1. Basic ODEs

   (a) seperatable: $y' = \frac{F(x)}{G(y)}$
   $\Rightarrow \frac{dy}{dx} = \frac{F(x)}{G(y)} \Rightarrow \int G(y)dy = \int F(x)dx$

   (b) Linear: $y' + p(x)y = q(x)$
   Integrating Factor: $e^{\int p(x)dx}$
   $\Rightarrow (e^{\int p(x)dx}y)' = e^{\int p(x)dx}q(x)$
   $\Rightarrow e^{\int p(x)dx}y = \int e^{\int p(x)dx}q(x)dx$

   (c) $ay'' + by' + cy = 0$ (constant coefficient)
   characteristc EQ: $ar^2 + br + c = 0$
   $\Delta > 0, y = C_1e^{r_1x} + C_2e^{r_2x}$
   $\Delta = 0, y = C_1e^{rx} + C_2xe^{rx}$
   $\Delta > 0, r = p \pm qi : y = e^{ax}[c_1cos(bx) + c_2sin(bx)]$

2. Fourier Series
   Given $F(x), x \in [-L, L]$ write $F(x)$ in a series:

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos(\frac{n\pi x}{L}) + \sum_{n=1}^{\infty} b_n sin(\frac{n\pi x}{L})$$

   where $a_n, b_n$ are constants.

   (a) Orthogonality Relations
   $\int_{-L}^{L} sin(\frac{n\pi x}{L})cos(\frac{m\pi x}{L}) = 0$
   $\int_{-L}^{L} cos(\frac{n\pi x}{L})cos(\frac{m\pi x}{L}) = 0(m \neq n), L(m = n)$
   $\int_{-L}^{L} sin(\frac{n\pi x}{L})sin(\frac{m\pi x}{L}) = 0(m \neq n), L(m = n)$

   (b) $a_n = \frac{1}{L}\int_{-L}^{L} F(x)cos(\frac{n\pi x}{L})dx$
   $b_n = \frac{1}{L}\int_{-L}^{L} F(x)sin(\frac{n\pi x}{L})dx, n \in [0, \infty], n \in \mathbf{Z}$

   (c) Convergece Statement of F.S.
   F.S. convergence to the "periodic extension" of F(x) whever F(x) is continous and to the average of $\frac{f(x^+)+f(x^-)}{2}$ at every point.

   (d) F.S.S and F.C.S of $F(x)$ on $[0, L]$:
   F.C.S = $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cos\frac{n\pi x}{L}, a_n = \frac{2}{L}\int_0^L F(x)cos(\frac{n\pi x}{L})dx$
   F.S.S = $\sum_{n=1}^{\infty} b_n sin\frac{n\pi x}{L}, a_n = \frac{2}{L}\int_0^L F(x)cos(\frac{n\pi x}{L})dx$
   F.C.S $\rightarrow$ even extension, F.S.S $\rightarrow$ odd extension