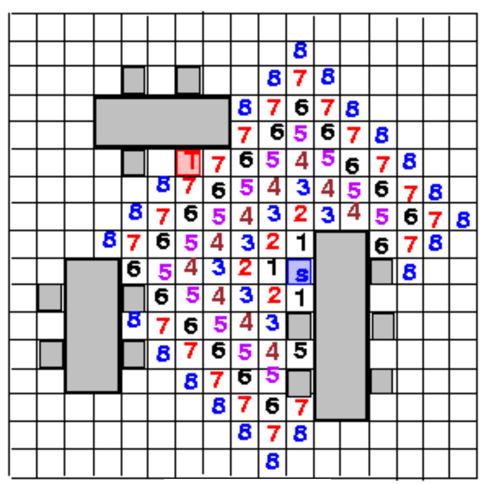
Unit 6: Global and Maze Routing

- Course contents:
  - Maze routing
  - Global routing
  - Routing trees
- Readings
  - W&C&C: Chapter 12
  - S&Y: Chapters 5 and 6



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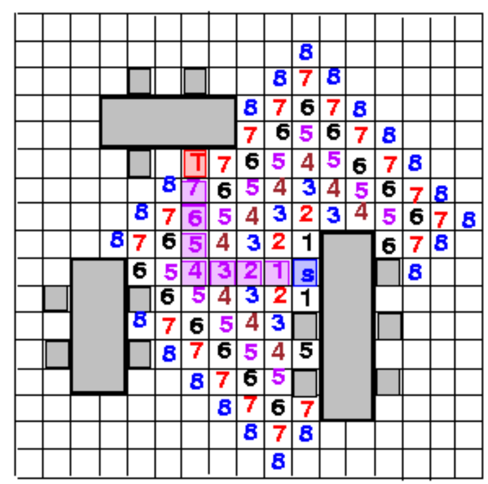


Retrace

Courtesy of Prof. Y.-W. Chang

Routing

placement

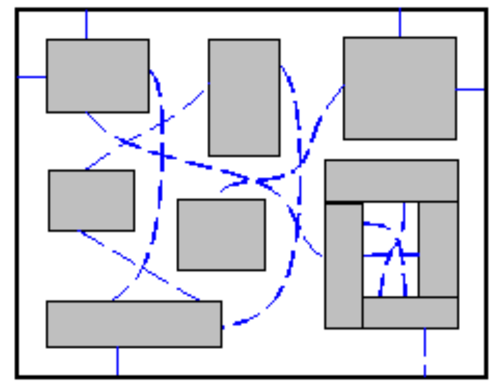
- Generates a "loose" route for each net.
- Assigns a list of routing regions to each net without specifying the actual layout of wires.

global routing

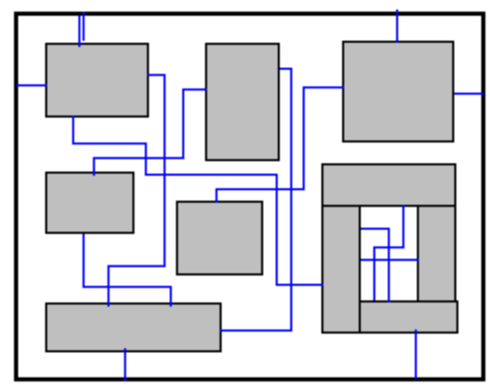
detailed routing

- Finds the actual geometric layout of each net within the assigned routing regions.

compaction



Global routing

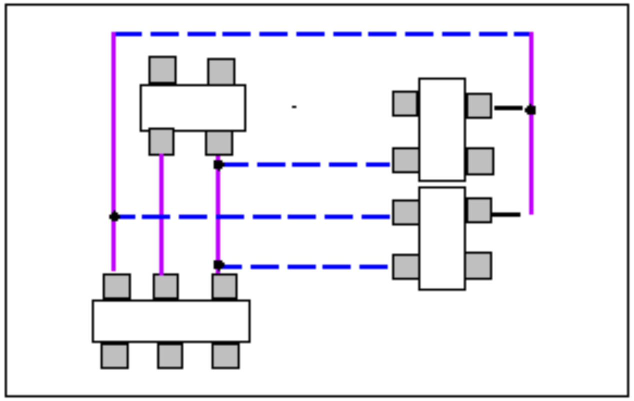


Detailed routing

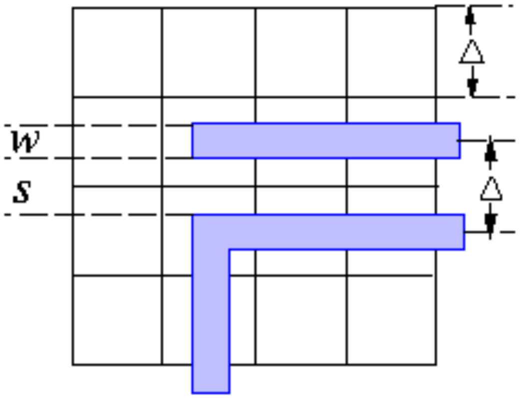
Courtesy of Prof. Y.-W. Chang

Routing Constraints

- 100% routing completion + area minimization, under a set of constraints:
  - Placement constraint: usually based on fixed placement
  - Number of routing layers
  - Geometrical constraints: must satisfy design rules
  - Timing constraints (performance-driven routing): must satisfy delay constraints
  - Crosstalk?
  - Design for manufacturability, reliability, and printability?



Two−layer routing



Geometrical constraint

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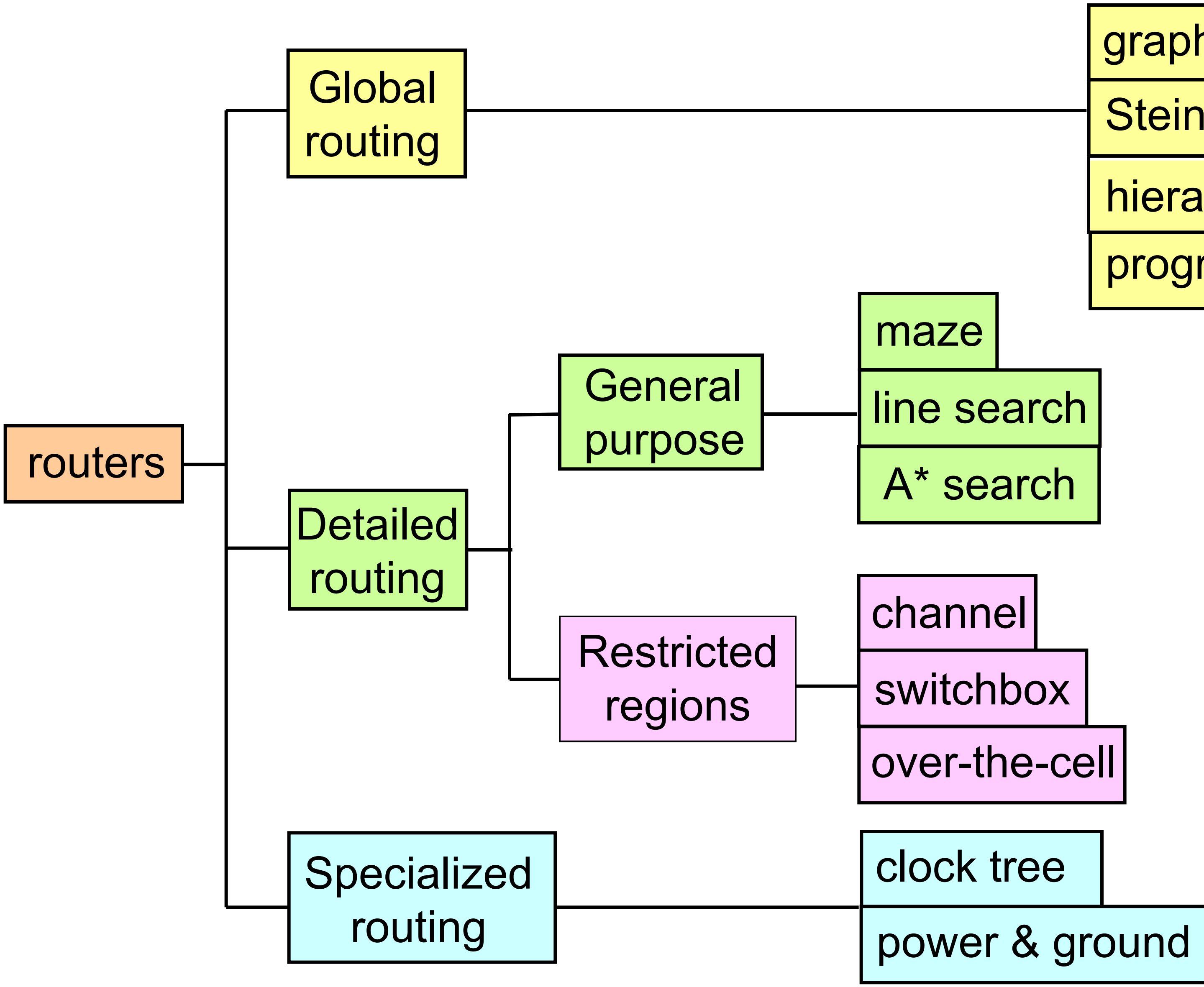
Classification of Routers



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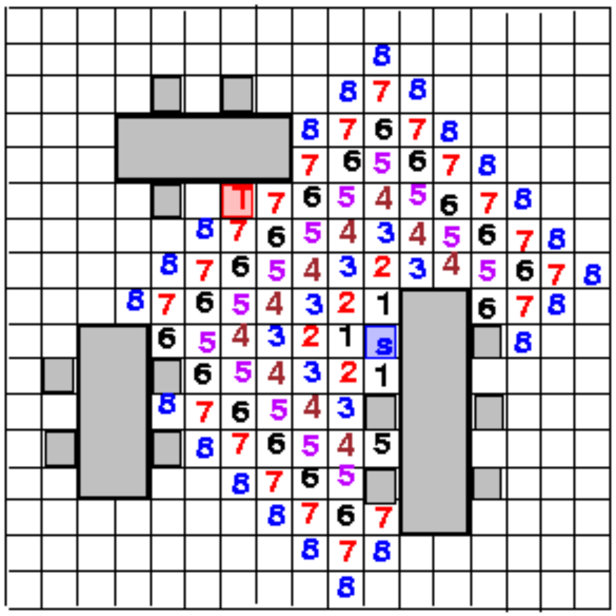
Maze Router: Lee Algorithm

- Lee, "An algorithm for path connection and its application," *IRE Trans. Electronic Computer*, EC-10, 1961.
- Discussion mainly on single-layer routing
- **Strengths**
  - Guarantee to find connection between 2 terminals if it exists.
  - Guarantee minimum path.
- **Weaknesses**
  - Requires large memory for dense layout
  - Slow
- Applications: global routing, detailed routing

Lee Algorithm

- Find a path from *S* to *T* by “wave propagation”.



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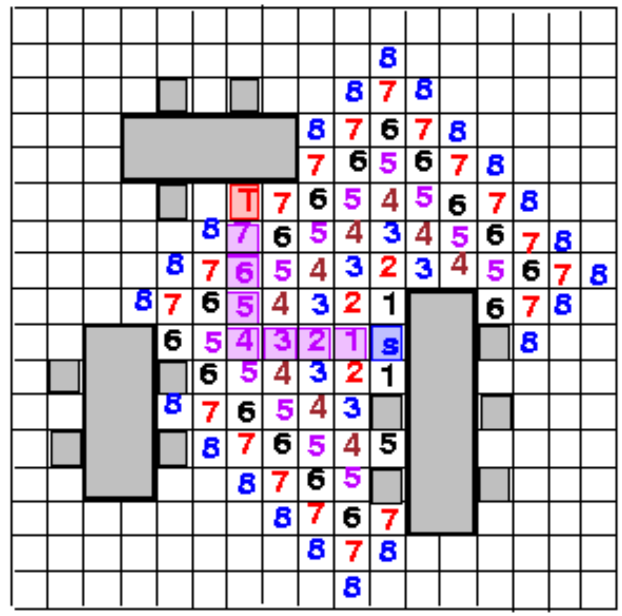
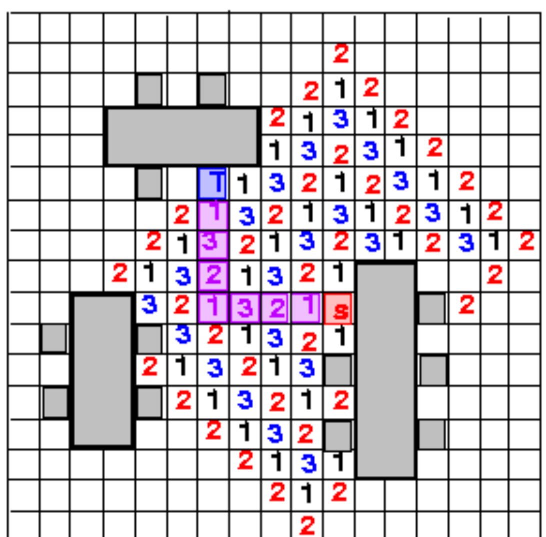


Retrace

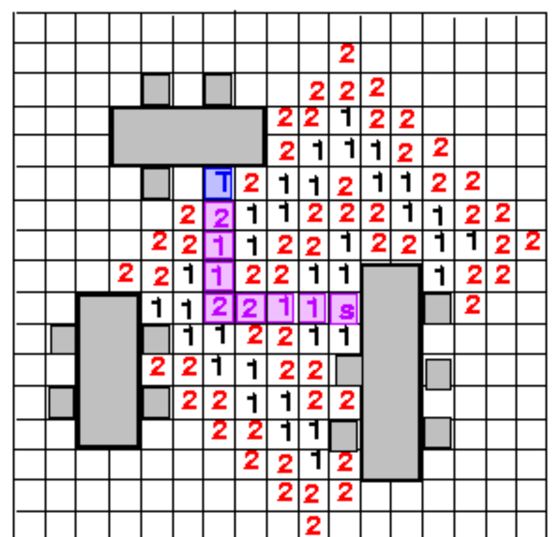
- Time & space complexity for an $M \times N$ grid: $O(MN)$ (**huge!**)

Reducing Memory Requirement

- Akers's Observations (1967)
  - Adjacent labels for *k* are either *k*-1 or *k*+1.
  - Want a labeling scheme such that each label has its preceding label different from its succeeding label.
- Way 1: coding sequence 1, 2, 3, 1, 2, 3, …; states: 1, 2, 3, *empty*, *blocked* (3 bits required)
- Way 2: coding sequence 1, 1, 2, 2, 1, 1, 2, 2, …; states: 1, 2, *empty*, *blocked* (need only 2 bits)



Sequence: 1, 2, 3, 1, 2, 3, ...



Sequence: 1, 1, 2, 2, 1, 1, 2, 2, ...

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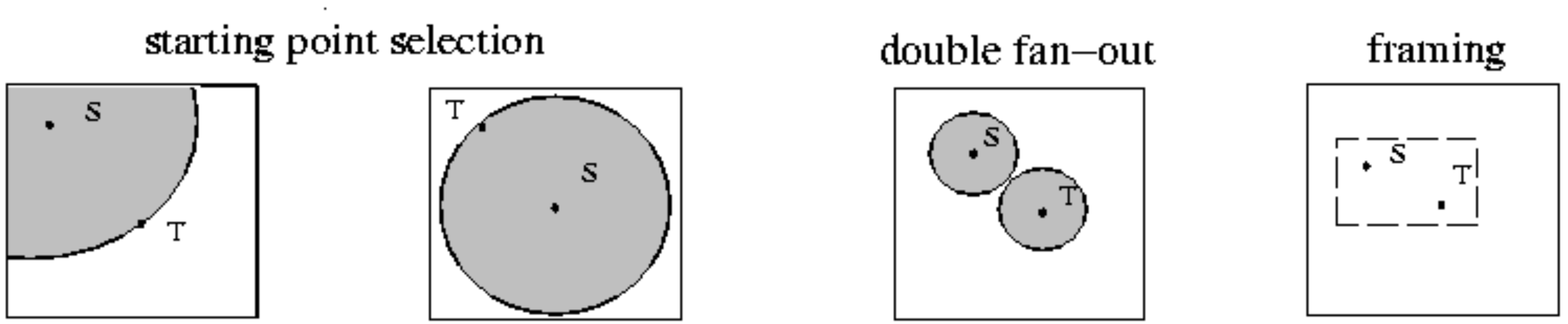
Reducing Running Time

- Starting point selection: Choose the point farthest from the center of the grid as the starting point.
- Double fan-out: Propagate waves from both the source and the target cells.
- Framing: Search inside a rectangle area 10--20% larger than the bounding box containing the source and target.
  - Need to enlarge the rectangle and redo if the search fails.



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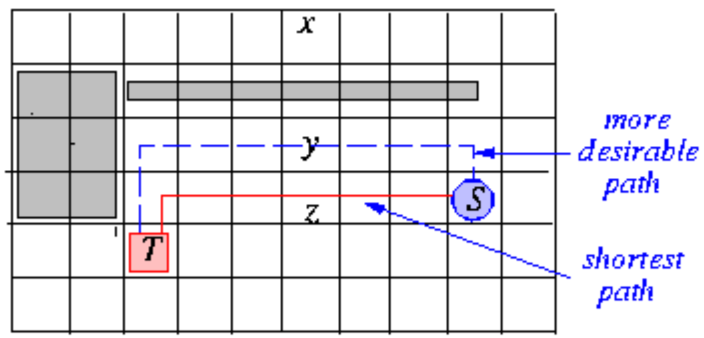
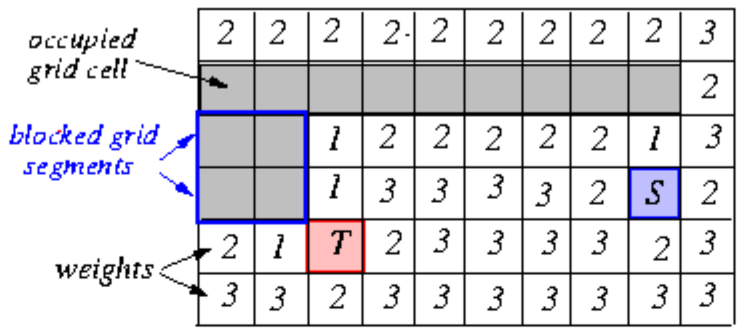
Routing on a Weighted Grid

- Motivation: finding more desirable paths
- *weight*(*grid cell*) = *# of unblocked grid cell segments* - 1





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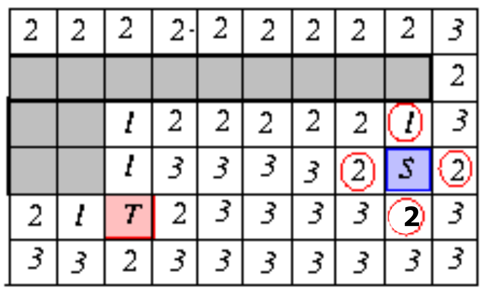
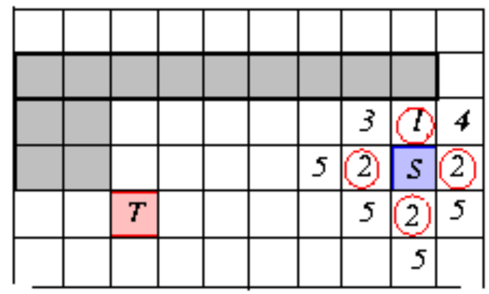
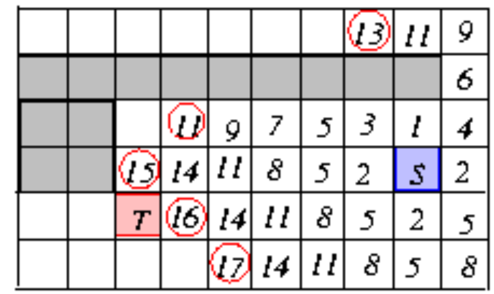
Example Routing on a Weighted Grid

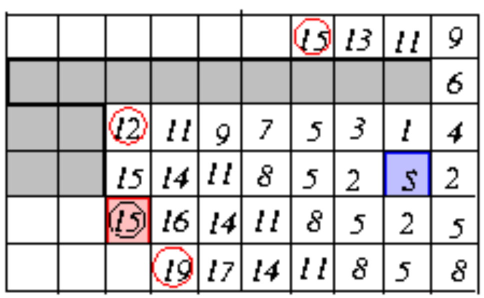


initialize cell weights

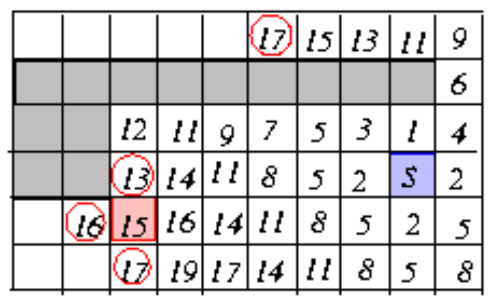




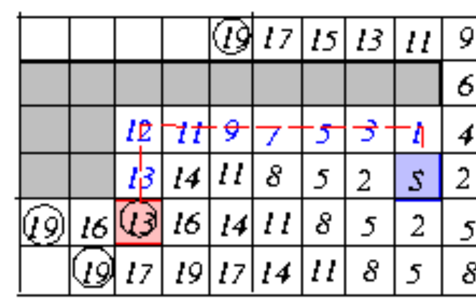
wave propagation



first wave reaches the target



finding other paths



min−cost path found

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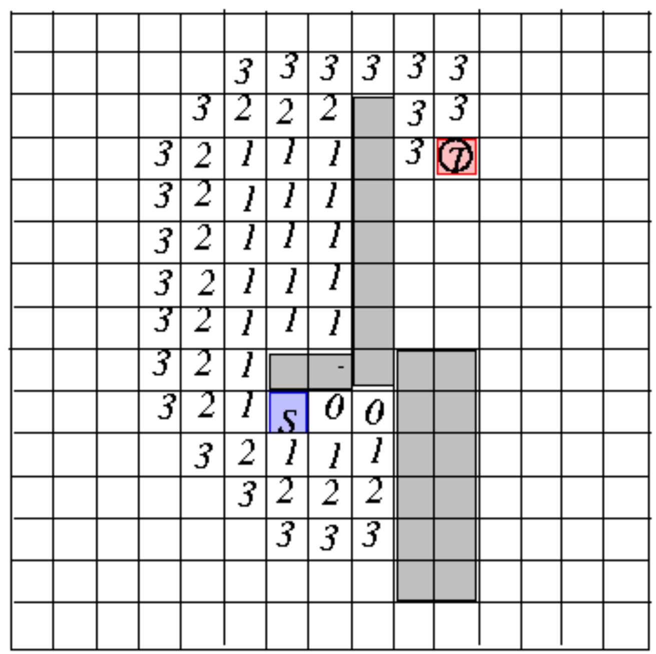
Hadlock's Algorithm

- Hadlock, "A shortest path algorithm for grid graphs," *Networks*, 1977.
- Uses detour number (instead of labeling wavefront in Lee's router)
  - Detour number, $d(P)$: # of grid cells directed **away from** its target on path P.
  - $MD(S, T)$: the Manhattan distance between S and T.
  - Path length of P, $l(P)$: $l(P) = MD(S, T) + 2\, d(P)$.
  - $MD(S, T)$ fixed! $\Rightarrow$ Minimize $d(P)$ to find the shortest path.
  - For any cell labeled i, label its adjacent unblocked cells **away from** T i+1; label i otherwise.
- Time and space complexities: $O(MN)$, but substantially reduces the # of searched cells.
- Finds the shortest path between S and T.

