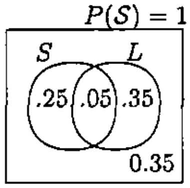
MTH U681 Quiz 1 Answers May 16, 2019

Name ____________________

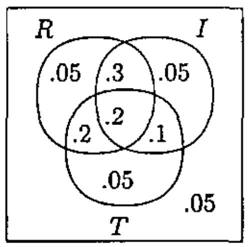
(1) The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work. Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

(A) 0.05 (B) 0.12 (C) 0.18 (D) 0.25 (E) 0.35 **Answer # 1:** $\boxed{\text{A}}$

$P(S) = 1$



(2) A survey is made to determine the number of households having electric appliances in a certain city. It is found that 75% have radios (R), 65% have irons (I), 55% have electric toasters (T), 50% have (IR), 40% have (RT), 30% have (IT), and 20% have all three. Find the following proportions.



(a) Of those households that have a toaster, find the proportion that also have a radio.

$$P(R|T) = \frac{P(R \cap T)}{P(T)} = \frac{0.40}{0.55} = \frac{8}{11} = 72\%$$

(b) Of those households that have a toaster but no iron, find the proportion that have a radio.

$$P(R|T \cap I') = \frac{P(R \cap T \cap I')}{P(T \cap I')} = \frac{0.20}{0.25} = \frac{4}{5} = 80\%.$$

(3) *You are given $P[A' \cap B'] = 0.3$ and $P[A \cup B'] = 0.9$. Determine $P[A]$.*

(A) 0.2 (B) 0.3 (C) 0.4 (D) 0.6 (E) 0.8 **Answer # 3:** $\boxed{\text{D}}$

$$\begin{aligned} 0.9 = P(A \cup B') &= P((A' \cap B)') = 1 - P(B \cap A') \\ \Rightarrow \ P(B \cap A') &= 0.1 \\ \text{since } P(A) &= P(A \cup B) - P(B \cap A') \\ \Rightarrow \ P(A) &= 0.7 - 0.1 = 0.6. \end{aligned}$$

(4) An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44. Calculate the number of blue balls in the second urn.

(A) 4 (B) 20 (C) 24 (D) 44 (E) 64 **Answer # 4:** $\boxed{\text{A}}$

Let x : number of blue balls in second urn.

$$\begin{aligned} P(\text{Same color}) &= P(\text{1st red and 2nd red}) + P(\text{1st blue and 2nd blue}) \\ &= P(\text{1st red}).P(\text{2nd red}) + P(\text{1st blue}).P(\text{2nd blue}) \\ 0.44 &= \frac{4}{10} \cdot \frac{16}{x+16} + \frac{6}{10} \cdot \frac{x}{x+16} \\ x = 4. \end{aligned}$$

MATH 4681 Quiz 2 Answers May 23, 2019

Name ________________________

(1) The moment generating function for the random variable X is $M_X(t) = Ae^t + Be^{2t}$. You are given that $\mathrm{Var}[X] = \frac{2}{9}$ and $A < \frac{1}{2}$. Find $E[X]$.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{4}{3}$ (E) $\frac{5}{3}$

Answer # 1: E

$$\begin{aligned}
M_X(t) &= Ae^t + Be^{2t} \;\Rightarrow\; M_X(0) = 1 = A + B \;\Rightarrow\; B = 1 - A \\
M'_X(t) &= Ae^t + 2Be^{2t} \;\Rightarrow\; M'_X(0) = E[X] = A + 2B \;\Rightarrow\; E[X] = 2 - A \\
M''_X(t) &= Ae^t + 4Be^{2t} \;\Rightarrow\; M''_X(0) = E[X^2] = A + 4B \;\Rightarrow\; E[X^2] = 4 - 3A \\
\mathrm{Var}[X] = \frac{2}{9} &= 4 - 3A - (2-A)^2 = -A^2 - 7A \\
A^2 + 7A - \frac{2}{9} &= 0 \;\Rightarrow\; A = \frac{1}{3} \text{ or } (A = \frac{2}{3}, \text{ rejected}) \\
\text{then } E[X] &= 2 - \frac{1}{3} = \frac{5}{3}.
\end{aligned}$$

(2) The model chosen for a discrete, integer-valued, non-negative random variable N with mean 2 is a binomial distribution with n trials and probability of success p on each trial. Various combinations of n and p are considered, and $P(N = 0)$ is calculated. Find $\lim_{n\to\infty} P(N = 0)$.

