2 A single fair die is rolled 10 times. What is the probability of getting

(a) exactly 2 sixes?

$$P(X=2) = C(10,2)\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8 = 0.2907.$$

(b) at least 2 sixes?

$$\begin{aligned} P(X \geq 2) &= 1 - P(X<2) \\ &= 1 - \left[\left(\frac{5}{6}\right)^{10} + 10\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9\right] \\ &= 0.5155 \end{aligned}$$

4 A company produces light bulbs of which 2% are defective.

(a) If 50 bulbs are selected for testing, what is the probability that exactly 2 are defective?

$$P(\text{exactly 2 defective}) = C(50,2)(0.02)^2(0.98)^{48} = 0.1858.$$

(b) If a distributor gets a shipment of a 1,000 bulbs, what are the mean and the variance of the number of defective bulbs?

$$\mu = 1000(0.02) = 20; \quad \sigma^2 = 1000(0.02)(0.98) = 19.6.$$

6 In a large population 10% of the people have type B+ blood. At a blood donation center 20 people donate blood. What is the probability that

(a) exactly 4 of these have B+ blood?

$$P(\text{4 have B+ blood}) = C(20,4)(0.10)^4(0.90)^{16} = 0.0898.$$

(b) at most 3 have B+ blood?

$$\begin{aligned} P(\text{at most 3 have B+ blood}) &= (0.90)^{20} + C(20,1)(0.10)(0.90)^{19} \\ &\quad + C(20,2)(0.10)^2(0.90)^{18} + C(20,3)(0.10)^3(0.90)^{17} \\ &= 0.8670. \end{aligned}$$

10 For a binomial random variable X with $n=2$ and $P(S)=p$, show that

(a) $E[X] = 2p$

For a binomial random variable with $n=2$ and $P(S)=p$ we have

k	0	1	2
$p(k)$	$(1-p)^2$	$2p(1-p)$	p^2

$$E[X] = 0(1-p)^2 + 1(2p)(1-p) + 2p^2 = 2p.$$

(b) $\mathrm{Var}[V] = 2p(1-p)$?

$$\begin{aligned} \mathrm{Var}[X] &= (0-2p)^2(1-p)^2 + (1-2p)^2(2p)(1-p) + (2-2p)^2p^2 \\ &= 2p(1-p)[2p(1-p) + (1-2p)^2 + 2p(1-p)] \\ &= 2p(1-p). \end{aligned}$$

12 In a hospital ward there are 16 paitients, 4 of whom have AIDS. A doctor is assigned to 6 of these paitients at random. What is the probability that he gets 2 of the AIDS paitients?

$$P(\text{2 with AIDS}) = \frac{C(4,2)\ C(12,4)}{C(16,8)} = 0.3709.$$

16 An auto insurance company has determined that the average number of claims against the comprehensive coverage of a policy is 0.6 per year. What is the probability that a policyholder will file

(a) 1 claim in a year

$$P(X = 1) = 0.6\ e^{-0.6} = 0.3293.$$

(b) more than 1 claim in a year?

$$P(X > 1) = 1 - e^{-0.6} - 0.6\ e^{-0.6} = 0.1219.$$

17 A city has an intesection where accidents have occurred at an average rate 1.5 per year. What is the probability that in a year there will be

(a) 0 accident in a year?

$$P(X = 0) = e^{-1.5} = 0.2231.$$

(b) 1 accident in a year?

$$P(X = 1) = 1.5\ e^{-1.5} = 0.3347.$$

(c) 2 accidents in a year?

$$P(X = 2) = \frac{1.5^2\ e^{-1.5}}{2} = 0.2510.$$

24 An experiment consists of drawing a card at random from a standard deck and replacing it. If this experiment is done repeatedly, what is the probability that

(a) the first heart apears on the fifth draw?

The number of initial failures is 4, and $p = \frac{1}{4},\ q = \frac{3}{4}$.

$$P(X = 4) = \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) = 0.0791.$$

(b) the first ace apears on the tenth draw?

The number of initial failures is 9, and $p = \frac{1}{13},\ q = \frac{12}{13}$.

$$P(X = 4) = \left(\frac{12}{13}\right)^9 \left(\frac{1}{13}\right) = 0.0374.$$

34 A contestant on a game show selects a ball from an urn containing 25 balls numbered from 1 to 25. His prize is \$1,000 times the number of the ball selected. If X is the random variable for the amount he wins, find the mean and standard deviation of X.

Let Y be the number on the ball chosen. Y is a discrete uniform random variable, with $n = 25$ and $p(n) = \frac{1}{25},\ n = 1, 2, \ldots, 25$, and $X = 1000\ T$.

$$\begin{aligned} E[X] &= 1000\ E[Y] = \frac{1000(26)}{2} = \$13,000. \\ \mathrm{Var}[X] &= 1000^2\ \mathrm{Var}[Y] = \frac{1000^2(25^2 - 1)}{12} = 1000^2\ (52) = \sigma^2 \\ \sigma &= \$7,211.10. \end{aligned}$$

38 A hospital receives $\frac{1}{5}$ of its flu vaccine shipments from Company X and the remainder of its shipments from other companies. Each shipment contains a very large number of vaccine vials. For Company Xs shipments, 10% of the vials are ineffective. For every other company, 2% of the vials are ineffective. The hospital tests 30 randomly selected vials from a shipment and finds that one vial is ineffective. What is the probability that this shipment came from Company X?

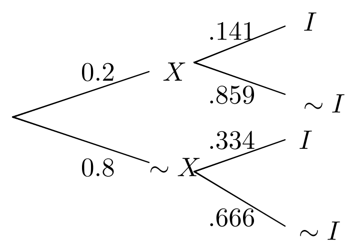
This is a Bayes theorem problem, with a binomial probability omponent. Let X denote the event that the shipment came from company X, and I denote the event that exactly one vial out of 30 tested is ineffictive. We are asked to find $P(X|I)$.

Note that if the shipment is from company X, the number of defectives in 30 components is a binomial random variable with $n = 30$ and $p = 0.1$. The probability of one defective in a batch of 30 from X is

$$P(I|X) = C(30,1)(0.1)(0.9)^{29} = 0.141.$$

Similarly if the shipment is from company X, the number of defectives in 30 components is a binomial random variable with $n = 30$ and $p = 0.02$. The probability of one defective in a batch of 30 from X is

$$P(I|\sim X) = C(30,1)(0.02)(0.98)^{29} = 0.334.$$



$$P(X|I) = \frac{P(X \cap I)}{P(I)} = \frac{0.2\ 0.141}{0.2\ 0.141 + 0.8\ 0.334} = 0.0955.$$