Hadlock's Algorithm (cont'd)

- $d(P)$: # of grid cells directed **away from** its target on path P.
- $MD(S, T)$: the Manhattan distance between S and T.
- Path length of P, $l(P)$: $l(P) = MD(S, T) + 2d(P)$.
- $MD(S, T)$ fixed! $\Rightarrow$ Minimize $d(P)$ to find the shortest path.
- For any cell labeled i, label its adjacent unblocked cells **away from** T i+1; label i otherwise.



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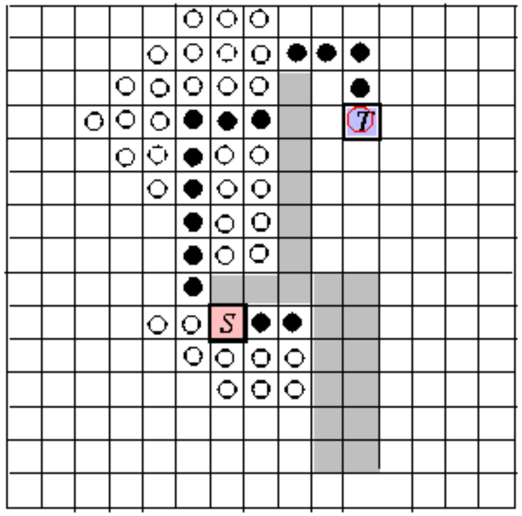
Soukup's Algorithm

- Soukup, "Fast maze router," DAC-78.
- Combined breadth-first and depth-first search.
  - Depth-first (**line**) search is first directed toward target *T* until an obstacle or *T* is reached.
  - Breadth-first (Lee-type) search is used to "bubble" around an obstacle if an obstacle is reached.
- Time and space complexities: $O(MN)$, but 10--50 times faster than Lee's algorithm.
- Find **a** path between *S* and *T*, but may not be the shortest!



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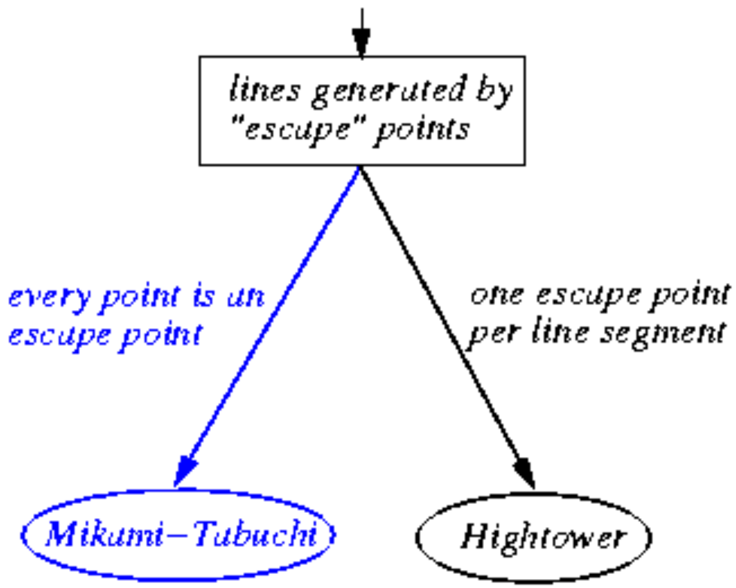
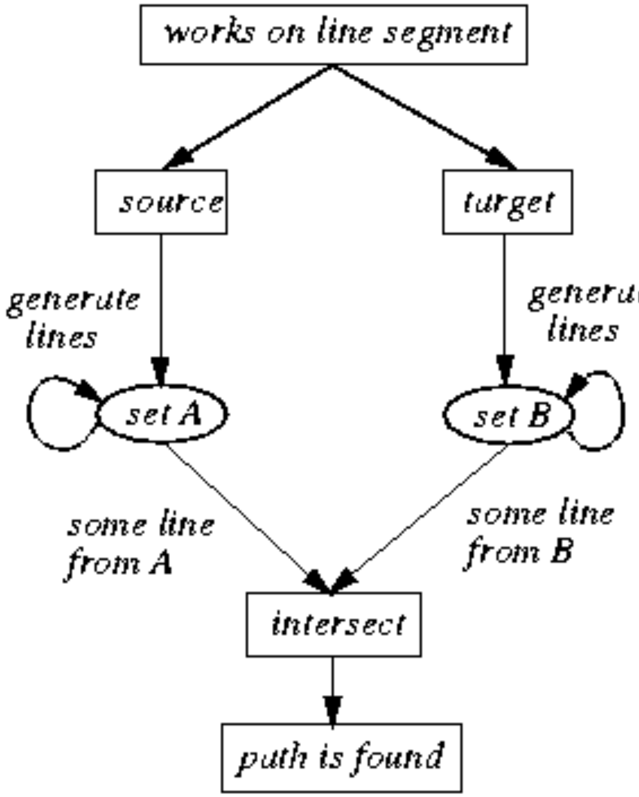
Features of Line-Search Algorithms

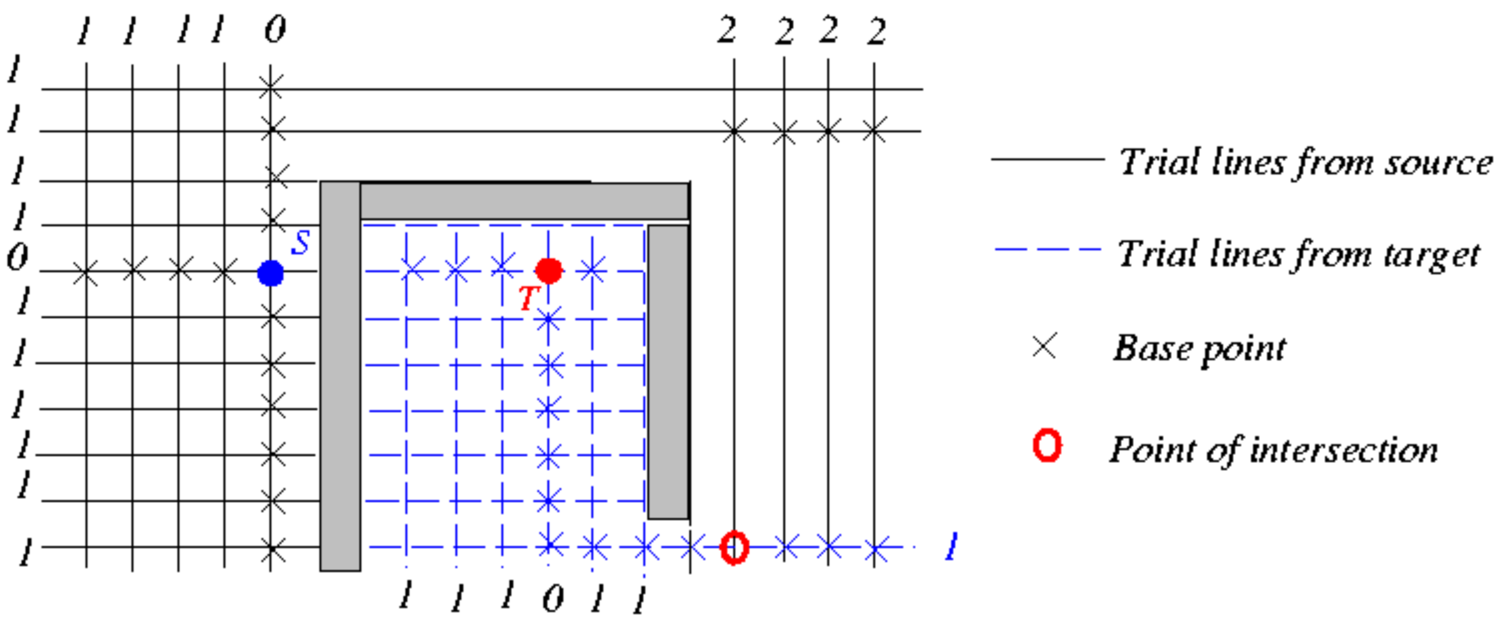




- Time and space complexities: $O(L)$, where L is the # of line segments generated.

Mikami-Tabuchi's Algorithm

- Mikami & Tabuchi, “A computer program for optimal routing of printed circuit connectors,” *IFIP*, H47, 1968.
- Every grid point is an escape point.



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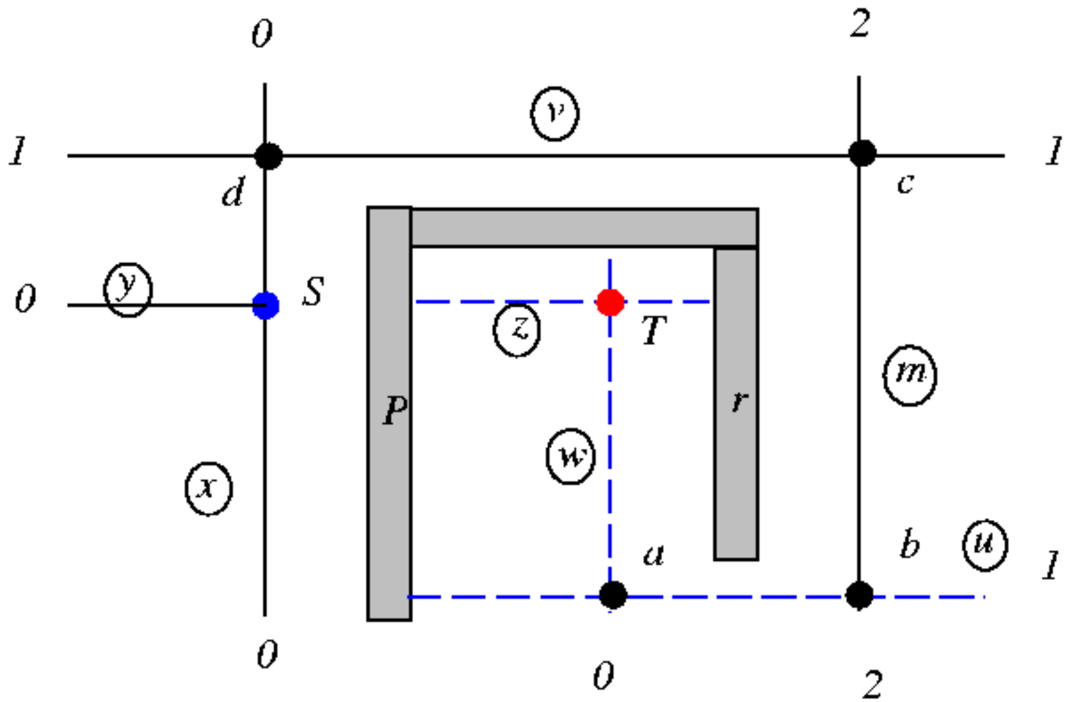
Hightower's Algorithm

- Hightower, “A solution to line-routing problem on the continuous plane,” DAC-69.
- A single escape point on each line segment.
- If a line parallels to the blocked cells, the escape point is placed just past the endpoint of the segment.



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Comparison of Algorithms

	Maze routing			Line search	
	Lee	Soukup	Hadlock	Mikami	Hightower
Time	$O(MN)$	$O(MN)$	$O(MN)$	$O(L)$	$O(L)$
Space	$O(MN)$	$O(MN)$	$O(MN)$	$O(L)$	$O(L)$
Finds path if one exists?	yes	yes	yes	yes	no
Is the path shortest?	yes	no	yes	no	no
Works on grids or lines?	grid	grid	grid	line	line

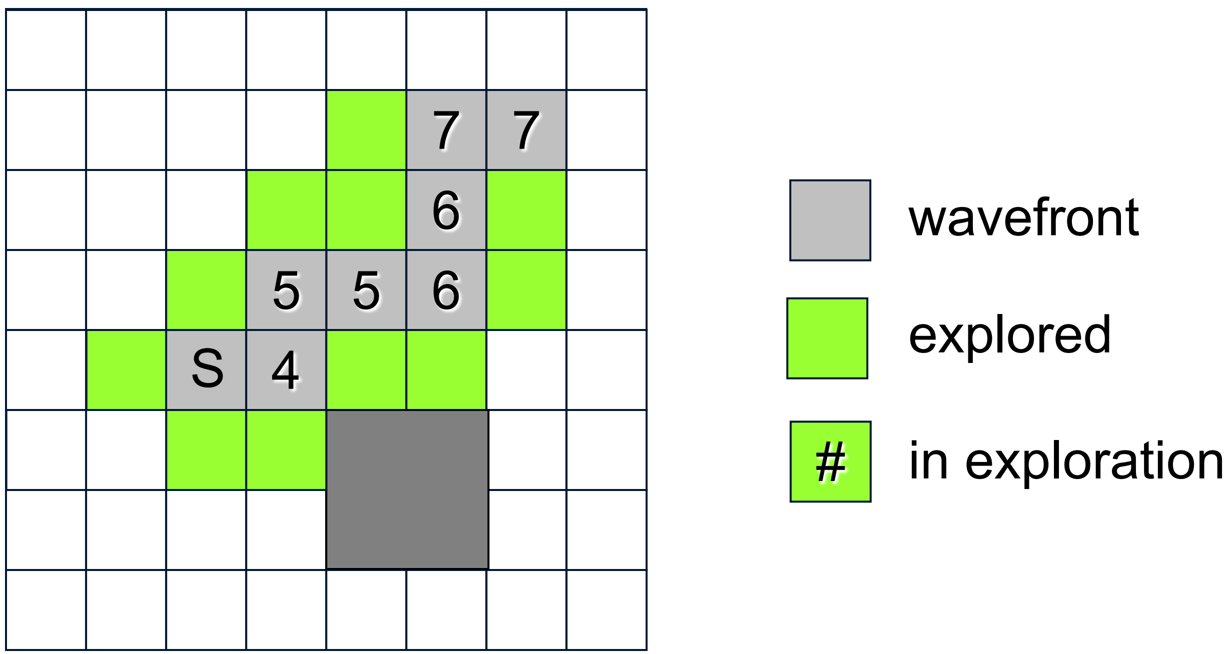
- Soukup, Mikami, and Hightower all adopt some sort of line-search operations $\Rightarrow$ cannot guarantee shortest paths.

A*-Search Routing

- Maze search is also called **blind search** since it searches the routing region in a blind way.
- A*-search is also called the **best-first search**
  - Uses function $f(x) = g(x) + h(x)$ to evaluate the cost of a path x
    - $g(x)$: the cost from the source to the current node of x
    - $h(x)$: the *estimated* cost from the current node of x to the target
- A*-search first searches the routes that is most likely to lead towards the target.
  - BFS is a special case of A*-search where $h(x) = 0$ for all x
- Good property:
  - If $h(x)$ is *admissible* (never overestimates the actual cost from the current node to the target), then A*-search is optimal

Example of A*-Search Routing

- Cost function for a path x: $f(x) = g(x) + h(x)$
  - $g(x)$: the label from the source S to the current node of x (*i.e.*, the lable used in maze routing)
  - $h(x)$: $\max(\mathrm{dist}_x(T, x), \mathrm{dist}_y(T, x))$



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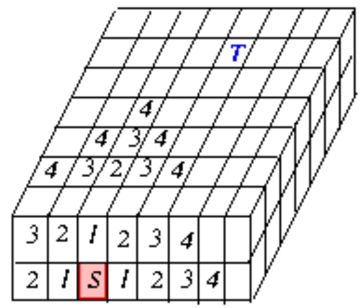
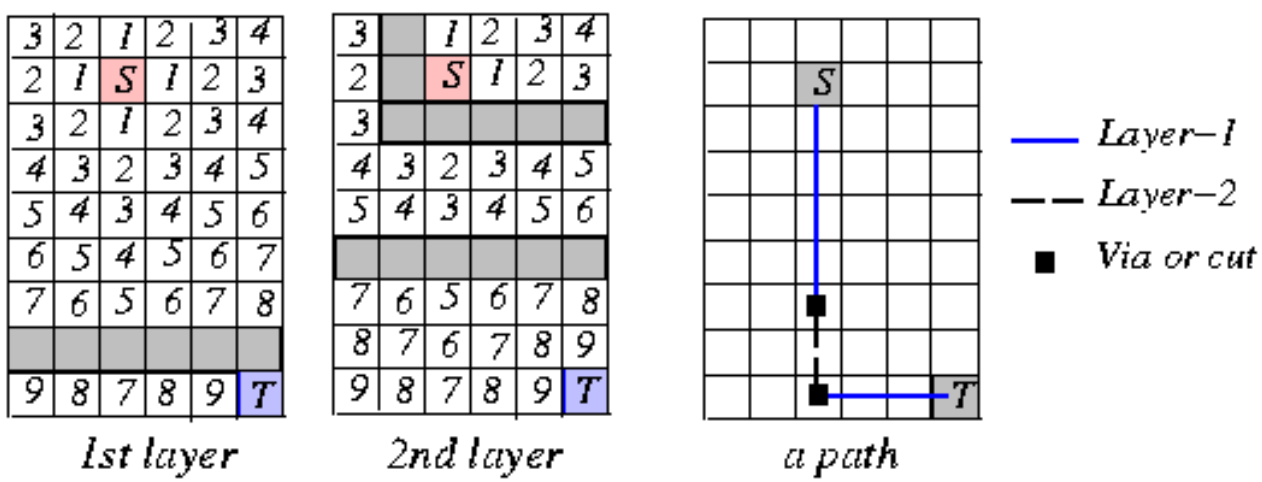
Multi-layer Routing

- 3-D grid:

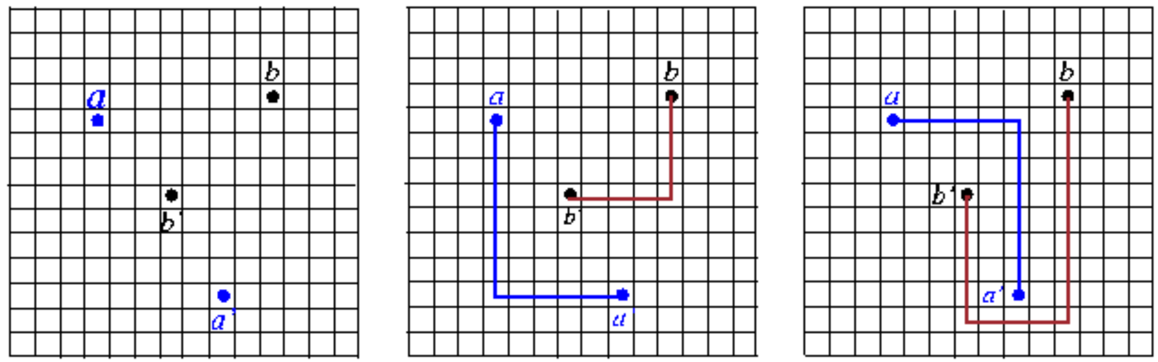


- Two planner arrays:
  - Neglect the weight for inter-layer connection through via.
  - Pins are accessible from both layers.

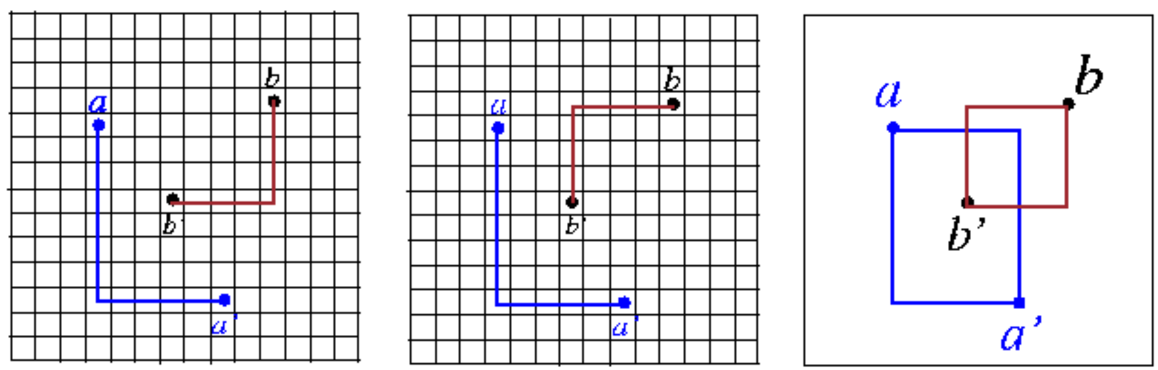


Net Ordering

- Net ordering greatly affects routing solutions.
- In the example, we should route net *b* before net *a*.



route net a before net b



route net b before net a

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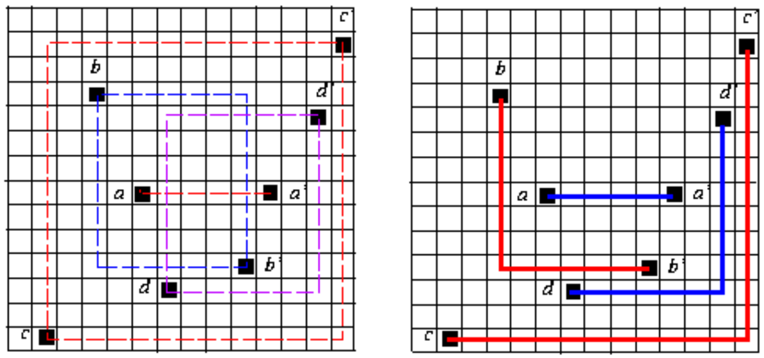
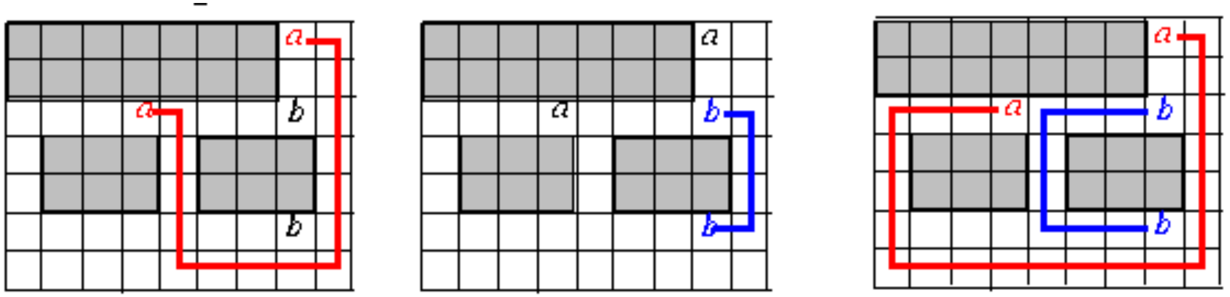
Net Ordering (cont'd)

- Order the nets in the ascending order of the # of pins within their bounding boxes.
- Order the nets in the ascending (or descending??) order of their lengths.
- Order the nets based on their timing criticality.



routing ordering: a (0) −> b (1) −> d (2) −> c (6)

- A mutually intervening case:



a prevents routing of b *b prevents routing of a* *a feasible routing*

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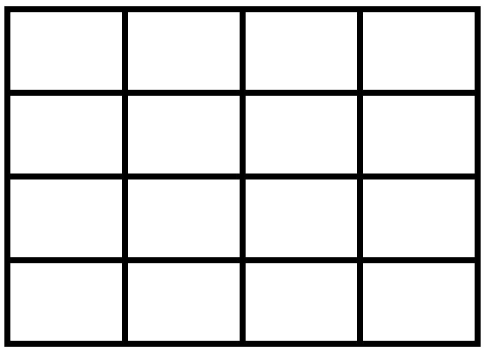
Rip-Up and Re-routing

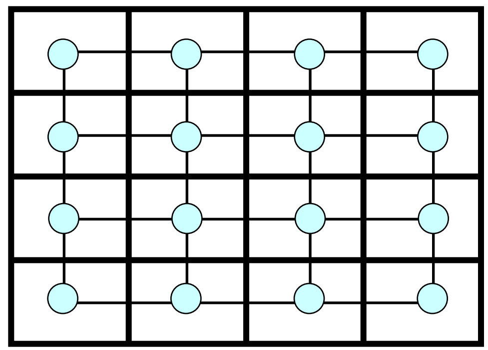
- Rip-up and re-routing is required if a global or detailed router fails in routing all nets.
- Approaches: the manual approach? the automatic procedure?
- Two steps in rip-up and re-routing
  1. Identify bottleneck regions, rip off some already routed nets.
  2. Route the blocked connections, and re-route the ripped-up connections.
- Repeat the above steps until all connections are routed or a time limit is exceeded.

