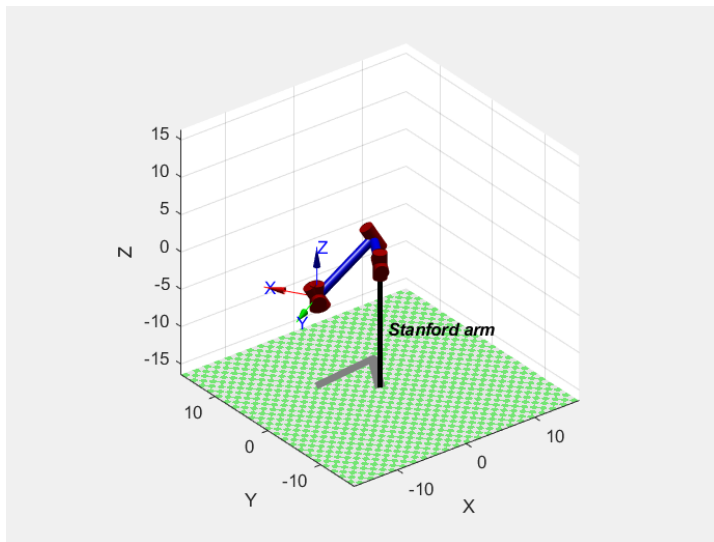


擴充功能：

Matlab Robotics ToolBox:

- 操作方式先需下載官方套件
- 參閱：<http://petercorke.com/wordpress/toolboxes/robotics-toolbox>
- 安裝完成後：
 - 於 RVCx/rvctools startup_rvc.m 執行
 - 在執行 Stanford_model_sim.m
- 於 Command line 輸入 Kinematics: joint variables
 - Ex: [100 100 10 100 100 100]
- Plot



機器人學 Project 1

介面說明

使用 Maple 作矩陣代數數學運算

使用 Matlab R2019a 開發

1. 打開 kinematics 或 inverse kinematics 儲存文件
2. RUN
3. 預設參數:

➤ kinematics 文件檔:

- ◆ $\text{joint_variables} = [100, 100, 100, 100, 100]$
- ◆ $d3 = 10$

➤ inverse_kinematics 文件檔:

- ◆ $N = [0.0805 \ 0.3571 \ 0.9306];$
- ◆ $O = [-0.3572 \ 0.8819 \ -0.3076];$
- ◆ $A = [-0.9306 \ -0.3076 \ 0.1986];$
- ◆ $P = [-7.9883 \ 8.5915 \ -1.7365];$

Matlab GUIDE 開發輸入介面

Kinematics:

Input here

Inverse kinematics:

press (IK)

OK2

Joint Variables

every row is one solution for joint variables.

press (kinematics)

Cartesian Point

x y z, RPY (degree), RPY(radian)

程式架構說明

Kinematics:

- 輸入Joint variables, degree to radian
- 參考kinematics table 求出各軸的旋轉矩陣A1-A6
- 求出總旋轉矩陣 $T6=A1*A2*A3*A4*A5*A6$ 相對應之NOAP
- 由NOAP直接看出px, py, pz. RPY代入以下公式解

i. $\phi = \tan^{-1} \left[\frac{a_y}{a_x} \right] \text{ or } \tan^{-1} \left[\frac{a_y}{a_x} \right] + 180^\circ$

ii. $\theta = \tan^{-1} \left(\frac{s\theta}{c\theta} \right) = \tan^{-1} \left[\frac{c\phi a_x + s\phi a_y}{a_z} \right]$

iii. $\psi = \tan^{-1} \frac{s\psi}{c\psi} = \tan^{-1} \left\{ \frac{-s\phi n_x + c\phi n_y}{-s\phi o_x + c\phi o_y} \right\}$

Inverse Kinematics:

- 輸入NOAP
- 參考kinematics table 求出各軸的旋轉矩陣A1-A6
- 設定總共四組解
- 使用代數法求出theta 1共有兩組解
- 依序計算其分別對應之theta2-6, 以及d3