(A) e^{-2} (B) e^{-1} (C) 0 (D) $\frac{1}{2}$ (E) 1

Answer # 2: A

$$\begin{aligned}
E[X] &= np = 2 \;\Rightarrow\; p = \frac{2}{n} \\
P(N=0) &= (1-p)^n = \left(1 - \frac{2}{n}\right)^n \\
\lim_{n\to\infty} P(N=0) &= \lim_{n\to\infty}\left(1 - \frac{2}{n}\right)^n = e^{-2}
\end{aligned}$$

(3) A company prices its hurricane insurance using the following assumptions:
- (a) In any calendar year, there can be at most one hurricane.
- (b) In any calendar year, the probability of a hurricane is 0.05.
- (c) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the companys assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(A) 0.06 (B) 0.19 (C) 0.38 (D) 0.62 (E) 0.92

Answer # 3: E

This is a binomial distribution problem. There are n=20 independent trials in 20 years, with p = Pr(hurricane) = .05 in each year. The binomial random variable X is the number of hurricanes in 20 years. We are asked to find

$$\begin{aligned}
P(X < 3) &= P(X=0) + P(X=1) + P(X=2) \\
&= 0.95^{20} + \binom{20}{1}(0.95)^{19}(0.05)^1 + \binom{20}{2}(0.95)^{18}(0.05)^2 \\
&= 0.92452
\end{aligned}$$

(4) An insurer uses the Poisson distribution with mean 4 as the model for the number of warranty claim per month on a particular product. Each warranty claim results in a payment of 1 by the insurer. Find the probability that the total payment by the insurer in a given month is less than one standard deviation above the average monthly payment.

(A) 0.9 (B) 0.8 (C) 0.7 (D) 0.6 (E) 0.5 **Answer # 4:** $\boxed{}$

Average monthly payment, (λ) is 4, variance, (σ^2) is 4 (variance of Poisson equal to mean).
Probability that the total payment is less than $\lambda + \sigma = 4 + 2 = 6$ is

$$P(N < \lambda + \sigma) = P(N < 6) = e^{-4}\left[1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} + \frac{4^4}{4!} + \frac{4^5}{5!}\right] = 0.785.$$

(5) As part of he underwriting process for insurance, each prospective policyholder is tested for high blood pressure. Let X represent the number of tests completed when the first person with high blood pressure is found. The expected value of X is 12.5. Calculate the probability that the sixth person tested in the first one with high blood pressure.

(A) 0.000 (B) 0.053 (C) 0.080 (D) 0.316 (E) 0.394 **Answer # 5:** $\boxed{\text{B}}$

This is a geometric distribution, the mean is $\frac{1}{p} = 12.5 \quad \to \quad p = 0.08$ (prob. of success). The probability that the first success occurs on the 6^{th} trial is

$$(1-p)^5 p = (0.92)^5\ (0.08) = 0.0527$$

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(1) Let X have a distribution with 75^{th} percentile equal to $\frac{1}{3}$ and density function equal to

$$f(x) = \begin{cases} \lambda\, e^{-\lambda x} & \text{for } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find λ.

(A) $\ln(\frac{64}{27})$ (B) $\frac{4}{3}\ \ln(\frac{3}{2})$ (C) $\ln 12$ (D) $\ln 64$ (E) $\ln 81$

Answer # 1: $\boxed{\text{D}}$

$$\int_0^{\frac{1}{3}} \lambda e^{-\lambda x}\, dx = 0.75 \;\Rightarrow\; 1 - e^{-\frac{\lambda}{3}} = 0.75 \;\Rightarrow\; \lambda = \ln 64.$$

(2) Let X be a normal random variable with mean 0 and variance $a > 0$. Calculate $P(X^2 < a)$.

