1. From a standard deck of cards a single card is drawn. Let E be the event that the card is a red face card. List the outcomes in the event E.

   The red suits are hearts (H) and diamond (D), and the face cards kings (K), queens (Q) and jacks (J). Thus the outcomes are KH, QH, JH, KD, QD and JD.

2. An urn contains balls numbered from 1 to 25. A ball is selected and its number noted.

   (a) What is the sample space for this experiment?

   $\mathcal{S}$ consists of the positive integers from 1 to 25.

   (b) If E is the event that the number is odd, what are the outcomes in E?

   E consists of the odd integers from 1 to 25.

3. Two dice are rolled. How many outcomes have a sum of: (a) 7; (b) 8; (c) 11; (d) 7 or 11?

   Count the number of ordered pairs with the desire sum in the list in the answer For Exercise 2-4. For example, the only two pairs which sum to 11 are (5,6) and (6,5), so the answer to part (c) is 2.

4. Let $\mathcal{S}$ be the sample space for drowning a ball from an urn containing balls numbered from 1 to 25, and E be the event the number is odd. What are the outcomes in $\sim E$?

   The outcomes in $\sim E$ are the even (not odd) integers between 1 and 25.

5. Consider the insurance company that insures against loss due to fire. Let A be the event the loss is strictly between \$1,000 and \$100,000, and B the event the loss is strictly between \$50,000 and \$500,000. What are the events in $A \cup B$ and $A \cap B$?

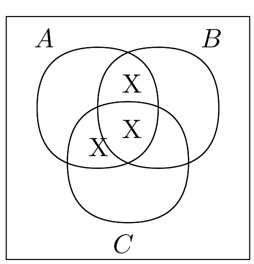
   $A \cup B$ is all rational numbers in (1,000, 500,000).
   $A \cap B$ is all rational numbers in (50,000, 100,000).

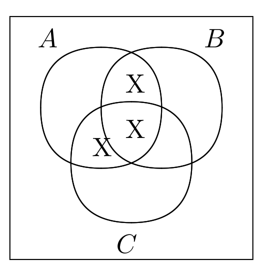
6. In an experiment of tossing two dice, let E be the event the sum of the dice is 6 and F be the event both dice show the same number. List the outcomes in the events $E \cup F$ and $E \cap F$.

   $E = \{(1,5),(2,4),(3,3),(4,2),(5,1)\}$
   $F = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$
   $E \cup F = \{(1,5),(2,4),(3,3),(4,2),(5,1),(1,1),(2,2),(4,4),(5,5),(6,6)\}$
   $E \cap F = \{(3,3)\}$.

7. Verify the two distributive laws by drawing appropriate Venn diagrams.
   To verify (2.1) draw a Venn diagram for $A \cap (B \cup C)$ and the second one showing $(A \cap B)$ and $(A \cap C)$ and observe that the first is the union of the second and third.

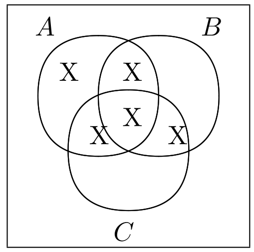
   $A \cap (B \cup C)$ $(A \cap B) \cup A \cap C)$

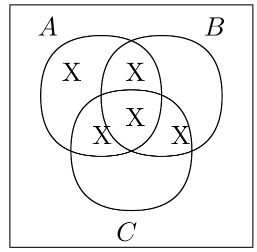




To verify (2.2) draw a Venn diagram for $A \cup (B \cap C)$ and the second one showing $(A \cup B)$ and $(A \cup C)$ and observe that the first is the intersection of the second and third.

$$A \cup (B \cap C) \qquad\qquad (A \cup B) \cap A \cup C)$$





8. Let M be the set of students in a large university who are taking a mathematics class and E be the set taking an economics class.

   (a) Give a verbal statement of the identity

   $$\sim (M \cup E) = \sim M \cap \sim E.$$

   See verbal statements in Answers to the Exercises.

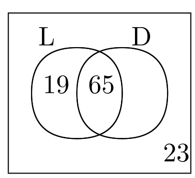
   (b) Give a verbal statement of the identity

   $$\sim (M \cap E) = \sim M \cup \sim E.$$

   See verbal statements in Answers to the Exercises.

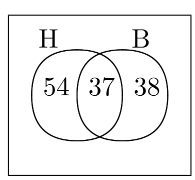
9. A company has 134 employees. There are 84 who have been with the company more than 10 years and 65 of those are college graduates. There are 23 who do not have college degrees and have been with the company less than 10 years. How many employees are college graduates?

   Let L be the set of those with the company more than 10 years and D be the set of those with college degrees. The given data can be used to fill in the Vinn diagram below. Then $n(L \cup D) = 134 - 23 = 111$. Then $n(D) = n(L \cup D) - 19 = 111 - 19 = 92$.



