THE FOUR BASIC S-L BVP'S

EQUATION: $f'' + \lambda f = 0$, $f = f(x)$, $0 \le x \le L$

(1) BOUNDARY CONDS. $f(0) = 0$ $f(L) = 0$

e-values $\lambda_n = \frac{n^2 \pi^2}{L^2}$ $n = 1, 2, \ldots$

e-functions $f_n \sim \sin \frac{n\pi x}{L}$

(2) BOUNDARY CONDS. $f'(0) = 0$ $f'(L) = 0$

e-values $\lambda_n = \frac{n^2 \pi^2}{L^2}$ $n = 0, 1, 2, \ldots$

e-functions $f_n \sim \cos \frac{n\pi x}{L}$ * note n starts at 0

BOUNDARY CONDS. $f(0) = 0$ $f'(L) = 0$

e-values $\lambda_n = \left[\frac{(2n-1)\pi}{2L}\right]^2$ $n = 1, 2, \ldots$

e-functions $f_n \sim \sin \frac{(2n-1)\pi}{2L} \cdot x$

) BOUNDARY CONDS. $f'(0) = 0$ $f(L) = 0$

e-values $\lambda_n = \left[\frac{(2n-1)\pi}{2L}\right]^2$

e-functions $f_n \sim \cos \frac{(2n-1)\pi}{2L} \cdot x$

Regular S-L problems

Eqn $\quad (p f')' + q f + \lambda \sigma f = 0 \qquad a < x < b$

.dy conds $\quad \begin{cases} K_1 f(a) + K_2 f'(a) = 0 \\ K_3 f(b) + K_2 f'(b) = 0 \end{cases}$

p, q, σ are fuctions of x
p, q, σ are required to be cont on $[a, b]$
p' is required to be cont on $[a, b]$
p ad σ are required to be positive
K_1, K_2, K_3, K_4 are constants

S-L Theory says

1) There are always an infinite number of real e-vals for the prolem $\lambda_1 < \lambda_2 < \dots < \lambda_n \dots$ so that $\lim_{n \to \infty} \lambda_n = \infty$.

2) Eigenfuctions corresponding to distinct e-values are orthogonal with respect to σ on $[a, b]$. This means

$$\int_a^b f_i(x) f_j(x) \sigma(x) \, dx = 0 \quad \text{for all } i \neq j.$$

Every p.w. differentiable function $g(x)$ on $[a, b]$ can be "written" as a sum of a complete set of e-fuctions:

$$g(x) \sim \sum_{n=1}^{\infty} a_n f_n(x) \quad - \text{ a generalized F.S. of } g(x)$$

The coefficients are computed as follows

$$a_n = \frac{\int_a^b g(x) \cdot f_n(x) \sigma(x) \, dx}{\int_a^b \sigma(x) f_n^2(x) \, dx}$$