(A) 0.34 (B) 0.42 (C) 0.68 (D) 0.84 (E) 0.90

Answer # 2: $\boxed{\text{C}}$

$$\begin{aligned} P(X^2 < a) &= P(-\sqrt{a} < X < \sqrt{a}) \\ &= P(\frac{-\sqrt{a} - 0}{\sqrt{a}} < X < \frac{\sqrt{a} - 0}{\sqrt{a}}) \\ &= P(-1 < Z < 1) = 0.68. \end{aligned}$$

(3) Let $X_i,\ i = 1, 2, \ldots, 10$ be independent random variables, each being uniform distributed over $(0,\ 1)$. Calculate $P\left(\sum_1^{10} X_i \ > \ 7\right)$.

(A) 0.0319 (B) 0.9681 (C) 0.0139 (D) 0.9861 (E) 0.6981

Answer # 3: $\boxed{\text{C}}$

Since $E[X_i] = \frac{1}{2}$, $\text{Var}[X] = \frac{1}{12}$ we have by the CLT that

$$\begin{aligned} P\left(\sum_1^{10} X_i \ > \ 7\right) &= P\left(\frac{\sum_1^{10} X_i - 5}{\sqrt{\frac{10}{12}}} \ > \ \frac{7-5}{\sqrt{\frac{10}{12}}}\right) \\ &= P(Z > \ 2.2) \\ &= 1 - P(Z < 2.2) = 0.0139. \end{aligned}$$

(4) [Extra Credit] Two independent random variables X and Y have the following pdf's

$$f(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise,} \end{cases} \qquad \text{and} \qquad f(y) = \begin{cases} ky & \text{for } 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find k, $F(x)$ and $F(y)$.

$$\int_0^1 ky\, dy = 1 \Rightarrow k = 2.$$

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } 0 \le x < 1 \\ 1 & \text{for } x \ge 1, \end{cases} \qquad \text{and} \qquad F(y) = \begin{cases} 0 & \text{for } y < 0 \\ y^2 & \text{for } 0 \le y < 1 \\ 1 & \text{for } y \ge 1. \end{cases}$$

(b) Suppose that $Z = \max(X, Y)$. Explain why $P(Z \le z) = P(X \le z)\ P(Y \le z)$ for all z.

$$F(z) = P(Z \le z) = P(\max(X, Y) \le z) = P(X \le z \text{ and } Y \le z) = P(X \le z)\ P(Y \le z)$$

Since X and Y are independent.

(c) Find $F(z)$, the cdf of Z.

$$F(z) = \begin{cases} 0 & \text{for } z < 0 \\ z^3 & \text{for } 0 \le z < 1 \\ 1 & \text{for } z \ge 1, \end{cases}$$

(d) find $f(z)$, the pdf of Z.

$$f(z) = \begin{cases} 3z^2 & \text{for } 0 \le z < 1 \\ 0 & \text{otherwise.} \end{cases}$$

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1. Use the z-table, find the value of z that satisfies the following proberties
   (a) $P(Z \leq z) = 0.8238 \Rightarrow z = 0.93$
   (b) $P(|Z| \leq z) = 0.95$

$$\begin{aligned} 0.95 &= P(|Z| \leq z) = P(-z \leq Z \leq z) \\ &= P(Z \leq z) - P(Z \leq -z) \\ &= P(Z \leq z) - P(Z \geq z) \\ &= P(Z \leq z) - [1 - P(Z \leq z)] \\ &= 2P(Z \leq z) - 1 \Rightarrow \\ P(Z \leq z) &= 0.975 \Rightarrow z = 1.96. \end{aligned}$$

2. If Y is lognormal and X, the normally distributed exponent with $\mu = 5.2$ and $\sigma = 0.80$, what is $P(100 \leq Y \leq 500)$?

$$\begin{aligned} P(100 \leq Y \leq 500) &= P(\ln 100 \leq X \leq \ln 500) \\ &= P\left(\frac{\ln 100 - 5.2}{0.8} \leq X \leq \frac{\ln 100 - 5.2}{0.8}\right) \\ &= P(-0.74 \leq X \leq 1.27) = 0.6684. \end{aligned}$$

3. X is a random variable with pdf

$$f(x) = kx(1-x)^{n-1}, \qquad 0 \leq x \leq 1.$$

Find k.

