

Covalence Typing Rules

JAD GHALAYINI and NEEL KRISHNASWAMI

CCS Concepts: • Mathematics of computing → Solvers; • Theory of computation → Type theory; Logic and verification; Higher order logic; Equational logic and rewriting; • Computing methodologies → Symbolic and algebraic manipulation.

Additional Key Words and Phrases: E-graphs, extensional MLTT

ACM Reference Format:

Jad Ghalayini and Neel Krishnaswami. 2025. Covalence Typing Rules. 1, 1 (October 2025), 3 pages. <https://doi.org/XXXXXXX.XXXXXXX>

1 Syntax

TODO 1: copy over from Lean

TODO 2: lore about capture-avoiding substitution and closure

2 Sugar

$$\begin{aligned} \Pi x : A. B &:= \Pi A. B_x & \Sigma x : A. B &:= \Sigma A. B_x & \epsilon x : A. \varphi &:= \epsilon A. \varphi_x \\ \text{ite}(x : \varphi) a b &:= \text{ite}(\varphi) a_x b_x \end{aligned}$$

$$\begin{aligned} 2 &:= \mathcal{U}_0 & \perp &:= 0 & \top &:= 1 & \neg \varphi &:= (\varphi = \perp) & \varphi \wedge \psi &:= \Sigma \varphi. \psi^\uparrow & \varphi \vee \psi &:= \text{ite}(\varphi) \top \psi^\uparrow \\ \varphi \Rightarrow \psi &:= \Pi \varphi. \psi^\uparrow & \forall A. \varphi &:= \Pi A. \varphi & \exists A. \varphi &:= \|\Sigma A. \varphi\| \\ A \times B &:= \Sigma A. B^\uparrow & A + B &:= \Sigma \varphi : 2. \text{ite}(\varphi) A^\uparrow B^\uparrow & \iota_1 a &:= (\perp, a) & \iota_2 b &:= (\top, b) \end{aligned}$$

3 Rules

3.1 Sugar

$$\frac{\Gamma \vdash a \equiv a : A}{\Gamma \vdash a : A} \text{ has-ty-def} \qquad \frac{\Gamma \vdash * : 1}{\Gamma \text{ ok}} \text{ ok-def}$$

3.2 Well-formedness

$$\frac{}{\cdot \text{ ok}} \text{ nil-ok} \qquad \frac{\Gamma \text{ ok} \quad x \notin \text{dom}(\Gamma) \quad \Gamma \vdash A \equiv A : \mathcal{U}_\ell}{\Gamma, x : A \text{ ok}} \text{ cons-ok}$$

Authors' Contact Information: [Jad Ghalayini, jeg74@cl.cam.ac.uk](mailto:jeg74@cl.cam.ac.uk); [Neel Krishnaswami, nk480@cl.cam.ac.uk](mailto:nk480@cl.cam.ac.uk).

Permission to make digital or hard copies of all or part of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM must be honored. Abstracting with credit is permitted. To copy otherwise, or republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee. Request permissions from permissions@acm.org.

© 2025 ACM.

