1 Let X be the random variable for the number of heads obtained when three fair coin are tossed. What is the probability function for X?

Number of heads x	0	1	2	3
Number of outcomes	1	3	3	1
$P(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

2 Ten cards are face down in a row on a table. Exacly one of them is an ace. You turn the cards over one at a time, moving from left to right. Let X be the random variable for the number of cards turned before the ace is turned over. What is the probability function for X?

$$\begin{aligned}
P(X=0) &= P(\text{first card is the ace}) = \frac{1}{10}, \\
P(X=1) &= P(\text{second card is the ace} \mid \text{first was not the ace}).P(\text{first not the ace}) \\
&= \frac{9}{10}.\frac{1}{9} = \frac{1}{10} \\
P(X=2) &= \frac{9}{10}.\frac{8}{9}.\frac{1}{8} = \frac{1}{10}, \text{ etc.}
\end{aligned}$$

5 Let X be the random variable for the sum obtained by rolling two fair dice. What is $E[X]$?

x	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$E[X] = \sum x\ p(x) = \frac{2.1+3.2+4.3+5.4+6.5+7.6+8.5+9.4+10.3+11.2+12}{36} = 7$$

10 A fair die is rolled repeatedly. Let X be the random variable for the number of times a fair die is rolled before a six appears. Find $E[X]$. %endeqnarray*

Geometric Distribution (We'll cover it latter in class)

$$E[X] = \frac{1-p}{p} = \frac{1-1/6}{1/6} = 5$$

12 If X is the random variable for the sum obtained by rolling two fair dice. What is Var$[X]$?

$$\begin{aligned}
E[X] &= \sum x\ p(x) \\
&= \frac{2.1+3.2+4.3+5.4+6.5+7.6+8.5+9.4+10.3+11.2+12}{36} = 7 \\
E[X^2] &= \sum x^2\ p(x) \\
&= \frac{2^2.1+3^2.2+4^2.3+5^2.4+6^2.5+7^2.6+8^2.5+9^2.4+10^2.3+11^2.2+12^2}{36} = 54.833 \\
\text{Var}[X] &= E[X^2] - E^2[X] \\
&= 54.833 - 49 = 5.833.
\end{aligned}$$

14 Verify Equation $\sigma_{aX} = |a|.\sigma_X$.
We proved in class that $\text{Var}[aX] = a^2\text{Var}[X]$. Take the square root of both sides you get $\sigma_{aX} = |a|.\sigma_X$.

16 An auto insurance company has 15,000 policyholders with comprehensive automobile coverage. In the past year 11,425 filed no claims, 3,100 filed one claim, 385 filed two claims, and 90 filed three claims. What are the mean and the standard deviation for the number of claims filed by a policyholder?

$$\begin{aligned}
\mu &= \frac{0.11,425+1.3100+2.385+3.90}{15,000} = 0.276 \\
\sigma^2 &= \frac{11,425(0-0.276)^2+3100(1-0.276)^2+385(2-0.276)^2+90(3-0.276)^2}{15,000} = 0.0287157 \\
\sigma &= 0.53587.
\end{aligned}$$

17 A marketing company polled 50 people at a mall about the number of movies they had seen in the previous month. The results of this poll are as follows:

Number of movies	0	1	2	3	4	5	6	7	8
Number of viewers	3	5	6	9	11	7	5	3	1

What are the sample mean and sample standard deviation for the number of movies seen by an individual in a month?

$$\begin{aligned}
\bar{x} &= \frac{0.3 + 1.5 + 2.6 + 3.9 + 4.11 + 5.7 + 6.5 + 7.3 + 8.1}{50} = 3.64 \\
\sum f(n-\bar{x})^2 &= 3(0-3.64)^2 + 5(1-3.64)^2 + 6(2-3.64)^2 + 9(3-3.64)^2 + 11(4-3.64)^2 \\
&\quad +7(5-3.64)^2 + 5(6-3.64)^2 + 3(7-3.64)^2 + 1(8-3.64)^2 = 96.3683 \\
s &= \sqrt{\frac{\sum f(n-\bar{x})^2}{49}} = \sqrt{\frac{96.3683}{49}} \\
&= 1.9667.
\end{aligned}$$