Graph Models: Global Routing Graph

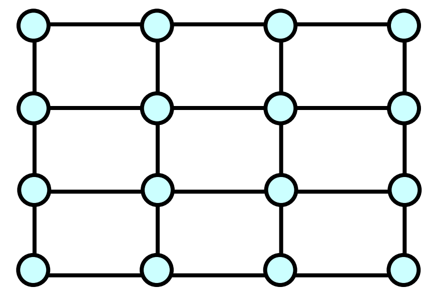
- Each cell is represented by a vertex.
- Two vertices are joined by an edge if the corresponding cells are adjacent to each other.



Partitioned Layout



Resource Modeling



Global Routing Graph

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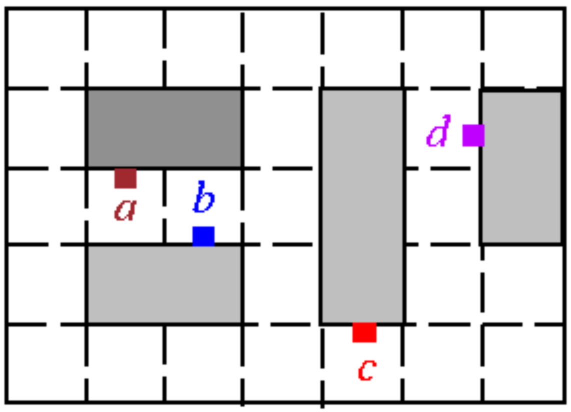
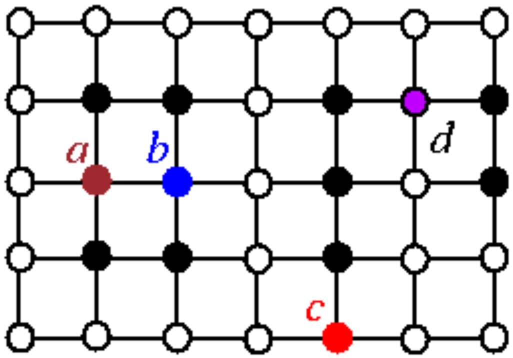
Graph Models for Global Routing: Grid Graph

- Each cell is represented by a vertex.
- Two vertices are joined by an edge if the corresponding cells are adjacent to each other.
- The occupied cells are represented as filled circles, whereas the others are as clear circles.





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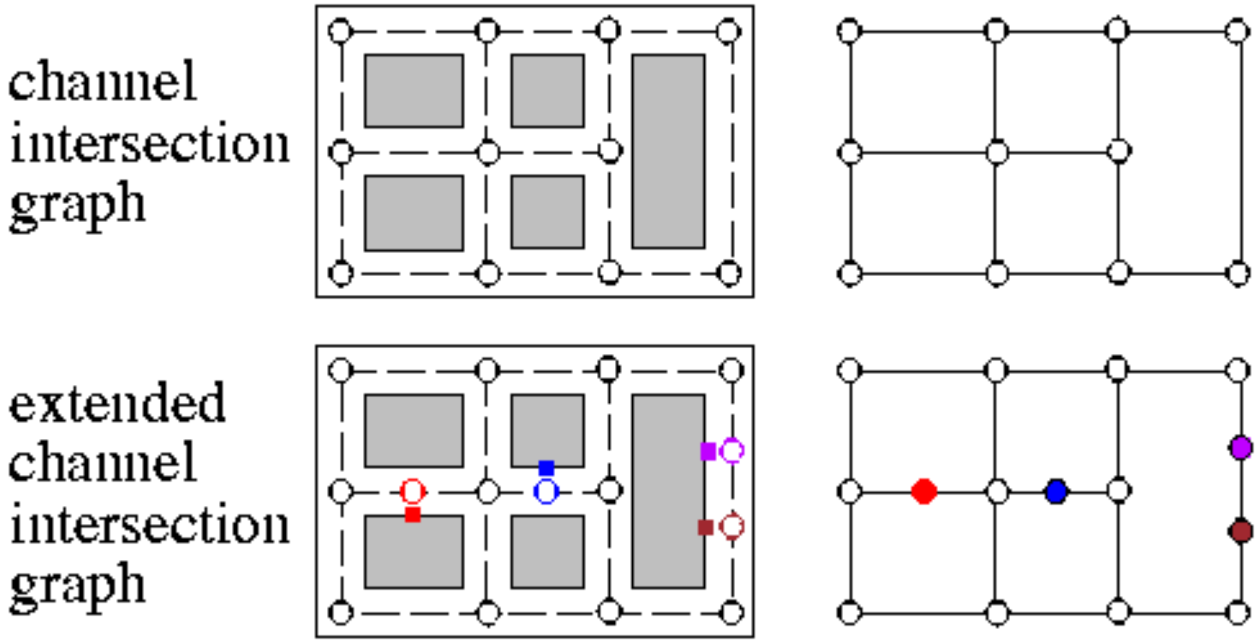
Graph Model: Channel Intersection Graph

- Channels are represented as edges.
- Channel intersections are represented as vertices.
- Edge weight represents channel capacity.
- Extended channel intersection graph: terminals are also represented as vertices.



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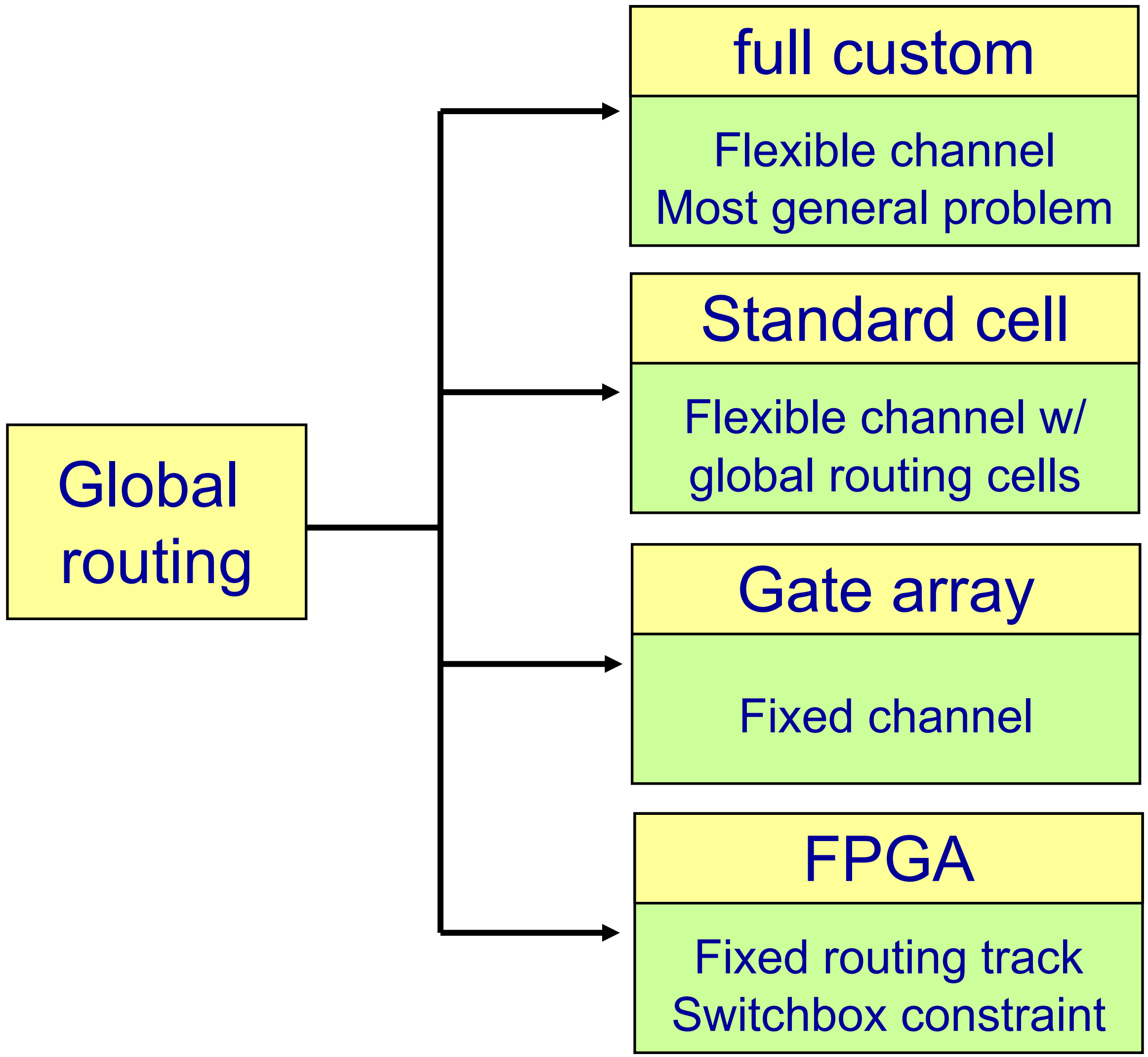
Global-Routing Problem

- Given a netlist N={N_1, N_2, …, N_n }, a routing graph $G = (V, E)$, find a Steiner tree T_i for each net N_i, $1 \le i \le n$, such that $U(e_j) \le c(e_j)$, $\forall\ e_j \in E$ and $\sum_{i=1}^{n} L(T_i)$ is minimized, where
  - $c(e_j)$: capacity of edge e_j
  - x_{ij}=1 if e_j is in T_i; x_{ij} = 0 otherwise
  - $U(e_j) = \sum_{i=1}^{n} x_{ij}$: # of wires that pass through the channel corresponding to edge e_j
  - $L(T_i)$: total wirelength of Steiner tree T_i.

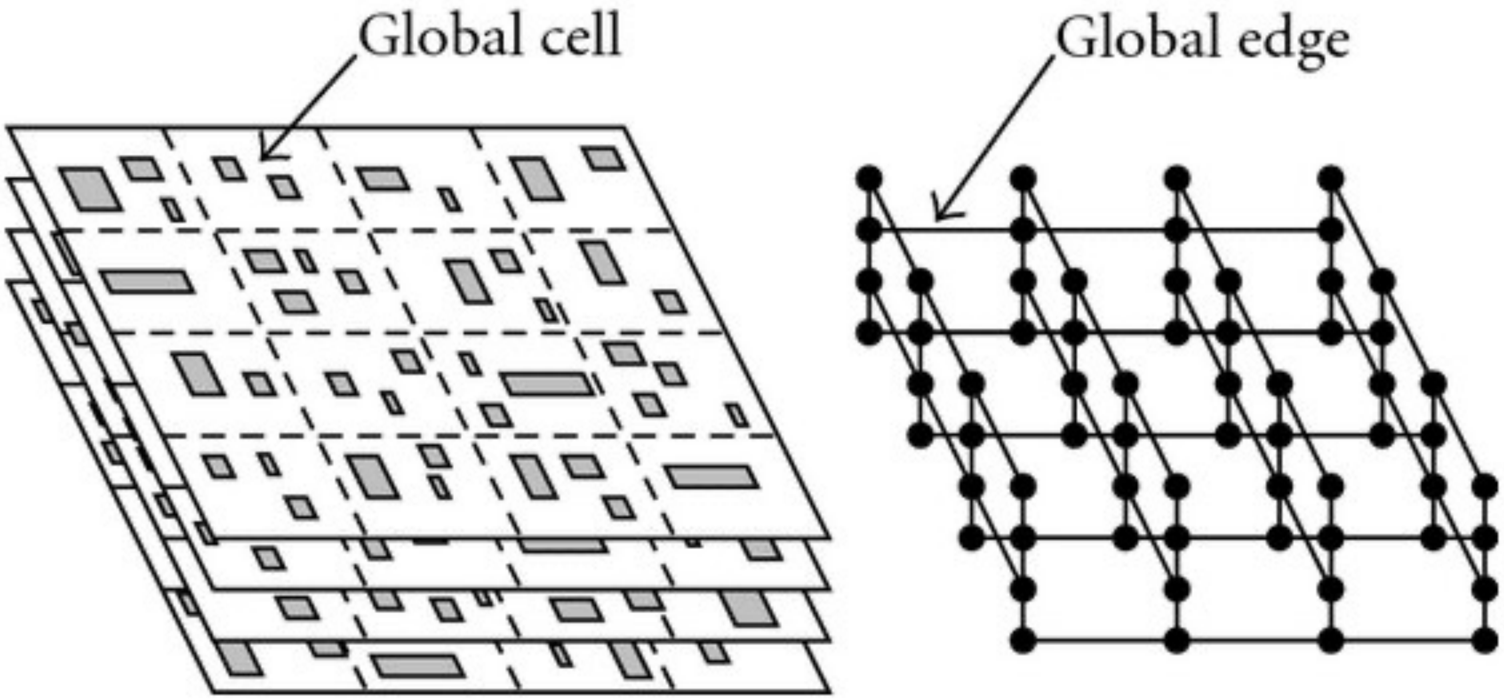
- For high-performance, the maximum wirelength ($\max_{i=1}^{n} L(T_i)$) is minimized (or the longest path between two points in T_i is minimized).

Global Routing in different Design Styles



Global Routing in Cell-based Design

- Objective
  - Minimize the maximum wire length.
- For high performance,
  - Minimize the maximum path length.



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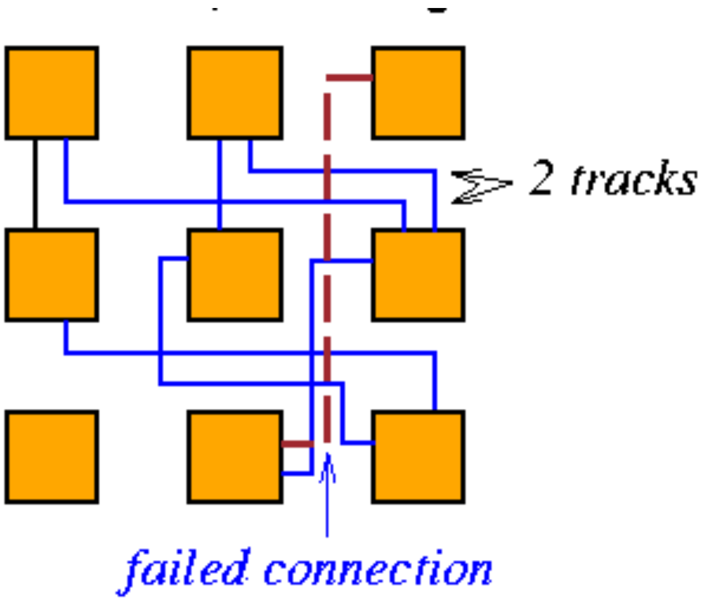
Global Routing in Gate Array

- Objective
  - **Guarantee 100% routability.**
- For high performance,
  - Minimize the maximum wire length.
  - Minimize the maximum path length.

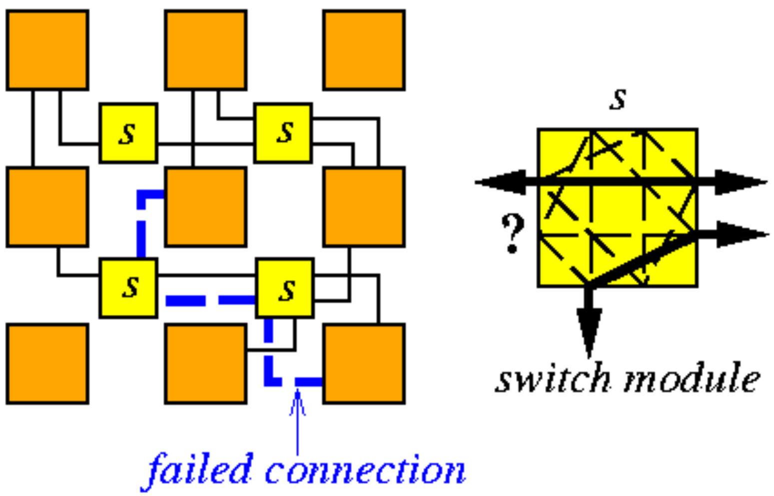


Each channel has a capacity of 2 tracks.

Courtesy of Prof. Y.-W. Chang

Global Routing in FPGA

- Objective
  - Guarantee 100% routability.
  - Consider **switch-module architectural constraints.**
- For performance-driven routing,
  - **Minimize # of switches used.**
  - Minimize the maximum wire length.
  - Minimize the maximum path length.

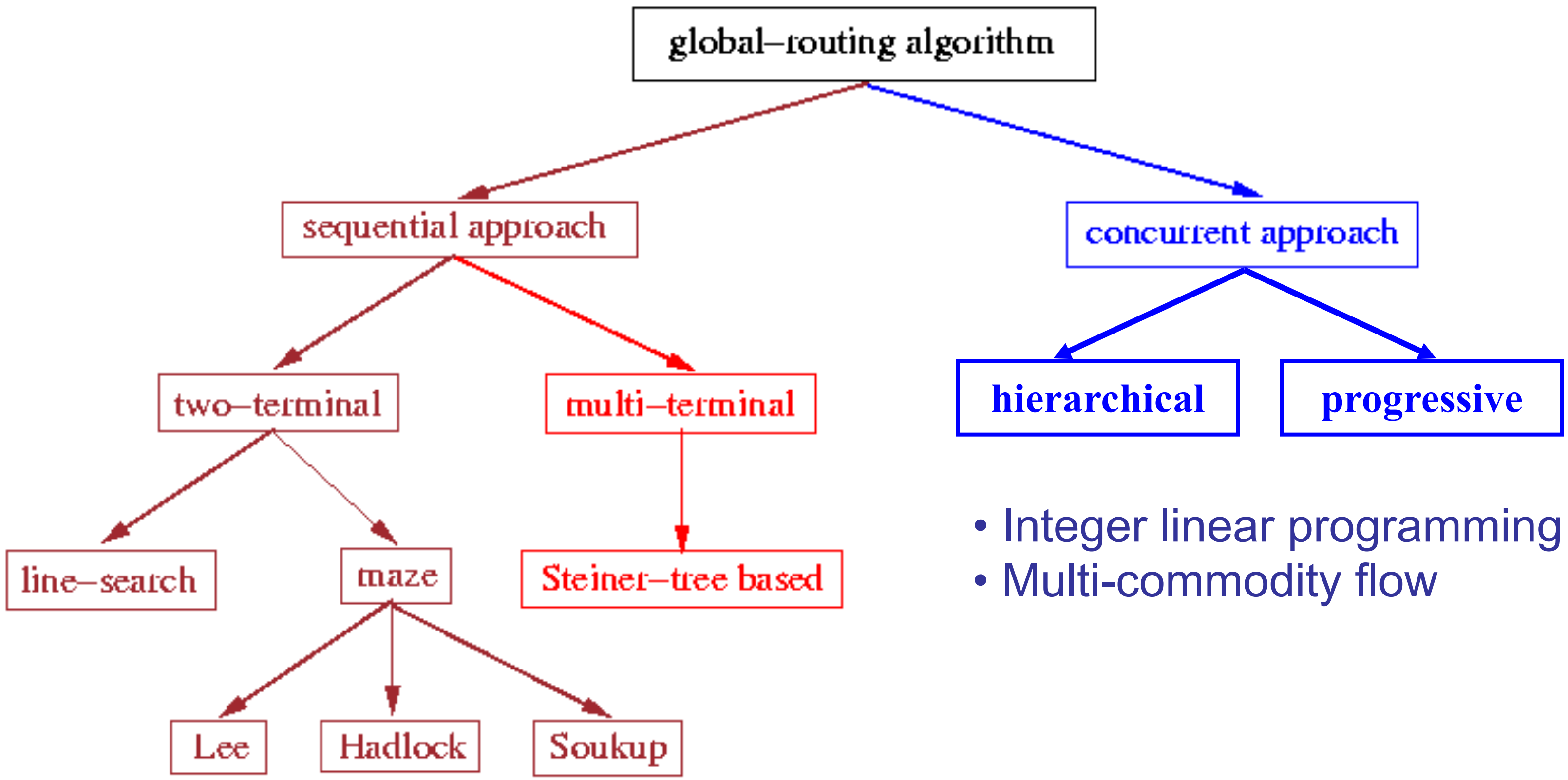


Each channel has a capacity of 2 tracks.

Courtesy of Prof. Y.-W. Chang

Classification of Global-Routing Algorithm

- **Sequential approach:** Assigns priority to nets; routes one net at a time based on its priority (net ordering?).
- **Concurrent approach:** All nets are considered at the same time (complexity?)



- Integer linear programming
- Multi-commodity flow

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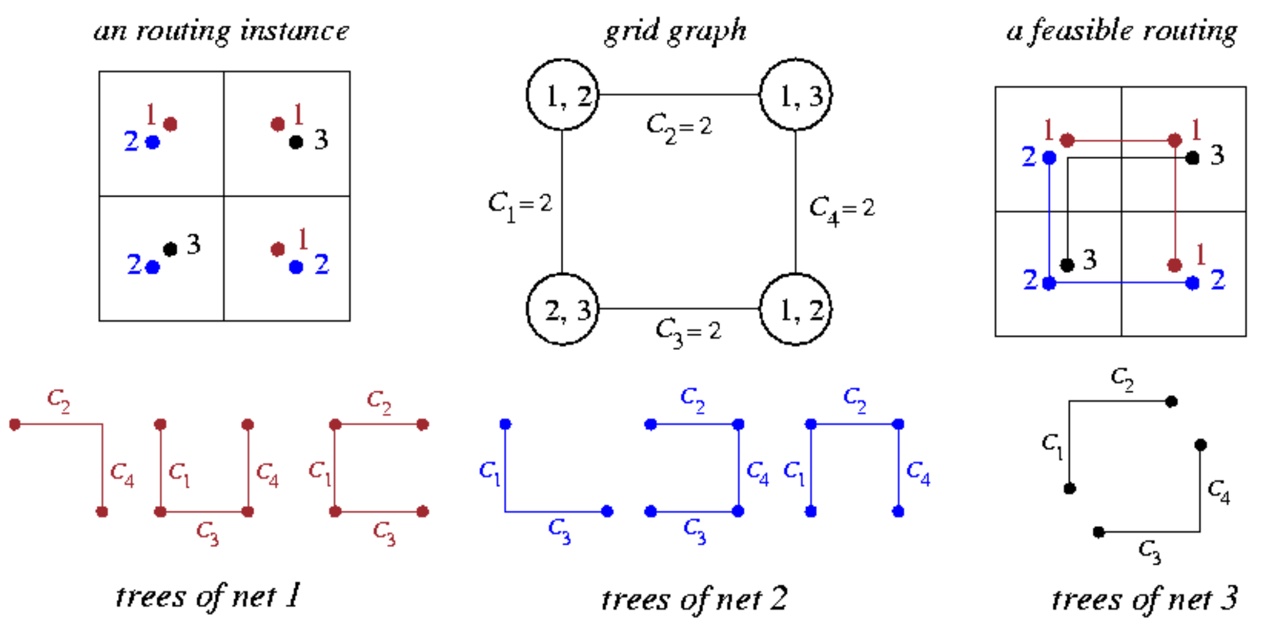
Global Routing by Integer Programming

- Suppose that for each net i, there are n_i possible trees t_1^i, t_2^i, …, t_{ni}^i to route the net.
- Constraint I: For each net i, only one tree t_j^i will be selected.
- Constraint II: The capacity of each cell boundary c_i is not exceeded.
- Minimize the total tree cost.
- **Question:** Feasible for practical problem sizes?
  - **Key:** Hierarchical approach!



