MTH 4581: Fall 2018: Prof. C. King

Homework 6

Reading: Notes on Poisson Process, notes on Queues.

Due date: Thursday November 8

Problems:

1) Consider a discrete time, single server queue with states $\{0, 1, 2, 3\}$ (so there are at most 3 customers in the system at any time). At each time step the number of customers can change by 0, 1 or -1, depending on whether a new customer arrives or a current customer completes their service. The system is modeled by a Markov chain with these transition probabilities:

$$\begin{aligned} &p_{01} = p_{12} = p_{23} = p, \\ &p_{10} = p_{21} = p_{32} = q, \\ &p_{11} = p_{22} = r = 1 - q - p, \\ &p_{00} = 1 - p, \quad p_{33} = 1 - q \end{aligned}$$

a) Find the stationary vector of the chain (your answer will depend on p, q). Hint: solve the reversibility equations to find w.

b) Using your answer in part **(a)**, find
(i) the probability that the system is empty
(ii) the probability that the queue is empty
(iii) the probability that the queue is full
(iv) E$[N]$ = the mean number in the system
(v) E$[N_q]$ = the mean number in the queue
(vi) λ_a = the arrival rate
(vii) λ_d = the departure rate

c) Apply Little's Law to the answers from part **(c)** to find
(i) E$[T]$ = the mean time spent in the system
(ii) E$[T_q]$ = the mean time spent in the queue
(iii) E$[S]$ = the mean time spent in service

d) Find the probability that the system is full for three successive time steps.

2) The law offices of Dewey, Cheathem and Howe receive two types of phone calls from clients. Happy clients call at the rate $\lambda_1 = 5$ per hour, and angry clients call at the rate $\lambda_2 = 15$ per hour. Assume happy and angry calls arrive as independent Poisson processes.
(i) Find the probability that at least 3 calls arrive in the next 5 minutes.
(ii) Find the probability that at least 2 of the next 10 calls are from happy clients.

3) On average 2 customers arrive per hour at the Tuff–luk service center, and service lasts on average 25 minutes for each customer. Assuming this system is described by a M/M/1 queue, find
(i) the probability that there are 3 customers waiting in the queue
(ii) the mean length of the queue
(iii) the mean time a customer spends waiting in the queue
(iv) the probability that a customer spends more than 3 hours waiting in the queue.

4) The office support company Shred-Itall has three machines available at all times. On average 5 customers arrive per hour, and the average service time per customer is 33 minutes. Assuming this system is described by a M/M/3 queue, find
(i) the probability that the queue is empty
(ii) the mean length of the queue
(iii) the mean time a customer spends waiting in the queue