MATH 4581.

Assignment 7 : SOLUTIONS

(1) $X(t) = A + t \qquad A \sim U[0,1]$

$\Rightarrow E[A] = \frac{1}{2} \quad E[A^2] = \frac{1}{3}$

$VAR[A] = \frac{1}{12}.$

$m(t) = E[X(t)] = \frac{1}{2} + t$

$R(t,s) = E[X(t)X(s)] = E[(A+t)(A+s)]$

$= \frac{1}{3} + \frac{1}{2}(s+t) + st.$

$C(t,s) = R(t,s) - m(t)m(s) = \frac{1}{3} + \frac{1}{2}(s+t) + st - (\frac{1}{2}+t)(\frac{1}{2}+s)$

$= \frac{1}{12}$

(2) $A, B \sim N(0,1).$

$X(t) = A + tB.$

$m(t) = E[X(t)] = 0$

$R(t,s) = E[X(t)X(s)] = E[(A+tB)(A+sB)]$

$= E[A^2] + st\,E[B^2]$

$= 1 + st.$

$C(t,s) = 1 + st.$

(3) Wager 1: buy stock

outcome	prob.	cost	future value	return
150	p	100	150	50
30	$1-p$	100	30	-70

Wager 2: buy option

outcome	prob	cost	future value	return
150	p	c	30	$30-c$
30	$1-p$	c	0	$-c$

No arbitrage $\Rightarrow$ $E[R_1] = 50p - 70(1-p) = 0$

$E[R_2] = (30-c)p - c(1-p) = 0$

$\Rightarrow$ $c = \$ \frac{35}{2} = \$ 17.50$

Risk-free prob. vector $\vec{p} = \left(\frac{7}{12}, \frac{5}{12}\right)$.

(4) Wager 1: buy stock

outcome	prob.	cost	future value	return
200	p	100	200	100
120	q	100	120	20
30	r	100	30	-70

Wager 2: buy option

outcome	prob.	cost	future value	return
200	p	25	90	65
120	q	25	10	-15
30	r	25	0	-25

Wager 3: buy the option

outcome	prob	cost	future value	return
200	p	c	110	110 − c
120	q	c	30	30 − c
30	r	c	0	−c.

No arbitrage:

$$E[R_1] = 100p + 20q - 70r = 0$$

$$E[R_2] = 65p - 15q - 25r = 0$$

$$E[R_3] = (110 - c)p + (30 - c)q - c(r) = 0.$$

$$p + q + r = 1.$$

$$\Rightarrow \quad c = \$\,\frac{145}{4} \simeq \$\,36.25$$

Risk-free prob. vector $\vec{p} = \left(\frac{31}{128}, \frac{41}{128}, \frac{56}{128}\right)$