

$$\begin{cases} f'' + \lambda f = 0 \\ f'(0) - f(0) = 0, \quad f(1) = 0 \end{cases}$$

chapter 3.1 #3

Find e-val's & e-funct's

Determine e-val's graphically

solu:  $\lambda = 0 \Rightarrow f = ax + b \quad f' = a$

$$f'(0) - f(0) = 0 \Rightarrow a - b = 0 \Rightarrow f = ax + a \quad f' = a$$

$$f(1) = 0 \Rightarrow 2a = 0 \Rightarrow a = 0 \Rightarrow b = 0 \Rightarrow \lambda = 0 \text{ not an e-val}$$

$$\lambda > 0 \quad f = c_1 \cos \sqrt{\lambda} x + c_2 \sin \sqrt{\lambda} x \quad f' = -c_1 \sqrt{\lambda} \sin \sqrt{\lambda} x + c_2 \sqrt{\lambda} \cos \sqrt{\lambda} x$$

$$f'(0) - f(0) = 0 \Rightarrow c_2 \sqrt{\lambda} \cos \sqrt{\lambda} - c_1 = 0$$

$$\Rightarrow c_1 = c_2 \sqrt{\lambda}$$

$$\Rightarrow f = c_2 \sqrt{\lambda} \cos \sqrt{\lambda} x + c_2 \sin \sqrt{\lambda} x = c_2 (\sqrt{\lambda} \cos \sqrt{\lambda} x + \sin \sqrt{\lambda} x)$$

$$f(1) = 0 \Rightarrow c_2 (\sqrt{\lambda} \cos \sqrt{\lambda} + \sin \sqrt{\lambda}) = 0$$

$$\Rightarrow \tan \sqrt{\lambda} = -\sqrt{\lambda}$$

$\lambda < 0$ ; let  $\lambda = -a^2$  with  $a > 0$

$$f = c_1 \cosh ax + c_2 \sinh ax$$

$$f' = c_1 a \sinh ax + c_2 a \cosh ax$$

$$f'(0) - f(0) = 0 \Rightarrow c_2 a - c_1 = 0$$

$$c_1 = c_2 a$$

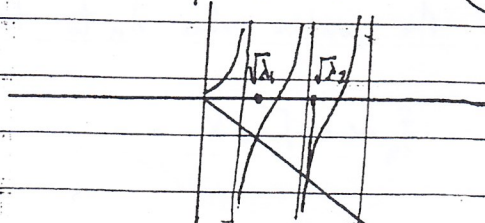
$$\Rightarrow f = c_2 a \cosh ax + c_2 \sinh ax = c_2 (a \cosh ax + \sinh ax)$$

$$f(1) = 0 \Rightarrow \tanh a = -a \quad \text{no solutions}$$

so the e-val's of the problem are the positive sol's  $\lambda_1, \lambda_2, \dots$  to  $\tan \sqrt{\lambda} = -\sqrt{\lambda}$

and the e-funct's are

$$f_n \sim \sqrt{\lambda_n} \cos \sqrt{\lambda_n} x + \sin \sqrt{\lambda_n} x$$



$$\sqrt{\lambda_1} \approx 2.03$$

$$\sqrt{\lambda_2} \approx 4.91$$

$$\sqrt{\lambda_n} \approx \frac{2n-1}{2} \pi \quad n \text{ large}$$

§3.1.1 #4 Find e-values and e-functions

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$$f'' + \lambda f = 0$$

$$f'(0) = 0$$

$$f'(1) + f(1) = 0$$

$$\lambda = 0 \Rightarrow f = a + bx \Rightarrow f' = b$$

$$f'(0) = 0 \Rightarrow b = 0 \Rightarrow f = a \Rightarrow f' = 0$$

$$f'(1) + f(1) = 0 \Rightarrow a = 0, a = 0, b = 0 \Rightarrow \lambda = 0 \text{ not an e-val}$$

$$\lambda > 0 \Rightarrow f = c_1 \cos \sqrt{\lambda} x + c_2 \sin \sqrt{\lambda} x \quad f' = -c_1 \sqrt{\lambda} \sin \sqrt{\lambda} x + c_2 \sqrt{\lambda} \cos \sqrt{\lambda} x$$

$$f'(0) = 0 \Rightarrow c_2 = 0 \Rightarrow f = c_1 \cos \sqrt{\lambda} x \quad f' = -c_1 \sqrt{\lambda} \sin \sqrt{\lambda} x$$

$$f'(1) + f(1) = 0 \Rightarrow -c_1 \sqrt{\lambda} \sin \sqrt{\lambda} + c_1 \cos \sqrt{\lambda} = 0 \Rightarrow \tan \sqrt{\lambda} = \frac{1}{\sqrt{\lambda}}$$

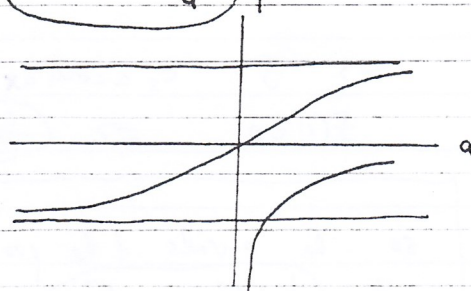
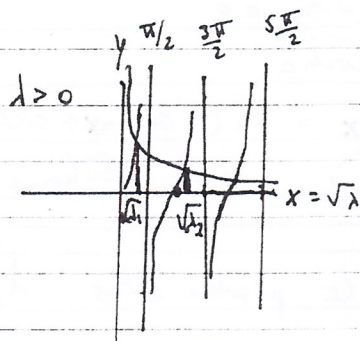
$$\lambda < 0, \lambda = -a^2, a > 0 \Rightarrow f = c_1 \cosh ax + c_2 \sinh ax$$

$$f' = c_1 a \sinh ax + c_2 a \cosh ax$$

$$f'(0) = 0 \Rightarrow c_2 = 0 \Rightarrow f = c_1 \cosh ax \quad f' = c_1 a \sinh ax$$

$$f'(1) + f(1) = 0 \Rightarrow c_1 a \sinh a + c_1 \cosh a = 0$$

$$\Rightarrow \tanh a = -\frac{1}{a}$$



estimates  $\sqrt{\lambda_1} \approx .86 \quad \sqrt{\lambda_2} \approx 3.4$

$$\sqrt{\lambda_n} \approx (n-1)\pi \quad n \text{ large}$$

no intersections

e-values The e-values of the problem are the positive solns  $\lambda_n$  to  $\tan \sqrt{\lambda} = \frac{1}{\sqrt{\lambda}}$

e-functions The e-functions of the problem are  $f_n \approx \cos \sqrt{\lambda_n} x$