1. Basic ODEs

   (a) seperatable: $y' = \frac{F(x)}{G(y)}$
   $\Rightarrow \frac{dy}{dx} = \frac{F(x)}{G(y)} \Rightarrow \int G(y)dy = \int F(x)dx$

   (b) Linear: $y' + p(x)y = q(x)$
   Integrating Factor: $e^{\int p(x)dx}$
   $\Rightarrow (e^{\int p(x)dx}y)' = e^{\int p(x)dx}q(x)$
   $\Rightarrow e^{\int p(x)dx}y = \int e^{\int p(x)dx}q(x)dx$

   (c) $ay'' + by' + cy = 0$ (constant coefficient)
   characteristc EQ: $ar^2 + br + c = 0$
   $\Delta > 0, y = C_1e^{r_1x} + C_2e^{r_2x}$
   $\Delta = 0, y = C_1e^{rx} + C_2xe^{rx}$
   $\Delta < 0, r = p \pm qi : y = e^{px}[c_1cos(qx) + c_2sin(qx)]$

   (d) $ay'' + by' + cy = f(x)$
   $y = y_c + y_p$, y_c is solution to homogeneous DIFF EQ
   $f(x)$:a polynomial in x or single sin/cos function
   $y_p = x^k$(a polynomial of the same degree), k: # char eq's zero roots (0,1,2)
   $f(x) = e^{ax}$(a polynomial in x)
   $y_p = x^ke^{ax}$(same degree), k: # char eq's roots $= a$ (0,1,2)
   $f(x) = e^{ax}cos(bx)$(poly in x) or $e^{ax}sin(bx)(apolyinx)$
   $y_p = x^ke^{ax}[$(poly in x)$cos(bx) +$ (poly in x)$sin(bx)]$
   k: # char eq's root $= a \pm bi$ (0,1)

2. Laplace Transform
   Def: $\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$

$$Def : H_a(t) = H(t-a) = \begin{cases} 1, & t \geq a \\ 0, & t < a \end{cases} \tag{1}$$

   Def: $erf(x) = \frac{2}{\pi}\int_0^x e^{-u^2}du$, $erfc(x) = 1 - erf(x)$

$f(t) = \mathcal{L}^{-1}F(t)$	$F(t) = \mathcal{L}f(s)$
$f^{(n)}(t)$ (nth derivative)	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0$
$H(t-a)f(t-a)$	$e^{-as}F(s)$
$e^{at}f(t)$	$F(s-a)$
$(f*g)(t)$	$F(s)G(s)$
1	$\frac{1}{s}$ $(s > 0)$
t^n (n is positive integer)	$\frac{n!}{s^{n+1}}$ $(s > 0)$
e^{at}	$\frac{1}{s-a}$ $(s > a)$
$sin(at)$	$\frac{a}{s^2+a^2}$ $(s > 0)$
$cos(at)$	$\frac{s}{s^2+a^2}$ $(s > 0)$
$sinh(at)$	$\frac{a}{s^2-a^2}$ $(s > \|a\|)$
$cosh(at)$	$\frac{s}{s^2-a^2}$ $(s > \|a\|)$
$\delta(t-a)$ $(a \geq 0)$	e^{-as}
t^ne^{at}	$\frac{n!}{(s-a)^{n+1}}$
$e^{at}sinbt$	$\frac{b}{((s-a)^2+b^2)}$
$e^atcosbt$	$\frac{s-a}{((s-a)^2+b^2)}$
$\text{erfc}(\frac{a}{2\sqrt{t}})$	$\frac{1}{s}e^{-a\sqrt{s}}$
$-a\sqrt{\frac{t}{\pi}}e^{-\frac{a^2}{4t}} + (\frac{1}{2}a^2 + t)\text{erfc}\frac{a}{2\sqrt{t}}$	$\frac{1}{s^2}e^{-a\sqrt{s}}$

Example:
$\mu_{tt}(x,t) = \mu_{xx}(x,t) + 1$, $\mu(x,0) = \mu_t(x,0) = -1, \mu(0,t) = t$
$\mathcal{L}\mu_{tt} = s^2U - s\mu(x,0) - \mu_t(x,0) = s^2U + 1$
$\mathcal{L}\mu_{xx} + 1 = U_{xx} + \frac{1}{s}$
$\Rightarrow U'' - s^2U = 1 - \frac{1}{s}$ (x is variable, s is scalar)
$U_c = k_1e^{sx} + k_2e^{-sx}, k_1 = 0$
$U_p = \frac{1-\frac{1}{s}}{-s^2} = \frac{1}{s^3} - \frac{1}{s^2}$
$\mathcal{L}\mu(0,t) = \mathcal{L}t = \frac{1}{s} = U(0,s)$
$\Rightarrow k_2 + \frac{1}{s^3} - \frac{1}{s^2} = \frac{1}{s^2}$
$\Rightarrow U = (\frac{2}{s^2} - \frac{1}{s^3})e^{-sx} - \frac{1}{s^2} + \frac{1}{s^3}$
$\mu = \mathcal{L}^{-1}(U) = H(t-x)[2(t-x) - \frac{1}{2}(t-x)^2] - t + \frac{1}{2}t^2$