(A) $\frac{1}{n(n+1)}$ (B) $n(n+1)$ (C) n (D) $n+1$ (E) $\frac{n+3}{n+1}$

Answer # 3: B

$$\int_0^1 kx(1-x)^{n-1}\, dx = 1 \rightarrow k = n(n+1).$$

Note: $1 = \beta(2,n) = k\,\frac{\Gamma(2)\,.\,\Gamma(n)}{\Gamma(n+2)} = k\,\frac{1!\,(n-1)!}{(n+1)!} = k\,\frac{1}{(n+1)n} \rightarrow k = n(n+1).$

4. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} 2x^{-2} & \text{for } x \geq 2 \\ 0 & \text{othewise.} \end{cases}$$

Determine the density function of $Y = \frac{1}{X-1}$ for $0 < y \leq 1$.

(A) $\frac{1}{y^2}$ (B) $\frac{2}{(y+1)^2}$ (C) $\frac{2}{(y+1)}$ (D) $2\left(\frac{y}{y+1}\right)^2$ (E) $2\left(\frac{y+1}{y}\right)^2$

Answer # 4: B

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P\left(\frac{1}{X-1} \leq y\right) = P\left(X \geq 1 + \frac{1}{y}\right) \\ &= \int_{\frac{y+1}{y}}^{\infty} 2x^{-2}\, dx = \frac{2y}{y+1} \\ f(y) &= F_Y'(y) = \frac{2}{(y+1)^2}. \end{aligned}$$

5. Let X_1, X_2, X_3 be independent random variables, each having a uniform distribution over (0, 1). Let M =minimum(X_1, X_2, X_3). Show that the distribution function of M, $F_M(x)$, is given by

$$F_M(x) = 1 - (1-x)^3, \qquad 0 \le x \le 1.$$

$$\begin{aligned}
F_M(x) &= P(M \le x) = P(\min(X_1, X_2, X_3) \le x) \\
&= 1 - P(\min(X_1, X_2, X_3) \ge x) \\
&= 1 - P(X_1 \ge x).P(X_2 \ge x).P(X_3 \ge x) \quad \text{X's are independent} \\
&= 1 - [1 - P(X_1 \le x)].[1 - P(X_2 \le x)].[1 - P(X_3 \le x)] \\
&= 1 - (1-x)^3.
\end{aligned}$$

since $P(X_i \le x) = F_{X_i}(x) = x, \;\; i = 1, 2, 3.$

6. [Extra credit] X has an exponential distribution with mean 1. Y is defined to be the conditional distribution of $X - 2$ given that $X > 2$, so for instance, for $c > 0$, we have $P(Y > c) = P(X - 2 > c \mid X > 2)$. What is the distribution of Y?

   (A) Exponential with mean 1
   (B) Exponential with mean 2
   (C) Exponential with mean $\frac{1}{2}$
   (D) Exponential with mean e
   (E) Exponential with mean e^2

Answer # 6: $\boxed{\text{A}}$

For $c > 0$, we have

$$\begin{aligned}
P(Y > c) &= P(X - 2 > c \mid X > 2) \\
&= P(X > 2 + c \mid X > 2) \\
&= \frac{P((X > 2 + c) \cap (X > 2))}{P(X > 2)} \\
&= \frac{P(X > 2 + c)}{P(X > 2)} = \frac{1 - F_X(2 + c)}{1 - F_X(2)} \\
&= \frac{e^{-(2+c)}}{e^{-2}} = e^{-c} \quad \text{so} \\
F_Y(c) &= 1 - e^{-c} \\
F_Y(y) &= 1 - e^{-y} \;\; \Rightarrow \;\; f(y) = F_Y'(y) = e^{-y}
\end{aligned}$$

which is exponential with mean 1.

1. Let X and Y be a discrete random variables with joint probability function

$$P(x,y) = \begin{cases} \frac{2^{x-y+1}}{9} & \text{for x=1, 2 and y=1,2} \\ 0 & \text{otherwise} \end{cases}.$$

Calculate $E[\frac{X}{Y}]$.

