28 Use the z-table in Appendex A, find the value of z that satisfies the following proberties

(a) $P(Z \le z) = 0.8238$, so $z = 0.93$.

(b) $P(Z \le z) = 0.0287$, so $z = -1.90$.

(c) $P(Z \ge z) = 0.9115 = 1 - P(Z \le z) \;\Rightarrow\; P(Z \le z) = 0.0885$, so $z = -1.35$.

(d) $P(Z \ge z) = 0.1660 = 1 - P(Z \le z) \;\Rightarrow\; P(Z \le z) = 0.8340$, s0 $z = 0.97$.

(e) $P(|Z| \ge z) = 0.10$

By symmetry of the standard normal density function, $P(Z \ge z) = P(Z \le -z)$.

$$P(|Z| \ge z) = P(Z \ge z) + P(Z \le -z) = 2P(Z \ge z) = 2 - 2P(Z \le z) = 0.10 \;\Rightarrow\; P(Z \le z) = 0.95,$$

so $z = 1.645$.

(f) $P(|Z| \le z) = 0.95$

$$0.95 = P(|Z| \le z) = P(-z \le Z \le z) = P(Z \le z) - P(Z \le -z) = 2P(Z \le z) - 1 \;\Rightarrow\; P(Z \le z) = 0.975,$$

so $z = 1.96$.

29 Let z be the standard normal random variable. If $z > 0$ and $F_z(z) = \alpha$, what are $F_z(-z)$ and $P(-z \le Z \le z)$?Let $Z > 0$ and $F_z(z) = \alpha = P(Z \le z)$. By symmetry of standard normal density function:

$$\begin{aligned}
F_z(-z) &= P(Z \le -z) = P(Z \ge z) \\
&= 1 - P(Z \le z) \\
&= 1 - \alpha \\
P(-z \le Z \le z) &= P(Z \le z) - P(Z \le -z) \\
&= \alpha - (1 - \alpha) \\
&= 2\alpha - 1.
\end{aligned}$$

31 An insurance company has 5000 policies and assumes these policies are all independent. Each policy is governed by the same distribution with a mean of \$495 and a variance of \$30,000. What is the probability that the total claims for the year will be less than \$2,500,000?

Let S be the total claims on the 5000 policies. Then S has a normal distribution with $\mu = 5000(495)$, $\sigma^2 = 5000(30,000)$, and $\sigma = 12,247.44$.

$$\begin{aligned}
P(S \le 2,500,000) &= P\left(Z \le \frac{2,500,000 - 2,475,000}{12,247.44}\right) \\
&= P(Z \le 2.04) = 0.9793.
\end{aligned}$$

32 A company manufactures engines. Specifications require that the length of a certain rod in this engine be between 7.48 cm. and 7.52 cm. The lengths of the rods produced by their supplier have a normal distribution with a mean of 7.505 cm. and a standard deviation of 0.01 cm.

(a) What is the probability that one of these rods meets these specifications?

Let X be the length of the rod. X is normally distributed with mean of 7.505 and standard deviation of 0.01.

$$\begin{aligned} P(7.48 \leq X \leq 7.52) &= P\left(\frac{7.48 - 7.505}{0.01} \leq Z \leq \frac{7.52 - 7.505}{0.01}\right) \\ &= P(-2.5 \leq Z \leq 1.5) = 0.9270. \end{aligned}$$

(b) If a worker selects 4 of these rods at random, what is the probability that at least 3 of them meets these specifications?

$$P(X \geq 3) = 4(0.927)^3(0.073) + (0.927)^4 = 0.9711.$$

36 If $Y = e^X$, where X is a normal random variable with $\mu = 5$ and $\sigma = 0.40$, what are $E[Y]$ and $\mathrm{Var}[Y]$?

Let $Y = e^X$, where X is normal with $\mu = 5$ and $\sigma = 0.40$.

$$\begin{aligned} E[Y] &= e^{\mu + \frac{1}{2}\sigma^2} = e^{5.08} = 160.77 \\ \mathrm{Var}[X] &= E[Y]^2\ (e^{\sigma^2} - 1) = e^{10.16}(e^{16} - 1)4484.96. \end{aligned}$$

37 If Y is lognormal and X, the normally distributed exponent, has parameters $\mu = 5.2$ and $\sigma = 0.80$, what is $P(100 \leq Y \leq 500)$?

Let $Y = e^X$, where X is normal with $\mu = 5.2$ and $\sigma = 0.8$.

$$\begin{aligned} P(100 \leq Y \leq 500) &= P(\ln 100 \leq X \leq \ln 500) = P\left(\frac{\ln 100 - 5.2}{0.8} \leq Z \leq \frac{\ln 500 - 5.2}{0.8}\right) \\ &= P(-0.74 \leq Z \leq 1.27) = 0.6684. \end{aligned}$$

38 The claim severity random variable for an insurance company is lognormal, and the normally distributed exponent has mean 6.8 and standard deviation 0.6. What is the probability that a claim is greater than \$1750?

Let $Y = e^X$, where X is normal with $\mu = 6.8$ and $\sigma = 0.60$.

$$\begin{aligned} P(Y \geq 1750) &= 1 - P(X \leq \ln 1750) = 1 - P\left(Z \leq \frac{\ln 1750 - 6.8}{0.6}\right) \\ &= 1 - P(Z \leq 1.11) = 0.1335. \end{aligned}$$

56 The time to a failure of a component in an electronic device has an exponential distribution with a median of four hours. Calculate the probability that the component will work without failing for at least five hours.

Let X be the time until failure of the device. We are asked to find $P(X \geq 5) = 1 - F_X(5)$.
We are not given the parameter λ for the exponential, but we can use the given information about the median to find it. The cumulative distribution for the exponential is $F_X(x) = 1 - e^{-\lambda x}$. By definition of the median m,

$$F_X(m) = 0.50.$$

Since $m = 4$ we have

$$F_X(m) = F_X(4) = 1 - e^{-4\lambda} = 0.50 \quad \Rightarrow \quad \lambda = 0.17329.$$

Thus $P(X \geq 5) = 1 - P(X \leq 5) = F_X(5) = e^{-5\lambda} = 0.42045.$

59 The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year. What portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year?

Let T be the time in days until the first accident for s high-risk driver. We are asked to find

$$P(T \leq 80) = F_T(80).$$

We know that $F_T(t) = 1 - e^{-\lambda t}$ but we do not know λ. That can be found using the other given information

$$0.30 = F_T(50) = 1 - e^{-50\lambda} \quad \Rightarrow \quad \lambda = 0.0071335.$$

Thus $P(T \leq 80) = F_T(80) = 1 - e^{-80\lambda} = 0.4348.$