

1. Definitions:

- **Group 5 Rules:**

- (a) Closed under binary operation
- (b) associative: $(ab)c = a(bc)$
- (c) identity: $\exists e \in G, ea = ae = a \forall a \in G$
- (d) inverse: $\forall a \in G, \exists! a^{-1} \text{ s.t. } a^{-1}a = aa^{-1} = e$
- (e) commutative $a, b \in G, ab = ba$.

1,2: semigroup

1,2,3: monoid

1,2,3,4: group

1,2,3,4,5: Abelian group

- **Subgroup:** H is a subgroup of G if

- $H \subseteq G$
- H is a group

CHECK a SUBGROUP:

- $H \subseteq G$ (subset)
- $e \in H$ (non empty)
- $\forall a, b \in H, ab \in H$ (closed)
- $\forall a \in H, a^{-1} \in H$

Proper subgroup: subgroup H that is not $H \neq G$

- **Order:**

Order of a group: $|G| = \#$ of elements in the group

Order of an element: $g \in G, |g| = \text{smallest positive integer } n, \text{ s.t. } x^n = e$

- $\langle x \rangle := \{ x^n \mid n \in \mathbb{Z} \}$

- **Conjugate:** $x, g \in G$, conjugate of x by g : gxg^{-1}

Conjugate class of $x := \{ gxg^{-1} \mid \forall g \in G \}$

- **ISOMORPHISMS of GROUP:** a function $f : G \rightarrow G'$ is called isomorphism if:

- (a) $f(xy) = f(x)f(y)$
- (b) f is one to one (injective)
- (c) f is onto (surjective)

- **Cyclic:** $\exists a \in G, \text{ s.t. } \langle a \rangle = G$