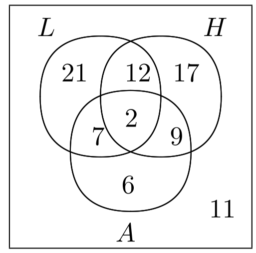
10. In a survey of 185 university students, 91 were taking a history course, 75 were taking a biology course, and 37 were taking both. How many were taking a course in exactly one of these subjects?

    The given data can be used to fill in Venn diagram below. The number of students taking exactly one of those courses is $54 + 38 = 92$.



11. An insurance agent sells life, health and auto insurance. During the year she met with 85 potential clients. Of these, 42 purchased life insurance, 40 health insurance, 24 auto insurance, 14 both life and health, 9 both life and auto, 11 both health and auto, and 2 purchased all three. How many of these potential clients purchased

    Using the given information we get the following Venn diagram



    (a) The number with no policies is 11.
    (b) The number with only health policies is 17.
    (c) The number with exactly one policy is $21 + 17 + 6 = 44$.
    (d) The number with life or health but not auto insurance is $21 + 12 + 17 = 50$.

12. In purchasing a car, a women has the choice of 4 body styles, 15 color combinations, and 6 accessory packages. In how many ways can she select her car?

    Number of ways to select car is $4 \ . \ 15 \ . \ 6 = 360$.

13. An experiment has two stages. The first stage consists of drawing a card from a standard deck. If the card is red, the second stage consists of tossing a coin. If the card is black, the second stage consists of rolling a die. How many outcomes are possible?

    Number of outcomes is $26 \ . \ 2 + 26 \ . \ 6 = 208$.

14. An arrangement of 4 letters from the set $\{A, B, C, D, E, F\}$ is called a (four-letter) word from that set. How many four-letter words are possible if repetitions are allowed? How many four-letter words are possible if repetitions are not allowed?

    If repetition are allowed, the number of words is $6^4 = 1296$.
    If repetition are not allowed, the number of words is $P(6, 4) = 360$.

15. A row contains 12 chairs. In how many ways can 7 people be seated in these chairs?

    The number of seating arrangements (in order) is $P(12, 7) = 3,991,680$.

16. A club of 30 members has three officers: president, secretary and treasurer. In how many ways can these offices be filled?

    The number of ways to fill the offices is $P(30, 3) = 24,360$.

17. Eight people are to be seated in a row of eight chairs. In how many ways can these people be seated if two of them insist on seating next to each other?

    First choose a pair of adjacent seats, which can be done in 7 ways, and the two people can be seated in those seats in 2 ways. Then arrange the 6 remaining people in remaining 6 chairs. This can be done in 6! ways, so the number of seating arrangements is $7 \ . \ 2 \ . \ 6! = 10,080$.

18. How many 5-card (poker) hands are possible from a deck of 52 cards?

The number of hands is $C(52,5) = 2,598,960$. (Order is irrelevant.)

19. In a class of 15 boys and 13 girls, the teacher wants a cast of 4 boys and 5 girls for a play. In how many ways can she select the cast?

Number of ways to pick cast is $C(15,4)\ C(13,5) = 1,756,755$. (Choose 4 boys and then 5 girls.)

20. How many different ways are there to arrange the letters in the word MISSISSIPPI?

Number of distinguishable arrangement of MISSISSIPPI is $\frac{11!}{4!\ 4!\ 2!} = 34,650$.

21. A company have 9 analysts. It has a major project which has been divided into 3 subprojects, and it assigns 3 analysts to each task. In how ways can this be done?

Number of ways to assign the analysis is $\frac{9!}{(3!)^3} = 1,680$, since this constitutes a partition of the group.

22. Expand $(2s-t)^4$.

$(2s-t)^4 = (2s)^4 + 4(2s)^3(-t) + 6(2s)^2(-t)^2 + 4(2s)(-t)^3 + (-t)^4$.

23. Prove the Binomial Theorem. (Hint: How many ways can you get the term $x^{n-k}y^k$ from the product of n factors, each of which is (x+y)?).

An $x^{n-k}y^k$ is obtained by selecting k of the $(x+y)$ factors from which to take y and take x from the remaining $n-k$ factors. This can be done in $C(n,k)$ ways.

24. An auto insurance company has 10,000 policyholders. Each policyholder is classified as

   (a) young or old;
   (b) male or female; and
   (c) married or single.

   Of these policyholders, 3,000 are young, 4,600 are male, and 7,000 are married. The policyholders can also be classified as 1,320 young males, 3,010 married males, and 1,400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

Let Y denote the event that the policyholder is young, M the event that he is male and H the event that the policy holder is married. We need the count from the shaded region X. The intersection of all three circles contains the 600 young married males, and $n(Y \cap H \cap M) = 600$. Since there are 1,320 young males, $n(Y \cap M) = 1,320$ and the remaining of $Y \cap M$ will contain 720 young single males. Similarly, since there are 1,400 young married persons, $n(Y \cap M) = 1,400$ and the remaining of $Y \cap M$ will contain 800 young married females. Then the shaded region X contains $3,000 - (800 + 600 + 720) = 880$ young single females.