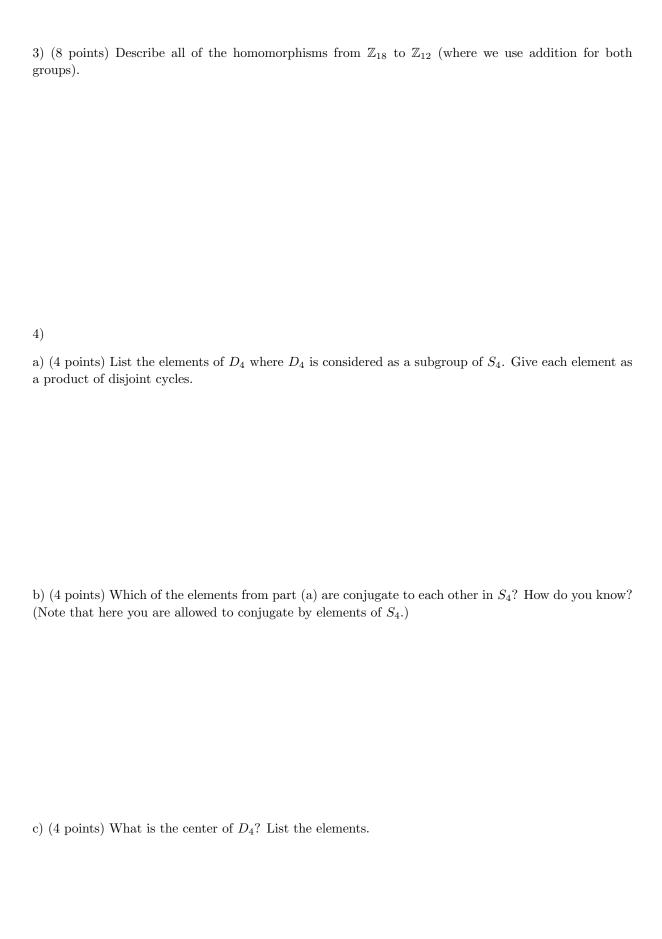
1) (8 points) Suppose we have a set G and function $*: G \times G \to G$ (a binary operation on G), where the value of the function on a pair (g_1, g_2) is denoted by $g_1 * g_2$.

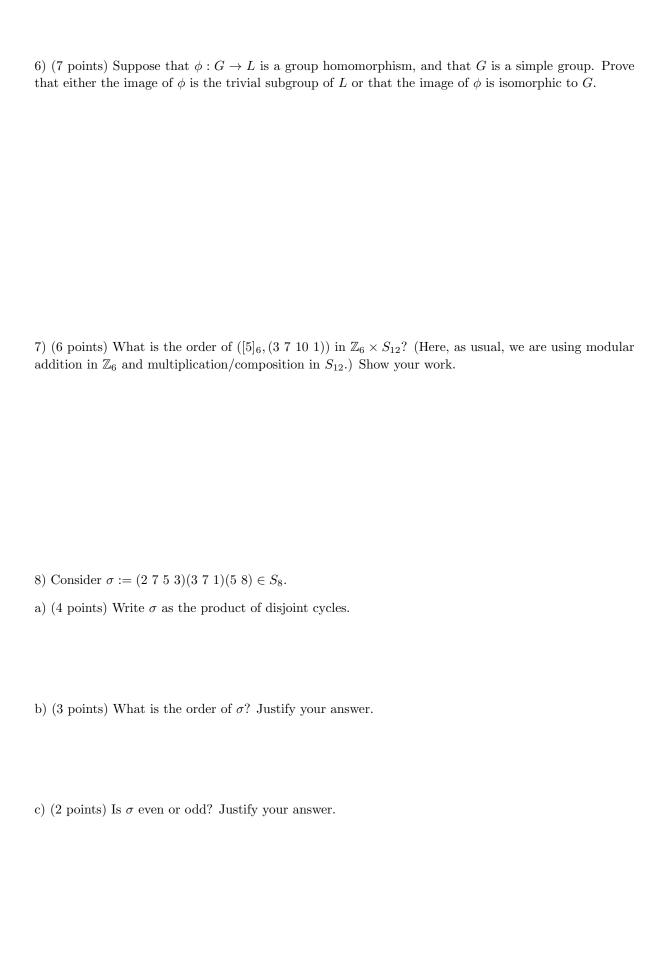
G and * must satisfy what conditions for (G,*) to be a group?

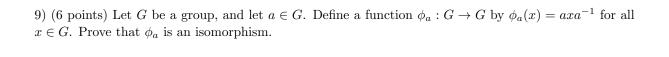
- 2) Suppose that G and L are groups (where we use multiplicative notation for both), and that $\phi: G \to L$ is a function.
- a) (4 points) Define what is means for ϕ to be a homomorphism.

b) (6 points) Suppose that ϕ is a homomorphism. Prove that the image, im ϕ , of ϕ is a subgroup of L.



| 5) Let G be a group of order 21. Suppose H is a subgroup of G. a) (3 points) What are the possible orders of H? |
|---|
| b) (4 points) Must a group of order 21 have subgroups of all of the orders you gave in part (a)? Justify your answer. |
| c) (5 points) Show that G must have a proper non-trivial normal subgroup. |
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10) (7 points) A group G of order 121 acts on the symmetric group S_7 . Prove that there must be at least one fixed point of the action.

