

MTH 4581: Fall 2018: Prof. C. King

Homework 7

Reading: Notes on Stochastic Processes, notes on Arbitrage.

Due date: Wednesday November 28

Problems:

1) Let $X(t) = A + t$, where A is randomly chosen from the interval $[0, 1]$ (that is, A is the uniform random variable on $[0, 1]$). Find the mean $m(t)$, the autocorrelation $R(t, s)$, and the autocovariance $C(t, s)$ of the stochastic process $X(t)$.

2) Let A and B be two independent standard normal random variables (with mean 0 and variance 1). Define the stochastic process $X(t) = A + tB$. Find the mean $m(t)$, the autocorrelation $R(t, s)$ and the autocovariance $C(t, s)$ of the stochastic process $X(t)$.

3) Let the current price of the stock be \$100 per unit, and suppose in the future it can change to either \$150 or \$30. What should be the price \$c of an option to buy one unit of stock for \$120 in the future (assuming we do not want any arbitrage opportunity, and assuming that shares and the option may be bought now)? Give the risk-free probability vector.

4) Let the current price of the stock be \$100, and suppose in the future it can change to either \$200, \$120, or \$30. Assume that we have three possibilities in the present moment:

1. Buy shares of stock
2. Buy options to buy the stock for \$110 in the future. This option costs \$25.
3. Buy options to buy the stock for \$90 in the future. This option costs \$c.

If we do not want arbitrage, what should be the price \$c? Give the risk-free probability vector.