- **Bijection**:
  Function $f : X \to Y$ is bijection if f is both surjection(on to) and injection (one to one)
  Proposition:

  1. $f : X \to Y$ is bijection $\Leftrightarrow$
     $\exists g : Y \to X$ s.t. $g \circ f = id_x, f \circ g = id_y$ ($id_x \to$ identity)
  2. Composition Properties:
     ∘ Composition of two injective functions is injective.
     ∘ Composition of two surjective functions is surjective.
     ∘ Composition of two bijective functions is bijective.

- **Permutation:**
  Permutation on set X is a bijection $f : X \to X$
  If $X = \{1, 2, \ldots, n\}$ then, $S_n := \{\text{all permutation on } X\}$ **Proposition:**

  1. if $f : X \to X$ is a permutation then $\exists f^{-1} : X \to X$ which is also permutation.
  2. composition of two permutation is again a permutation.

- **Group 5 Rules**:

  1. Closed under binary operation
  2. associative: (ab)c = a(bc)
  3. identity: $\exists e \in G, ea = ae = a \forall a \in G$
  4. inverse: $\forall a \in G, \exists! a^{-1} s.t. a^{-1}a = aa^{-1} = e$
  5. commutative $a, b \in G, ab = ba$.

  1,2: semigroup
  1,2,3: monoid
  1,2,3,4: group
  1,2,3,4,5: Abelian group

- **Subgroup**: H is a subgroup of G if

  – $H \subseteq G$
  – H is a group

  CHECK a SUBGROUP:

  – $H \subseteq G$ (subset)
  – $e \in H$ (non empty)
  – $\forall a, b \in H, ab \in H$ (closed)
  – $\forall a \in H, a^{-1} \in H$

  Proper subgroup: subgroup H that is not $H \neq G$

- **Order**:
  Order of a group: $|G|$ = # of elements in the group
  Order of an element: $g \in G, |g|$ = **smallest positive integer** $n,\ s.t.\ x^n = e$

- $< x >:= \{ x^n \mid n \in \mathbb{Z}\}$
- **Conjugate**: $x, g \in G$, conjugate of x by g: gxg^{-1}
  Conjugate class of x:= $\{gxg^{-1} \mid \forall g \in G\}$
- **ISOMORPHISMS of GROUP**: a function $f : G \to G'$ is called isomorphism if:
  1. $f(xy) = f(x)f(y)$
  2. f is one to one (injective)
  3. f is onto (surjective)
- **Cyclic**: $\exists a \in G,\ s.t.\ < a >= G$
- **Center of Group**:
  Center of a Group $G : Z(G) := \{Z \in G | gz = zg,\ \forall g \in G\}$
  Proposition:
  1. $Z(G)$ is asubgroup of G.
  2. If G is abelian, then $Z(G) = G$
- **External direct product of Groups:**
  Group G, H, Define $G \times H := \{(x, y) \mid x \in G, y \in H\}$
  $(x_1, y_1)(x_2, y_2) = (x_1 x_2, y_1 y_2)$
  Proposition:
  1. $e_{G\times H} = (e_G, e_H)$
  2. $(x, y)^{-1} = (x^{-1, y^{-1}})$
  3. $|(x, y)| = LCM(|x|, |y|)$
- **Internal product of groups:**
  Group G has subgroup H, K. Defind $HK := \{xy | x \in H, y \in K\}$
  NOTE: HK is not always a subgroup.
  Proposition:
  1. H, K are subgroup of G.
     Suppose $x^{-1}yx \in K, \forall x \in H, y \in K$ Then HK is a subgroup of G.
     Corollary: H, K are subgroup of abelien group G, then HK is a subgroup of G.
- **Group Homomorphisms:**
  $f : G \to G'$ if $f(xy) = f(x)f(y) \forall x, y \in G$
  Compared to isomorphism, we don't need bijection.
- **Kernal and Image:**
  $f : G \to G'$, Define:
  Kerf $:= \{g \in G \mid f(g) = e'_G\}$
  Imf $:= \{y \in G' \mid \exists x \in G, s.t.\ f(x) = y\} \equiv \{f(x) \mid x \in G\}$
  Proposition:
  1. $f : G \to G'$ be group homomorphism:
     - kerf is a subgroup of G
     - Imf is a subgroup of G'