數學運算說明

[Kinematics運算部分]

目的為求 Noap, 以及 RPY angles

1. Noap=T6
2. RPY angles:

$$\text{if } \theta \neq 0, \quad \phi = \tan^{-1} \frac{ay}{ax} \text{ or } \tan^{-1} \frac{ay}{ax} + 180$$

$$\text{if } c\theta = az, \quad \theta = \tan^{-1} \frac{s\theta}{c\theta} = \tan^{-1} \left\{ \frac{c\phi ax + s\phi ay}{az} \right\}$$

$$\psi = \tan^{-1} \frac{s\psi}{c\psi} = \tan^{-1} \left\{ \frac{-s\phi nx + c\phi ny}{-s\phi ox + c\phi oy} \right\}$$

3. 使用 Maple 作代數運算, 此部分可用於接下來的 Inverse Kinematics 手寫部分的矩陣運算並當檢查用.

restart :

with(linalg) :

$A1 := \text{Matrix}([[c1, 0, -s1, 0], [s1, 0, c1, 0], [0, -1, 0, 0], [0, 0, 0, 1]])$

$$\begin{bmatrix} c1 & 0 & -s1 & 0 \\ s1 & 0 & c1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A2 := \text{Matrix}([[c2, 0, s2, 0], [s2, 0, -c2, 0], [0, 1, 0, 6.375], [0, 0, 0, 1]])$

$$\begin{bmatrix} c2 & 0 & s2 & 0 \\ s2 & 0 & -c2 & 0 \\ 0 & 1 & 0 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A3 := \text{Matrix}([[c3, -s3, 0, 0], [s3, c3, 0, 0], [0, 0, 1, d3], [0, 0, 0, 1]])$

$$\begin{bmatrix} c3 & -s3 & 0 & 0 \\ s3 & c3 & 0 & 0 \\ 0 & 0 & 1 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A4 := \text{Matrix}([[c4, 0, -s4, 0], [s4, 0, c4, 0], [0, -1, 0, 0], [0, 0, 0, 1]])$

$$\begin{bmatrix} c4 & 0 & -s4 & 0 \\ s4 & 0 & c4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A5 := \text{Matrix}([[c5, 0, s5, 0], [s5, 0, -c5, 0], [0, 1, 0, 0], [0, 0, 0, 1]])$

$$\begin{bmatrix} c5 & 0 & s5 & 0 \\ s5 & 0 & -c5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A6 := \text{Matrix}([[c6, -s6, 0, 0], [s6, c6, 0, 0], [0, 0, 1, 0], [0, 0, 0, 1]])$

$$\begin{bmatrix} c6 & -s6 & 0 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T56 := \text{multiply}(A5, A6)$$

$$\begin{bmatrix} c5 c6 & -c5 s6 & s5 & 0 \\ s5 c6 & -s5 s6 & -c5 & 0 \\ s6 & c6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T46 := \text{multiply}(A4, A5, A6)$$

$$\begin{bmatrix} c4 c5 c6 - s4 s6 & -c4 c5 s6 - s4 c6 & c4 s5 & 0 \\ s4 c5 c6 + c4 s6 & -s4 c5 s6 + c4 c6 & s4 s5 & 0 \\ -s5 c6 & s5 s6 & c5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T36 := \text{multiply}(A3, A4, A5, A6)$$

$$\begin{bmatrix} (c3 c4 - s3 s4) c5 c6 + (-c3 s4 - s3 c4) s6 & -(c3 c4 - s3 s4) c5 s6 + (-c3 s4 - s3 c4) c6 & (c3 c4 - s3 s4) s5 & 0 \\ (s3 c4 + c3 s4) c5 c6 + (c3 c4 - s3 s4) s6 & -(s3 c4 + c3 s4) c5 s6 + (c3 c4 - s3 s4) c6 & (s3 c4 + c3 s4) s5 & 0 \\ -s5 c6 & s5 s6 & c5 & d3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T26 := \text{multiply}(A2, A3, A4, A5, A6)$$

