Name:

Due date: Wednesday, June 12.

2 Suppose the amount of a single loss for an insurance policy has density function $f(x) = 0.001\, e^{-0.001x}$, for $x > 0$. If this policy has a \$300 per claim deductible and also has a payment cap of \$1500 per claim, what is the expected amount of a single claim for this policy?

4 Let X be the random variable which is uniformaly distributed over the interval $[a, b]$. Find $M_X(t)$.

5 Find $E[X]$ for the random variable in the previous problem using its moment generating function.

6 Let X be the random variable whose density function is given by

$$f(x) = \begin{cases} 2(1-x) & \text{for } 0 \le x \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find $M_X(t)$.

7 Let X be the random variable whose density function is given by

$$f(x) = \begin{cases} 2(1-x) & \text{for } 0 \le x \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Find $E[X]$ using the moment generating function. (Note: the derivative of $M_X(t)$ is not defined at 0, but you can take the limit as t approaches 0 to find $E[X]$. This is a much more difficult way to find $E[X]$ than direct integration for this particular density function.)

8 If the moment generating function of X is $\left(\frac{2}{2-t}\right)^5$, identify the random variable X.

11 Let X be a normal random variable with parameters μ and σ. Use the moment generating function for X to find $E[X^2]$. Then show that $\mathrm{Var}[X] = \sigma^2$.

12 Let X uniformaly distributed over $[0, 1]$ and $Y = e^X$. Find $F_Y(y)$ and $f_Y(y)$.

13 Let X be a random variable with density function given by $f_X(x) = 3\,x^{-4}$, for $x \geq 1$ (Pareto with $\alpha = 3,\ \beta = 1$), and let $Y = \ln X$. Find $F_Y(y)$.

21 An auto insurance company issues a comprehencive policy with a \$200 deductible. Last year 90 percent of the policyholders filed no claims (either no damage or damage less than the deductable). For the 10 percent who filed claims, the claim amount had a Pareto distribution with $\alpha = 3$ and $\beta = 200$. If X is a random variable of the amount paid by the insurer, what is $F(x)$, for $x \geq 0$?

29 A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made. At what level must x be set if the expected payment made under this insurance is to be 1000?

30 A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Determine $E[X]$.

35 The time, T, that a manufacturing system is out of operation has cumulative distribution function

$$F(t) = \begin{cases} 1 - \left(\frac{2}{t}\right)^2 & \text{for } \ t > 2 \\ 0 & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = T^2$. Determine the density function of Y, for $y > 4$.