**MTH 4581: Fall 2018: Prof. C. King**

**Homework 4**

**Reading:** Grinstead and Snell, Chapter 11.

**Due date:** Thursday October 18

**Problems:**

**1)** Grinstead and Snell: p.422, Problem # 7. [Write down the modified transition matrix in canonical form, find $Q$ and $R$, and compute $N$ and $B$]

**2)** Grinstead and Snell: p.422, Problem # 9. [Write down the transition matrix in canonical form, find $Q$ and $R$, compute $N$ and $B$, and use your result to answer the question]

**3)** We solved the Gambler's Ruin problem in class. Recall the setup: the state space is $\{0, 1, 2, \ldots, N\}$, and at each step the gambler moves one step left or right with probability 1/2. Both 0 ('ruin') and $N$ ('fortune') are absorbing. We found that the probability of reaching fortune before ruin, starting at $X_0 = k$, is

$$p_k = \frac{k}{N}$$

Now let $g_k$ be the expected number of steps for the gambler to reach either ruin or fortune, starting at $X_0 = k$. Analyze this in the same way, by conditioning on the first step to get a difference equation for $g_k$ in terms of $g_{k+1}$ and $g_{k-1}$. Look for a solution of the form $A + Bk + Ck^2$, and use the boundary conditions to determine the constants.

**4)** Grinstead and Snell: p.442, Problem # 3.

**5)** Grinstead and Snell: p.442, Problem # 5.

**6)** A fair die is continually rolled, and the sum modulo 5 is computed at each step, so

$$X_n = (R_1 + R_2 + \cdots + R_n) \text{ modulo } 5$$

where $R_k$ is the result of the $k^{th}$ roll.

a) Write down the transition matrix for this chain.

b) Compute

$$\lim_{n\to\infty} \mathrm{P}(X_n = 1)$$

c) Find the mean first return time starting in state 4.