HW1,

1) a) $g(-t)$  b) $e^t g(t)$  c) $g(3t)$  d) $e^{bt} g(at)$

2) mgf: $g(t) = E[e^{tX}] = \sum\limits_{k=0}^{\infty} e^{tk} \dfrac{1}{ek!} = \dfrac{1}{e} e^{e^t}$

$g'(t) = \dfrac{1}{e} e^{e^t} \cdot e^t$ $\qquad\qquad\qquad g'(0) = E[X] = 1$

$g''(t) = \dfrac{1}{e} e^{e^t} e^{2t} + \dfrac{1}{e} e^{e^t} e^t$ $\qquad g''(0) = E[X^2] = 2$

$\Rightarrow$ $VAR[X] = 1$

3) $g(t) = E[e^{tX}] = \displaystyle\int_0^{\infty} e^{tx} 9x \, e^{-3x} = \dfrac{9}{(3-t)^2}$ $\qquad (t < 3)$

$g'(t) = \dfrac{18}{(3-t)^3}$ $\qquad g'(0) = \dfrac{2}{3}$

$g''(t) = \dfrac{3(18)}{(3-t)^4}$ $\qquad g''(0) = \dfrac{2}{3}$ $\qquad E[X] = \dfrac{2}{3}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad VAR[X] = \dfrac{2}{9}.$

4) a) $M_W(t) = M_{X+Y}(t) = M_X(t) M_Y(t) = (1-2t)^{-5}$

b) $W \sim \chi^2(10)$

c) $M_V(t) = M_X(t) M_Y(-t) = (1-4t^2)^{-5/2}$

d) No.