1. 01.22

(a) Definition: 7.1
Function $f : X \to Y$ is bijection if f is both surjection(on to) and injection (one to one)

(b) Theorem: 7.2
$f : X \to Y$ is bijection $\Leftrightarrow$
$\exists g : Y \to X$ s.t. $g \circ f = id_x, f \circ g = id_y$ (id_x means identity)
Such g is called the inverse of f. Denoted by f^{-1}

(c) Recall:
∘ Composition of two injective functions is injective.
∘ Composition of two surjective functions is surjective.
∘ Composition of two bijective functions is bijective.

(d) Definition: 7.4 Permutation:
Permutation on set X is a bijection $f : X \to X$

(e) prop 7.5
  i. if $f : X \to X$ is a permutation then $\exists f^{-1} : X \to X$ which is also permutation.
  ii. composition of two permutation is again a permutation.

(f) Definition: 7.6
if $X = \{1, 2, \dots, n\}$ then, $S_n := \{\text{all permutation on } X\}$

(g) EX 7.7
$\alpha = (1\ 2\ 3\ 4\ 5) \to (3\ 4\ 1\ 2\ 5)$
$\beta = (1\ 2\ 3\ 4\ 5) \to (5\ 1\ 4\ 3\ 2)$
Find $\alpha\beta$ (composition of α and β), α^{-1}
Solution:
$(\alpha\beta)(1) = \alpha(\beta(1)) = \alpha(5) = 5$
$(\alpha\beta)(2) = \alpha(\beta(2)) = \alpha(1) = 3$
$(\alpha\beta)(3) = \alpha(\beta(3)) = \alpha(4) = 2$
$(\alpha\beta)(4) = \alpha(\beta(4)) = \alpha(5) = 1$
$(\alpha\beta)(5) = \alpha(\beta(5)) = \alpha(2) = 4$
Then, $\alpha\beta = (1\ 2\ 3\ 4\ 5) \to (5\ 3\ 2\ 1\ 4)$

$\alpha^{-1} = (3\ 4\ 1\ 2\ 5) \to (1\ 2\ 3\ 4\ 5)$
rearrange:
$\alpha^{-1} = (1\ 2\ 3\ 4\ 5) \to (3\ 4\ 1\ 2\ 5)$

(h) Homework: 2.1 9(b)
$g : \mathbf{Z}_8 \Rightarrow \mathbf{Z}_1 2, g([x]_8) = [6x]_1 2$ show that g is well defined.
Solution:

Proof. Suppose $[x]_8 = [x']_8$, WTS $g([x]_8) = g([x']_8)$
Let $[x]_8 = [x']_8$
$\Rightarrow x \equiv x'(mod\ 8)$
$\Rightarrow 8|(x - x')$

$\Rightarrow x - x' = 8 * q$ for some $q \in \mathbf{Z}$
$x = 8 \cdot q + x'$
By definition of g, $g([x]_8) = [6x]_{12}$
Then, $g([x]_8) = [6(8q + x')]_{12} = [48q + 6x']_{12}, g([x']_8) = [6x']_{12}$
WTS $[48q + 6x']_{12} = [6x']_{12}$
Enough to show: $12|(48q + 6x' - 6x')$
Since $48q + 6x' - 6x' = 48q = 12 \cdot 4 \cdot q$
$\Rightarrow 12|12 \cdot 4 \cdot q$
$\Rightarrow 12|(48q + 6x' - 6x')$
$\Rightarrow g([x]_8) = g([x']_8)$ $\square$