Name:

Due date: Wednesday, May 29.
Reading: Chapter 6: $1, 6, 9, 12,$ chapter 7: $12, 13, 14, 17.$

- Chapter 6

1 In a year, a policyholder with an insurance company has no claims with probability 0.69, one claim with probability 0.23, two claims with probability 0.07, and three claims with probability 0.01. If X is the random variable for the number of claims, find

(a) $E[500X + 50]$

(b) $E[X^2]$

(c) $E[X^3]$

6 Use the moment generating function for Poisson distribution to verify that $E[X] = \mathrm{Var}[X] = \lambda$.

9 Let X be a discrete random variable with $p = \frac{1}{n}$ for $x = 1, 2, \ldots, n$. (X is a discrete uniform random variable.)

(a) Show that the moment generating function for X is $M_X(t) = \dfrac{1}{n}\displaystyle\sum_{x=1}^{n} e^{xt}$.

(b) $E[X]$ and $\mathrm{Var}[X]$.

12 If X is a binomial random variable with $p = 0.6$ and $n = 8$, and if $Y = 3X + 4$, what is $M_Y(t)$?

- Chapter 7

12 The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ with probability density function f, where $f(x)$ is proportional to $(10 + x)^{-2}$. Calculate the probability that the lifetime of the machine part is less than 6.

13 An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x) = \begin{cases} \frac{2.5\ (200)^{2.5}}{x^{3.5}} & \text{for } \ x > 200 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the difference between the 25^{th} and 75^{th} percentiles of X.

14 An insurance company's monthly claims are modeled by a continuous, positive random variable X, whose probability density function is proportional to $(1+x)^{-4}$, where $0 < x < \infty$. Determine the company's expected monthly claims.

17 An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} 3x^{-4} & \text{for } \ x > 1 \\ 0 & \text{otherwise.} \end{cases}$$

Given that a randomly selected home is insured for at least 1.5, what is the probability that it is insured for less than 2?