1. (a) Solution:

we write $P(X=x)=\theta^x(1-\theta)^{(1-x)}$
$P(D|\theta)=P(X_1=x_1,X_2=x_2\dots,X_N=x_N)$
$=\prod_{x=1}^{N}\theta_i^x(1-\theta)^{(1-x_i)}$
$=\theta^{N_1}(1-\theta)^{N_0}$
Let $J(\theta)=logP(D|\theta)=N_1log(\theta)+N_0log(1-\theta)$
$\frac{\partial J(\theta)}{\partial\theta}=\frac{N_1}{\theta}-\frac{N_0}{1-\theta}$. Let $\frac{\partial J(\theta)}{\partial\theta}=0$.
Then $\hat{\theta}=\frac{N_1}{N_0+N_1}=\frac{N_1}{N}$
Therefore, the maximum likelihood solution is $\hat{\theta}=\frac{N_1}{N}$

(b) Solution:

$$p(D|\theta)\times p(\theta)=\begin{cases}0.2\cdot 0.6^{N_1}\cdot 0.4^{N_0} & \theta=0.6\\ 0.8\cdot 0.8^{N_1}\cdot 0.2^{N_0} & \theta=0.8\\ 0 & \text{otherwise}\end{cases}$$

$\frac{P(D|0.6)P(0.6)}{P(D|0.8)P(0.8)}=\frac{1}{4}(\frac{3}{4})^{N_1}(2)^{N_0}=3^{N_1}4^{-N_1-1}2^{N_0}=2^{N_1log_2^3}2^{-2N_1-2}2^{N_0}=2^{N_0-(2-log_2^3)N_1-2}$
Therefore, when $\frac{P(D|0.6)P(0.6)}{P(D|0.8)P(0.8)}\geq 1$:
$N_0-(2-log_2^3)N_1-2\geq 0\Rightarrow N_0\geq(2-log_2^3)N_1+2$

Therefore, $\hat{\theta}=\begin{cases}0.6 & N_0\geq(2-log_2^3)N_1+2\\ 0.8 & N_0<(2-log_2^3)N_1+2\end{cases}$

2. (a) Solution:

$$\theta_j^y=\begin{cases}\frac{3}{7} & j=0\\ \frac{4}{7} & j=1\end{cases}$$

$$\theta_{\bar{x}_\ell|j}^{x_\ell|y}=\begin{cases}\frac{1}{3} & x_1=1,j=0\\ \frac{2}{3} & x_1=0,j=0\\ \frac{1}{3} & x_2=1,j=0\\ \frac{2}{3} & x_2=0,j=0\\ \frac{1}{2} & x_1=1,j=1\\ \frac{1}{2} & x_1=0,j=1\\ \frac{1}{2} & x_2=1,j=1\\ \frac{1}{2} & x_2=0,j=1\end{cases}$$

(b) Solution:

$P(y=0|x_1=0,x_2=1)$
$=\frac{P(x_1=0,x_2=1|y=0)P(y=0)}{P(x_1=0,x_2=1)}$
$=\frac{P(x_1=0|y=0)P(x_2=1|y=0)P(y=0)}{P(x_1=0,x_2=1)}$
$=\frac{\theta_{0|0}^{x_1|y}\theta_{1|0}^{x_2|y}\theta_0^y}{P(x_1=0,x_2=1)}$
$=\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{3}{7}/\frac{2}{7}$
$=\frac{1}{3}$