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An Integer-Programming Example

Boundary	t_1^1	t_2^1	t_3^1	t_1^2	t_2^2	t_3^2	t_1^3	t_2^3
B1	0	1	1	1	0	1	1	0
B2	1	0	1	0	1	1	1	0
B3	0	1	1	1	1	0	0	1
B4	1	1	0	0	1	1	0	1

- $g_{i,j}$: cost of tree $t_j^i \Rightarrow$ $g_{1,1}$=2, $g_{1,2}$=3, $g_{1,3}$=3, $g_{2,1}$= 2, $g_{2,2}$=3, $g_{2,3}$=3, $g_{3,1}$=2, $g_{3,2}$=2.

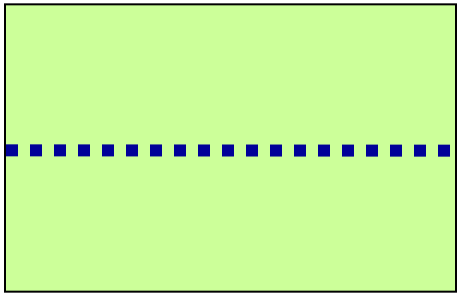
Minimize $2x_{1,1} + 3x_{1,2} + 3x_{1,3} + 2x_{2,1} + 3x_{2,2} + 3x_{2,3} + 2x_{3,1} + 2x_{3,2}$
subject to

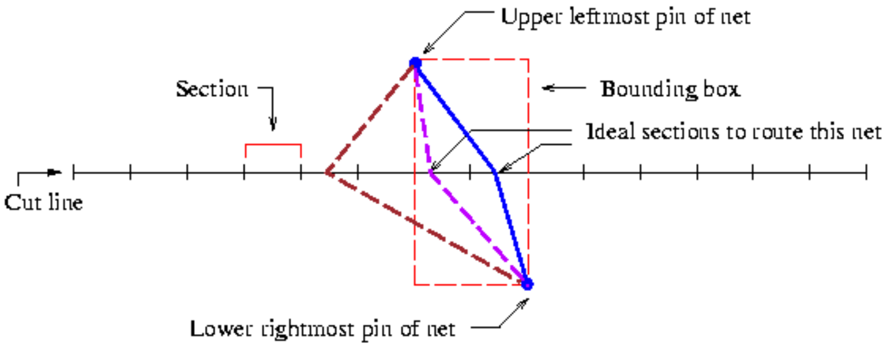
$$
\begin{aligned}
x_{1,1} + x_{1,2} + x_{1,3} &= 1 \quad (Constraint\ I : t^1) \\
x_{2,1} + x_{2,2} + x_{2,3} &= 1 \quad (Constraint\ I : t^2) \\
x_{3,1} + x_{3,2} &= 1 \quad (Constraint\ I : t^3) \\
x_{1,2} + x_{1,3} + x_{2,1} + x_{2,3} + x_{3,1} &\leq 2 \quad (Constraint\ II : B1) \\
x_{1,1} + x_{1,3} + x_{2,2} + x_{2,3} + x_{3,1} &\leq 2 \quad (Constraint\ II : B2) \\
x_{1,2} + x_{1,3} + x_{2,1} + x_{2,2} + x_{3,2} &\leq 2 \quad (Constraint\ II : B3) \\
x_{1,1} + x_{1,2} + x_{2,2} + x_{2,3} + x_{3,2} &\leq 2 \quad (Constraint\ II : B4) \\
x_{i,j} &= 0, 1, 1 \leq i, j \leq 3
\end{aligned}
$$

Courtesy of Prof. Y.-W. Chang

Hierarchical Global Routing

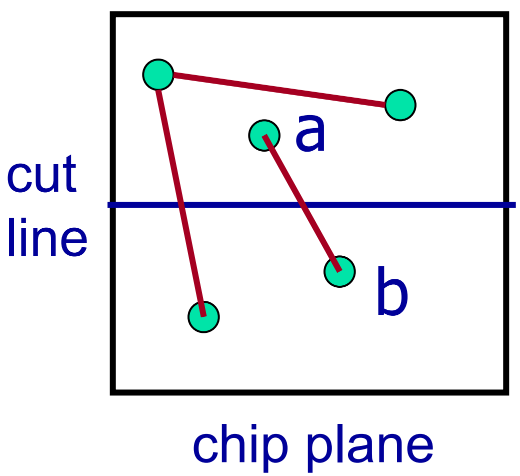
- Marek-Sadowska, “Router planner for custom chip design,” ICCAD-86.
- At each level of the hierarchy, an attempt is made to minimize the cost of nets crossing cut lines.
- At the lowest level of the hierarchy, the layout surface is divided into $R \times R$ grid regions with boundary capacity equal to C tracks.
- Let R_l be the # of grid regions of a given cut line l; a cut line can be divided into $M = R_l / C$ sections.
- Global routing can be formulated as a linear assignment problem:
  - $x_{i,j} = 1$ if net i is assigned to section j; $x_{i,j} = 0$ otherwise.
  - Each net crosses the cut line exactly once: $\sum_{j=1}^{M} x_{ij} = 1, 1 \leq i \leq N.$
  - Capacity constraint of each section: $\sum_{i=1}^{N} x_{ij} \leq C, 1 \leq j \leq M.$
  - w_{ij}: cost of assigning net i to section j. Minimize $\sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij} x_{ij}.$

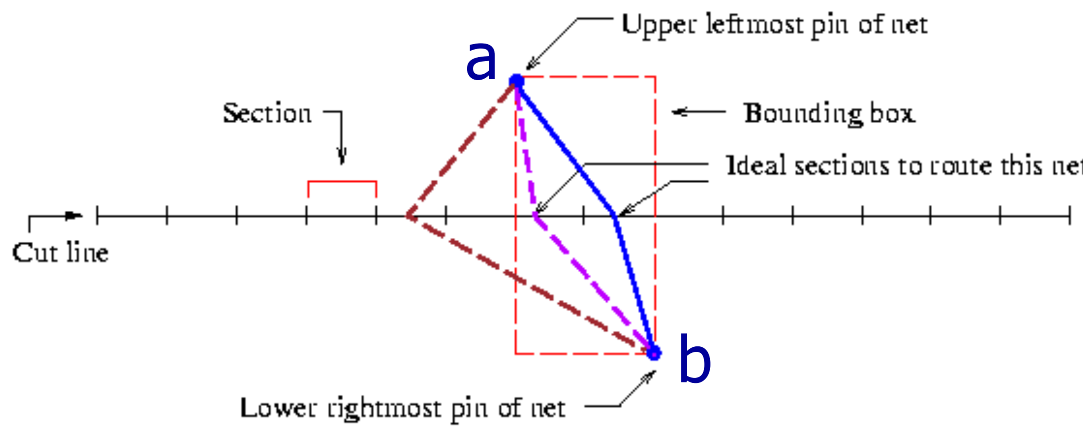




Hierarchical Global Routing (cont'd)

- Global routing can be formulated as a linear assignment problem:
  - $x_{i,j}$ = 1 if net i is assigned to section j; $x_{i,j}$ = 0 otherwise.
  - Each net crosses the cut line exactly once: $\sum_{j=1}^{M} x_{ij} = 1, 1 \leq i \leq N.$
  - Capacity constraint of each section: $\sum_{i=1}^{N} x_{ij} \leq C, 1 \leq j \leq M.$
  - w_{ij}: cost of assigning net i to section j. Minimize $\sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij} x_{ij}.$

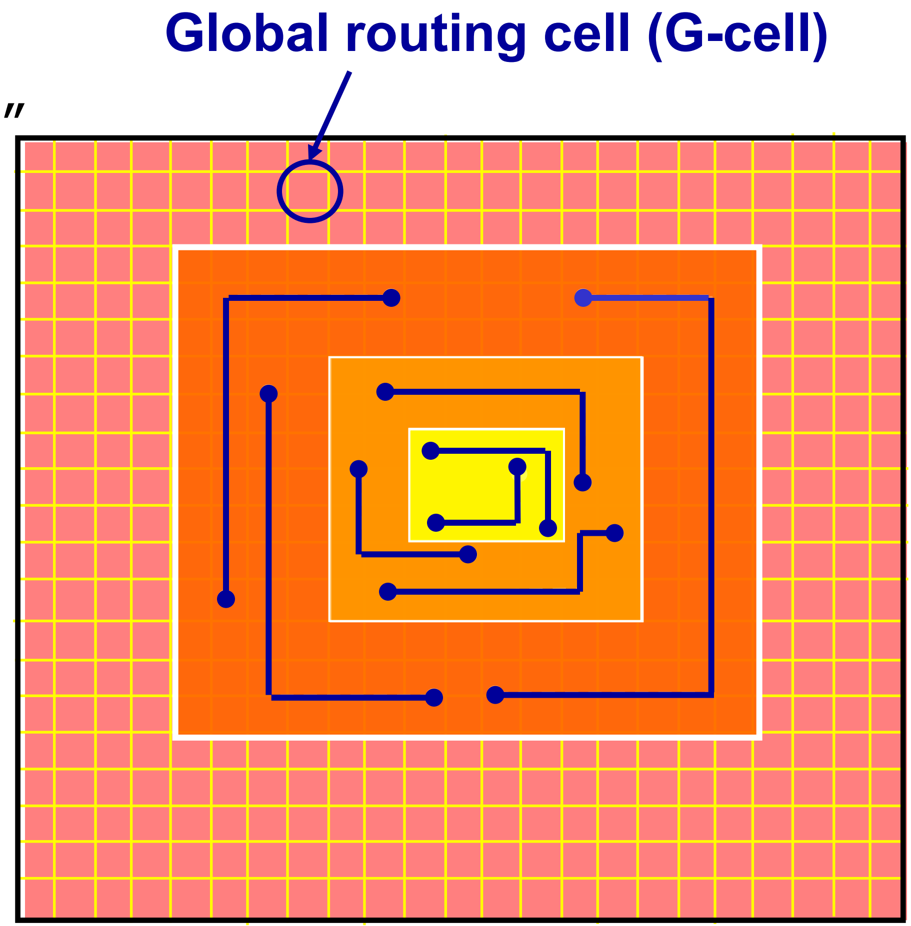




BoxRouter: Progressive Global Routing

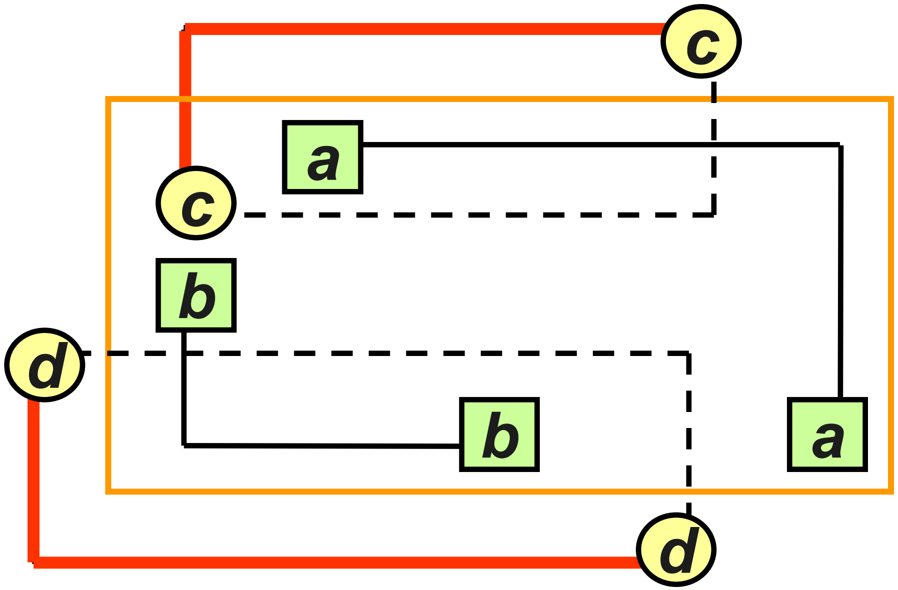
- Cho & Pan, "BoxRouter: A new global router based on box expansion and progressive ILP," DAC-2006.
  - Slides modified from the DAC presentation

1. Incremental box expansion
   - From the most congested region (擒賊先擒王)
2. Progressive ILP
   - With adaptive maze routing
3. Post routing
   - Rerouting w/o rip-up



- Try to minimize the total routing overflow

Motivation for Box Expansion



- Different resource management for wires inside/outside of a box
- Box expansion pushes/diffuses congestion outwards progressively

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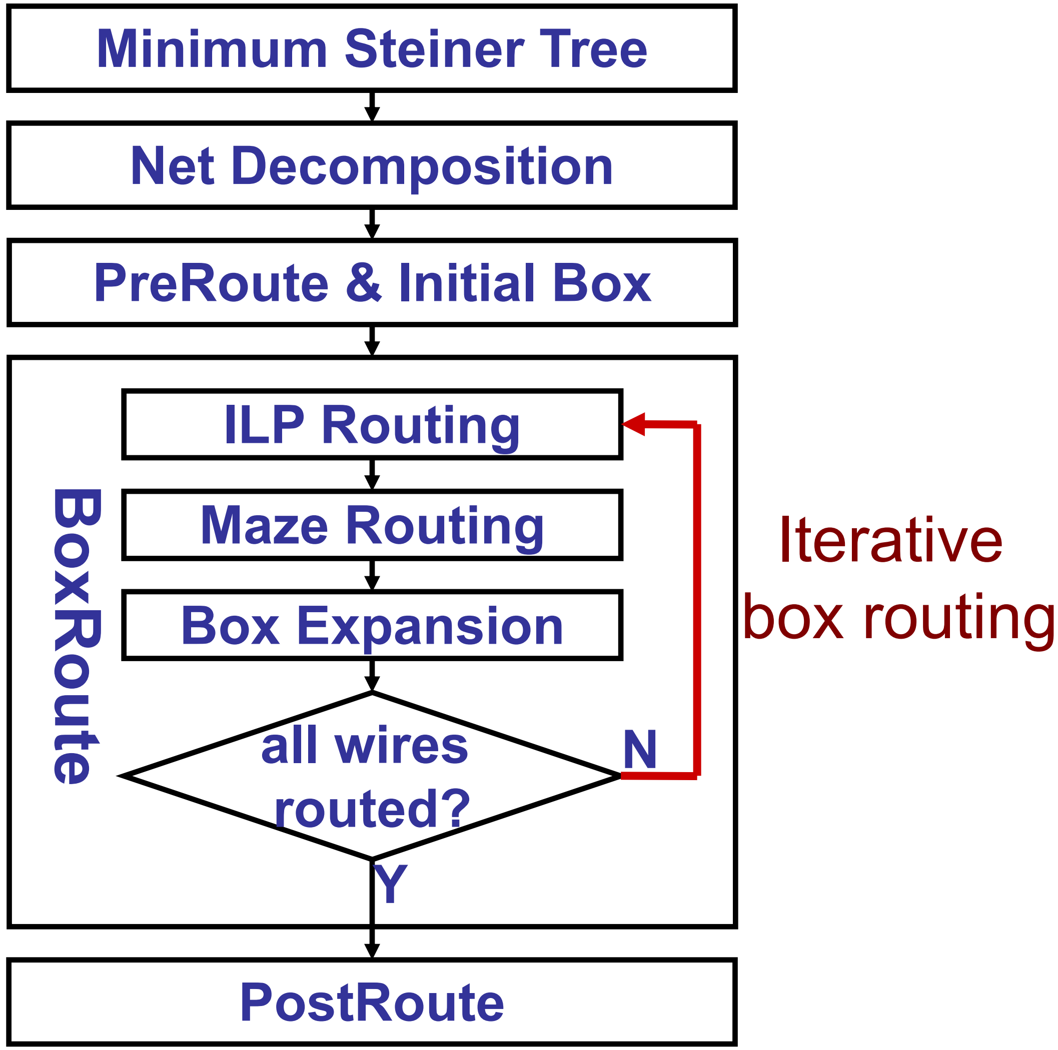
Overall Flow of BoxRouter



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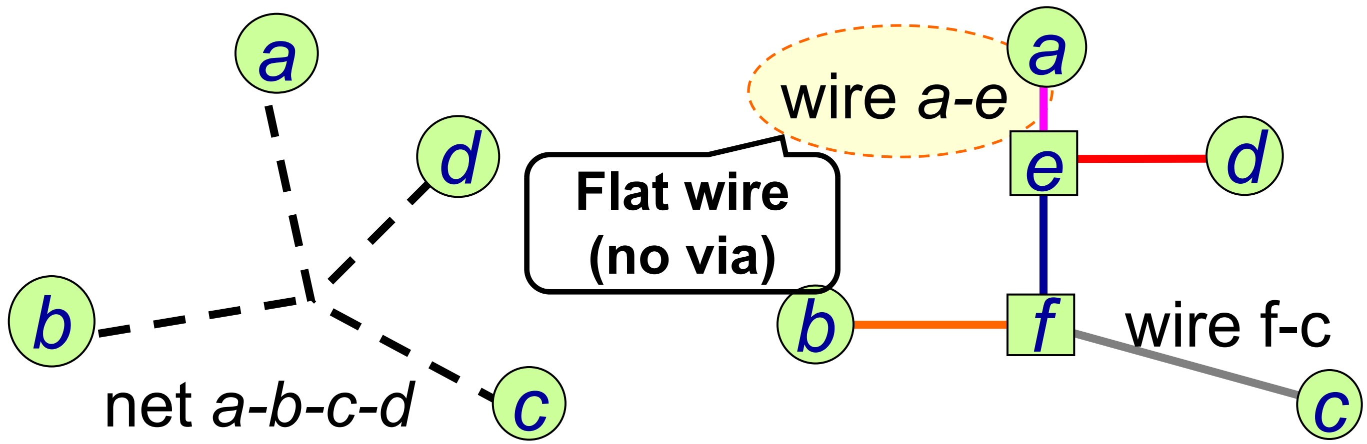
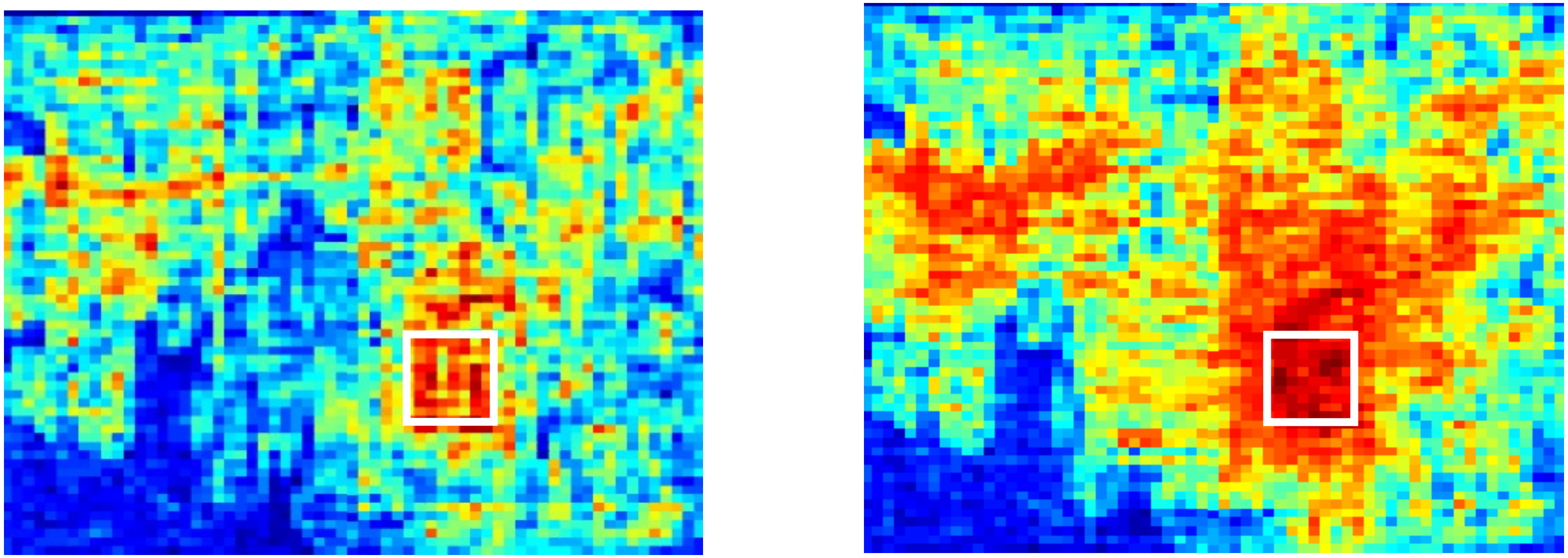
Minimal Steiner Tree & Net Decomposition



- Minimum Steiner Tree
  - FLUTE [ICCAD-04], Spanning graph [ISPD-03], Geosteiner 3.1
  - Decompose into 2-pin wires
  - Each wire becomes an atomic routing object

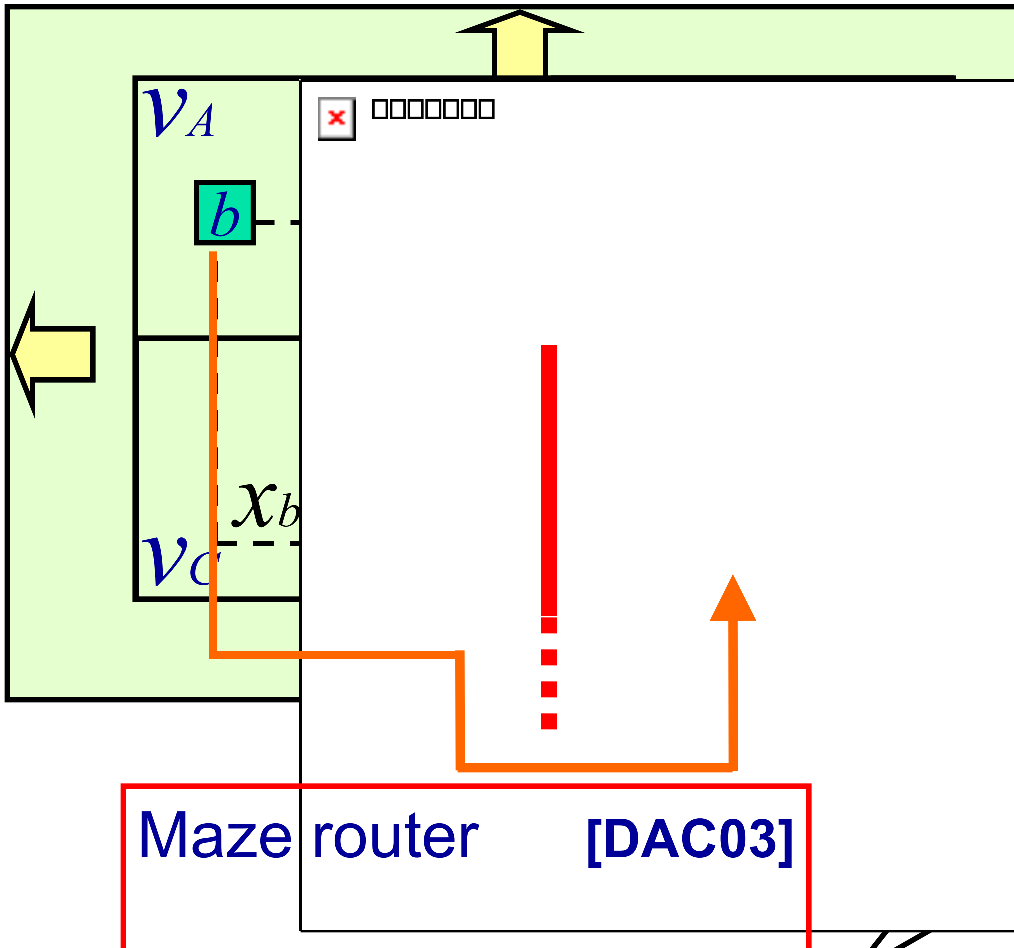
PreRouting

- Preroute as many flat wires as possible
  - 60% of wirelength can be prerouted
- Benefits
  - Overall congestion captured
  - Runtime improved
- Starting “Box” of BoxRouting



