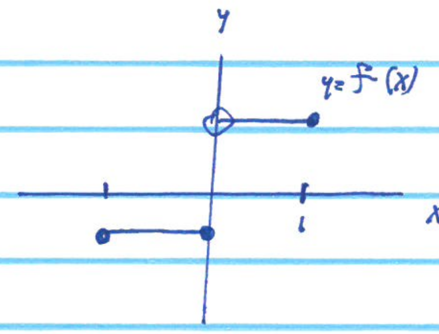
F.S. exercise



2.1 #3

$$f(x) = \begin{cases} -2 & -1 \le x \le 0 \\ 3 & 0 < x \le 1 \end{cases}$$

Find the F.S. of $f(x)$

Soln: $L = 1$ $\quad a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \int_{-1}^{1} f(x)\,dx = 1$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx = \int_{-1}^{1} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$= \int_{-1}^{0} -2\cos n\pi x\,dx + \int_{0}^{1} 3\cos n\pi x\,dx = 0$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx = \int_{-1}^{0} -2\sin n\pi x\,dx + \int_{0}^{1} 3\sin n\pi x\,dx =$$

$$\frac{2}{n\pi}\cos n\pi x\Big|_{-1}^{0} - \frac{3}{n\pi}\cos n\pi x\Big|_{0}^{1} = \frac{2}{n\pi} - \frac{2}{n\pi}\cos n\pi - \frac{3}{n\pi}\cos n\pi + \frac{3}{n\pi}$$

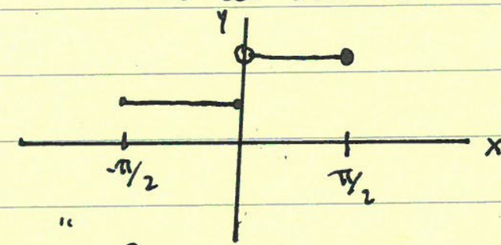
$$= \frac{5}{n\pi}\left(1 - \cos n\pi\right)$$

$$f(x) \sim \frac{1}{2} + \sum_{n=1}^{\infty} \frac{5}{n\pi}\left(1 - \cos n\pi\right)\sin n\pi x$$

2.1 #4

$$f(x) = \begin{cases} 1 & -\frac{\pi}{2} \le x \le 0 \\ 2 & 0 < x \le \frac{\pi}{2} \end{cases}$$

Write and sketch the F.S.



Sol $\quad L = \frac{\pi}{2}$

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = \frac{1}{\pi/2} \cdot \text{"area"} = 3$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx = \frac{2}{\pi}\left[\int_{-\pi/2}^{0} \cos 2nx\,dx + \int_{0}^{\pi/2} 2\cos 2nx\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{2n}\sin 2nx\Big|_{-\pi/2}^{0} + \frac{1}{n}\sin 2nx\Big|_{0}^{\pi/2}\right] = 0$$

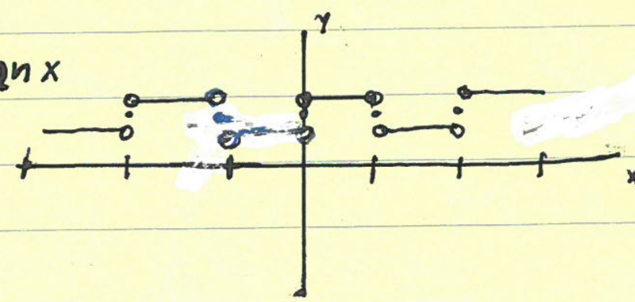
$$b_n = \frac{2}{\pi}\left[\int_{-\pi/2}^{0} \sin 2nx\,dx + \int_{0}^{\pi/2} 2\sin 2nx\,dx\right]$$

$$= \frac{2}{\pi}\left[-\frac{1}{2n}\cos 2nx\Big|_{-\pi/2}^{0} - \frac{1}{n}\cos 2nx\Big|_{0}^{\pi/2}\right]$$

$$= \frac{2}{\pi}\left[-\frac{1}{2n} + \frac{1}{2n}\cos n\pi - \frac{1}{n}\cos n\pi + \frac{1}{n}\right]$$

$$= \frac{2}{\pi}\left[\frac{1}{2n} - \frac{1}{2n}\cos n\pi\right] = \frac{1}{n\pi}\left[1 - \cos n\pi\right]$$

$$f(x) = \frac{3}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi}\left[1 - \cos n\pi\right]\sin 2nx$$

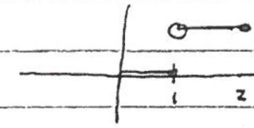


FCS al FSS problems

Chapter 2.2

#1 $f(x) = \begin{cases} 0 & 0 \le x \le 1 \\ 1 & 1 < x \le 2 \end{cases}$ Write al sketch its FCS al FSS.