(A) $\frac{9}{2}$ (B) $\frac{5}{4}$ (C) $\frac{4}{3}$ (D) $\frac{25}{18}$ (E) $\frac{5}{3}$

Answer # 1: D

$$E[\frac{X}{Y}] = \sum_{x=1}^{2}\sum_{y=1}^{2} \frac{x}{y}\, P(x,y) = 1.\frac{2}{9} + \frac{1}{2}\,.\frac{1}{9} + 2.\frac{4}{9} + 1.\frac{2}{9} = \frac{25}{15}.$$

2. Let X and Y be independent random variables with $\mu_X = 1$, $\mu_Y = -1$, $\sigma_X^2 = \frac{1}{2}$, and $\sigma_Y^2 = 2$. Calculate $E[(X+1)^2(Y-1)^2]$.

(A) 1 (B) $\frac{9}{2}$ (C) 16 (D) 17 (E) 27

Answer # 2: E

$$\begin{aligned} E[(X+1)^2(Y-1)^2] &= E[(X+1)^2]\ E[(Y-1)^2] \\ E[(X+1)^2] &= E[X^2] + 2E[X] + 1 \ \text{ and since} \\ \sigma_X^2 &= E[X^2] - E[X]^2 \ \Rightarrow\ E[X^2] = \sigma_X^2 + E[X]^2 = \frac{3}{2}, \ \text{ and then} \\ E[(X+1)^2] &= \frac{9}{2}. \ \text{ Similarly} \\ E[(Y-1)^2] &= E[Y^2] - 2E[Y] + 1 = 6. \ \text{ So} \\ E[(X+1)^2(Y-1)^2] &= \frac{9}{2}.\,6 = 27. \end{aligned}$$

3. The distribution of Smith's future lifetime is X, an exponential random variable with mean α, and the distribution of Brown's future lifetime is Y, an exponential random variable with mean β. Smith and Brown have future lifetimes that are independent of one another. Find the probability that Smith outlives Brown.

(A) $\frac{\alpha}{\alpha+\beta}$ (B) $\frac{\beta}{\alpha+\beta}$ (C) $\frac{\alpha-\beta}{\alpha}$ (D) $\frac{\beta-\alpha}{\beta}$ (E) $\frac{\alpha}{\beta}$

Answer # 3: A

$P(Y < X) = \int_0^\infty \int_y^\infty f_X(x)\, f_Y(y)\, dx\, dy$ (since X and Y are independent, the joint density function of X and Y is the product of the two separate density functions). The density function of X is $\frac{1}{\alpha}\, e^{-\frac{x}{\alpha}}$, and of Y is $\frac{1}{\beta}\, e^{-\frac{x}{\beta}}$, so that

$$P(Y < X) = \int_0^\infty \int_y^\infty \frac{1}{\alpha}\, e^{-\frac{y}{\alpha}}\, \frac{1}{\beta}\, e^{-\frac{x}{\beta}}\, dx\, dy = \int_0^\infty \frac{1}{\beta}\, e^{-\frac{y}{\beta}}\, e^{-\frac{y}{\alpha}}\, dy = \frac{\frac{1}{\beta}}{\frac{1}{\alpha}+\frac{1}{\beta}} = \frac{\alpha}{\alpha+\beta}.$$

4. If $f(x,y) = k(x^2+y^2)$ is the density function for the joint distribution of the continuous random variables X and Y defined over the unit square bounded by the points $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$,

(a) Find k and $P(X+Y \geq 1)$.

$$\begin{aligned}
1 &= \int_0^1 \int_0^1 k(x^2+y^2)\ dy\ dx \\
&= k \cdot \frac{2}{3} \quad \Rightarrow \quad k = \frac{3}{2} \\
P(X+Y \geq 1) &= \int_0^1 \int_{1-x}^1 \frac{3}{2}(x^2+y^2)\ dy\ dx \\
&= \frac{3}{4}.
\end{aligned}$$

(b) Find the marginal distributions of X and Y.

$$\begin{aligned}
f_X(x) &= \int_0^1 f(x,y)\ dy \\
&= \int_0^1 \frac{3}{2}(x^2+y^2)\ dy \\
&= \frac{3}{2}x^2 + \frac{1}{2} \qquad \text{for } 0 \leq x \leq 1. \\
f_Y(y) &= \int_0^1 f(x,y)\ dx \\
&= \int_0^1 \frac{3}{2}(x^2+y^2)\ dx \\
&= \frac{3}{2}y^2 + \frac{1}{2} \qquad \text{for } 0 \leq y \leq 1.
\end{aligned}$$