Courtesy of Prof. Y.-W. Chang

BoxRouting in Action



$$max: \; x_{b1} + x_{b2} + x_{f1} + x_{f2} + x_{h1} + x_{h2}$$

$$s.t: \; x_{b1}, x_{b2}, x_{f1}, x_{f2}, x_{h1}, x_{h2} \in \{0,1\}$$

$$x_{b1} + x_{f1} + x_{h1} \le C_{AB}$$

$$x_{b1} + x_{h1} \le C_{BD}$$

$$x_{b2} + x_{h2} \le C_{AC}$$

$$x_{b2} + x_{h2} \le C_{CD}$$

routing cap. constraint

$$x_{b1} + x_{b2} \le 1$$

$$x_{f1} \le 1, x_{f2} = 0$$

$$x_{h1} + x_{h2} \le 1$$

solution constraint

$$x_{b1} = x_{b2} = 0$$

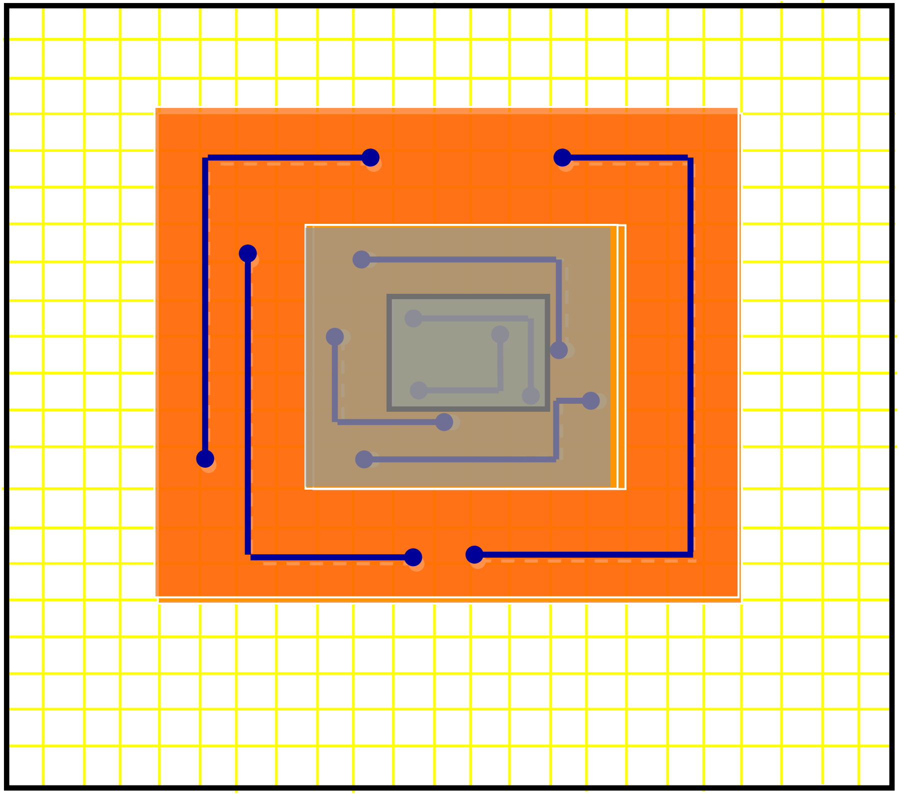
$$x_{f1} = 1$$

$$x_{h1} = 1$$

Courtesy of Prof. Y.-W. Chang

Progressive ILP with Box Expansion

- ILP typically incurs very high time complexity
  - How to reduce the time complexity?
- Key: Progressive ILP
  - Incorporate the solution from inner boxes
  - Solve it between two consecutive boxes



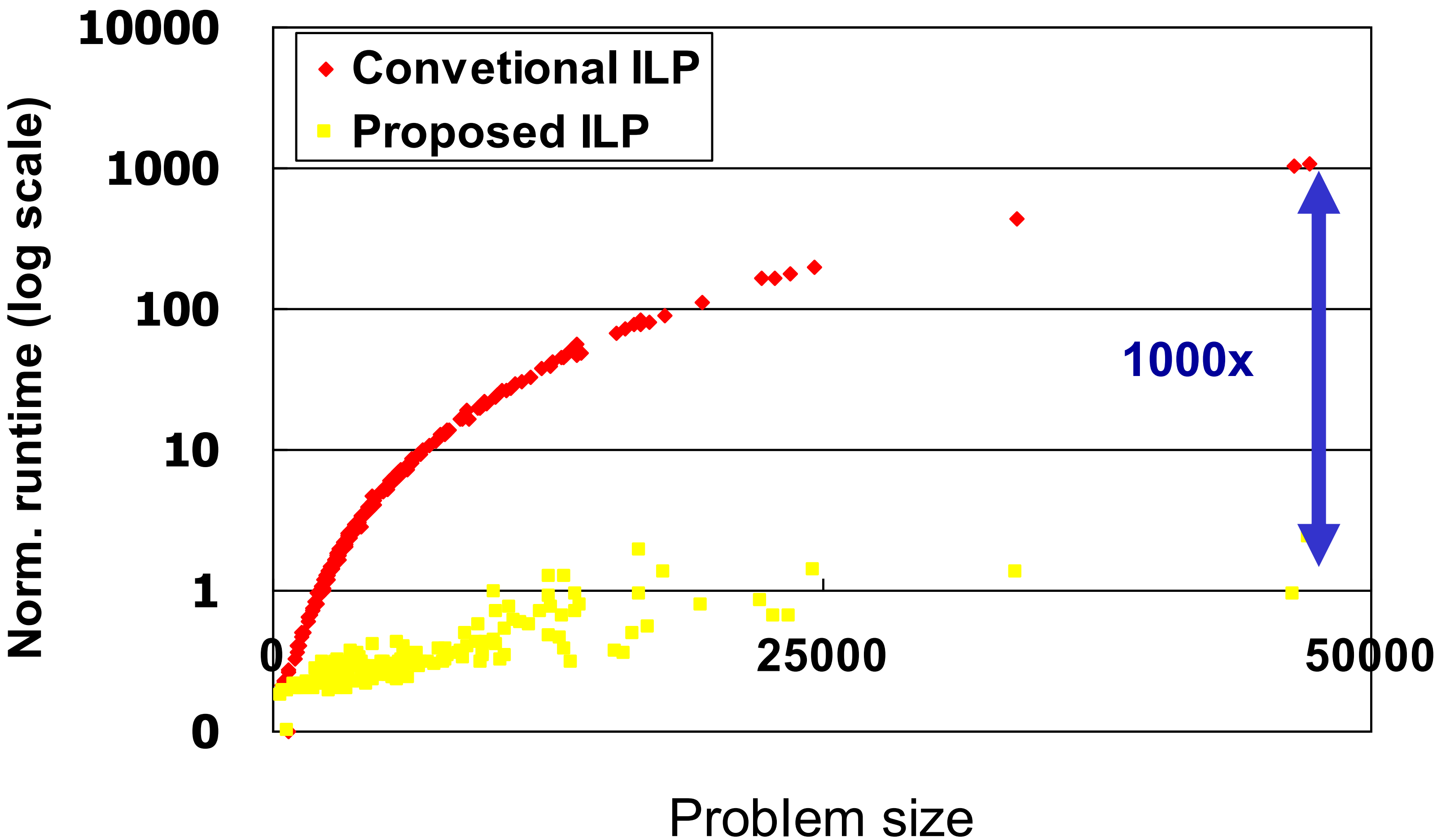
Courtesy of Prof. Y.-W. Chang

Proposed ILP vs. Conventional ILP

Proposed ILP		Conventional ILP	
$max: \sum \{x_{i1} + x_{i2}\}$		$min: D$	$\forall i \in W_{box}$
$s.t: x_{i1}, x_{i2} \in \{0,1\}$	$\forall i \in W_{box}$	$s.t: x_{i1}, x_{i2} \in \{0,1\}$	$\forall i \in W_{box}$
$x_{i1} + x_{i2} \leq 1$	$\forall i \in W_{box}$	$x_{i1} + x_{i2} = 1$	$\forall i \in W_{box}$
$x_{i2} = 0$	$\forall i \in W_{box} \cap W_{flat}$	$x_{i2} = 0$	$\forall i \in W_{box} \cap W_{flat}$
$\sum_{e \in x_{ij}} x_{ij} \leq C_e$	$\forall i \in W_{box}$	$\sum_{e \in x_{ij}} x_{ij} \leq D$	$\forall i \in W_{box}$

- Proposed ILP formulation
  - Maximize # routed nets (min. total overflow)
  - Work together with maze router
- Conventional ILP formulation
  - Minimize max overflow [ICCAD-01]

Proposed ILP vs. Conventional ILP



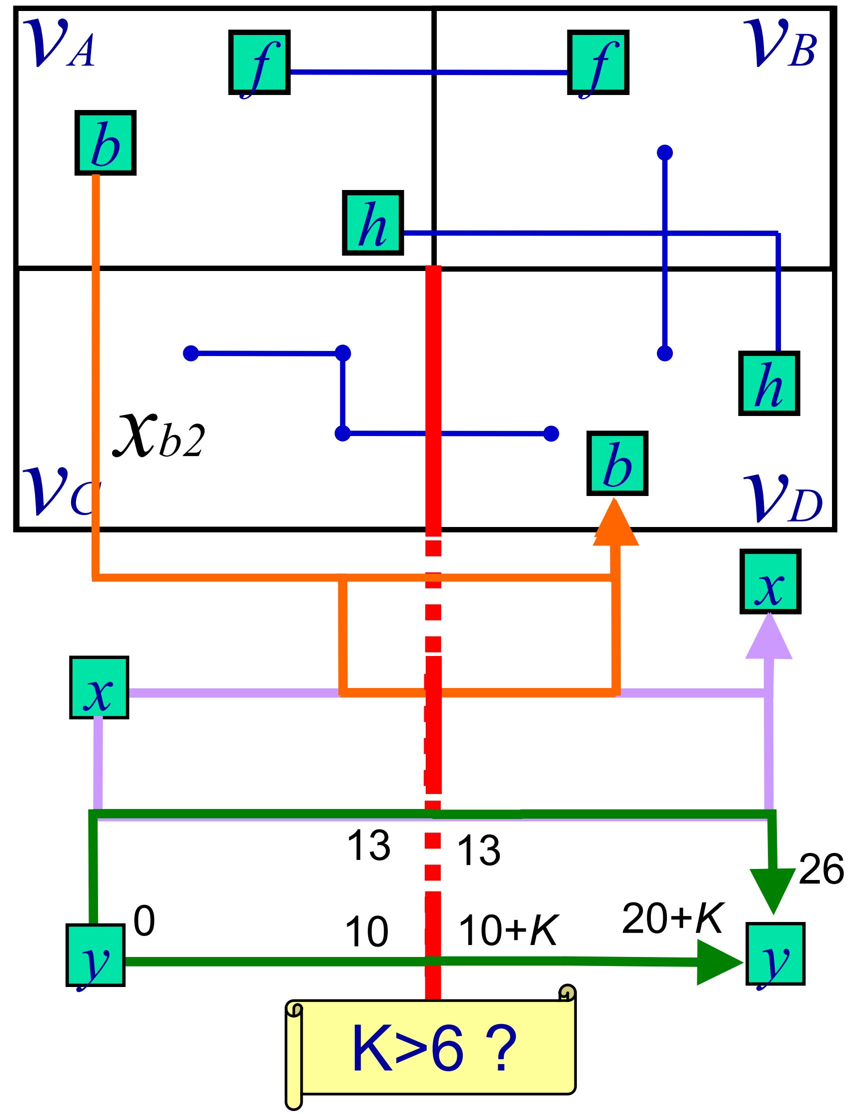
GNU Linear Programming Kit 4.10, Industrial circuit (7mm x 7mm)

Courtesy of Prof. Y.-W. Chang

PostRouting

- Reroute w/o rip-up other nets
  - In the same order as box expansion
  - Enhance routing path

- Smooth tradeoff
  - Between wirelength and congestion
  - Controlled by a parameter *K*



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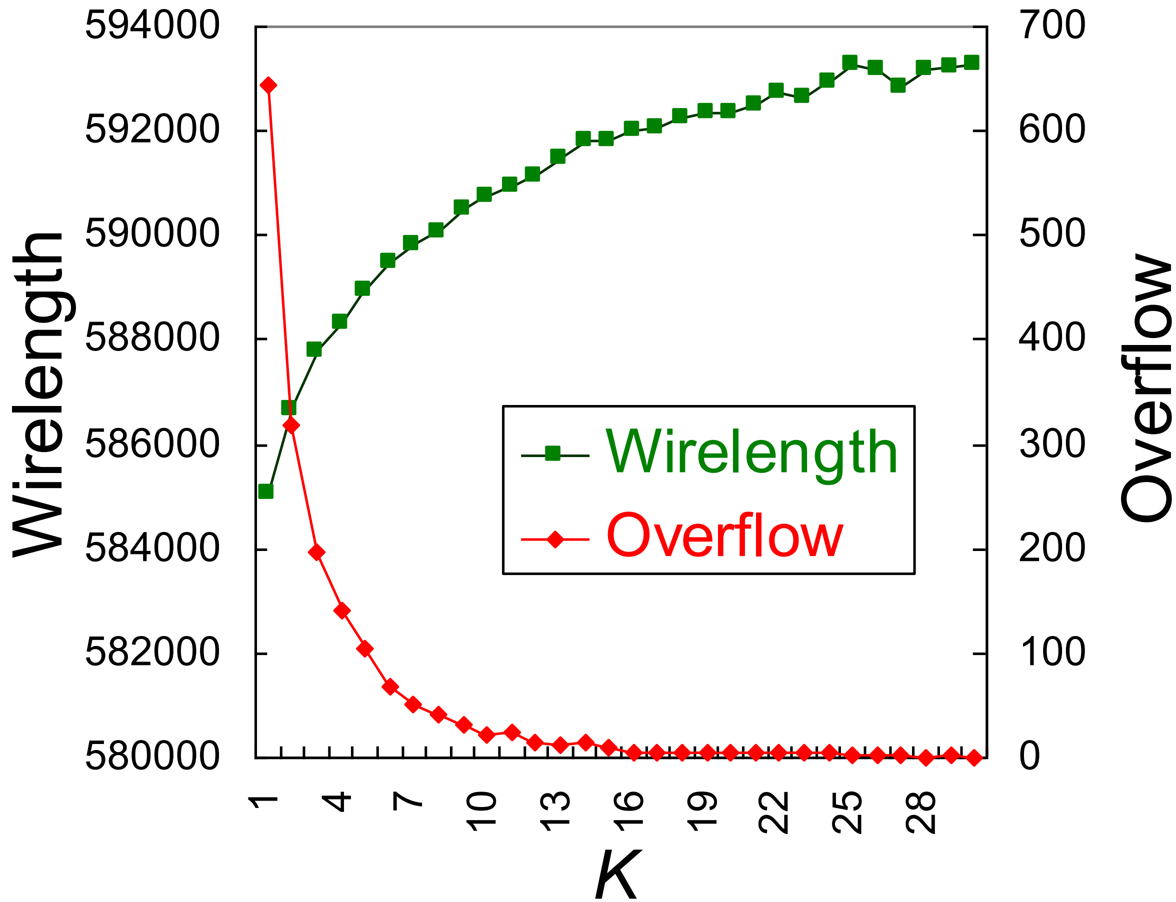
PostRoute



- Smooth tradeoff: *K* ↑ => overflow↓, wirelength↑

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INR (Iterative Negotiation-based Rip-up/rerouting)

- Proposed in **PathFinder** [McMurchie and Ebeling, FPGA'95] to reduce overflows
- Spreads the congested wires iteratively
- At the (i)-th iteration, the cost of a global edge *e*:

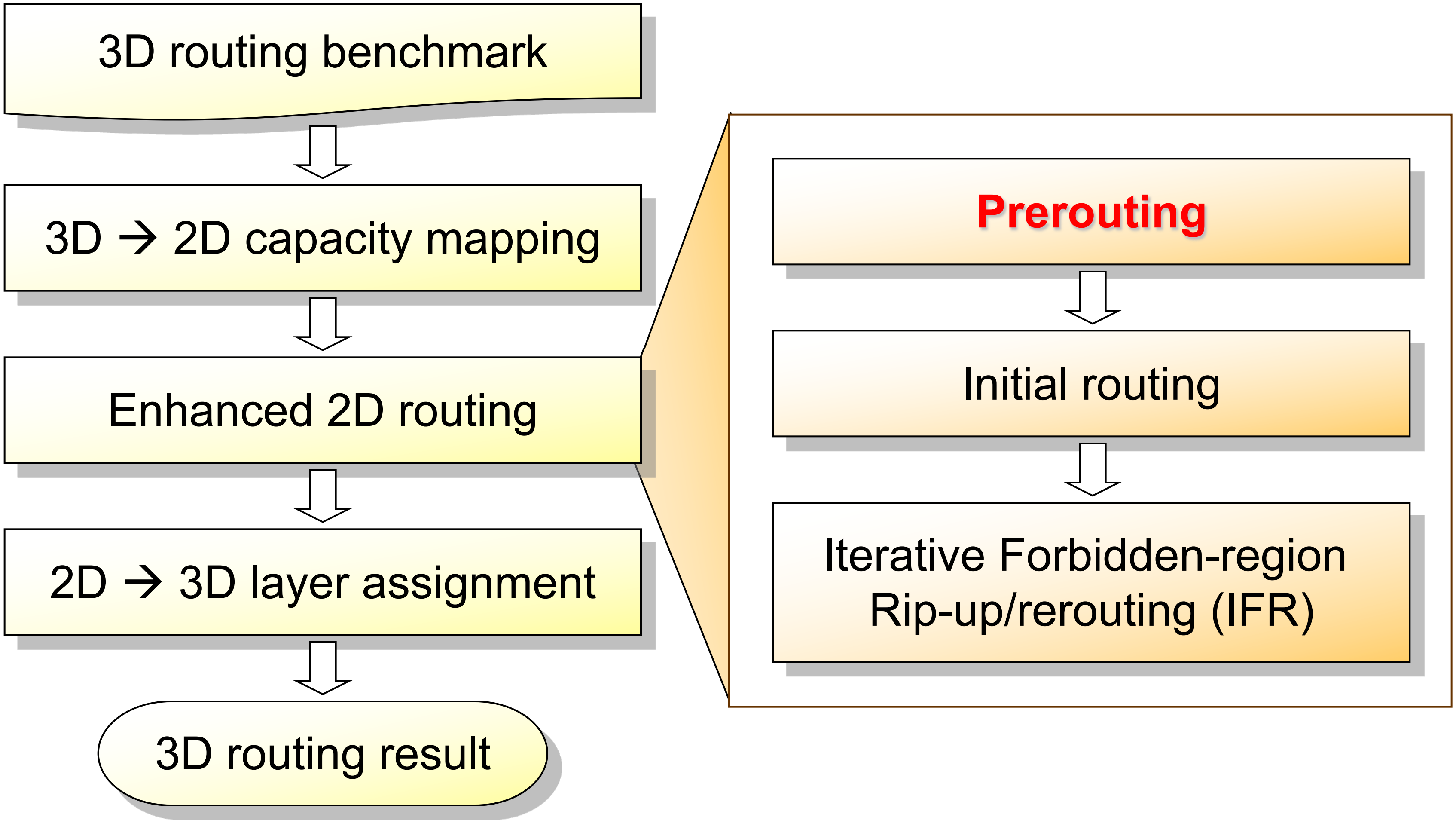
$$(b_e + h_e^{(i)}) \cdot p_e$$

  - b_e: base cost of using *e*,
  - p_e: # of nets passing *e*,
  - $h_e^{(i)}$: historical cost on *e*, $h_e^{(i)} = \begin{cases} h_e^{(i-1)} + 1 & \text{if } e \text{ has overflow} \\ h_e^{(i-1)} & \text{otherwise} \end{cases}$

- Also used by recent academic global routers, Archer [ICCAD'07], BoxRouter [ICCAD'07], FastRoute [ICCAD'06, ASPDAC'07, TCAD'08], FGR [ICCAD'07], NTHU-Route [ASPDAC'08, ICCAD'08]
- INR may get stuck as the number of iterations increases [Ozdal, ICCAD'07] [Gao et al., ASPDAC'08]

NTUgr Global Routing Flow

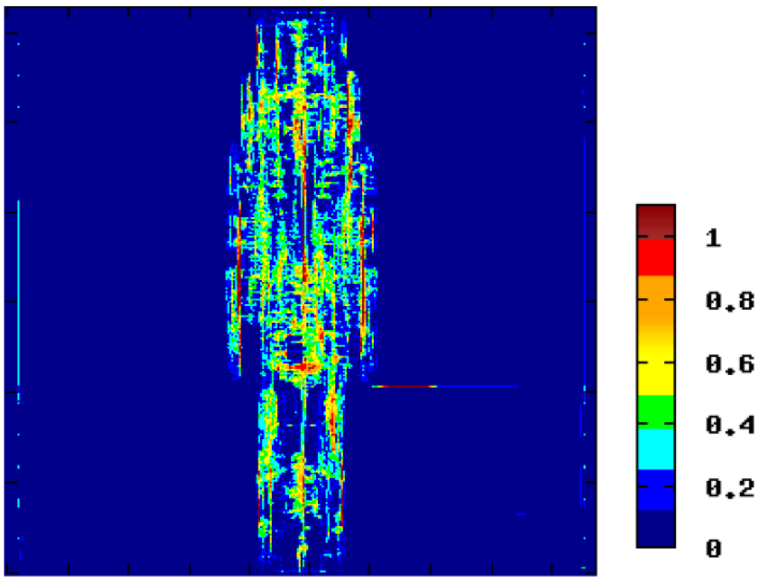
- Hsu, Chen & Chang, ICCAD-08, ASP-DAC-09, TCAD-10



Prerouting

1. Congestion-hotspot historical cost pre-increment
   - Identify the high-pin-density tiles (#pin exceeds total tile capacity)
   - Increase the historical cost lying around these tiles by 10
     - To avoid other nets passing through these congested tiles
2. Small bounding-box area routing
   - Route the less-flexibility nets with smaller bounding-box area

Prerouting of newblue3 (49.22% routed nets, 74374 overflows)



