9.2.1 #2 $u_{tt} = u_{xx} - 1$

$u(0,t) = t + 1$

$u(x,0) = 2 \qquad u_t(x,0) = 0$

Soln.   Let $U(x,s) = \mathcal{L}[u(x,t)] \qquad \Rightarrow$

$\qquad s^2 U - s u(x,0) - u_t(x,0) = U'' - \dfrac{1}{s}$

$\qquad s^2 U - 2s = U'' - \dfrac{1}{s}$

$\qquad U'' - s^2 U = -2s + \dfrac{1}{s}$

$U_c = h_1 e^{sx} + h_2 e^{-sx} \qquad\qquad U_p = \dfrac{2}{s} - \dfrac{1}{s^3}$

$U = h_2 e^{-sx} + \dfrac{2}{s} - \dfrac{1}{s^3}$

$U(0,s) = \mathcal{L}[u(0,t)] = \dfrac{1}{s^2} + \dfrac{1}{s} \qquad \Rightarrow$

$\dfrac{1}{s^2} + \dfrac{1}{s} = h_2 + \dfrac{2}{s} - \dfrac{1}{s^3} \qquad \Rightarrow \qquad h_2 = -\dfrac{1}{s} + \dfrac{1}{s^2} + \dfrac{1}{s^3}$

$U = \left(-\dfrac{1}{s} + \dfrac{1}{s^2} + \dfrac{1}{s^3} \right) e^{-sx} + \dfrac{2}{s} - \dfrac{1}{s^3}$

$\mathcal{L}^{-1}\left(-\dfrac{1}{s} + \dfrac{1}{s^2} + \dfrac{1}{s^3} \right) = -1 + t + \dfrac{1}{2}t^2 \qquad \Rightarrow$

$u(x,t) = \left(-1 + (t-x) + \dfrac{1}{2}(t-x)^2 \right) H(t-x) + 2 - \dfrac{1}{2}t^2$

9.2.1 #3

$$u_{tt} = 4u_{xx} + e^{-3t} \qquad u_x(0,t) = 2t \qquad u(x,0) = 0 \qquad u_t(x,0) = -1$$

$$U \text{ bd at } \infty.$$

Soln $\quad s^2 U - s\, u(x,0) - u_t(x,0) = 4 U'' + \dfrac{1}{s+3}$

$$s^2 U + 1 = 4 U'' + \dfrac{1}{s+3}$$

$$U'' - \dfrac{s^2}{4} U = \dfrac{s+2}{4(s+3)}$$

$$U'(0,s) = \mathcal{L}\left[u_x(0,t) \right] = \dfrac{2}{s^2}$$

$$U_c = c_1 e^{-\frac{s}{2}x}$$

$$U_p = A \implies -\dfrac{s^2}{4} A = \dfrac{s+2}{4(s+3)} \implies A = -\dfrac{s+2}{s^2(s+3)}$$

$$U = c_1 e^{-\frac{s}{2}x} - \dfrac{s+2}{s^2(s+3)}$$

p.f. $\quad \dfrac{s+2}{s^2(s+3)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{s+3} \implies \begin{cases} A = \frac{1}{9} \\ B = \frac{2}{3} \\ C = -\frac{1}{9} \end{cases}$

$$U = c_1 e^{-\frac{s}{2}x} - \left(\dfrac{1}{9s} + \dfrac{2}{3s^2} - \dfrac{1}{9(s+3)} \right)$$

$$U' = c_1 \cdot \left(-\dfrac{s}{2} e^{-\frac{s}{2}x} \right) \qquad U'(0,s) = \dfrac{2}{s^2} \implies c_1 = \dfrac{-4}{s^3}$$

$$U = -\dfrac{4}{s^3} e^{-\frac{s}{2}x} - \left(\dfrac{1}{9s} + \dfrac{2}{3s^2} - \dfrac{1}{9(s+3)} \right)$$

$$\mathcal{L}^{-1}\left[-\dfrac{4}{s^3} \right] = -2t^2 \implies$$

$$u(x,t) = -2\left(t - \dfrac{x}{2} \right)^2 H\left(t - \dfrac{x}{2} \right) - \left[\dfrac{1}{9} + \dfrac{2}{3} + - \dfrac{1}{9} e^{-3t} \right]$$