

MTH 4581: Fall 2018: Prof. C. King

Homework 8

Reading: Notes on Brownian motion and Black-Scholes.

Due date: Wednesday December 5.

Problems:

1) Let $X(t)$ be a geometric Brownian motion with initial value $X(0) = 1$, drift parameter $\mu = 2$, and variance $\sigma^2 = 3$. Compute the autocorrelation $R(3, 1) = E[X(3)X(1)]$.

2) Assume that the stock price $X(t)$ is modeled by a geometric Brownian motion with drift parameter $\mu = -1$ per year, and variance parameter $\sigma^2 = 12$ per year. The current price is \$50.

- a) What is the probability that after 1 year the price is greater than or equal to \$60?
- b) What is the probability that this happens after 1.5 years?

3) The current price of a stock is \$100, and we assume that the price can be modeled by geometric Brownian motion $X(t)$ with a drift parameter of 3% per year with a variance parameter of 2% per year. Assume that the annual interest rate is 4% and suppose that we want to sell an option to buy the stock for \$120 in 1.5 years.

- a) What should be the initial price of the option if we do not want an arbitrage opportunity?
- b) What should be the price of the option after one year if the stock price has risen to \$140, and again assuming we do not want an arbitrage opportunity?