A Generalized F.S. for the problem

$$\begin{cases} f'' + \lambda f = 0 & f = f(x) \quad 0 \le x \le 1 \\ f'(0) = 0 & f(1) - f'(1) = 0 \end{cases}$$

Summary: We found one negative e-value $\lambda_1 = -a_1^2$. Its e-function is

$$f_1 \sim \cosh a_1 x \qquad a_1 \approx 1.2$$

where a_1 is the positive solution to $\tanh a = \frac{1}{a}$

The positive e-vals we found were $\lambda_2, \lambda_3, \ldots,$ solutions to $\tan\sqrt{\lambda} = -\frac{1}{\sqrt{\lambda}}$.

The e-functions are $f_n \sim \cos\sqrt{\lambda_n}\, x \qquad n = 2, 3 \ldots$

$\lambda_2 \approx 7.8 \qquad \lambda_3 \approx 37.2 \ldots$

Write a given $g(x) \qquad 0 \le x \le 1$

in a generalized F.S. in these e-functions

$g(x) =$

$c_n =$

B Another SL BVP

$f'' + \lambda f = 0 \quad , \quad 0 \le x \le 1$

$f'(0) + f(0) = 0 \qquad f(1) = 0$

Find e-vals and e-functions

Soln:

$\lambda = 0 \Rightarrow f = \alpha x + \beta \quad , \quad f' = \alpha$

$f'(0) + f(0) = 0 \Rightarrow \alpha + \beta = 0$
$f(1) = 0 \Rightarrow \alpha + \beta = 0$
$\Rightarrow \beta = -\alpha$

$\Rightarrow$

$\lambda > 0 \qquad f = c_1 \cos\sqrt{\lambda}\, x + c_2 \sin\sqrt{\lambda}\, x \quad ; \quad f' = -c_1\sqrt{\lambda}\sin\sqrt{\lambda}\, x + c_2\sqrt{\lambda}\cos\sqrt{\lambda}\, x$

$f'(0) + f(0) = 0 \Rightarrow c_1 + c_2\sqrt{\lambda} = 0 \Rightarrow c_1 = -c_2\sqrt{\lambda}$

$\Rightarrow f = c_2\left(-\sqrt{\lambda}\cos\sqrt{\lambda}\, x + \sin\sqrt{\lambda}\, x\right)$

$f(1) = 0 \Rightarrow \tan\sqrt{\lambda} = \sqrt{\lambda}$

$\lambda < 0 \quad \lambda = -a^2 \quad a > 0 \Rightarrow f = c_1 \cosh ax + c_2 \sinh ax$

$f' = c_1 a \sinh ax + c_2 a \cosh ax$

$f'(0) + f(0) = 0 \Rightarrow c_1 + c_2 a = 0 \Rightarrow c_1 = -c_2 a$

$\Rightarrow f = c_2 \left(-a\cosh ax + c_2 \sinh ax\right)$

$f(1) = 0 \Rightarrow \tanh a = a$