$$\begin{bmatrix} ((c2 c3 c4 - c2 s3 s4) c5 - s2 s5) c6 + (-c2 c3 s4 - c2 s3 c4) s6 & -((c2 c3 c4 - c2 s3 s4) c5 - s2 s5) s6 + (-c2 c3 s4 - c2 s3 c4) c6 & (c2 c3 c4 - c2 s3 s4) s5 + s2 c5 & s2 d3 \\ ((s2 c3 c4 - s2 s3 s4) c5 + c2 s5) c6 + (-s2 c3 s4 - s2 s3 c4) s6 & -((s2 c3 c4 - s2 s3 s4) c5 + c2 s5) s6 + (-s2 c3 s4 - s2 s3 c4) c6 & (s2 c3 c4 - s2 s3 s4) s5 - c2 c5 & -c2 d3 \\ (s3 c4 + c3 s4) c5 c6 + (c3 c4 - s3 s4) s6 & -(s3 c4 + c3 s4) c5 s6 + (c3 c4 - s3 s4) c6 & (s3 c4 + c3 s4) s5 & 6.375 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T6 := \text{multiply}(A1, A2, A3, A4, A5, A6)$$

N:

$$\begin{bmatrix} (((c1 c2 c3 - s1 s3) c4 + (-c1 c2 s3 - s1 c3) s4) c5 - c1 s2 s5) c6 + (-c1 c2 c3 - s1 s3) s4 + (-c1 c2 s3 - s1 c3) c4) s6 \\ (((s1 c2 c3 + c1 s3) c4 + (-s1 c2 s3 + c1 c3) s4) c5 - s1 s2 s5) c6 + (-s1 c2 c3 + c1 s3) s4 + (-s1 c2 s3 + c1 c3) c4) s6 \\ ((-s2 c3 c4 + s2 s3 s4) c5 - c2 s5) c6 + (s2 c3 s4 + s2 s3 c4) s6 \\ 0. \end{bmatrix}$$

O:

$$\begin{aligned}
& -(((c1c2c3 - s1s3)c4 + (-c1c2s3 - s1c3)s4)c5 - c1s2s5)s6 + (-c1c2c3 - s1s3)s4 + (-c1c2s3 - s1c3)c4)c6 \\
& -(((s1c2c3 + c1s3)c4 + (-s1c2s3 + c1c3)s4)c5 - s1s2s5)s6 + (-s1c2c3 + c1s3)s4 + (-s1c2s3 + c1c3)c4)c6 \\
& \quad -((-s2c3c4 + s2s3s4)c5 - c2s5)s6 + (s2c3s4 + s2s3c4)c6 \\
& \quad 0.
\end{aligned}$$

A,P:

$$\begin{aligned}
& ((c1c2c3 - s1s3)c4 + (-c1c2s3 - s1c3)s4)s5 + c1s2c5 \quad c1s2d3 - 6.375s1 \\
& ((s1c2c3 + c1s3)c4 + (-s1c2s3 + c1c3)s4)s5 + s1s2c5 \quad s1s2d3 + 6.375c1 \\
& \quad (-s2c3c4 + s2s3s4)s5 + c2c5 \quad c2d3 \\
& \quad 0. \quad 1.
\end{aligned}$$

[Inverse Kinematics運算部分]

目的為求各軸的：θ1, θ2, θ4, θ5, θ6 and d3

此部分使用Handwriting並掃描

兩種算法比較

	Algebraic	Geometric
Pros	It's easier to calculate by computer step by step.	It's understandable in 3D space
Cons	Have to check all solutions to be right after calculate	<ol style="list-style-type: none"> 1. The solution is not unique and is not continuous. 2. Complicated for high DOF