ACM XXXX-XXXX/2025/10-ART

<https://doi.org/XXXXXXX.XXXXXXX>

3.3 Congruence

$$\begin{array}{c}
\frac{\Gamma \text{ ok} \quad \Gamma(x) = A}{\Gamma \vdash x \equiv x : A} \text{var} \quad \frac{\Gamma \text{ ok}}{\Gamma \vdash \mathcal{U}_\ell \equiv \mathcal{U}_\ell : \mathcal{U}_{\ell+1}} \text{univ} \quad \frac{\Gamma \text{ ok}}{\Gamma \vdash \mathbf{0} \equiv \mathbf{0} : \mathcal{U}_\ell} \text{empty} \\
\frac{\Gamma \text{ ok}}{\Gamma \vdash \mathbf{1} \equiv \mathbf{1} : \mathcal{U}_\ell} \text{unit} \quad \frac{\Gamma \vdash a \equiv a' : A \quad \Gamma \vdash b \equiv b' : A}{\Gamma \vdash a = b \equiv a' = b' : \mathbf{2}} \text{eqn} \\
\frac{\Gamma \vdash A \equiv A' : \mathcal{U}_m \quad \forall x \notin L.\Gamma, x : A \vdash B^x \equiv B'^x : \mathcal{U}_n \quad \text{imax}(m, n) \leq \ell}{\Gamma \vdash \Pi A.B \equiv \Pi A'.B' : \mathcal{U}_\ell} \text{pi} \\
\frac{\Gamma \vdash A \equiv A' : \mathcal{U}_m \quad \forall x \notin L.\Gamma, x : A \vdash B^x \equiv B^x : \mathcal{U}_n \quad \forall x \notin L.\Gamma, x : A \vdash b^x \equiv b'^x : B^x}{\Gamma \vdash \lambda A.b \equiv \lambda A'.b' : \mathcal{U}_\ell} \text{abs} \\
\frac{\Gamma \vdash A \equiv A : \mathcal{U}_m \quad \forall x \notin L.\Gamma, x : A \vdash B^x \equiv B^x : \mathcal{U}_n \quad \Gamma \vdash f \equiv f' : \Pi A.B \quad \Gamma \vdash a \equiv a' : A}{\Gamma \vdash f a \equiv f' a' : B^a} \text{app} \\
\frac{\Gamma \vdash A \equiv A' : \mathcal{U}_m \quad \forall x \notin L.\Gamma, x : A \vdash B^x \equiv B'^x : \mathcal{U}_n \quad m \leq \ell \quad n \leq \ell}{\Gamma \vdash \Sigma A.B \equiv \Sigma A'.B' : \mathcal{U}_\ell} \text{sigma} \\
\frac{\Gamma \vdash A \equiv A : \mathcal{U}_m \quad \forall x \notin L.\Gamma, x : A \vdash B^x \equiv B^x : \mathcal{U}_n \quad \Gamma \vdash a \equiv a' : A \quad \Gamma \vdash b \equiv b' : B^a}{\Gamma \vdash (a, b) \equiv (a', b') : \Sigma A.B} \text{pair} \\
\frac{\Gamma \vdash A \equiv A : \mathcal{U}_m \quad \forall x \notin L.\Gamma, x : A \vdash B^x \equiv B^x : \mathcal{U}_n \quad \Gamma \vdash p \equiv p' : \Sigma A.B}{\Gamma \vdash \pi_1 p \equiv \pi_1 p' : A} \text{fst} \\
\frac{\Gamma \vdash A \equiv A : \mathcal{U}_m \quad \forall x \notin L.\Gamma, x : A \vdash B^x \equiv B^x : \mathcal{U}_n \quad \Gamma \vdash p \equiv p' : \Sigma A.B}{\Gamma \vdash \pi_2 p \equiv \pi_2 p' : B^{\pi_1 p}} \text{snd} \\
\frac{\Gamma \vdash \varphi \equiv \varphi' : \mathbf{2} \quad \forall x \notin L.\Gamma, x : \varphi \vdash a^x \equiv a'^x : A \quad \Gamma \vdash A \equiv A : \mathcal{U}_\ell \quad \forall x \notin L.\Gamma, x : \neg \varphi \vdash b^x \equiv b'^x : A}{\Gamma \vdash \text{ite}(\varphi) a b \equiv \text{ite}(\varphi) a' b' : A} \text{dite} \quad \frac{\Gamma \vdash A \equiv A' : \mathcal{U}_\ell}{\Gamma \vdash \|A\| \equiv \|A'\| : \mathbf{2}} \text{trunc} \\
\frac{\Gamma \vdash A \equiv A' : \mathcal{U}_\ell \quad \Gamma \vdash \|A\| \equiv \top : \mathbf{2} \quad \forall x \notin L.\Gamma, x : A \vdash \varphi^x \equiv \varphi'^x : \mathbf{2}}{\Gamma \vdash \epsilon A.\varphi \equiv \epsilon A'.\varphi' : A} \text{choose} \\
\frac{\Gamma \text{ ok}}{\Gamma \vdash \mathbb{N} \equiv \mathbb{N} : \mathcal{U}_\ell} \text{nats} \quad \frac{\Gamma \text{ ok}}{\Gamma \vdash \mathbf{0} \equiv \mathbf{0} : \mathbb{N}} \text{zero} \quad \frac{\Gamma \vdash n \equiv n' : \mathbb{N}}{\Gamma \vdash s n \equiv s n' : \mathbb{N}} \text{succ} \\
\frac{\forall x \notin L.\Gamma, x : \mathbb{N} \vdash C^x \equiv C'^x : \mathcal{U}_\ell \quad \Gamma \vdash z \equiv z' : C^0 \quad \forall x \notin L.\Gamma, x : \mathbb{N} \vdash s \equiv s' : C^x \rightarrow C^{s x}}{\Gamma \vdash \text{rec}_{\mathbb{N}}\{C\} z s \equiv \text{rec}_{\mathbb{N}}\{C'\} z' s' : \Pi \mathbb{N}.C} \text{rec}_{\mathbb{N}} \\
\frac{\Gamma \vdash a \equiv a' : A \quad \Gamma \vdash A \equiv A' : \mathcal{U}_\ell}{\Gamma \vdash [a \in A] \equiv [a' \in A'] : \mathbf{2}} \text{has-ty}
\end{array}$$