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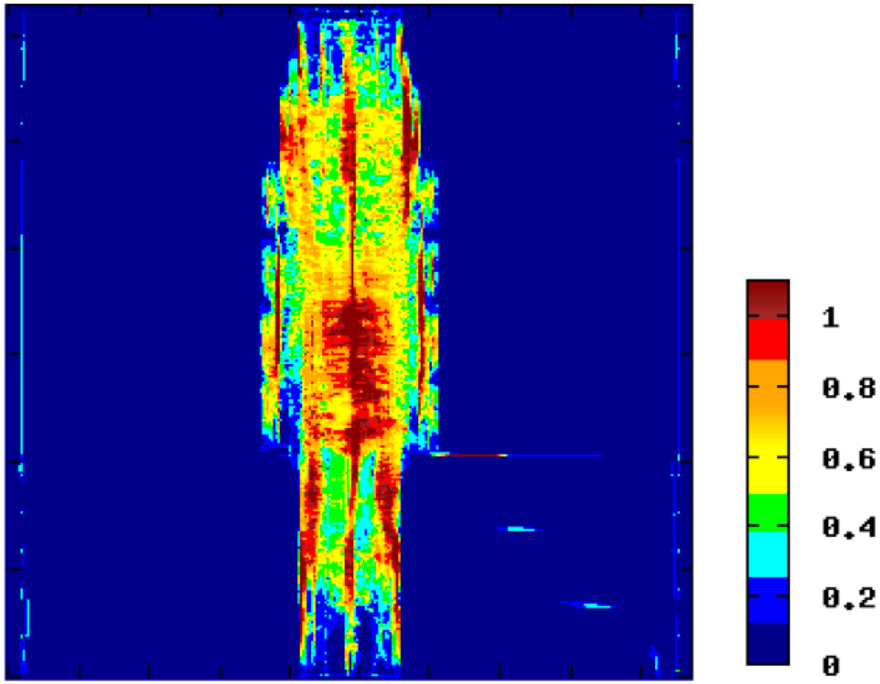
Initial Routing

- The first stage completing all nets in the whole chip
- Apply **iterative monotonic routing** until the overflow improvement is less than 5% of the previous iteration

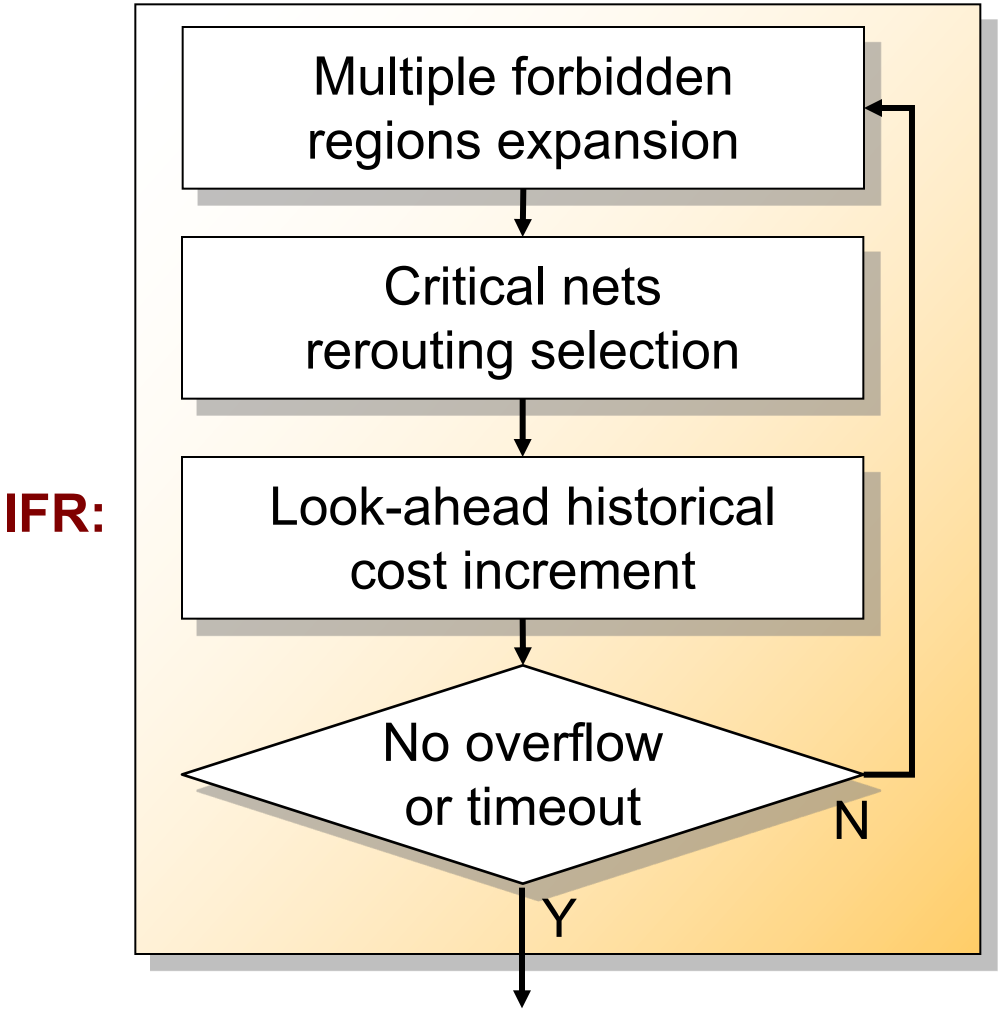
Initial routing of newblue3 (100% routed nets, 306082 overflows)



Courtesy of Prof. Y.-W. Chang

Iterative Forbidden-region Rip-up/rerouting (IFR)

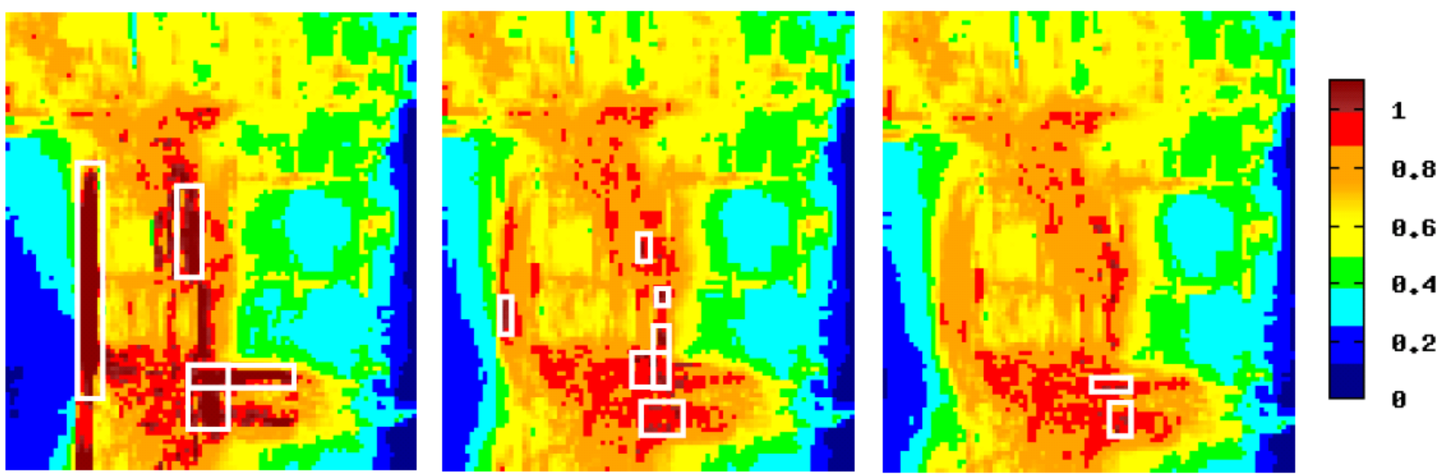
- An enhanced flow over the traditional INR
- Perform iteratively until no overflow or timeout



Courtesy of Prof. Y.-W. Chang

Multiple Forbidden-Regions Construction

- At each iteration of IFR, new **forbidden regions** are constructed from the most congested regions
  - Initially contains two adjacent tiles w.r.t. the most congested edge
  - Expand the region until the average congestion of each boundary is smaller than a threshold (overlap is allowed)
- Apply a special cost metric for nets in forbidden regions
  - Introducing new overflows within these regions is almost forbidden by incurring a large penalty



(a) (b) (c)

Forbidden-region routing of adaptec5

Cost Considering Forbidden Regions

- The cost function of a global edge *e*:

$$\text{cost}(e) = \begin{cases} P_n \cdot (d_e / c_e) & \text{if } e \in \text{forbidden regions and is congested} \\ b_e \cdot (1 + h_e^{(i)}) & \text{otherwise} \end{cases}$$

P_n : forbidden region penalty (=1000 in NTUgr)

d_e : routing demand of e

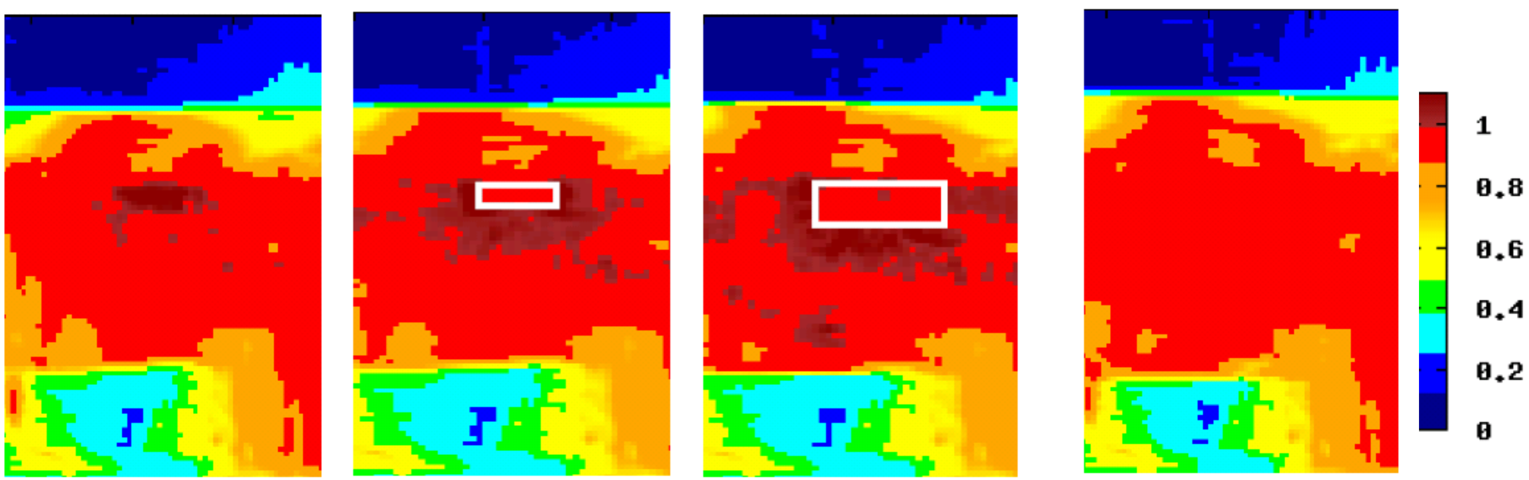
c_e : routing capacity of e

$h_e^{(i)}$: historical cost of e

$$b_e = \begin{cases} 1/(c_e - d_e) & \text{if } d_e < c_e \\ \xi & \text{if } d_e = c_e \quad \text{(penalized base cost of } e\text{)} \\ \xi + d_e / c_e & \text{if } d_e > c_e \quad (\xi = 3 \text{ in NTUgr}) \end{cases}$$

Region Propagation Leveling

- Applied when # of overflows stops decreasing (get stuck at the local optima)
  - Stop creating new forbidden regions
  - Expand all forbidden regions at the previous iteration simultaneously



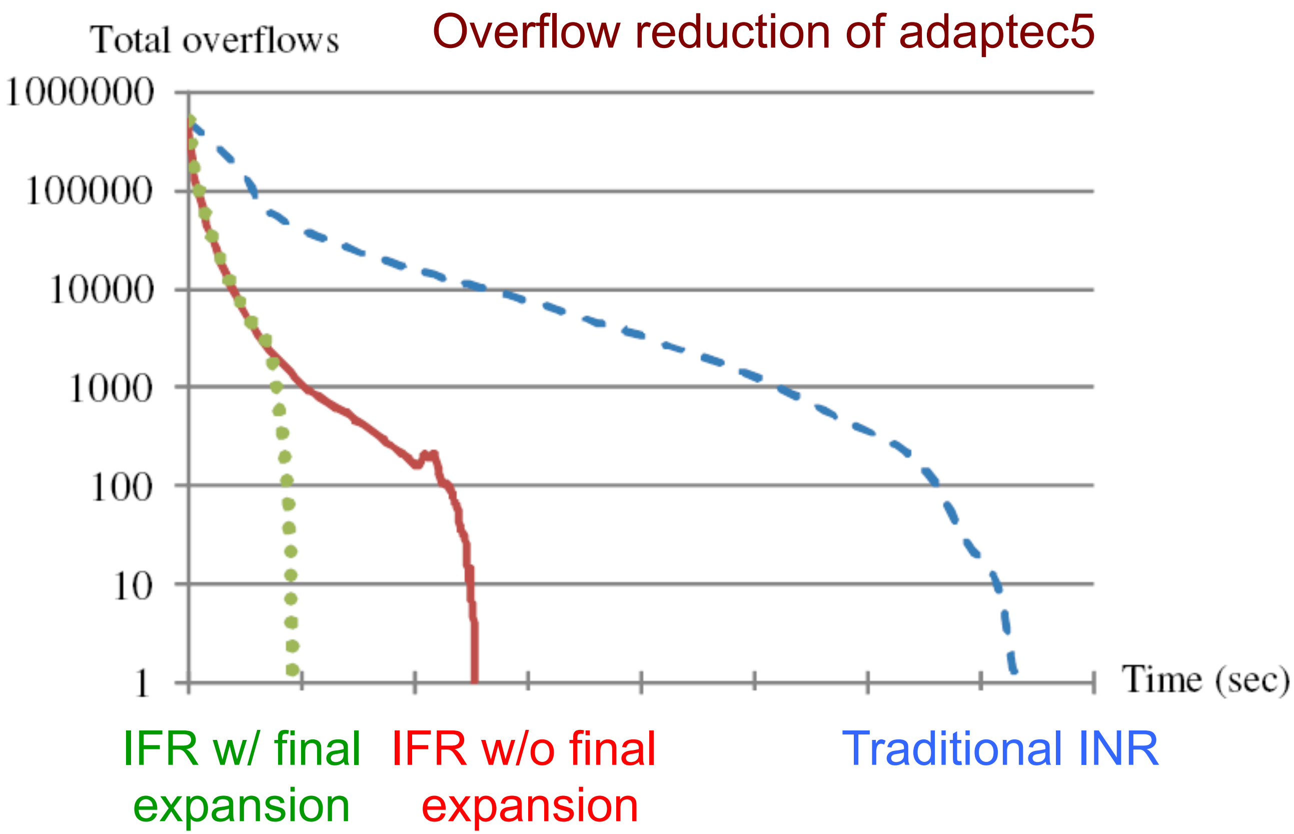
(i)-th iteration (i+1)-th iteration (i+2)-th iteration final iteration

Forbidden-region routing of bigblue3

Courtesy of Prof. Y.-W. Chang

Final Expansion of Forbidden Regions

- Applied when # of overflows < 0.5% of initial overflow
- Expand the forbidden region to the whole routing graph to quickly reduce the remaining overflows



Courtesy of Prof. Y.-W. Chang

Comparisons of Congested Region Handling

	BoxRouter	NTHU-Route 1.0	NTUgr (Ours)
Terminology	Box	Congested region	Forbidden region
Shape	Rectangular	Rectangular	Rectilinear
# of regions	Single box	Single region	Multiple regions
Objective	Performing progressive ILP	Selecting rerouting nets	Performing different cost functions
Simultaneous expansion	No	No	Yes

Courtesy of Prof. Y.-W. Chang

Critical Nets Rerouting Selection

- To speed up the rip-up/rerouting process
- Only rip-up/reroute the **critical nets** in each iteration
- The critical nets are those nets with overflows or small remaining capacity:

$$\min_{e \in n}\{c_e - d_e\} \leq 1$$

c_e : routing capacity of e

d_e : routing demand of e

Courtesy of Prof. Y.-W. Chang

Look-Ahead Historical Cost Increment

- For the **near-overflow global edges** (those edges would have overflow if more *N* demands are added), increase their historical cost in advance

$$h_e^{(i)} = \begin{cases} h_e^{(i-1)} + 1 & \text{if } d_e + N > c_e \ (e \text{ is near-overflow}) \\ h_e^{(i-1)} & \text{otherwise} \end{cases}$$

$N \geq 1$: The look-ahead historical-cost update scheme

- Setting $N = 1$ in NTUgr results in better quality and with about 2x runtime speedup

Results on ISPD'08 Benchmarks

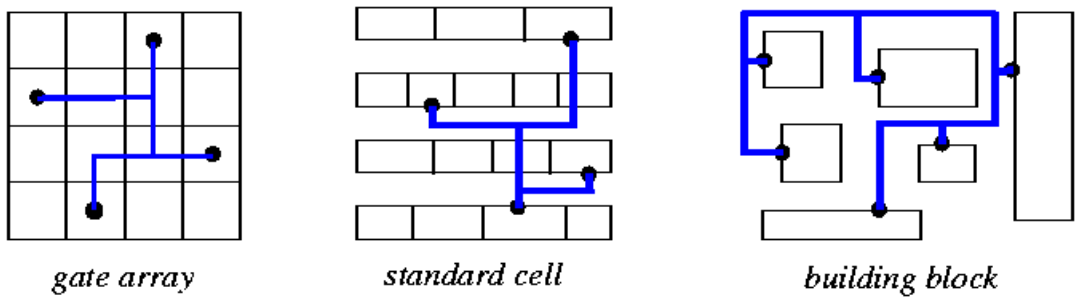
- Compared with the winners of ISPD'08 global routing contest
  - Runtime is averagely the same as that of NTHU-Route 2.0
  - Overflow is better than FastRoute 3.0

Circuit	NTHU-Route 2.0				FastRoute 3.0				NTUgr (Ours)			
	Overflow	Max Overflow	ISPD'08 WL (e^5)	CPU (min)	Overflow	Max Overflow	ISPD'08 WL (e^5)	CPU (min)	Overflow	Max Overflow	ISPD'08 WL (e^5)	CPU (min)
adaptec1	0	0	53.5	7.9	0	0	55.5	1.8	0	0	57.4	4.5
adaptec2	0	0	52.3	1.7	0	0	53.1	0.4	0	0	53.7	1.1
adaptec3	0	0	131.1	8.0	0	0	133.3	1.7	0	0	135.0	4.4
adaptec4	0	0	121.7	2.3	0	0	122.2	0.6	0	0	123.7	1.2
adaptec5	0	0	155.6	17.2	0	0	160.9	4.7	0	0	159.9	15.3
bigblue1	0	0	56.3	9.8	0	0	58.3	3.9	0	0	60.0	18.1
bigblue2	0	0	90.6	9.9	142	4	98.2	11.2	0	0	91.2	248.3
bigblue3	0	0	130.8	4.3	0	0	131.7	3.1	0	0	133.5	4.0
bigblue4	182	2	230.8	125.6	206	2	243.4	41.3	188	8	242.8	413.1
newblue1	0	0	46.5	5.1	76	2	49.0	12.9	6	2	49.3	977.5
newblue2	0	0	75.7	1.1	0	0	76.3	0.3	0	0	76.9	0.6
newblue3	31454	204	106.5	129.1	31650	734	109.3	31.5	31024	408	188.3	884.0
newblue4	152	2	129.9	67.1	226	4	135.7	9.9	142	2	143.8	1118.1
newblue5	0	0	231.7	14.2	0	0	241.2	5.5	0	0	244.9	20.5
newblue6	0	0	177.0	13.6	0	0	186.6	4.0	0	0	186.6	21.3
newblue7	68	2	353.6	140.6	588	6	358.6	189.9	310	2	372.2	1445.5
Comp.	-	-	1.00	3.48	-	-	1.03	1.00	-	-	1.04	3.01

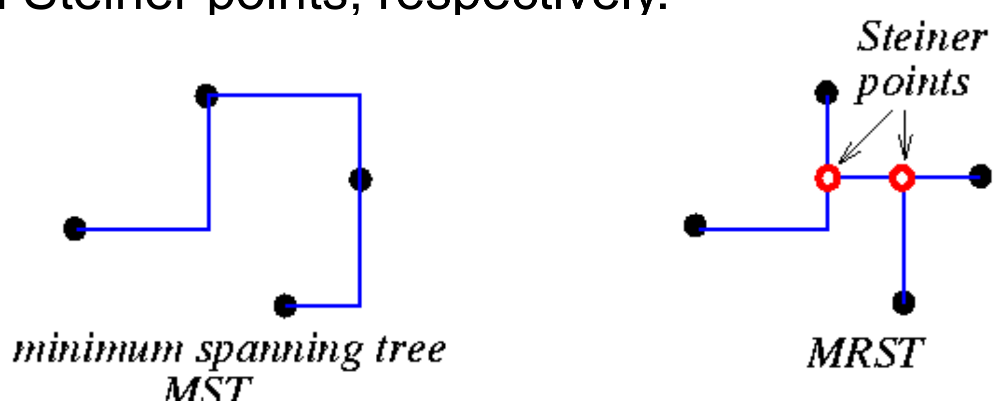
Courtesy of Prof. Y.-W. Chang

The Routing-Tree Problem

- **Problem:** Given a set of pins of a net, interconnect the pins by a "routing tree."



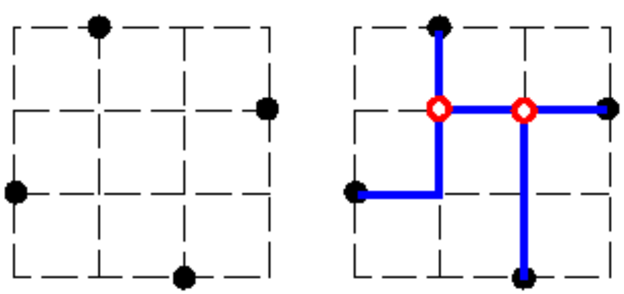
- **Minimum Rectilinear Steiner Tree (MRST) Problem:** Given *n* points in the plane, find a minimum-length tree of rectilinear edges which connects the points.
- $MRST(P) = MST(P \cup S)$, where P and S are the sets of original points and Steiner points, respectively.



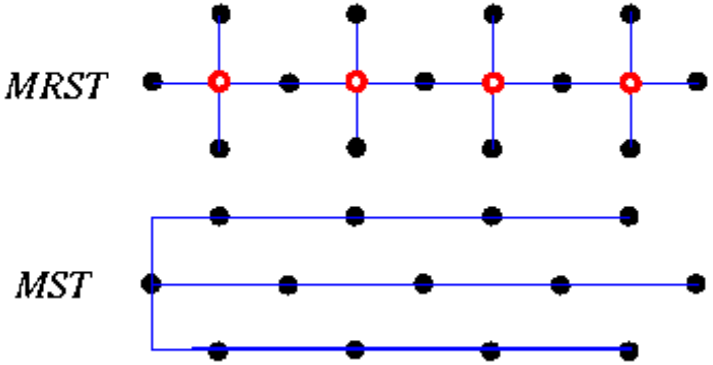
Courtesy of Prof. Y.-W. Chang

Theoretic Results for the MRST Problem

- **Hanan's Thm:** There exists an MRST with all Steiner points (set *S*) chosen from the intersection points of horizontal and vertical lines drawn points of *P*.
  - Hanan, "On Steiner's problem with rectilinear distance," *SIAM J. Applied Math.*, 1966.
- **Hwang's Theorem:** For any point set *P*, $\frac{Cost(MST(P))}{Cost(MRST(P))} \leq \frac{3}{2}$.
  - Hwang, "On Steiner minimal tree with rectilinear distance," *SIAM J. Applied Math.*, 1976.
- Better approximation algorithm with the performance bound 61/48
  - Foessmeier *et al*, "Fast approximation algorithm for the rectilinear Steiner problem," Wilhelm Schickard-Institut für Informatik, TR WSI-93-14, 93.



