

MTH 4581: Fall 2018: Prof. C. King

Homework 2

Reading: notes on moment generating functions; review of t -distribution; tables for t -distribution, F -distribution and chi-square distribution.

Due date: Thursday September 20

Problems:

1) Let X be a chi-square random variable with 7 degrees of freedom. Find z so that

$$P(X \geq z) = 0.05$$

2) Let X be a chi-square random variable with n degrees of freedom, where $n \geq 1$ is an integer. Find the largest value of n so that

$$P(X \geq 20) \leq 0.05$$

3) Let X, Y, Z be independent chi-square random variables with 7, 9, 12 degrees of freedom respectively. Find z so that

$$P(X + Y + Z \leq z) = 0.99$$

4) Let X, Y be independent chi-square random variables with 7, 3 degrees of freedom respectively. Let $F = (3X)/(7Y)$. Find z so that

$$P(F \geq z) = 0.1$$

5) Let T be a student t -distribution with 5 degrees of freedom. Find z so that

$$P(-1.476 \leq T \leq z) = 0.85$$

6) Ten independent measurements of a normal random variable give 62, 52, 68, 23, 45, 27, 42, 83, 56, 40. The mean and variance of the random variable are unknown. Use the t -distribution to test the hypothesis that the mean is 55 (use a two-sided test at significance level $\alpha = 0.05$). Find the p -value of the data.

7) A random variable can take values $\{1, 2, 3, 4\}$. The observed frequencies from 200 measurements are $\{85, 70, 25, 20\}$ respectively. The null hypothesis is

$$H_0 : p_1 = 0.4, p_2 = 0.3, p_3 = 0.2, p_4 = 0.1$$

and the alternative hypothesis is that these are not the probabilities. Use goodness of fit to test H_0 at the 99% significance level (state the decision rule, test the data, and state your conclusion). Also calculate the p -value of the data.