Fall 2018 Math 4581   **Final Review**

Note: Problems 1-12 are from the Fall 2017 Final.

1. A study is looking to see which method works best to heal a leg injury. They break patients into four groups with the following results (in days until recovery):

Physical Therapy and drug A (group PTA): 19.4, 18.7, 19, 19.9, 19.3, 19.7, 20.2
PT and placebo (group PTP): 20.3, 20.6, 20.9, 21.8, 21.1, 21.6, 22.1
Drug A (group A): 21, 21.9, 22.6, 22.1, 21.3
Placebo (group P): 20.9, 21.7, 21.1, 22

a) Test at the 5% level of significance to see if the means are all the same. Give the p-value, s_p and decide if you accept or reject the null hypothesis.

b) Test at the 5% level of significance to see if the PT groups are different from the placebo group (use the contrast: $H_0 : \frac{\mu_{pta} + \mu_{ptp}}{2} = \mu_p$). Show your work.

2. Given the data below:

x	19.4	18.7	19	19.9	19.3	19.7	20.2
y	20.3	20.6	20.9	21.8	21.1	21.6	22.1

a) Test at the 5% level to see if the slope of the regression line is 0. Give the p-value and decide if you accept or reject the null hypothesis. Also give the regression line and s.

b) Find a 95% confidence interval for the mean value of y when x=23. Show your work.

3. The model for the number of hits a site gets is Poisson with a mean of 1.5 hits per minute. A sample finds the following:

Hits (in a minute)	0	1	2	More
Actual	27	27	24	22
Expected				

a) fill in the expected numbers.

b) find X^2 and the critical value, then decide if the model is correct at the 5% level of significance. Show your work.

4. Given the transition matrix (the states are: 1,2,3) $P = \begin{pmatrix} .5 & .3 & .2 \\ .4 & .4 & .2 \\ .3 & .3 & .4 \end{pmatrix}$

a) Find the fixed vector w.

b) Find the mean time to go from state 1 to state 3 for the first time.

5. Given the transition matrix (the states are: 1,2,3,4) $P = \begin{pmatrix} .5 & .1 & .1 & .3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ .4 & .2 & .1 & .3 \end{pmatrix}$

a) Rewrite the matrix in canonical form and give Q.

b) Find the fundamental matrix N.

c) If we start in state 4, find the probability that we end in state 2.

6. If we have the M/M/2 system where 3 jobs arrive per hour on average and each job takes 30 minutes to finish,
a) find p_0, p_1, and p_2. Show your work.

b) find E(N) and E(T)

7. A store has three employees who each can serve 1 customer a minute. The arrival rate depends on the number of people **in line**:
if there is no one in line then 2 people arrive per minute;
The store can only hold 4 people so if anyone else arrives they're turned away.

a) Find the steady state probabilities. Show your work.

b) Find E(N), E(N_q), E(W), and E(T). Show your work.

8. Find the moment generating function, $M_X(t)$, for $f_X(x) = 2x$ for $0 \le x \le 1$. Show your work and given any restrictions on t.

9. If $f_{X,Y}(x,y) = \dfrac{1}{2}x(y+1)$ for $0 < x < 2$, $0 < y < 2-x$,

a) find $f_Y(y|x)$. Show your work.

b) Set up the integral for the regression equation.

10. If $f_X(x) = \dfrac{1}{9}x^2$ for $0 \le x \le 3$ and $Y = 10 - X^2$, then find the cdf for Y.

11. The current price of a stock is \$50 and we assume it can be modeled by geometric Brownian motion with a drift parameter of 2.5% per year and a variance parameter of 1% per year. If the interest rate is 3% per year and we want to sell an option to buy the stock for \$55 in 2 years, what should be the initial price of the option for there not to be an arbitrage opportunity? Show your work.

12. Suppose the current price of a share of stock is \$100 and it will change to either \$40 or \$300 in the future.

a) Find the price, \$c, to buy a unit of an option to buy the stock for \$160 in the future so there is not an arbitrage opportunity. Show your work and find p=P(stock changes to \$40).

b) We buy x shares of stock and y options. If c=\$10, then find values of x and y that guarantee that you make money.

13. On average I get 40 emails a day, where 15 of them are spam. If this is Poisson:
a) Find the probability that 3 of the next 5 emails are spam. Give your answer to three decimal places.

b) Find the probability that I get 3 emails in the next hour. Give your answer to three decimal places.

14. Let $X(t) = t - 2a$ where A has the density $f_A(a) = \dfrac{3}{8}a^2$ for $0 \le a \le 2$.
a) Find $F_{X(t)}(x)$.   **b)** Find the mean.
c) Set up the integral for the autocorrelation.

15. For the random variable X with the following pdf, $f_X(x) = \dfrac{1}{3}e^{-\frac{1}{3}(x-4)}$ for $x \ge 4$

a) Show that the moment generating function $M_X(t)$ is: $M_X(t) = \dfrac{e^{4t}}{1-3t}$ and give the domain of t where $M_X(t)$ converges.

b) Use $M_X(t)$ to find E(X).

16. A bucket has 3 marbles that are either red or white. At each step you will randomly take out a marble and then toss a coin–if the toss is a T then you put in a red marble and if it's an H you put in a white marble. You stop once the bucket has only white marbles.
a) Give the transition matrix where the states are the number of white marbles in the bucket.

b) Find the probability of going from having 2 red marbles in the bucket to having 1 red marble in 3 steps.

c) Find the expected number of steps until you stop if there are originally 3 red marbles in the bucket.

17. For M/M/3 where 6 messages arrive each minute on average and it takes 20 seconds to process each:
a) solve for the steady state probabilities p_0, p_1, p_2, and p_3.
b) using the fact that $F_W(x|W > 0) = 1 - e^{-c\mu(1-(\rho/c))x}$, find $P(W < .5)$.

16. $B(t)$ is Brownian motion with $\sigma = 2.5$.
a) Find $P(B(1) < 3)$ **b)** Find $P(B(4) - B(1) > 5, B(8) - B(4) < -4)$