1. Computations:
   (a) Find gcd(77,49).
   (b) Express gcd(77,49) as $\alpha \cdot 77 + \beta \cdot 49$=gcd(77,49).
   (c) Find an integer b so that $b \cdot 7 \equiv 1(mod\ 10)$
   (d) Find an integer b so that $b \cdot 7 \equiv 1(mod\ 10)$
   (e) Find an integer b so that $b \cdot 137 \equiv 1(mod\ 532)$
   (f) Find an integers b so that $b \cdot 138 \equiv 1(mod\ 532)$
   (g) Find an integer b so that $b \cdot 7 \equiv 1(mod\ 10)$
   (h) Find d=gcd(177,48) and express d as $\alpha \cdot 177 + \beta \cdot 48 =$ d.
   (i) Express gcd(177,49) as $\alpha \cdot 177 + \beta \cdot 49$=gcd(177,49).

2. Computations (mod 15). Always express your answer in the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$. Make sure that you show enough work.
   (a) $5 \cdot 2 \equiv$ ____ $(mod\ 15)$
   (b) $5 \cdot 6 \equiv$ ____ $(mod\ 15)$
   (c) $5 + 6 \equiv$ ____ $(mod\ 15)$
   (d) $9 \cdot 10 \equiv$ ____ $(mod\ 15)$
   (e) $5^2 \equiv$ ____ $(mod\ 15)$
   (f) $5^3 \equiv$ ____ $(mod\ 15)$
   (g) $5^4 \equiv$ ____ $(mod\ 15)$
   (h) $32 \cdot 6 \equiv$ ____ $(mod\ 15)$
   (i) $32^3 \equiv$ ____ $(mod\ 15)$
   (j) $151^7 \equiv$ ____ $(mod\ 15)$
   (k) $149^7 \equiv$ ____ $(mod\ 15)$
   (l) $1/7 \equiv$ ____ $(mod\ 15)$
   (m) $4/7 \equiv$ ____ $(mod\ 15)$

3. Solve the following congruences:
   (a) $5x \equiv 6(mod\ 7)$
   (b) $5x \equiv 6(mod\ 35)$
   (c) $5x \equiv 10(mod\ 35)$
   (d) $15x \equiv 6(mod\ 35)$
   (e) $15x \equiv 10(mod\ 35)$

(f) $153x \equiv 10(mod\ 35)$

4. Find the greatest common divisors gcd and least common multiples lcm for the following pairs of numbers:

   (a) $gcd(5,7) =$ ________ $lcm(5,7) =$ ________

   (b) $gcd(1,27) =$ ________ $lcm(1,27) =$ ________

   (c) $gcd(5^3 \cdot 7^2, 7 \cdot 11 \cdot 13^4) =$ ________________ $lcm(5^3 \cdot 7^2, 7 \cdot 11 \cdot 13^4) =$ ________________

   (d) $gcd(5^{123} \cdot 7^2, 5^2 \cdot 7^{11} \cdot 13^4) =$ ________________ $lcm(5^{123} \cdot 7^2, 5^2 \cdot 7^{11} \cdot 13^4) =$ ________________

   (e) $gcd(p^5, p^7) =$ ________ $lcm(p^5, p^7) =$ ________ (here p is a prime).

   (f) $gcd(10^5, 10^7) =$ ________ $lcm(10^5, 10^7) =$ ________

   (g) $gcd(56,77) =$ ________, $lcm(56,77) =$ ________

5. Find all the divisors of 15.

6. Find all the divisors of 27.

7. Find the prime factorization of 15.

8. Find the prime factorization of 27.

9. Find the prime factorization of 360.

10. True -False - Sometimes

    T F S - 5 has multiplicative inverse $(mod\ 11)$

    T F S - 6 has multiplicative inverse $(mod\ 11)$

    T F S - Let $a \in \mathbb{Z}_{>0}$. Then a has multiplicative inverse $(mod\ 11)$

    T F S - 5 has multiplicative inverse $(mod\ 10)$

    T F S - 6 has multiplicative inverse $(mod\ 10)$

    T F S - 7 has multiplicative inverse $(mod\ 10)$

    T F S - Let $a \in \mathbb{Z}_{>0}$. Then a has multiplicative inverse $(mod\ 10)$

    T F S - Let $10x \equiv 23(mod\ 41)$. There is a unique solution mod 41 for x.

    T F S - Let $10x \equiv 20(mod\ 40)$. There is a unique solution mod 40 for x.

    T F S - Let $10x \equiv 20(mod\ 40)$. There are 10 distinct solutions mod 40 for x.

    T F S - Let $10x \equiv c(mod\ 41)$. There is a unique solution mod 41 for x.

    T F S - Let $10x \equiv c(mod\ 40)$. There is a unique solution mod 40 for x.

    T F S - Let $10x \equiv c(mod\ 40)$. There are 10 distinct solutions mod 40 for x.

    T F S - Let $10x \equiv c(mod\ 40)$. There are no solutions mod 40 for x.

11. Make the table for the addition of equivalence classes (mod 8) which are given as

$$\mathbb{Z}_8 = \{[0]_8, [1]_8, [2]_8, [3]_8, [4]_8, [5]_8, [6]_8, [7]_8\}.$$

12. Make the table for the multiplication of equivalence classes (mod 8) which are given as

$$\mathbb{Z}_8 = \{[0]_8, [1]_8, [2]_8, [3]_8, [4]_8, [5]_8, [6]_8, [7]_8\}.$$

13. Make the table for the addition of equivalence classes (mod 3)

14. Make the table for the multiplication of equivalence classes (mod 3)

15. Examples

    (a) Give an example of a prime number.

    (b) Give an example of a number which is not prime number.

    (c) Give an example of two integers which are relatively prime.

    (d) Give an example of two integers which are not relatively prime.

    (e) Give an example of two integers a, b such that $gcd(a, b) = 12$.

    (f) Give an example of two integers a, b such that $lcm(a, b) = 12$.

    (g) Give an example of two integers a, b such that $lcm(a, b) = 12$.

    (h) Give an example of two integers a, b such that $lcm(a, b) = 5$.

    (i) Give an example of two integers a, b such that $gcd(a, b) = 5$.

    (j) Give an example of two integers a, b such that $gcd(a, b) = lcm(a, b)$.

    (k) Give an example of two integers a, b such that $[a]_7 + [b]_7 = [0]_7$.

    (l) Give an example of two integers $a, b \neq 0$ such that $[a]_6[b]_6 = [0]_6$.

HAVE FUN!