Wave Exs.

5.2.1 #3 Formal Soln.

$$u(x,t) = \sum_{n=1}^{\infty} \left(b_{1n} \cos n\pi t + b_{2n} \sin n\pi t \right) \sin n\pi x$$

$$u(x,0) = 2 \sin 3\pi x \implies 2 \sin 3\pi x = \sum_{n=1}^{\infty} b_{1n} \sin n\pi x$$

$\underline{n=3}$ $\quad b_{13} = 2 \quad$ all other b_{1n}'s $= 0$

$$u_t(x,0) = 2 \implies 2 = \sum_{n=1}^{\infty} n\pi b_{2n} \sin n\pi x$$

$$n\pi b_{2n} = \frac{\int_0^1 2 \sin n\pi x\,dx}{\int_0^1 \sin^2 n\pi x\,dx}$$

num. $\quad -\frac{2}{n\pi} \cos n\pi x \Big|_0^1 = -\frac{2}{n\pi} \cos n\pi + \frac{2}{n\pi}$

$$\implies b_{2n} = \frac{-4 \cos n\pi}{n^2\pi^2} + \frac{4}{n^2\pi^2}$$

$$u(x,t) = 2 \cos 3\pi t \sin 3\pi x + \sum_{n=1}^{\infty} \left(\frac{4}{n^2\pi^2} - \frac{4}{n^2\pi^2} \cos n\pi \right) \sin n\pi x \sin n\pi t$$

5.1.2 #4 Formal Soln: $u(x,t) = \sum_{n=1}^{\infty} (b_{1n} \cos n\pi t + b_{2n} \sin n\pi t) \sin n\pi x$

$u(x,0) = 1 \Rightarrow 1 = \sum_{n=1}^{\infty} b_{1n} \sin n\pi x$

$$b_{1n} = \frac{\int_0^1 \sin n\pi x\,dx}{\int_0^1 \sin^2 n\pi x\,dx} \qquad \text{num} = -\frac{1}{n\pi}\cos n\pi x \Big|_0^1 = -\frac{1}{n\pi}\cos n\pi + \frac{1}{n\pi}$$

$$\Rightarrow \quad b_{1n} = \frac{2}{n\pi} - \frac{2}{n\pi}\cos n\pi$$

$u_t(x,0) = x \Rightarrow x = \sum_{n=1}^{\infty} n\pi b_{2n} \sin n\pi x$

$$\Rightarrow \quad n\pi b_{2n} = \frac{\int_0^1 x \sin n\pi x\,dx}{\int_0^1 \sin^2 n\pi x\,dx} \qquad \begin{array}{ll} u = x & dv = \sin n\pi x \\ du = dx & v = -\frac{1}{n\pi}\cos n\pi x \end{array}$$

$$\text{num} \quad -\frac{x}{n\pi}\cos n\pi x \Big|_0^1 + \int_0^1 \frac{1}{n\pi}\cos n\pi x\,dx = -\frac{1}{n\pi}\cos n\pi$$

$$\Rightarrow \quad b_{2n} = -\frac{2}{n^2\pi^2}\cos n\pi$$

$$u(x,t) = \sum_{n=1}^{\infty}\left(\frac{2}{n\pi} - \frac{2}{n\pi}\cos n\pi\right)\cos n\pi t \sin n\pi x - \sum_{n=1}^{\infty} \frac{2\cos n\pi}{n^2\pi^2} \sin n\pi t \sin n\pi x$$