Hanan grid

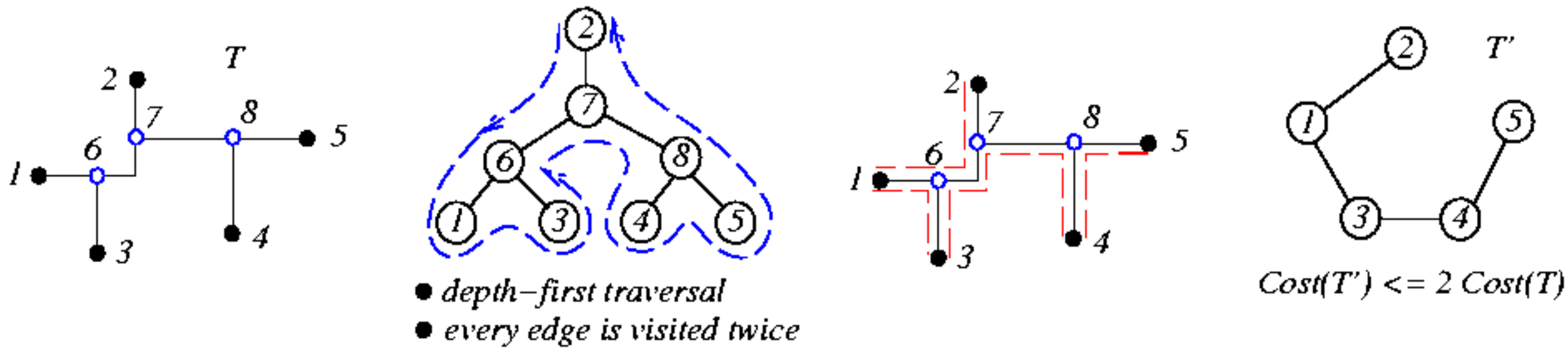


Cost(MST)/Cost(MRST) –> 3/2

Courtesy of Prof. Y.-W. Chang

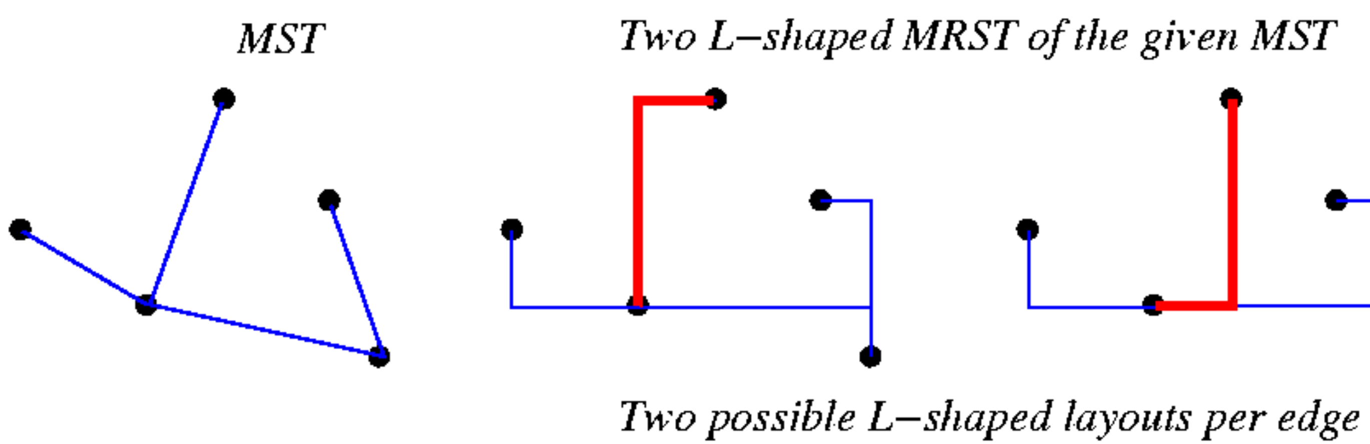
A Simple Performance Bound

- Easy to show that $\frac{Cost(MST(P))}{Cost(MRST(P))} \leq$ **2**
- Given any MRST *T* on point set *P* with Steiner point set *S*, construct a spanning tree *T'* on *P* as follows:
  1. Select any point in *T* as a root.
  2. Perform a depth-first traversal on the rooted tree *T*.
  3. Construct *T'* based on the traversal.



Coping with the MRST Problem

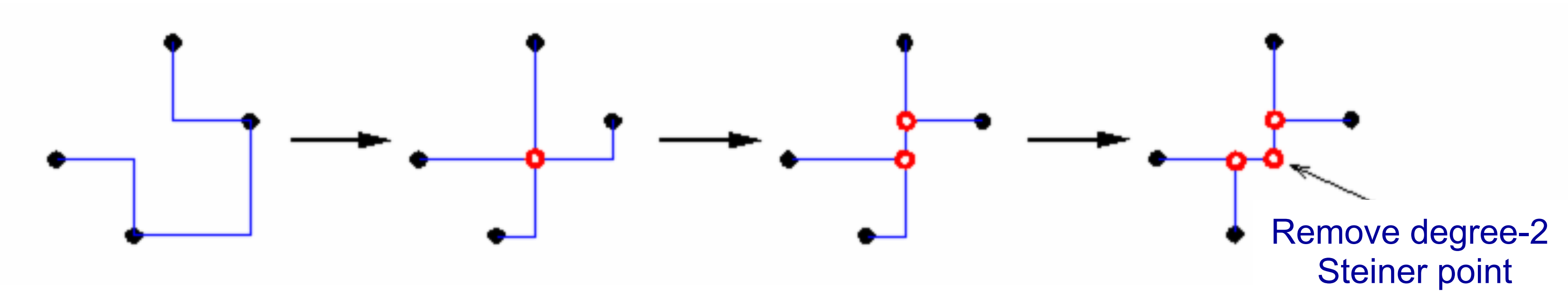
- Ho, Vijayan, Wong, “New algorithms for the rectilinear Steiner problem,” TCAD-90.
  1. Construct an MRST from an MST.
  2. Each edge is straight or L-shaped.
  3. Maximize overlaps by dynamic programming.
- About 8% smaller than *Cost*(*MST*).



Iterated 1-Steiner Heuristic for MRST

- Extended by Kahng & Robins, "A new class of Steiner tree heuristics with good performance: the iterated 1-Steiner approach," *ICCAD*, 1990.

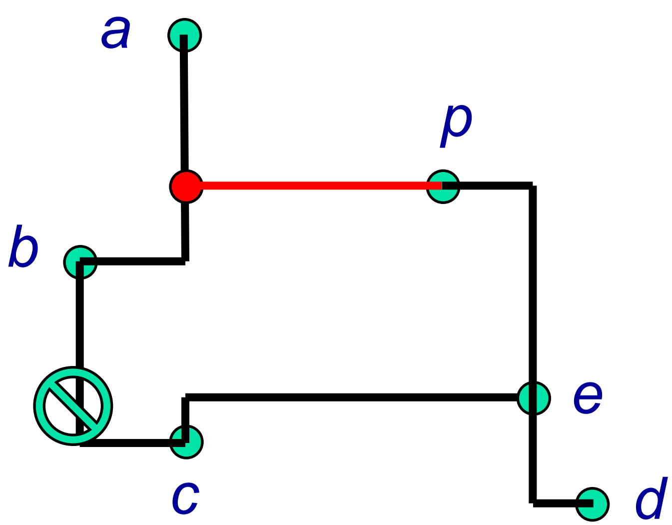
Algorithm: Iterated_1-Steiner(P)
P: set P of n points.
1 **begin**
2 $S \leftarrow \emptyset$;
    /* $H(P \cup S)$: set of Hanan points */
    /* $\Delta MST(A, B) = Cost(MST(A)) - Cost(MST(A \cup B))$ */
3 **while** ($Cand \leftarrow \{x \in H(P \cup S) \mid \Delta MST(P \cup S, \{x\}) > 0\} \neq \emptyset$) **do**
4   Find $x \in Cand$ which maximizes $\Delta MST(P \cup S, \{x\})$;
5     $S \leftarrow S \cup \{x\}$;
6   Remove points in S which have degree ≤ 2 in $MST(P \cup S)$;
7 **Output** $MST(P \cup S)$;
8 **end**



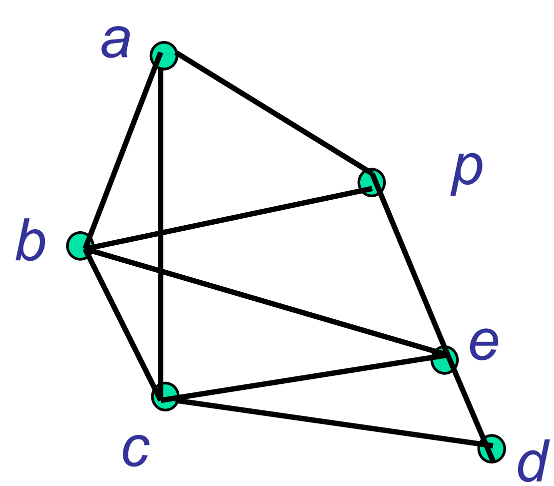
Courtesy of Prof. Y.-W. Chang

Steiner Minimal Trees Based on Spanning Graphs

- Zhou, "Efficient Steiner tree construction based on spanning graphs," ISPD-03 (TCAD-04)
- Select Borah *et al.*'s edge-substitution approach as the basis (TCAD-94)
  - Connect a point to an edge and delete the longest edge on the created cycle
- Enhance it with Zhou *et al.*'s spanning graph (ISPD-03)



Edge substitution

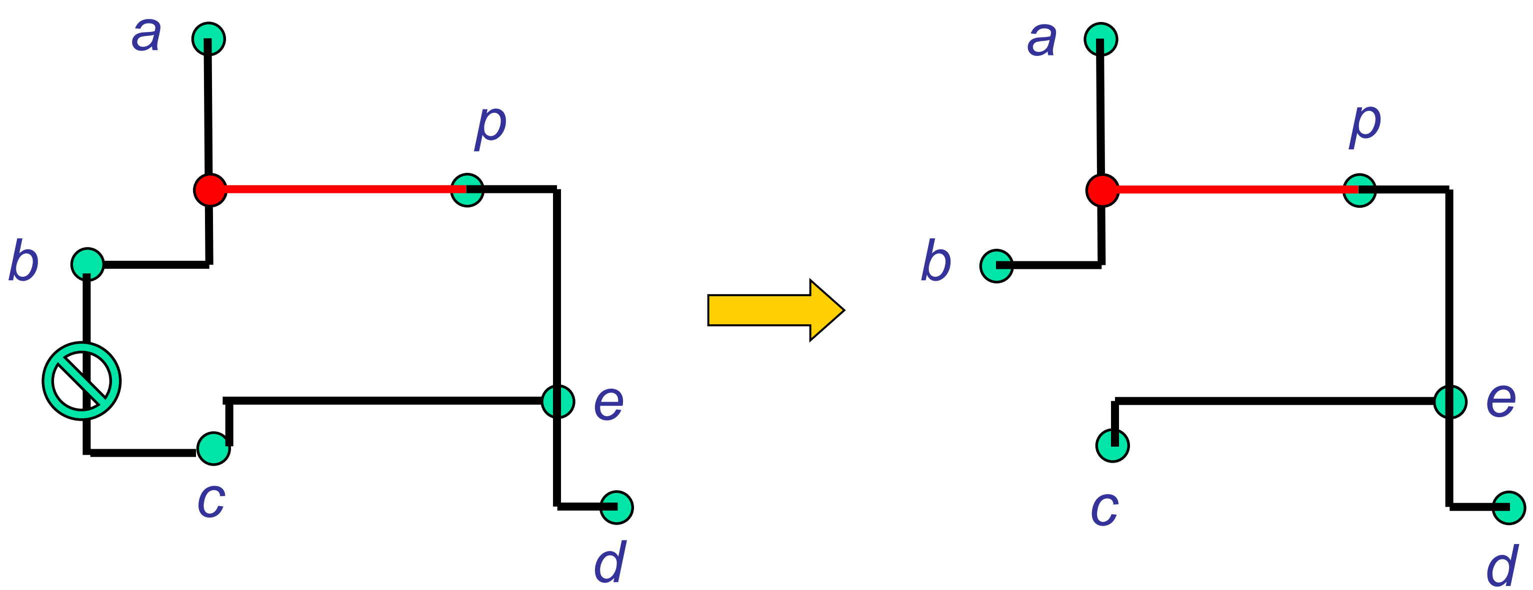


Spanning graph

Courtesy of Prof. Y.-W. Chang

Edge-Substitution Approach

- Borah, Owens, and Irwin's algorithm for the rectilinear SMT (TCAD-94)
  - Start with a minimal spanning tree
  - Iteratively consider connecting a point to a nearby edge (e.g., point *p* to edge (*a, b*))
  - Delete the longest edge on the cycle thus formed (e.g., edge (*b, c*))



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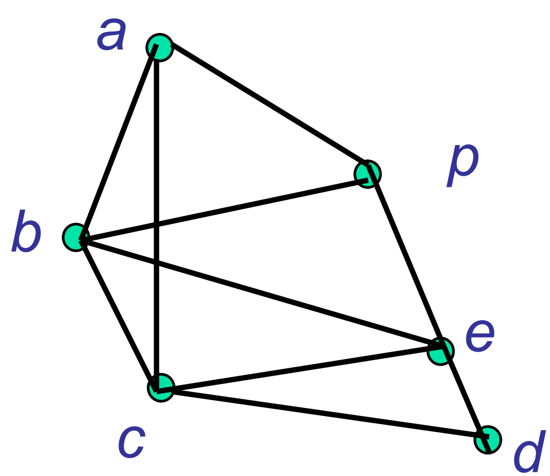
Implementing the Edge-Substitution Approach

- How to construct the initial spanning tree?
- How to find the edge-point pair candidates for connection?
- How to find the longest edge on the formed cycle?

Spanning graphs may answer the questions!!

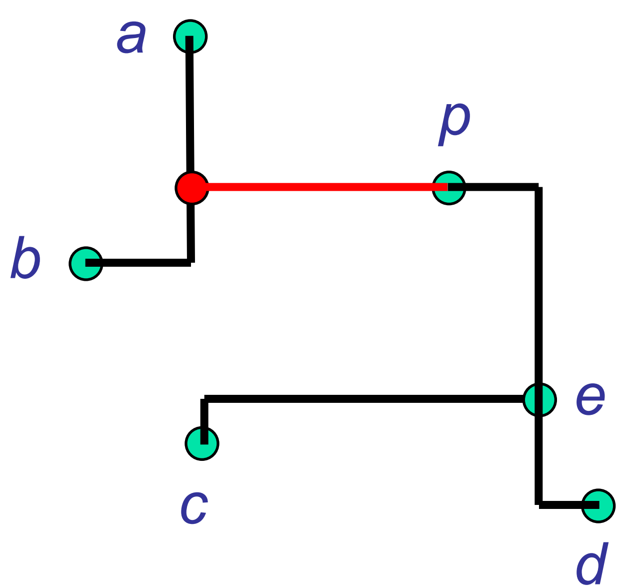


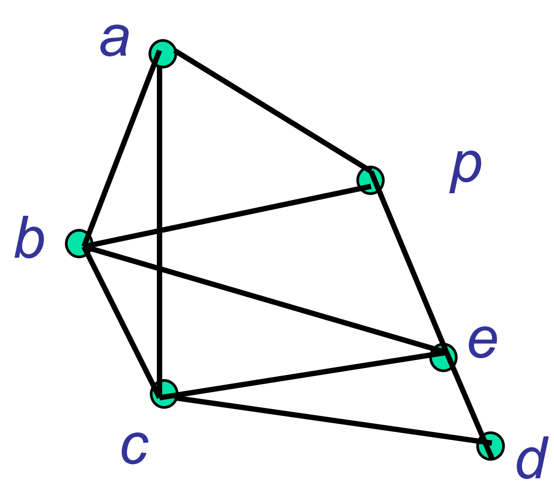
Spanning graph

Courtesy of Prof. Y.-W. Chang

Spanning Graph

- An MST on the graph is an MST of the given points
  - geometrical MST = spanning graph + graph MST
- Spanning graphs represent geometrical proximity information
  - Also provide candidates for point-edge connection
  - E.g., no edge (*b, d*) since *d* is blocked by edge (*c, e*)
- It is a sparse graph: $O(n)$ edges



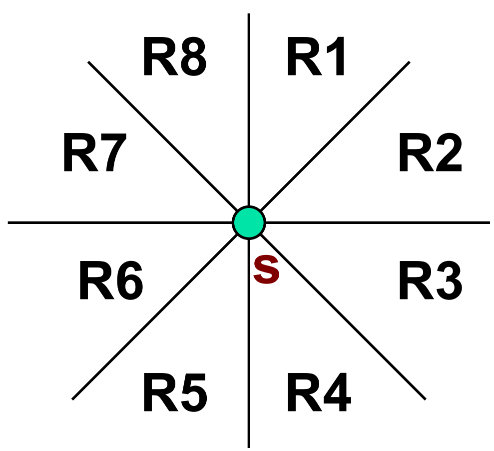


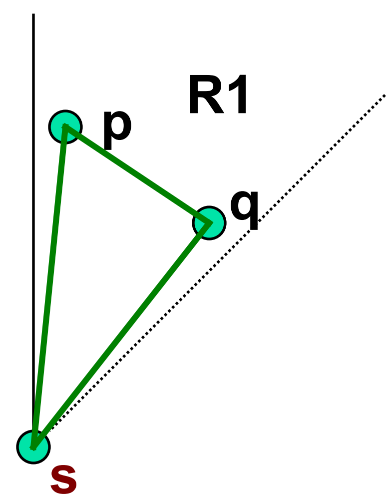
Spanning graph

Courtesy of Prof. Y.-W. Chang

Octal Partition for Finding a Spanning Graph

- Find a spanning graph in $O(n \lg n)$ time
  - Yao, SIAM J. Computing, 1982
- For each point *s*, the plane is divided into 8 octal regions
- ||pq|| < max(||ps||, ||qs||)
- Only the closest point in each region needs to be connected to *s*





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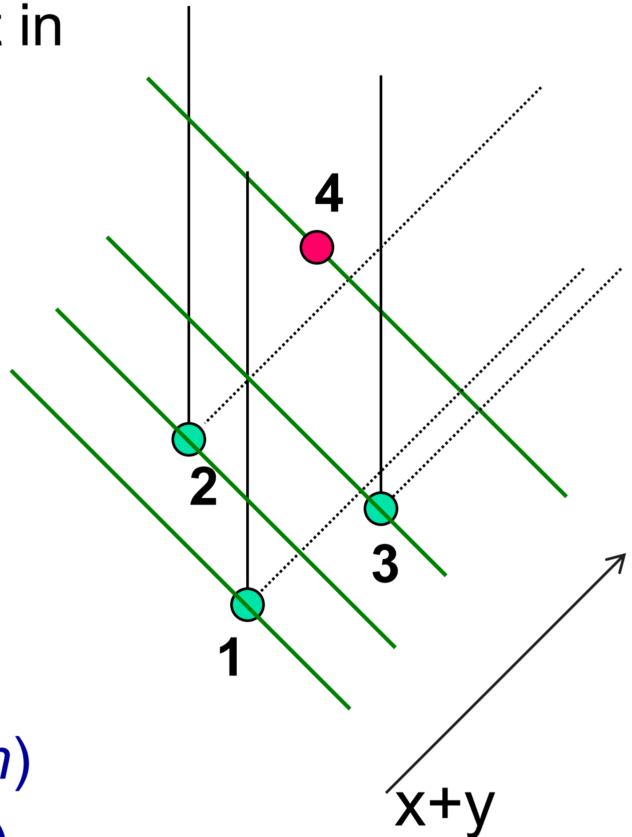
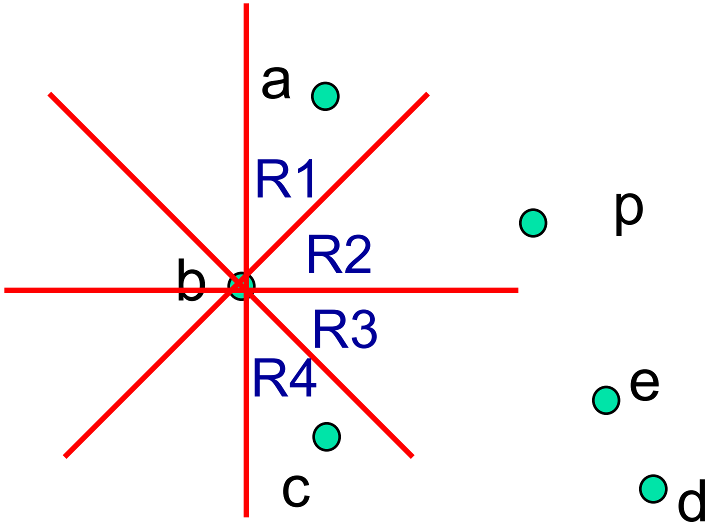
Finding Closest Points in R1

- Sweep points by increasing *x+y*
- Keep points waiting for closest point in *A* (active set)
- Checking current point with *A*
  - make connections
  - delete connected points
  - add current point in *A*
- Active set can be linearly order according to *x*
- Use a binary search tree
  - Finding an insertion place: O(lg *n*)
  - Deleting/inserting a point: O(lg *n*)
- Each point is inserted and deleted at most once: *O(n* lg *n*)

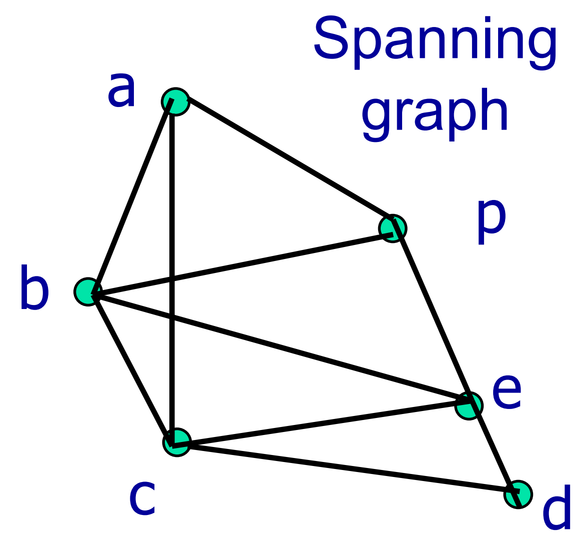


Finding the Neighbors



For each point, use the octal partition to find its neighbors.

