Upcoming

Quiz Friday 2/22

5.1.1, 5.1.2, 5.1.3 heat equation

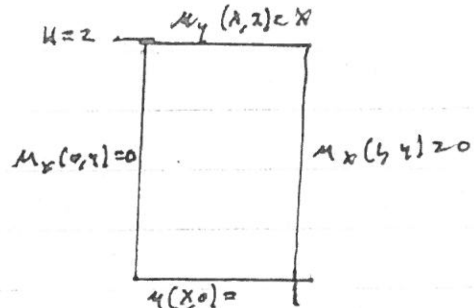
5.2.1 wave " , with fixed ends

graphing waves using d'Alembert's Solution

You may use the sheet of basic SL - BVP +
1 sheet of notes with anything you like on it.

Take home assignment - due on or before 3/1

Laplace in a Rectangle



§5.3.1 #. 4 $\quad u_x(0,y)=0 \qquad u_x(1,y)=0$

$u(x,0)=2-\cos\pi x \qquad u_y(x,2)=x$

Soln: translate boundary conds.

$X''+\lambda X=0 \qquad Y''-\lambda Y=0$

$X'(0)=0 \qquad X'(1)=0$

$\Rightarrow \lambda_n=n^2\pi^2 \quad X_n\sim\cos n\pi x \quad \Rightarrow \quad Y_0=a_0y+b_0$

$n=0,1,2,\dots \qquad n>0,\ Y_n=\sinh n\pi y+\delta_n\cosh n\pi(y-2)$

formal soln: $u(x,y)=a_0y+b_0+\sum_{n=1}^{\infty}\cos n\pi x\left(a_n\sinh n\pi y+b_n\cosh n\pi(y-2)\right)$

$u(x,0)=2-\cos\pi x \Longrightarrow$ by matching up $b_0=2 \quad b_1=\frac{-1}{\cosh 2\pi}$

$$u_y(x,y)=a_0+\sum_{n=1}^{\infty}\cos n\pi x\left(a_n\cdot n\pi\cosh n\pi y+b_n n\pi\sinh n\pi(y-2)\right)$$

$u_y(x,2)=x \Longrightarrow \boxed{x=a_0+\sum_{n=1}^{\infty}(a_n\cdot n\pi\cosh 2n\pi)\cos n\pi x}$

$$a_0=\frac{\int_0^1 x\,dx}{\int_0^1 dx}=\frac{1}{2} \qquad n\pi\cosh(2n\pi)\,a_n=\frac{\int_0^1 x\cos n\pi x\,dx}{\int_0^1\cos^2 n\pi x\,dx}$$

after integration and solving for a_n: $a_n=\frac{2}{n^3\pi^3\cosh 2n\pi}(\cos n\pi-1)$

Assembling the soln:

$$u(x,y)=\frac{1}{2}y+2+\sum_{n=1}^{\infty}\frac{2(\cos n\pi-1)}{n^3\pi^3\cosh 2n\pi}\cos n\pi x\sinh n\pi y-\frac{1}{\cosh 2\pi}\cos\pi x\cosh\pi(y-2)$$