Also, do the assigned HW problems. This is just in addition to HW.

ALWAYS JUSTIFY YOUR ANSWER!

Computations

1. Describe all group homomorphisms $\mathbb{Z}_{45} \to \mathbb{Z}_{10}$.
2. Describe all group homomorphisms $\mathbb{Z}_{45} \to \mathbb{Z}_{7}$.
3. Describe all group homomorphisms $\mathbb{Z}_{2} \to \mathbb{Z}_{4}$.
4. Describe all group homomorphisms $\mathbb{Z}_{2} \to S_3$.
5. Let $f : \mathbb{Z}_{18} \to \mathbb{Z}_{27}$ be given by $f(1) = 3$ and therefore $f(i) = 3i(mod 27)$.
   (a) Find $f(0)$=
   (b) Find $f(3)$=
   (c) Find $f(9)$=
   (d) Find $f(10)$=
   (e) Find $Im(f)$=
   (f) Find $Ker(f)$=
   (g) Is f onto? Justify your answer.
   (h) Is f one-to-one? Justify your answer.
6. Let $f : \mathbb{Z}_{45} \to \mathbb{Z}_{100}$ be given by $f(1) = 20$ and therefore $f(i) = 20i(mod 100)$.
   (a) Find $f(0)$=
   (b) Find $f(5)$=
   (c) Find $f(30)$=
   (d) Find $f(38)$=
   (e) Find $Im(f)$=
   (f) Find $Ker(f)$=
   (g) Is f onto? Justify your answer.
   (h) Is f one-to-one? Justify your answer.
7. Let $f : \mathbb{Z}_6 \to S_4$ be given by $f(1) = (124)$.
   (a) Find $f(0)$=
   (b) Find $f(2)$=

(c) Find $f(3)$=

(d) Find $f(4)$=

(e) Find $f(5)$=

(f) Find $Im(f)$=

(g) Find $Ker(f)$=

(h) Is f onto? Justify your answer.

(i) Is f one-to-one? Justify your answer.

8. Let $G = \mathbb{Z}_{12}$.

   (a) Find the subgroup $H = \langle 3 \rangle$.

   (b) Find all left cosets of H in G.

   (c) Find all right cosets of H in G.

   (d) Is H normal subgroup of G?

   (e) Find G/H.

   (f) Find $[G : H]$.

9. Let $G = S_4$.

   (a) Find the subgroup $H = \langle (1342) \rangle$.

   (b) Find all left cosets of H in G.

   (c) Find all right cosets of H in G.

   (d) Is H normal subgroup of G?

   (e) Find $[G : H]$.

10. Let $G = S_4$.

    (a) Find the subgroup A_4 of even permutations.

    (b) Find all left cosets of A_4 in G.

    (c) Find all right cosets of A_4 in G.

    (d) Is A_4 normal subgroup of G?

    (e) Find G/H.

    (f) Find $[G : H]$.

11. Consider the group $\mathbb{Z}_{15}^{\times}$.

    (a) Find the subgroup $H = \langle 2 \rangle$

    (b) Find all left cosets of H in G.

(c) Find all right cosets of H in G.

(d) Is H normal subgroup of G?

(e) Find G/H.

(f) Find $[G : H]$.

Theoretic Questions

12. Write the definition of *Normal Subgroup.*

13. Write the definition of *Left coset.*

14. Write the definition of *Subgroup.*

15. Write the definition of *Quotient group.*

16. Write the definition of *Kernel of a homomorphism.*

Proofs

17. Let $f : G \to G'$ be a group homomorphism. Let $g \in G$. Prove that $|f(g)|$ divides $|g|$.

18. Let $f : G \to G'$ be a group homomorphism. Prove that $f(e_G) = e_{G'}$.

19. Let $f : G \to G'$ be a homomorphism. Prove that $Im(f)$, the image of f is a subgroup of G'.

20. Let $f : G \to G'$ be a homomorphism. Prove that $Ker(f)$, the kernel of f is a subgroup of G.

21. Let $f : G \to G'$ be a homomorphism. Prove that $Ker(f)$ is normal subgroup of G.

22. Let H be a subgroup of group G. Prove that $(aH = H) \iff (a \in H)$.

23. Let H be a subgroup of group G. Let $a \in G$ and let $aHa^{-1} = \{aha^{-1} \mid h \in H\}$. Prove that aHa^{-1} is a subgroup of G.

24. Prove that the center of a group is normal subgroup, i.e. $Z(G)$ is normal subgroup in G.

True -False - Sometimes

25. True -False - Sometimes

    T F S - Let $\mathbb{Z}_n^\times$ is a subgroup of $\mathbb{Z}_n$.

    T F S - Let $G = (\mathbb{Z}_n, +_n)$, let H be a subgroup of G. Then H is normal subgroup.

    T F S - Let $G = S_7$, let H be a subgroup of G. Then H is normal subgroup.

    T F S - Let H be a subgroup of G. Let $a \in G$. Then $aH = Ha$.

    T F S - $(2\mathbb{Z}, +)$ is a normal subgroup of $(\mathbb{Z}, +)$.

T F S - $\langle 6 \rangle$ is a normal subgroup of $(\mathbb{Z}_9, +_9)$

T F S - Let H be a proper subgroup of S_3. Then H is normal subgroup.

T F S - All proper subgroups of S_4 are normal.

T F S - Let G be a group of order $|G| = 5$. Let $H < G$. Then $|H| = 4$.

T F S - Let G be a group of order $|G| = 5$. Let $H < G$. Then $|H| = 1$.

T F S - Let G be a group of order $|G| = 15$. Let $H < G$. Then $[G : H] = 5$.

T F S - Let G be a cyclic group of order $|G| = 15$. Let $H < G$. Then $|H| = 10$.

T F S - Let G be a group. Let $g \in G$. Then $\langle g \rangle$ is normal subgroup.

Examples

26. Give an example of a group and a subgroup which is not normal. Prove your statement.
27. Give an example of a group and a subgroup which is normal. Prove your statement.
28. Give an example of a non cyclic group and a subgroup which is normal. Prove your statement.
29. Give an example of a group homomorphism which is onto. Prove your statement.
30. Give an example of a group homomorphism which is not onto. Prove your statement.
31. Give an example of a group homomorphism which is one-to-one. Prove your statement.
32. Give an example of a group homomorphism $f : G \to G'$ such that $Ker(f) = \{e_G\}$. Prove your statement.
33. Give an example of a group homomorphism $f : G \to G'$ such that $Ker(f) \neq \{e_G\}$. Prove your statement.
34. Give an example of a group G and a subgroup H such that $[G : H] = 3$.
35. Give an example of a group G and a subgroup H such that $[G : H] = 2$.