

MAT454 Academic Offense Sheet

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A quick collection of useful facts, theorems, and definitions for complex analysis. May be incorrect, and is certainly incomplete. Use at your own risk!

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1 Basic Definitions and Theorems

Definition 1. The *differential* of f is given by

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \quad (1)$$

$$dz = dx + idy, \quad d\bar{z} = dx - idy \iff dx = \frac{1}{2}(dz + d\bar{z}), \quad dy = \frac{1}{2i}(dz - d\bar{z}) \quad (2)$$

$$\frac{\partial f}{\partial z} = \frac{1}{2} \left(\frac{\partial f}{\partial x} - i \frac{\partial f}{\partial y} \right), \quad \frac{\partial f}{\partial \bar{z}} = \frac{1}{2} \left(\frac{\partial f}{\partial x} + i \frac{\partial f}{\partial y} \right) \implies df = \frac{\partial f}{\partial z} dz + \frac{\partial f}{\partial \bar{z}} d\bar{z} \quad (3)$$

Definition 2 (Harmonic). We say a real or complex valued function $f(x, y)$ is **harmonic** if f is C^2 and

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \iff \frac{\partial^2 f}{\partial z \partial \bar{z}} = 0 \quad (4)$$

Proposition 1. Every real-valued harmonic function is, not necessarily everywhere but at least locally, the real part of a holomorphic function.

2 Useful Tools

3 Residues and Integrals

4 Elliptic Curves