3.4 Conversion

$$\begin{array}{c}
\frac{\Gamma \vdash a \equiv a' : A \quad \Gamma \vdash A \equiv A' : \mathcal{U}_\ell}{\Gamma \vdash [a \in A] \equiv \top : 2} \text{has-ty}_\top \quad \frac{\Gamma \vdash a \equiv a' : A \quad \Gamma \vdash A \equiv A' : \mathcal{U}_\ell}{\Gamma \vdash [a \in A] \equiv \top : 2} \text{has-ty}_\top \\
\frac{\Gamma \vdash \lambda A.b : \Pi A.B \quad \Gamma \vdash a : A \quad \Gamma \vdash (\lambda A.b) a : C \quad \Gamma \vdash b^a : C}{\Gamma \vdash (\lambda A.b) a \equiv b^a : C} \beta_{\text{app}} \\
\frac{\Gamma \vdash (a, b) : \Sigma A.B \quad \Gamma \vdash \pi_1(a, b) : A \quad \Gamma \vdash a : A}{\Gamma \vdash \pi_1(a, b) \equiv a : A} \beta_{\pi_1} \\
\frac{\Gamma \vdash (a, b) : \Sigma A.B \quad \Gamma \vdash \pi_2(a, b) : C \quad \Gamma \vdash b : C}{\Gamma \vdash \pi_2(a, b) \equiv b : C} \beta_{\pi_2} \\
\frac{\Gamma \vdash A : \mathcal{U}_\ell \quad \forall x \notin L.\Gamma, x : \top \vdash a^x : A \quad \Gamma \vdash \text{ite}(\top) a b : A \quad \Gamma \vdash a^* : A}{\Gamma \vdash \text{ite}(\top) a b \equiv a : A} \beta_\top \\
\frac{\Gamma \vdash A : \mathcal{U}_\ell \quad \forall x \notin L.\Gamma, x : \neg \perp \vdash b^x : A \quad \Gamma \vdash \text{ite}(\perp) a b : A \quad \Gamma \vdash b^* : B}{\Gamma \vdash \text{ite}(\perp) a b \equiv b : A} \beta_\perp \\
\frac{\forall x \notin L.\Gamma, x : \mathbb{N} \vdash C^x \equiv C'^x : \mathcal{U}_\ell \quad \Gamma \vdash z \equiv z' : C^0 \quad \forall x \notin L.\Gamma, x : \mathbb{N} \vdash s \equiv s' : C^x \rightarrow C^s x}{\text{TODO 3 : this}} \beta_0 \\
\frac{\text{TODO 3 : this}}{\text{TODO 4 : this}} \beta_s
\end{array}$$

3.5 Choice and Extensionality

TODO 5: this

3.6 Equivalence and Casting

TODO 6: this