Also, do the assigned HW problems. This is just in addition to HW.

ALWAYS JUSTIFY YOUR ANSWER!

Computations

1. Let $G = S_6$. Let $\alpha = (12)(46)$.
   (a) Find a conjugate of α.
   (b) How many elements are there in the conjugacy class of α?
2. Let $G = \mathbb{Z}_6$. Let $x = 2$.
   (a) Find a conjugate of x.
   (b) How many elements are there in the conjugacy class of x?
3. Let $G = \mathbb{Z}_6^\times$. Let $x = 5$.
   (a) Find a conjugate of x.
   (b) How many elements are there in the conjugacy class of x?
4. Let $G = \mathbb{D}_6 = \langle s, r \mid |s| = 2, |r| = 6, srs = r^5 \rangle$.
   (a) Find a conjugate of s.
   (b) How many elements are there in the conjugacy class of s?
5. Let $G = \mathbb{D}_6 = \langle s, r \mid |s| = 2, |r| = 6, srs = r^5 \rangle$.
   (a) Find a conjugate of r.
   (b) How many elements are there in the conjugacy class of r?
6. Let $G = \mathbb{D}_6 = \langle s, r \mid |s| = 2, |r| = 6, srs = r^5 \rangle$.
   (a) Find a conjugate of r.
   (b) How many elements are there in the conjugacy class of r?
7. Let $G = M_2(\mathbb{R})$ be the group of 2×2 matrices with entries in $\mathbb{R}$.
   Find the conjugate of matrix $X = \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}$ by $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.
8. Let $G = Gl_2(\mathbb{R})$ be the group of 2×2 (multiplicatively) invertible matrices with entries in $\mathbb{R}$.
   Find the conjugate of matrix $X = \begin{bmatrix} 1 & -2 \\ 3 & 6 \end{bmatrix}$ by $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.

9. Let $G = S_4$.
   (a) Find the subgroup $H = \langle (1342) \rangle$.
   (b) Find the conjugate of H by $\beta = (14)$.
10. Let $G = S_4$.
    (a) Find the subgroup A_4 of even permutations.
    (b) Find the conjugate of A_4 by $\beta = (14)$ in G.
11. Consider the group $\mathbb{Z}_{15}$.
    (a) Find the subgroup $H = \langle 3 \rangle$
    (b) Find the conjugate of H by 4.
12. Consider the group $\mathbb{Z}_{15}^{\times}$.
    (a) Find the subgroup $H = \langle 2 \rangle$
    (b) Find the conjugate of H by 4.
13. Consider the group $G = \mathbb{D}_6 = \langle s, r \mid |s| = 2, |r| = 6, srs = r^5 \rangle$.
    (a) Find the subgroup $H = \langle s \rangle$
    (b) Find the conjugate of H by the identity e
    (c) Find the conjugate of H by s
    (d) Find the conjugate of H by r
    (e) Find the conjugate of H by r^2
    (f) Find the conjugate of H by r^3
    (g) Find the conjugate of H by r^4
    (h) Find the conjugate of H by r^5
    (i) Find the conjugate of H by sr
    (j) Find the conjugate of H by sr^2
    (k) Find the conjugate of H by sr^3
    (l) Find the conjugate of H by sr^4
    (m) Find the conjugate of H by sr^5
    (n) Find the conjugacy class of H

Theoretic Questions

14. Write the definition of *Action of group on a set.*

15. Write the definition of *Orbit of a point.*

16. Write the definition of *Stabilizer.*

17. Write the definition of *Conjugacy class of an element.*

18. Write the definition of *Conjugacy class of a subgroup.*

19. Write the definition of gHg^{-1}.

Proofs

20. Let $H < G$ be a subgroup of G. Let $g \in G$. Prove that gHg^{-1} is a subgroup of G.

21. Let $H < G$ be a subgroup of G. Let $h \in H$. Prove that $hHh^{-1} = H$.

22. Let H be a subgroup of an abelian group G. Prove that H is normal subgroup of G.

23. Prove that the center of a group is normal subgroup, i.e. $Z(G)$ is normal subgroup in G.

24. Let H be a subgroup of a group G. Assume that H is contained in $Z(G)$, the center of G. Prove that H is a normal subgroup of G.

25. Let G be a group with $|G| = 25$ acting on a set X. What are the possible sizes of orbits?

26. Let G be a group with $|G| = 25$ acting on a set X with $|X| = 91$. Prove that there must be a fixed point.

27. Let G be a group with $|G| = 15$ acting on a set X with $|X| = 9$. Prove that there must be at least 3 orbits.

28. Suppose x is conjugate to y and y is conjugate to z. Prove that x is conjugate to z.

29. Prove that a factor group of a cyclic group is cyclic.

30. Prove that any subgroup of an Abelian group is normal subgroup.

True -False - Sometimes

31. True -False - Sometimes

    T F S - Let H be a subgroup of G. Then H is normal subgroup.

T F S - Let $G = (\mathbb{Z}_n, +_n)$, let H be a subgroup of G. Then H is normal subgroup.

T F S - Let H be a subgroup of G. Then $|H| = |gHg^{-1}|$ for all $g \in G$.

T F S - Let H be a subgroup of G. Then $H = gHg^{-1}$ for all $g \in G$.

T F S - Let H be a subgroup of an abelian group G. Then $H = gHg^{-1}$ for all $g \in G$.

T F S - Let G be a group of order $|G| = 5$ acting on a set X with $|X| = 10$. Then there is a fixed point.

T F S - Let G be a group of order $|G| = 5$. Let $H < G$. Then $|H| = 1$ or $|H| = 5$.

T F S - Let G be a group of order $|G| = 10$. Let $C(x)$ be a conjugacy class. Then $|C(x)| = 2$.

T F S - Let G be a cyclic group of order $|G| = 15$. Let $C(x)$ be a conjugacy class. Then $|C(x)| = 2$.

T F S - Let G be a group of order $|G| = 15$. Let $C(x)$ be a conjugacy class. Then $|C(x)| = 2$.

T F S - Let G be a cyclic group of order $|G| = 15$. Let $C(x)$ be a conjugacy class. Then $|C(x)| = 1$.

T F S - Let G be a group of order $|G| = 150$. Let $C(x)$ be a conjugacy class. Then $|C(x)| = 1$.

T F S - Let G be a group. Let $g \in G$. Then $\langle g \rangle$ is normal subgroup.

T F S - Let $G = D_5$. Let $g \in G$. Then $\langle g \rangle$ is normal subgroup.

T F S - Let $G = S_5$. Let $g \in G$. Then $\langle g \rangle$ is normal subgroup.

T F S - Let $G = \mathbb{Z}_5$. Let $g \in G$. Then $\langle g \rangle$ is normal subgroup.

Examples

32. Give an example of a group and an element x such that it's conjugacy class $C(x)$ has exactly one element. Prove your statement.

33. Give an example of a group and an element x such that it's conjugacy class $C(x)$ has more then one element. Prove your statement.

34. Give an example of a group G and a subgroup H which is not isomorphic to all of its conjugates.

35. Give an example of a group G and a subgroup H which is isomorphic to all of its conjugates.

36. Describe all conjugacy classes in D_5.

37. Describe all conjugacy classes in D_6.