

Name:

Due date: Wednesday, June 5.

Reading: Chapter 8: 2, 4, 9, 10, 11, 12, 18, 20, 24, 25.

- 2 Let T be the time from birth until death of a randomly selected member of a proportion. Assume that T has a uniform distribution on $[0, 100]$. Find $E[T]$ and $\text{Var}[T]$.

- 4 On a large construction site the lengths of pieces of lumber are rounded off to the nearest centimeter. Let X be the rounding error random variable (the actual length of a piece of lumber minus the rounded-off value). Suppose that X is uniformly distributed over $[-0.50, 0.50]$. Find

(a) $P(-0.10 \leq X \leq 0.20)$.

(b) $\text{Var}[X]$.

- 9 Tests on a certain machine part have determined that the mean time until failure of this part is 500 hours. Assume that the time T until failure of this part is exponentially distributed.

(a) What is the probability that one of these parts will fail within 300 hours.

(b) What is the probability that one of these parts will still be working after 900 hours.

- 10 If T has an exponential distribution with parameter λ , what is the median of T ?
- 11 For a certain population the time until death random variable T has an exponential distribution with mean 60 years.
- (a) What is the probability that a member of this population will die by age 50?
- (b) What is the probability that a member of this population will live to be 100?
- 12 If T is uniformly distributed over $[a, b]$, what is its failure rate?
- 18 Let T be a random variable whose distribution is exponential with parameter λ . Show that $P(T \geq a + b \mid T \geq a) = P(T \geq b)$.
- 20 Use the following two equations

$$\int_0^\infty x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{(n+1)}}, \text{ for } a > 0, \text{ and } n > -1 \quad (1)$$

$$\Gamma(n) = (n-1) \cdot \Gamma(n-1), \quad (2)$$

show that the mean of the gamma distribution with parameters α and β is $\frac{\alpha}{\beta}$.

- 24 A gamma distribution has a mean of 18 and a variance of 27. What are α and β for this distribution?
- 25 A gamma distribution has parameters $\alpha = 2$ and $\beta = 3$. Find
- (a) $F(x)$
- (b) $P(0 \leq X \leq 3)$
- (c) $P(1 \leq X \leq 2)$