

Summary of discrete distributions

Distribution	Parameters	$P(x)$	$E[X]$	$\text{Var}[X]$	$M_X(t)$
Uniform	$N > 0$, Integer	$\frac{1}{N}, x = 1, \dots, N$	$\frac{N+1}{2}$	$\frac{N^2-1}{12}$	$\frac{e^t(e^{Nt}-1)}{N(e^t-1)}$
Binomial	$n > 0$, $0 < p < 1$	$\binom{n}{x} p^x (1-p)^{n-x}$ $x = 0, 1, 2, \dots, n$	np	$np(1-p)$	$(1-p+pe^t)^n$
Poisson	$\lambda > 0$	$\frac{e^{-\lambda} \lambda^x}{x!}$ $x = 0, 1, 2, \dots$	λ	λ	$e^{\lambda(e^t-1)}$
Geometric	$0 < p < 1$	$(1-p)^x p$ $x = 0, 1, 2, \dots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^t}$
Negative Binomial	$r > 0$, $0 < p < 1$	$\binom{r+x-1}{x} p^r (1-p)^x$ $x = 0, 1, 2, \dots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{p}{1-(1-p)e^t} \right]^r$
Hypergeometric	$N > 0, 0 \leq r \leq N$ $1 \leq n \leq N$, integers	$\frac{\binom{N-r}{n-x} \binom{r}{x}}{\binom{N}{n}}$ $x \leq \min(n, r)$	$\frac{nr}{N}$	$\frac{nr}{N} \frac{N-r}{N} \frac{N-n}{N-1}$	$\sum_{x=0}^n \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}} e^{xt}$
Multinomial	n, p_1, \dots, p_k , $0 < p_i < 1$	$\frac{n!}{x_1! x_2! \dots x_k!} p_1^{x_1} \dots p_k^{x_k}$, $x_1 + x_2 + \dots + x_k = n$	$E[X_i] = np_i$	$\text{Var}[X] = np_i(1-p_i)$	

Summary of continuous distributions

Distribution	Parameters	$f(x)$	$E[X]$	$\text{Var}[X]$	$M_X(t)$
Uniform	$a < b$	$\frac{1}{b-a}, a \leq x \leq b$	$\frac{b+a}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
Normal	μ (any number), $\sigma^2 > 0$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$	μ	σ^2	$e^{\mu t + \frac{1}{2}\sigma^2 t^2}$
Exponential	$\frac{1}{\lambda} = \theta > 0$	$\frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0$	θ	θ^2	$\frac{1}{1-\theta t}$
Gamma	$k > 0, \theta > 0$	$\frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}}, x > 0$	$k\theta$	$k\theta^2$	$\left(\frac{1}{1-\theta t}\right)^k$
Chi-Square	k deg. of freedom.	$\frac{1}{\Gamma(\frac{k}{2}) 2^{\frac{k}{2}}} x^{\frac{k}{2}-1} e^{-\frac{x}{2}}, x > 0$	k	$2k$	$\left(\frac{1}{1-2t}\right)^{\frac{k}{2}}$
Pareto	$k > 0, \theta > 0$	$\frac{k}{\theta} \left(\frac{\theta}{x}\right)^{k+1}, k > 2, x \geq \theta > 0$	$\frac{k\theta}{k-1}$	$\frac{k\theta^2}{(k-2)(k-1)^2}$	
Lognormal	μ (any number), $\sigma^2 > 0$	$\frac{1}{y\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln y - \mu}{\sigma}\right)^2}, y > 0$	$e^{\mu + \frac{1}{2}\sigma^2}$	$(e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$	
Weibull	$k > 0, \lambda > 0$	$k\lambda x^{k-1} e^{-\lambda x^k}, x > 0$	$\frac{\Gamma(1+\frac{1}{k})}{\lambda^{\frac{1}{k}}}$	$\frac{\Gamma(1+\frac{2}{k})\Gamma(1+\frac{1}{k})^2}{\lambda^{\frac{2}{k}}}$	
Beta	$a > 0, b > 0$	$\frac{(a+b-1)!}{(a-1)!(b-1)!} x^{a-1} (1-x)^{b-1}, 0 < x < 1$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$	

- If X and Y are independent χ^2 distributions with n_1 and n_2 degrees of freedom, then $X + Y$ has χ^2 distribution with $n_1 + n_2$ degrees of freedom.
- If X_1, X_2, \dots, X_n is a random sample from a normal distribution $N(\mu, \sigma^2)$, then $Z = \sum_{i=1}^n \frac{(X_i - \mu)^2}{\sigma^2}$ is $\chi^2(n)$.
- If X_1, X_2, \dots, X_n is a random sample from a normal distribution $N(\mu, \sigma^2)$, then $Z = \sum_{i=1}^n \frac{(X_i - \bar{X})^2}{\sigma^2}$ is $\chi^2(n-1)$.
- If $X \sim N(0, 1)$, then $X^2 \sim \chi^2(1)$.

$$\Gamma(n) = \int_{-\infty}^{\infty} x^{n-1} e^{-x} dx$$

$$\Gamma(n) = (n-1)\Gamma(n-1) = (n-1)!, \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$\begin{aligned} \beta(a, b) &= \int_0^1 x^{a-1} (1-x)^{b-1} dx \\ &= \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \frac{(a-1)!(b-1)!}{(a+b-1)!} \end{aligned}$$