \frac{\text{②}}{\text{①}} = \text{atan2} \left( \frac{-S_1 a_x + C_1 a_y}{C_1 C_2 a_x + S_1 C_2 a_y - S_2 a_z} \right) \quad \#$$

表示同  $\text{atan2}(\text{前}, \text{後})$

$$x_o \begin{cases} C_1 S_2 o_x + S_1 S_2 o_y + C_2 o_z = S_5 S_6 \end{cases} \quad \text{--- ④}$$

$$x_n \begin{cases} C_1 S_2 n_x + S_1 S_2 n_y + C_2 n_z = -S_5 C_6 \end{cases} \quad \text{--- ⑤}$$

$$\theta_6 = \frac{\text{④}}{\text{⑤}} = \text{atan2} \left( \frac{-C_1 S_2 o_x - S_1 S_2 o_y - C_2 o_z}{C_1 S_2 n_x + S_1 S_2 n_y + C_2 n_z} \right)$$



$$A_4^{-1} A_3^{-1} A_2^{-1} A_1^{-1} T b = A_5 A b.$$

$$\begin{bmatrix} c_4 & s_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -s_4 & c_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 c_2 & c_2 s_1 - s_2 & 0 \\ -s_1 & c_1 & 0 & -d_2 \\ c_1 s_2 & s_1 s_2 & c_2 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n o a & p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} c_5 c_6 & -c_5 s_6 & s_5 & 0 \\ s_5 c_6 & -s_5 s_6 & c_5 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & a & p \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_4 c_1 c_2 - s_4 s_1 & c_4 c_2 s_1 + c_1 s_4 & -c_4 s_2 & -d_2 s_4 \\ -c_1 s_2 & -s_1 s_2 & -c_2 & d_3 \\ -s_4 c_1 c_2 - c_4 s_1 & -s_4 c_2 s_1 + c_1 c_4 & s_2 s_4 & -d_2 c_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n o a p \\ 0 0 0 1 \end{bmatrix}$$

$$\text{if } \times p \rightarrow \begin{cases} (c_4 c_1 c_2 - s_4 s_1) p_x + (c_4 c_2 s_1 + c_1 s_4) p_y - (c_4 s_2) p_z - d_2 s_4 = 0 \\ (-c_1 s_2) p_x - (s_1 s_2) p_y - c_2 p_z + d_3 = 0 \\ -(s_4 c_1 c_2 - c_4 s_1) p_x - (s_4 c_2 s_1 + c_1 c_4) p_y + (s_2 s_4) p_z - d_2 c_4 = 0 \end{cases}$$

$$\text{if } \times a \rightarrow \begin{cases} (c_4 c_1 c_2 - s_4 s_1) a_x + (c_4 c_2 s_1 + c_1 s_4) a_y - c_4 s_2 a_z = s_5 & \text{--- ①} \\ -c_1 s_2 a_x - s_1 s_2 a_y - c_2 a_z = -c_5 & \text{--- ②} \end{cases}$$

$$\therefore \theta_5 = \frac{\text{①}}{\text{②}} = \text{atan2} \left( \frac{(c_4 c_1 c_2 - s_4 s_1) a_x + (c_4 c_2 s_1 + c_1 s_4) a_y - c_4 s_2 a_z}{c_1 s_2 a_x + s_1 s_2 a_y + c_2 a_z} \right) \begin{matrix} \nearrow \text{前項} \\ \searrow \text{後項} \end{matrix}$$