9.1 exercise

Find $\mathcal{L}[f(t)]$

1) $f(t) = e^{-t}\sin 3t - 3t^4$

$$\mathcal{L}[f(t)] = \frac{3}{(s+1)^2+9} - 3\cdot\frac{4!}{s^5}$$

2) $f(t) = e^{4t}\cos 2t + 4(t-3)^3 H(t-3)$

$$\mathcal{L}[f(t)] = \frac{s-4}{(s-4)^2+4} + 4\,\frac{3!}{s^4}\,e^{-3s}$$

3) $f(t) = e^{-2t}(\cos t - 3\sin t) - 2t^2 H(t-1)$

$$\mathcal{L}[e^{-2t}\cos t] = \frac{s+2}{(s+2)^2+1}$$

$$\mathcal{L}[-3e^{-2t}\sin t] = -3\cdot\frac{1}{(s+2)^2+1}$$

$$t^2 = (t-1)^2 + 2(t-1) + 1$$

$$\Rightarrow -2t^2 H(t-1) = -2\left[(t-1)^2 H(t-1) + 2(t-1)H(t-1) + H(t-1)\right]$$

$$\mathcal{L}[-2t^2 H(t-1)] = -2\left[\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right]e^{-s}$$

$$\mathcal{L}[f(t)] = \frac{s+2}{(s+2)^2+1} - 3\,\frac{1}{(s+2)^2+1} - 2\left[\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}\right]e^{-s}$$

Some L.T. problems

9.1 #4 Given $f(t) = t^3 e^{t-2} - 3\sin^2\left(\frac{t}{2}\right)$. Find $\mathcal{L}[f(t)]$

Soln: $f(t) = e^{-2} \cdot t^3 e^t - 3\left(\frac{1}{2} - \frac{1}{2}\cos t\right)$

$$\Rightarrow \mathcal{L}[f(t)] = e^{-2} \cdot \frac{3!}{(s-1)^4} - 3\left(\frac{1}{2s} - \frac{1}{2} \cdot \frac{s}{s^2+1}\right)$$

#5 Given $F(s) = \dfrac{2s+1}{s^2-2s+26}$ Find $\mathcal{L}^{-1}(F(s))$

Sol: $$\frac{2s+1}{s^2-2s+26} = \frac{2s+1}{(s-1)^2+5^2} = \frac{2(s-1)}{(s-1)^2+5^2} + \frac{3}{5} \cdot \frac{5}{(s-1)^2+5^2}$$

$$\Rightarrow \mathcal{L}^{-1}(F(s)) = e^t\left[2\cos 5t + \frac{3}{5}\sin 5t\right]$$

#6 $F(s) = \dfrac{3s+2}{s^2+6s+25} - \dfrac{2s}{(s-1)^2} e^{-s}$ Find $\mathcal{L}^{-1}(F(s))$

Soln: $$F(s) = \frac{3s+2}{(s+3)^2+4^2} - \left[\frac{2(s-1)}{(s-1)^2} + \frac{2}{(s-1)^2}\right] e^{-s}$$

$$= 3\frac{(s+3)}{(s+3)^2+4^2} - \frac{7}{4}\frac{4}{(s+3)^2+4^2} - \left[\frac{2}{s-1} + \frac{2}{(s-1)^2}\right] e^{-s}$$

$$\Rightarrow \mathcal{L}^{-1}(F(s)) = e^{-3t}\left(3\cos 4t - \frac{7}{4}\sin 4t\right) - 2\left[e^{t-1} + (t-1)e^{t-1}\right] H(t-1)$$