(c) Find the conditional density and conditional expectation and conditional variance of X given $Y = 0.3$.

$$\begin{aligned}
f(x \mid Y=0.3) &= \frac{f(x,0.3)}{f_Y(0.3)} = \frac{\frac{3}{2}\ (x^2+0.3^2)}{\frac{3}{2}\ (0.3)^2 + \frac{1}{2}} = \frac{\frac{3}{2}\ (x^2+0.09)}{0.635}. \\
E[X \mid Y=0.3] &= \int_0^1 x\ f(x \mid Y=0.3)\ dx = \int_0^1 x\ \frac{\frac{3}{2}\ (x^2+0.09)}{0.635}\ dx = 0.697. \\
E[X^2 \mid Y=0.3] &= \int_0^1 x^2\ f(x \mid Y=0.3)\ dx = \int_0^1 x^2\ \frac{\frac{3}{2}\ (x^2+0.09)}{0.635}\ dx = 0.543. \\
\mathrm{Var}[X \mid Y=0.3] &= = 0.543 - 0.697^2 = 0.057.
\end{aligned}$$

Math 4681 Quiz 5 Answers June 17, 2019

Name ______________________

1. The joint density function for the pair of random variables X and Y is

$$f(x,y) = \frac{1}{2}\, e^{-x} \sin y \quad \text{for } 0 < x < \infty,\ \ 0 < y < \pi.$$

Find $P[(X < 1) \cap (Y < \frac{\pi}{2})]$.

(A) $\frac{1-e^{-1}}{2}$ (B) $\frac{e-1}{2}$ (C) $\frac{2}{e-1}$ (D) $\frac{2}{1-e^{-1}}$ (E) $\frac{e}{\pi}$

Answer # 1: $\boxed{\text{A}}$

The joint density function can be written as $f(x,y) = \frac{1}{2}\, e^{-x} \sin y = (e^{-x}).(\frac{1}{2}\, \sin y) = g(x).h(y)$. Since the joint density is defined on a rectangular region and since it factors into the form $g(x).h(y)$, it follows that the marginal distributions of X and Y are independent. Therefore

$$P[(X<1)\cap(Y<\frac{\pi}{2})] = P(X<1)\ .P(Y<\frac{\pi}{2}) = \left(\int_0^1 e^{-x}\, dx\right).\left(\int_0^{\frac{\pi}{2}} \frac{1}{2}\, \sin y\, dy\right) = \frac{1-e^{-1}}{2}.$$

2. Consider two random variables X and Y where the standard deviation of Y is twice that of X. Given that $\rho_{XY} = 0.5$ and $\text{Var}[2X+Y] = 24$, calculate $\text{Var}[X+2Y]$.

(A) 18 (B) 26 (C) 36 (D) 42 (E) 48

Answer # 2: $\boxed{\text{D}}$

Given $\sigma_Y = 2\sigma_X$. Now,

$$\begin{aligned} \rho_{XY} &= \frac{\text{Cov[X, Y]}}{\sigma_X\ \sigma_Y} \\ 0.5 &= \frac{\text{Cov[X, Y]}}{2\sigma_X^2} \\ \text{Cov[X, Y]} &= \sigma_X^2. \end{aligned}$$

Thus as $\text{Var}[2X+Y] = 24$,

$$\begin{aligned} 4\text{Var}[X] + 4\text{Cov}[X,Y] + \text{Var}[Y] &= 24 \\ 4\sigma_X^2 + 4\sigma_X^2 + 4\sigma_X^2 &= 24 \\ \Rightarrow \quad \sigma_X^2 &= 2. \end{aligned}$$

Hence

$$\begin{aligned} \text{Var}[X+2Y] &= \text{Var}[X] + 4\text{Cov}[X,Y] + 4\text{Var}[Y] \\ &= \sigma_X^2 + 4\sigma_X^2 + 16\sigma_X^2 \\ &= 21\sigma_X^2 = 42. \end{aligned}$$

3. X is $\chi^2(50)$. Determine $P(40 < X < 60)$.

$$\begin{aligned} E[X] &= n = 50, \\ \text{var}[X] &= 2n = 100 \\ P(40 < X < 60) &= P(-1 < Z < 1) = 0.68. \end{aligned}$$