

2.2

#3

$$f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ -1 & 1 < x \leq 2 \end{cases} \quad \text{Write its FCS and FSS}$$

Soln  $L=2$

$$\text{FCS } a_0 = \frac{2}{L} \int_0^L f(x) dx = 0$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx = \int_0^1 \cos \frac{n\pi x}{2} dx - \int_1^2 \cos \frac{n\pi x}{2} dx$$

$$= \frac{2}{n\pi} \sin \frac{n\pi x}{2} \Big|_0^1 - \frac{2}{n\pi} \sin \frac{n\pi x}{2} \Big|_1^2 = \left( \frac{4}{n\pi} \sin \frac{n\pi}{2} \right)$$

$$f(x) \sim \sum_{n=1}^{\infty} \left( \frac{4}{n\pi} \sin \frac{n\pi}{2} \right) \cos \frac{n\pi x}{2}$$

$$\text{FSS } b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx = \int_0^1 \sin \frac{n\pi x}{2} dx - \int_1^2 \sin \frac{n\pi x}{2} dx$$

$$= \int_0^1 \sin \frac{n\pi x}{2} dx - \int_1^2 \sin \frac{n\pi x}{2} dx = -\frac{2}{n\pi} \cos \frac{n\pi x}{2} \Big|_0^1 + \frac{2}{n\pi} \cos \frac{n\pi x}{2} \Big|_1^2$$

$$= -\frac{2}{n\pi} \cos \frac{n\pi}{2} + \frac{2}{n\pi} + \frac{2}{n\pi} \cos n\pi - \frac{2}{n\pi} \cos \frac{n\pi}{2}$$

$$= \frac{2}{n\pi} (1 + \cos n\pi - 2 \cos \frac{n\pi}{2})$$

$$f(x) \sim \sum_{n=1}^{\infty} \frac{2}{n\pi} (1 + \cos n\pi - 2 \cos \frac{n\pi}{2}) \sin \frac{n\pi x}{2}$$

