MATH 4581 PRACTICE PROBLEMS FOR QUIZ 3

1). Consider a Markov chain on the integers $\{0, 1, 2, \ldots\}$ with transition probabilities

$$p_{k,k+1} = p, \quad p_{k,0} = q = 1 - p \qquad \text{for all } k \geq o$$

where $0 < p < 1$. Compute the stationary distribution of the chain, and find the mean first return time to state 1.

2). Suppose a bus arrives on average every 10 minutes. If this is a Poisson process:
a) find the probability that 3 buses arrive in the next 15 minutes.
b) find the probability that you have to wait at least 10 minutes for the next bus.
c) find the expected time until 5 buses have arrived

3). Suppose the mean time between arrivals at a station is 5 minutes. Assuming a Poisson process,
a) find the probability that at least 2 people arrive in the next minute.
b) find the probability that the time until the next arrival is more than 10 minutes.
c) find the expected time until the 100th arrival.
d) estimate the probability that more than 130 arrivals come in the next 10 hours.

4). Suppose that 2 people arrive per hour and service takes an average of 10 minutes per customer. Assuming it is a $M/M/1$ queue, find the mean number of customers in the system, the mean number in the queue, the mean time spent in queue, and the mean total time in the system.

5). Referring to Problem (4), find by how much the arrival rate needs to change in order to double the mean total time in the system.

6). Referring to Problem (4), find the median total time spent in the system.

7). A company has 5 phone lines that receive calls as a Poisson process, and a call is put on hold if all 5 lines are busy. If a telephone line gets a call on average every two minutes and calls last on average four minutes,

a) find the probability that a caller is put on hold.
b) find the mean number of calls in the system.
c) find the mean total time spent by a caller in the system.

8). We will compare an M/M/1 system with a M/M/3 system. Suppose that the number entering the system is Poisson with an average of 2 per minute while it takes 20 seconds to process a person for the M/M/1 system and 1 minute for the M/M/3 system.

a) find the mean number in the queue, the mean time spent in the queue, and the mean time spent in the system for the M/M/1 model.
b) find the mean number in the queue, the mean time spent in the queue, and the mean time spent in the system for the M/M/3 model.
c) which system better serves the needs of a customer? which system better serves the needs of the machine operator (the operator loses money whenever the service machines are idle)?