

# MAT454 Academic Offense Sheet

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A quick collection of useful facts, theorems, and definitions for complex analysis. May be incorrect, and is certainly incomplete. Use at your own risk!

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# 1 Basic Definitions and Theorems

For  $f = u + iv$  holomorphic, we have

$$2\frac{\partial f}{\partial \bar{z}} = \frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y} = 0 \iff \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \wedge \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad (1)$$

**Definition 1.** The *differential* of  $f$  is given by

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy = \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial \bar{z}}d\bar{z} \quad (2)$$

$$dz = dx + idy, \quad d\bar{z} = dx - idy \iff dx = \frac{1}{2}(dz + d\bar{z}), \quad dy = \frac{1}{2i}(dz - d\bar{z}) \quad (3)$$

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(\frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y}\right), \quad \frac{\partial f}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial f}{\partial x} + i\frac{\partial f}{\partial y}\right) \implies df = \frac{\partial f}{\partial z}dz + \frac{\partial f}{\partial \bar{z}}d\bar{z} \quad (4)$$

**Definition 2** (Harmonic). We say a real or complex valued function  $f(x, y)$  is **harmonic** if  $f$  is  $C^2$  and

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \iff \frac{\partial^2 f}{\partial z \partial \bar{z}} = 0 \quad (5)$$

**Proposition 1.** Every real-valued harmonic function is, not necessarily everywhere but at least locally, the real part of a holomorphic function.

## 2 Common Formulas

- Projection from the Riemann Sphere:

$$\pi : S^2 \setminus \{N\} \rightarrow \mathbb{C}, \pi(x, y, t) = \frac{x + iy}{1 - t} \quad (6)$$

## 3 Useful Tools

## 4 Residues and Integrals

## 5 Elliptic Curves