Soln FCS $L = 2$



$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{2}\cdot 1 = 1$$

$n \ge 1$ $$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = \int_1^2 \cos\frac{n\pi x}{2}\,dx = \frac{2}{n\pi}\sin\frac{n\pi x}{2}\Big|_1^2 = -\frac{2}{n\pi}\sin\frac{n\pi}{2}$$

$$f(x) \sim \frac{1}{2} + \sum_{n=1}^{\infty}\left(-\frac{2}{n\pi}\sin\frac{n\pi}{2}\right)\cos\left(\frac{n\pi x}{2}\right)$$

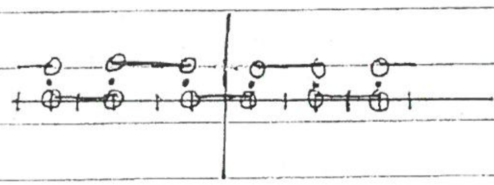
FSS $L = 2$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx = \int_1^2 \sin\frac{n\pi x}{2}\,dx = -\frac{2}{n\pi}\cos\frac{n\pi x}{2}\Big|_1^2$$

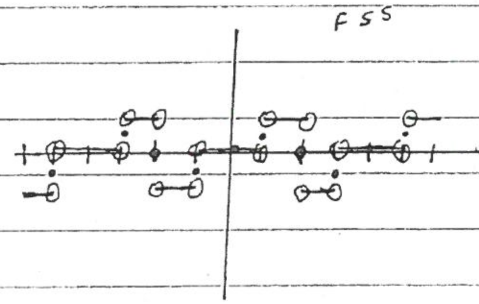
$$= \frac{2}{n\pi}\cos\frac{n\pi}{2} - \frac{2}{n\pi}\cos n\pi$$

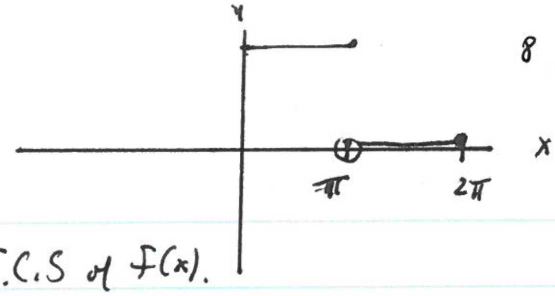
$$f(x) \sim \sum_{n=1}^{\infty}\frac{2}{n\pi}\left(\cos\frac{n\pi}{2} - \cos n\pi\right)\sin\frac{n\pi x}{2}$$

FCS



FSS





2.2 #2 $f(x) = \begin{cases} 2 & 0 \le x \le \pi \\ 0 & \pi < x \le 2\pi \end{cases}$

Find and graph the F.S.S and F.C.S of $f(x)$.

Soln: $L = 2\pi$

F.C.S. $a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\cdot 2\pi = 2$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}\,dx = \frac{1}{\pi}\int_0^{\pi} 2\cos\frac{nx}{2}\,dx = \frac{4}{n\pi}\sin\frac{nx}{2}\Big|_0^{\pi}$$

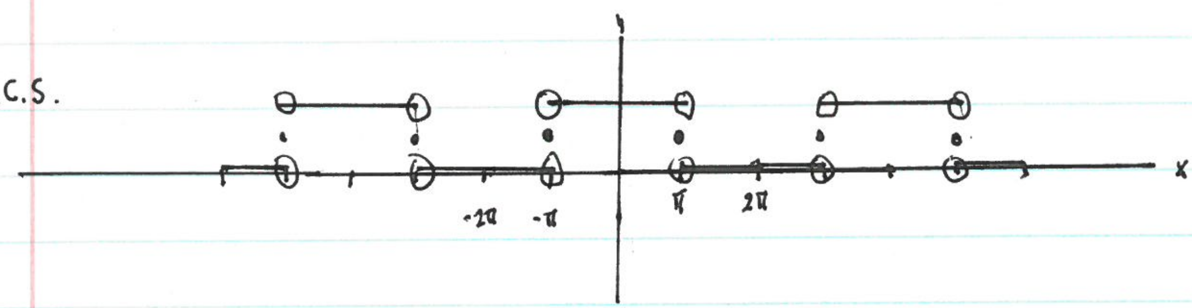
$= \frac{4}{n\pi}\sin\frac{n\pi}{2}$ => F.C.S is $f(x) = 1 + \sum_{n=1}^{\infty}\left(\frac{4}{n\pi}\sin\frac{n\pi}{2}\right)\cos\frac{nx}{2}$

F.S.S. $b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{2} = \frac{1}{\pi}\int_0^{\pi} 2\sin\frac{nx}{2}\,dx =$

$= -\frac{4}{n\pi}\cos\frac{nx}{2}\Big|_0^{\pi} = -\frac{4}{n\pi}\cos\frac{n\pi}{2} + \frac{4}{n\pi}$ $\Rightarrow$

F.S.S is $f(x) = \sum_{n=1}^{\infty}\left(\frac{4}{n\pi} - \frac{4}{n\pi}\cos\frac{n\pi}{2}\right)\sin\frac{nx}{2}$

F.C.S.



F.S.S.