Generalized F.S. exs

3.1.2

$$2x-1 = \sum_{n=1}^{\infty} c_n \sin \frac{2n-1}{2} x \qquad 0 \le x \le \pi$$

$$c_n = \frac{\int_0^\pi (2x-1) \sin \frac{(2n-1)}{2} x \, dx}{\int_0^\pi \sin^2 \frac{(2n-1)}{2} x \, dx}$$

$u = 2x-1 \qquad dv = \sin \frac{2n-1}{2} x$

$du = 2 \qquad v = \frac{-2}{2n-1} \cos \frac{2n-1}{2} x$

$$c_n = (2x-1)\left(\frac{-2}{2n-1} \cos \frac{2n-1}{2} x\right)\Bigg|_0^\pi + \frac{4}{2n-1}\int_0^\pi \cos \frac{(2n-1)}{2} x \, dx$$

$$= \frac{2}{2n-1} + \frac{8}{(2n-1)^2} \sin \frac{2n-1}{2} x \Bigg|_0^\pi$$

$$= -\frac{2}{2n-1} + \frac{8}{(2n-1)^2} \sin \frac{(2n-1)}{2}\pi$$

$$\text{denom} = \frac{L}{2} = \frac{\pi}{2}$$

$$2x-1 \sim \sum_{n=1}^{\infty}\left(-\frac{4}{(2n-1)\pi} + \frac{16}{(2n-1)^2\pi} \sin \frac{(2n-1)\pi}{2}\right) \sin \frac{(2n-1)x}{2}$$

Generalized F.S. examples

3.1.2 #10 Write $u(x) = x+1$ in a generalized F.S. in the e-functions $\cos\frac{(2n-1)\pi x}{2}$ $n=1,2,\ldots$ $0 \le x \le 1$

Soln: $$x+1 \sim \sum_{n=1}^{\infty} c_n \cos\frac{(2n-1)\pi x}{2} \qquad 0 \le x \le 1$$

$$c_n = \frac{\int_0^1 (x+1)\cos\frac{(2n-1)\pi}{2}x\,dx}{\int_0^1 \cos^2\frac{(2n-1)\pi}{2}x\,dx}$$

$u = x+1$
$du = dx$

$dv = \cos\frac{(2n-1)\pi x}{2}\,dx$

$v = \frac{2}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2}x$

num. $$(x+1)\frac{2}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2}x\Big|_0^1 - \int_0^1 \frac{2}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2}x\,dx$$

$$= \frac{4}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2} + \frac{4}{(2n-1)^2\pi^2}\cos\frac{(2n-1)\pi}{2}x\Big|_0^1$$

$$= \frac{4}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2} - \frac{4}{(2n-1)^2\pi^2}$$

denom: $\frac{1}{2}$

$$x+1 \sim \sum_{n=1}^{\infty}\left[\frac{8}{(2n-1)\pi}\sin\frac{(2n-1)\pi}{2} - \frac{8}{(2n-1)^2\pi^2}\right]\cos\frac{(2n-1)\pi x}{2}$$

Note $\sin\frac{(2n-1)\pi}{2} = (-1)^{n+1}$