2 If a fair coin is tossed 3 times. What is the probability of getting at least 1 head?

P(at least 1 head)=1-P(no heads)$= 1 - \frac{1}{8} = \frac{7}{8}$.

6 An insurance agent has 78 clients. Of these 45 have live insurance, 32 have auto insurance, and 16 have both types. What is the probability that a client chosen at random has neither life nor auto insurance?

Number of clients with either life or auto insurance is $54 + 32 - 16 = 61$. The probability a client has neither is $\frac{17}{78}$.

8 A computer company has a shipment of 40 computer components of which 5 are defective. If 4 components are chosen at random to be tosed, what is the probability that

(a) all are good?

P(all 4 good) $= \frac{C(35,4)}{C(40,4)} = \frac{52{,}360}{91{,}390} = 0.5279$.

(b) 2 are good and 2 are defective?

P(2 good and 2 defective) $= \frac{C(35,2)\ C(5,2)}{C(40,4)} = \frac{5{,}950}{91{,}390} = 0.0651$.

15 The odds for an event E are defined as the ratio $P(E)$ to $P(\sim E)$. Odds are generally written as the ratio of two integers, such as $5 : 4$, which is read "5 to 4". The odds against E are given by the reverse ratio (i.e., 4:5). If a pair of dice are rolled, what are

(a) the odds for a 7?

$P(7) = \frac{1}{6}$, $P(\sim 7) = \frac{5}{6}$. Odds for a 7 are $1 : 5$.

(b) the odds against an 11?

$P(11) = \frac{1}{18}$, $P(\sim 11) = \frac{17}{18}$. Odds against an 11 are $17 : 1$.

20 An auto insurance company finds that in the past 10 years 22% of its policyholders have filed liability claims, 37% have filed comprehensive claims, and 13% have filed both types of claims. What is the probability that a policyholder chosen at random has not filed a claim of either kind?

$$\begin{aligned}\text{P(no claim)} &= 1 - \text{P(liability or comprehensive)}\\ &= 1 - (0.22 + 0.37 - 0.13) = 0.54.\end{aligned}$$

26 Two cards are drawn from a standard deck without replacement. What is the probability that

(a) both are hearts?

$$\begin{aligned}\text{P(both hearts)} &= P(1^{st}\text{ heart})\ P(2^{nd}\text{ heart}|1^{st}\text{ heart})\\ &= \left(\frac{1}{4}\right)\cdot\left(\frac{12}{51}\right) = 0.0588.\end{aligned}$$

(b) neither is a heart?

$$\begin{aligned}\text{P(neither a heart)} &= P(1^{st}\text{ not a heart})\ P(2^{nd}\text{ not a heart}|1^{st}\text{ not a heart})\\ &= \left(\frac{3}{4}\right)\cdot\left(\frac{38}{51}\right) = 0.5588.\end{aligned}$$

(c) exactly one is a heart?

$$P(\text{exactly one heart}) = 1 - 0.0588 - 0.5588 = 0.3824.$$

36 A machine has two parts that could fail and have to be replaced. The probabilities of failure of parts A and B are 0.17 and 0.12, respectively. If failures of these parts are independent of each other, what is the probability that at least one of them will fail?

$$P(\text{at least one fails}) = 1 - P(\text{neither fails}) = 1 - (0.83).(0.88) = 0.2696.$$

40 An insurance company divides its policyholders into low-risk and high-risk classes. For the year, of those in the low-risk class, 80% had no claims, 15% had one claim, and 5% had two claims. Of those in the high-risk class, 50% had no claims, 30% had one claim, and 20% had two claims. Of the policyholders, 60% were in the low-risk class ans 40% in the high-risk class.

(a) If the policyholder had no claims in the year, what is the probability that he is in the low-risk class?

$$\begin{aligned} \text{P(0 claims)} &= (0.6).(0.8) + (0.4).(0.5) = 0.68 \\ \text{P(2 claims)} &= (0.6).(0.05) + (0.4).(0.02) = 0.11 \\ \text{P(low risk} \mid \text{0 claims)} &= \frac{0.48}{0.68} = 0.7059. \end{aligned}$$

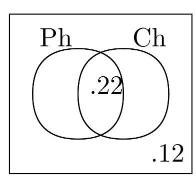
(b) If the policyholder had two claims in the year, what is the probability that he is in the high-risk class?

$$\text{P(high risk} \mid \text{2 claims)} = \frac{0.08}{0.11} = 0.7273.$$

47 You are given $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$. Determine $P(A)$.

$$\begin{aligned} P(B \cap A') &= 1 - P(A \cup B') = 0.1 \\ \text{and } P(A) &= P(A \cup B) - P(B \cap A') \\ &= 0.7 - 0.1 = 0.6. \end{aligned}$$

49 Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.



Given $P(Ch) = P(Ph) + 0.14$, where Ph : Physical therapist, Ch : Chiropractor.

$$\begin{aligned} P(Ph \cup Ch) &= 1 - P(Ph \cap Ch) = 0.88 \\ P(Ph) + P(Ch) - 0.22 &= 0.88 \\ P(Ph) + P(Ph) + 0.14 - 0.22 &= 0.88 \\ 2P(Ph) &= 0.96 \\ P(Ph) &= \frac{0.96}{2} = 0.48. \end{aligned}$$