MTH 4581: Fall 2018: Prof. C. King

Notes on Stochastic Processes

A stochastic process $\{X(t)\}$ is a family of random variables indexed by time t. The process is fully described by the (infinite) family of joint distributions

$$\{f_{X(t_1),\ldots,X(t_n)} \, : \, t_1 < \cdots < t_n, \; n = 1, 2, \ldots\}$$

The process is *stationary* if for all constant c, all $n \geq 1$ and all $t_1 < \cdots < t_n$ we have

$$f_{X(t_1+c),\ldots,X(t_n+c)} = f_{X(t_1),\ldots,X(t_n)}$$

Often it is very difficult to compute the joint distributions, and it may not be necessary to do so. The analogous situation for a single random variable is where knowledge of the mean and standard deviation is enough to answer a question. Similarly the stochastic process is partly described by its first and second moments. The first moment is the mean $m(t)$, which is a function of one variable:

$$m(t) = \mathrm{E}[X(t)]$$

The second moment is the autocorrelation $R(t, s)$ which is a function of two variables:

$$R(t, s) = \mathrm{E}[X(t)\, X(s)]$$

The autocovariance is then given by

$$C(t, s) = R(t, s) - m(t)\, m(s) = \mathrm{E}[(X(t) - m(t))(X(s) - m(s))]$$

The process is *wide-sense stationary* if for all constant c, and all t, s we have

$$\begin{aligned} m(t + c) &= m(t) \\ R(t + c, s + c) &= R(t, s) \end{aligned}$$