MAT454 Notes

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Definition 1 (n-dimensional complex projective space)**.** *We define*

$$\mathrm{P}^n(\mathbb{C}) = \mathbb{C}^{n+1} \backslash \{0\} / \sim$$

where

$$(x_0, ..., x_n) \sim (x_0', ..., x_n') \iff \exists \lambda \in \mathbb{C}, (x_0', ..., x_n') = (\lambda x_0, ..., \lambda x_n)$$

We denote the equivalence class of $(x_0, ..., x_n)$ *by* $[x_0, ..., x_n]$.

Definition 2 (Homogeneous coordinates)**.** *We define coordinate charts* $U_i = \{[x_0, ..., x_n] \in \mathrm{P}^n(\mathbb{C}) : x_i \neq 0\}$ *with affine coordinates* $U_i \to \mathbb{C}^n$*,*

$$[x_0, ..., x_n] \mapsto \left(\frac{x_0}{x_i}, ..., \frac{x_{i-1}}{x_i}, \frac{x_{i+1}}{x_i}, ..., \frac{x_n}{x_i}\right)$$

with inverse

$$(g_1, ..., g_n) \mapsto [g_1, ..., g_{i-1}, 1, g_{i+1}, .., g_n]$$

Using these coordinates, we have that $\mathrm{P}^n(\mathbb{C})$ has the structure of an n-dimensional complex manifold, as the transition mappings are rational.