

1). Find the moment generating functions for the random variables with the following distributions:

$$(a) \quad P(X = k) = \frac{16}{37} \left(\frac{3}{4}\right)^k \quad \text{for } k = 0, 1, 2$$

$$(b) \quad f_X(x) = \frac{1}{4} \quad \text{for } 1 \leq x \leq 5$$

2). The moment generating function for X is $M_X(t) = e^{5t}(1 - 4t^2)^{-1}$. Find $E[X]$.

3).

x	19	22	25	26	28	29	29	30
y	47	37	38	40	36	33	39	31

a) Test at the 5% level to see if the slope of the regression line is zero. Give the p -value and decide if you accept or reject the null hypothesis. Give the equation of the regression line and the value of s , the sample standard deviation.

b) Find a 99% confidence interval for the mean value of y when $x = 28$. Show your work.

4). Below are three groups of data (notice that the numbers in groups A, B are the same as those in Problem (3)):

A	19	22	25	26	28	29	29	30
B	47	37	38	40	36	33	39	31
C	25	36	31	28	37	29		

a) Test at the 5% level to see if the means are the same for all three groups. Give the p -value and the pooled standard deviation s_p , and decide if you accept or reject the null hypothesis.

b) Use the Bonferroni method to test at the 3% level of significance if the first and third groups have the same mean (note that the sample means are $\bar{x}_A = 26, \bar{x}_C = 31$). Show your work.

c) Use the contrast with $H_0 : 2\mu_B = \mu_A + \mu_C$ to test at the 5% level of significance whether the second group is different from the first and third groups.

5). A random number generator is supposed to have the distribution given in Problem (1), part (a). A sample from the generator finds these values:

x	0	1	2
Observed	25	15	10
Expected			

a) Fill in the expected numbers.

b) Find the value of χ^2 for a goodness of fit test. Decide at the 5% level of significance if the data has the supposed distribution. Show your work (either with a p -value or a critical value).

6).

	a	b	Total
c	60	30	
d	40	70	
Total			

	a	b	Total
c			
d			
Total			

a) The first table gives the observed joint values for two variables. Fill in the expected numbers in the second table assuming independence.

b) Find the value of χ^2 . Decide at the 5% level of significance if the variables are independent. Show your work (either with a p -value or a critical value).

7). The joint pdf of X and Y is

$$f_{X,Y}(x,y) = \frac{3}{2}xy \quad \text{for } 0 \leq x \leq 2, 0 \leq y \leq 2x$$

a) Find the conditional pdf $f_Y(y|x)$.

b) Set up the integral for the regression equation of Y on X . Do not evaluate the integral.

8). The following transition matrix describes a Markov chain on the states $\{1, 2, 3, 4\}$:

$$P = \begin{pmatrix} 0.4 & 0.3 & 0.2 & 0.1 \\ 0 & 1 & 0 & 0 \\ 0.3 & 0.2 & 0.3 & 0.2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a) Rewrite the matrix in canonical form and give Q and R .

b) Find the fundamental matrix N .

c) Find the probability to get absorbed in state 4 given that the chain starts in state 3. Show your work.

d) Find the expected number of steps to get absorbed given that the chain starts in state 3.

9). Suppose we move around the grid

1	2
4	3

 according to the following scheme: at each step we roll a fair die; if we roll 1 then we stay where we are; if we roll 2 or 3 then we move one space clockwise; else we move two spaces counter-clockwise.

a) Find the transition matrix of the chain.

b) Find the probability to move from 1 to 3 in three steps.

c) Find the stationary vector of the chain.

d) Find the mean first return time if we start in state 1.

e) Find the mean time to go from state 2 to state 1 for the first time.