Undetermined Coefficients - Review

$ay'' + by' + cy = g(x)$ $\quad y = y(x)$, a, b, c constants

Solve the complementary eqn $ay'' + by' + cy = 0$ for the general solution y_c.

Find any solution y_p to the given equation - this is a particular solution.

Then $y = y_c + y_p$ is the general solution to the given equation.

To find y_p in the "standard" cases.

a) If $g(x)$ is a polynomial of degree n let $y_p = A_n x^n + \cdots + A_1 x + A_0$, a general polynomial of degreen n. Substitute and solve for the coefficients

b) If $g(x)$ is an exponential with exponent αx let $y_p = Ae^{\alpha x}$. Substitute and solve for the coefficient.

c) If $g(x)$ is a linear combination of sines and cosines with frequency α let $y_p = A\cos\alpha x + B\sin\alpha x$. Substitute and solve for the coefficients.

This method doesn't always work - (turn over)

First, if $g(x)$ is a product of two or all three of the "types" in a), b) and c) set your y_p equal to a corresponding product.

The method may fail because of <u>duplication</u>

To see if there's duplication first get your y_c. Then set up the <u>standard</u> y_p.

If any of the individual terms in your y_p solve the complementary equation that term will substitute into the left hand side of the given equation to give zero and you won't be able to solve for the coefficient. Ie it "duplicates" the complementary solution

In that case modify the standard y_p by xy_p.

If xy_p also duplicates modify to $x^2 y_p$.