	a	b	c	d	e	p
R1		a				
R2		p	e			
R3	p	e	d			
R4	c	c			d	e



Courtesy of Prof. Y.-W. Chang

Rectilinear Steiner Minimal Tree Algorithm

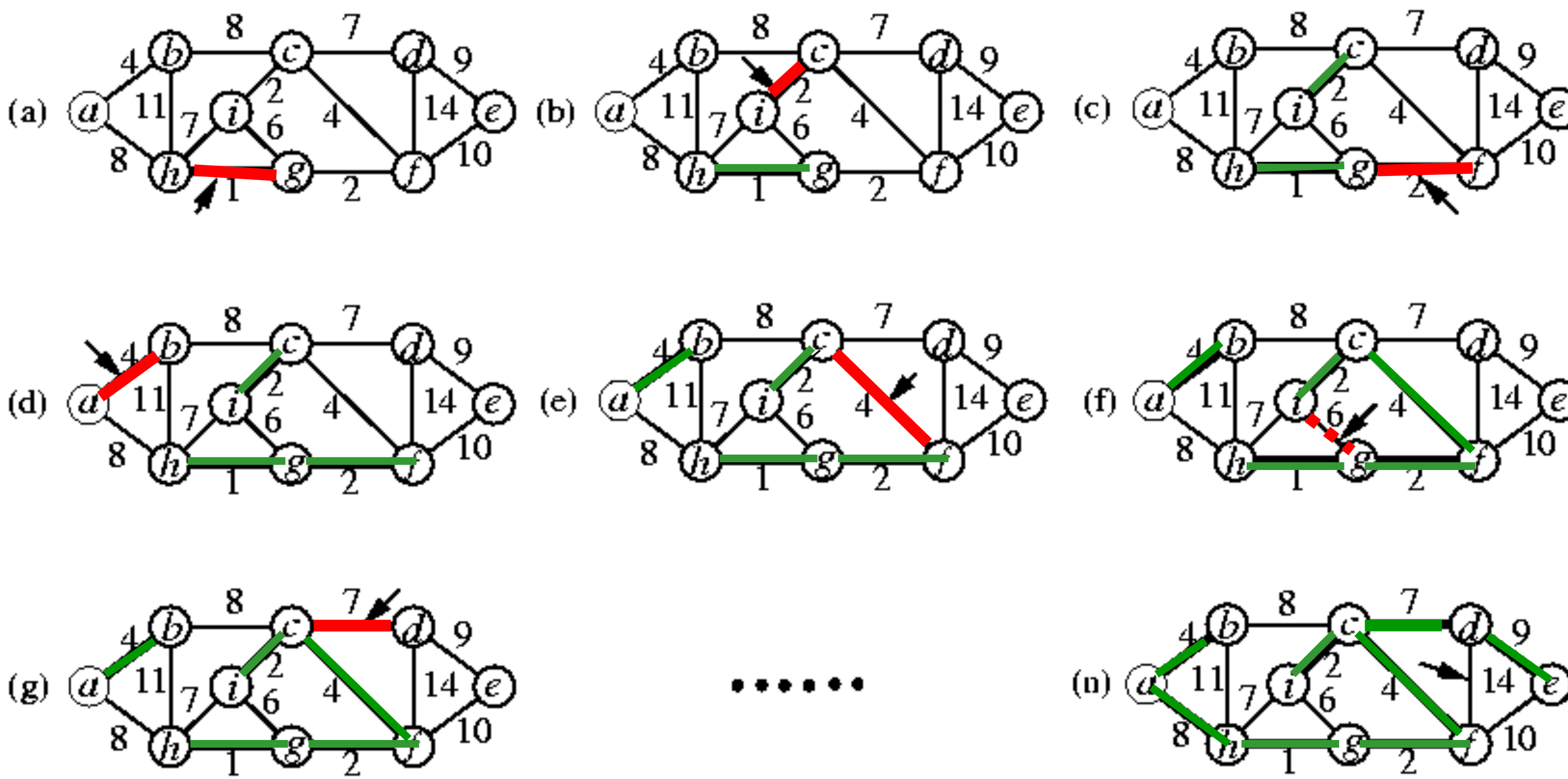
- Construct a rectilinear spanning graph
- Sort the edges in the graph
- Use Kruskal to build MST, at the same time
  - build the merging binary tree and
  - possible point-edge candidates (by the spanning graph)
- Use least common ancestors on the merging binary tree to find the longest edges in cycles
- Iteratively update point-edge connections

Kruskal's MST Algorithm

```
MST-Kruskal(G,w)
1. A ← ∅ ;
2. for  each vertex v ∈ V[G]
3.     Make-Set(v);
4. sort the edges of E[G] by nondecreasing weight w;
5. for  each edge (u,v) ∈ E[G], in order by nondecreasing weight
6.     if Find-Set(u) ≠ Find-Set(v)
7.         A ← A ∪ {(u, v)};
8.         Union(u,v);
9. return A;
```

- Add a safe edge at a time to the growing **forest** by finding an edge of least weight (from those connecting two trees in the forest).
- Time complexity: $O(E \lg E + V)$ (= $O(E \lg V + V)$, why?)
  - Lines 2--3: $O(V)$.
  - Line 4: $O(E \lg E)$.
  - Lines 5--8: $O(E)$ operations on the disjoint-set forest, so $O(E\,\alpha(E, V))$.

Example: Kruskal's MST Algorithm



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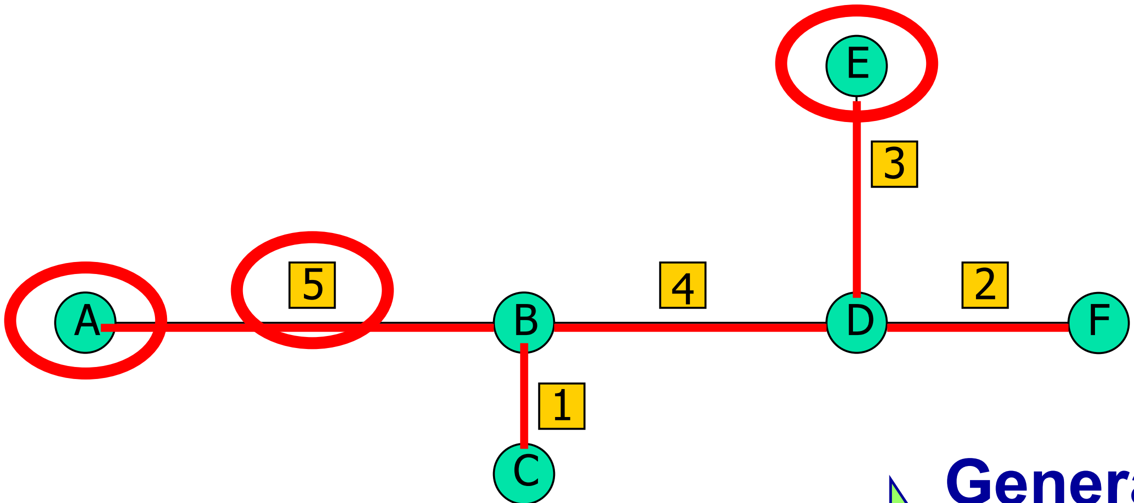
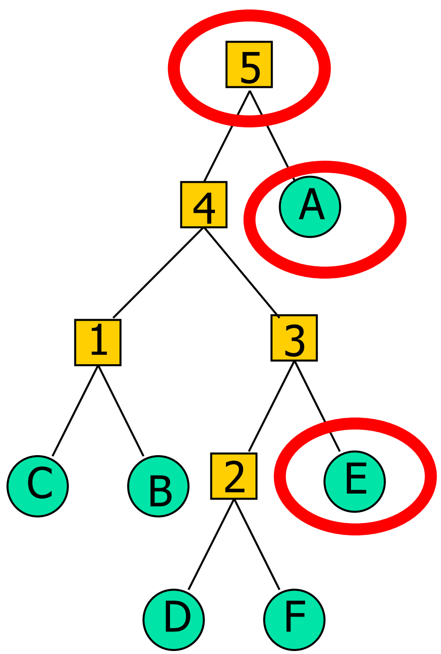
Finding the Longest Edge on a Cycle



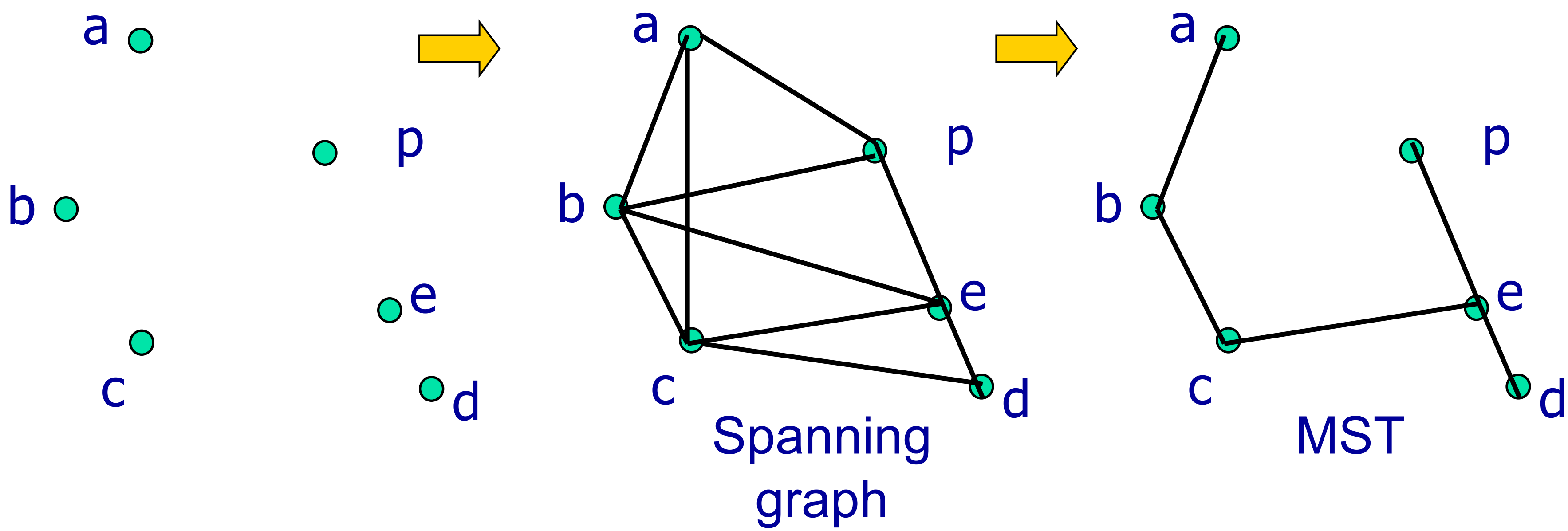


- Generate the merging binary tree corresponding to the minimal spanning tree
- The longest edge for this pair is the least common ancestor

Courtesy of Prof. Y.-W. Chang

Example Spanning Graph Construction

1. Construct a rectilinear spanning graph
2. Use Kruskal's algorithm to build MST
3. Update point-edge connections iteratively

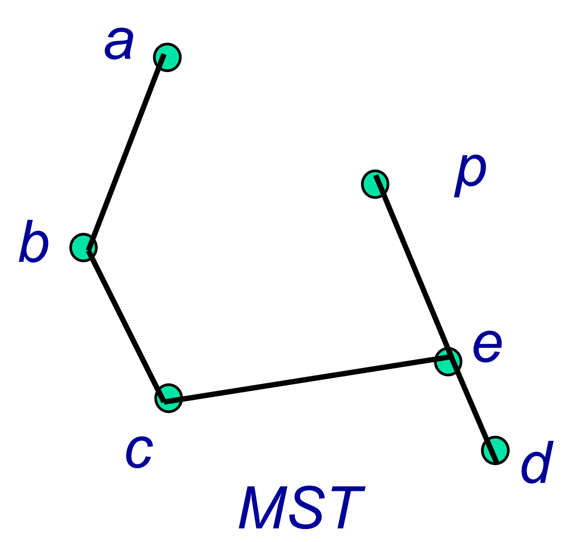


Courtesy of Prof. Y.-W. Chang

Point-Edge Connection Updating

1. Construct a rectilinear spanning graph
2. Use Kruskal's algorithm to build MST
3. Update point-edge connections iteratively

point	edge	delete edge	gain
a	(b,c)	(a,b)	3
c	(a,b)	(b,c)	3
p	(a,b)	(b,c)	3
p	(a,b)	(c,e)	2
c	(d,e)	(c,e)	1
e	(b,c)	(c,e)	1
p	(a,b)	(e,p)	1
p	(b,c)	(c,e)	1
p	(c,e)	(e,p)	1

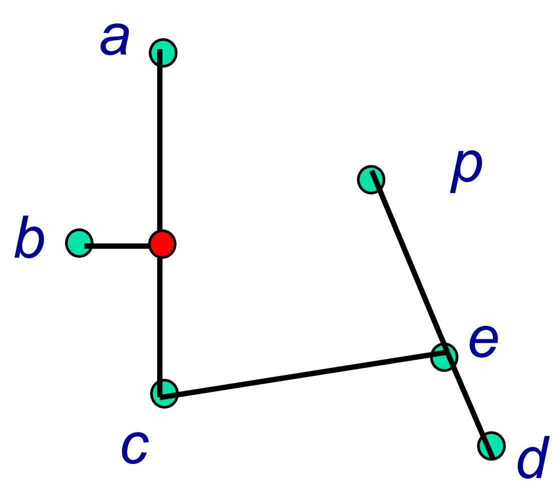


Building the table according to the spanning graph

Point-Edge Connection Updating (cont'd)

1. Construct a rectilinear spanning graph
2. Use Kruskal's algorithm to build MST
3. Update point-edge connections iteratively
   - Connect the point-edge pair with the max gain (e.g., point *a* to edge *(b, c)*)
   - Delete related point-edge candidates (point *c* to edge (*a, b*), etc.)

point	edge	delete edge	gain
a	(b,c)	(a,b)	3
c	(a,b)	(b,c)	3
p	(a,b)	(b,c)	3
P	(a,b)	(c,e)	2
c	(d,e)	(c,e)	1
e	(b,c)	(c,e)	1
p	(a,b)	(e,p)	1
p	(b,c)	(c,e)	1
p	(c,e)	(e,p)	1

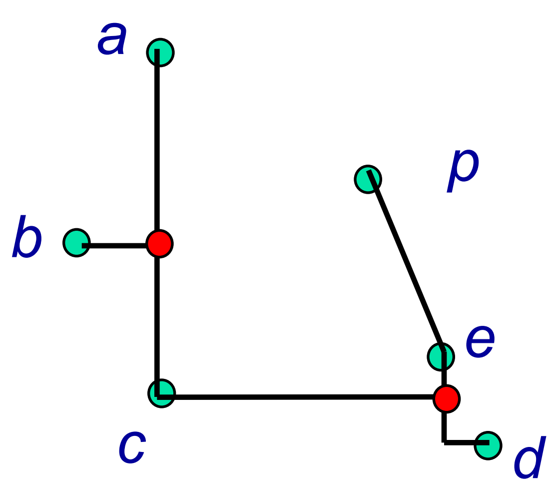


Point-Edge Connection Updating (cont'd)

1. Construct a rectilinear spanning graph
2. Use Kruskal's algorithm to build MST
3. Update point-edge connections iteratively

- Find the remaining point-edge pair with the max gain (point *c* to edge *(d, e)*)

point	edge	delete edge	gain
a	(b,c)	(a,b)	3
c	(a,b)	(b,c)	3
p	(a,b)	(b,c)	3
P	(a,b)	(c,e)	2
c	(d,e)	(c,e)	1
e	(b,c)	(c,e)	1
p	(a,b)	(e,p)	1
p	(b,c)	(c,e)	1
p	(c,e)	(e,p)	1



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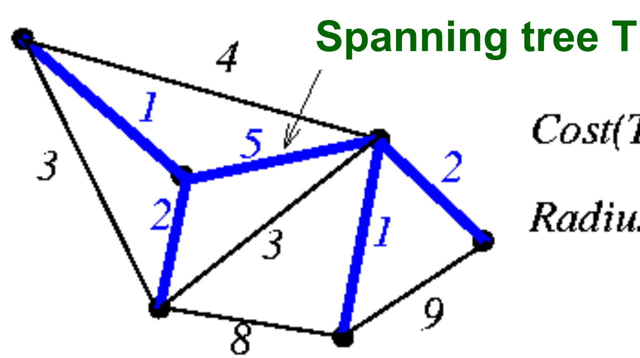
Bounded-Radius (-Diameter) Minimum Spanning Tree

- **Problem:** Given a parameter $\varepsilon \geq 0$ and a signal net with radius R (diameter D), find a minimum-cost spanning tree T with radius $r(T) \leq (1 + \varepsilon)R$ ($d(T) \leq (1 + \varepsilon)D$).
  - Awerbuch, *et al.*, "Cost-sensitive analysis of communication protocols," *ACM Symp. Principles of Distributed Computing*, 1990.
  - Cong, Kahng, Robins, Sarrafzadeh, Wong, "Performance-driven global routing for cell based IC's," ICCD-91 (& TCAD, June 1992).
- R (D): the maximum length among all shortest paths from the source (among every pair of nodes).
- MST (minimum spanning tree) $\leftrightarrow$ minimum cost; SPT (shortest path tree) $\leftrightarrow$ minimum radius.
- Question: How to find a spanning tree with a good trade-off between cost and radius?



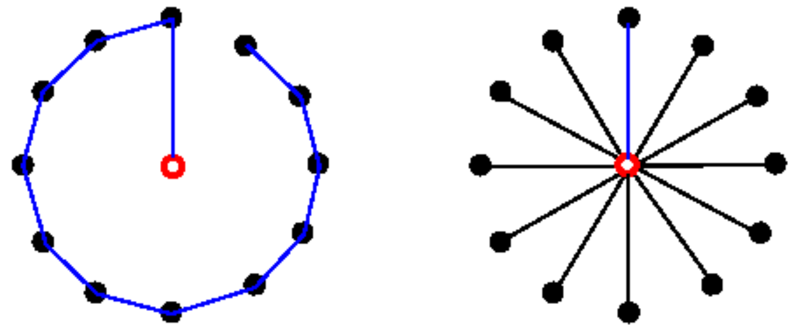
$Cost(T)=1+2+5+1+2=11$

$Radius(T)=max\{1+2,\ 1+5+1,\ 1+5+2\}=8$

MST vs. SPT Trade-off

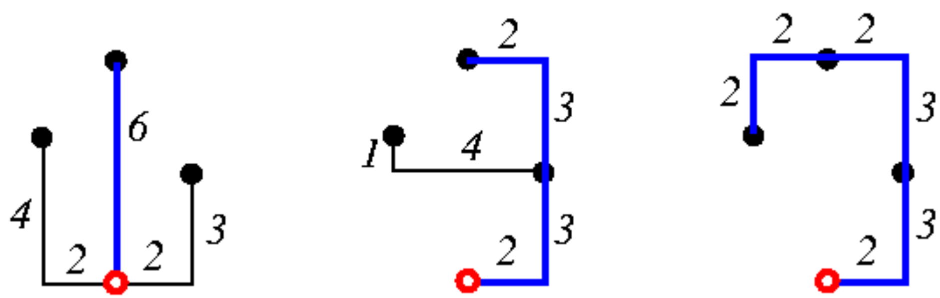
- *Cost*(*SPT*) may be $\Omega(|n|)$ times larger than *Cost*(*MST*).

Minimum spanning tree (MST) *Shortest path tree (SPT)*



$$\frac{Cost(SPT)}{Cost(MST)} = \Theta(n)$$

$r(T) <= (1 + \epsilon)R$



$\epsilon = 0$ *Cost = 17* *Radius = 6*

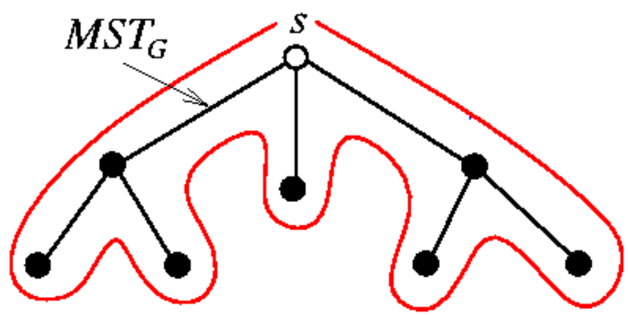
$\epsilon = 1$ *Cost = 15* *Radius = 10*

ϵ *= infinite* *Cost = 14* *Radius = 14*

Courtesy of Prof. Y.-W. Chang

Bounded Radius Bounded Cost Spanning Tree

```
Algorithm: Bounded-Radius_Bounded-Cost_Spanning_Tree(G)
begin
Compute MST_G and SPT_G;
Q ← MST_G;
L ← depth-first tour of MST_G;
S ← 0;
for i ← 1 to |L| - 1
    S ← S + cost(L_i, L_{i+1});
    if S ≥ ε • dist_G(s, L_{i+1}) then
        Q ← Q ∪ minpath_G(s, L_{i+1});
        S ← 0;
T = shortest path tree of Q;
end
```



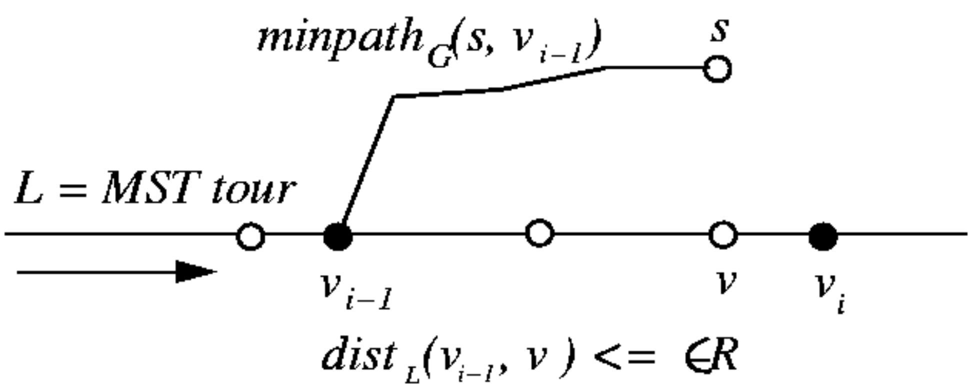
L = depth–first tour of MST

$cost(L) <= 2\ cost(MST_G)$

Bounded-Radius Bounded-Cost Spanning Tree

- For any weighted graph G and parameter ε, the routing tree T constructed by the algorithm has radius $r(T) \leq (1+\varepsilon)R$.
- v_{i-1}: the last node before v on L for which we added $minpath_G(s, v_{i-1})$ to Q.

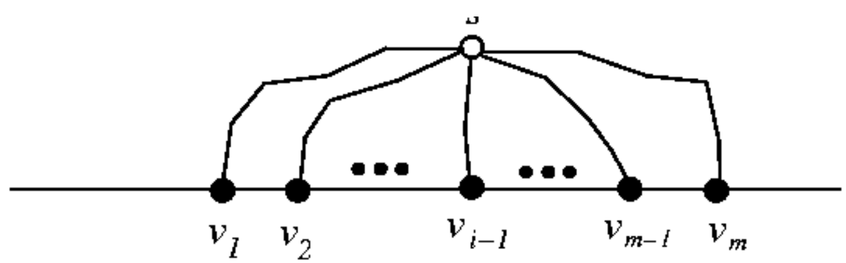
$$\begin{aligned} dist_T(s,v) &\leq dist_T(s,v_{i-1}) + dist_L(v_{i-1},v) \\ &\leq dist_G(s,v_{i-1}) + \epsilon R \\ &\leq R + \epsilon R \\ &= (1+\epsilon)R \end{aligned}$$



Bounded-Radius Bounded-Cost Spanning Tree

- For any weighted graph G and parameter ε, the routing tree T constructed by the algorithm has cost $cost(T) \le (1+2/\varepsilon)cost(MST_G)$.
- Let $v_1, v_2, \ldots, v_m$ be the set of nodes to which the algorithm added shortest paths from source s.

$$
\begin{aligned}
cost(T) &\le cost(MST_G) + \sum_{i=1}^{m} dist_G(s, v_i) \\
dist_L(v_{i-1}, v_i) &\ge \epsilon \cdot dist_G(s, v_i) \\
cost(T) &\le cost(MST_G) + \sum_{i=1}^{m} \frac{dist_G(v_{i-1}, v_i)}{\epsilon} \\
&\le cost(MST_G) + \frac{cost(L)}{\epsilon} \\
&\le cost(MST_G) + \frac{2 \cdot cost(MST_G)}{\epsilon} \\
&\le (1 + \frac{2}{\epsilon})cost(MST_G)
\end{aligned}
$$



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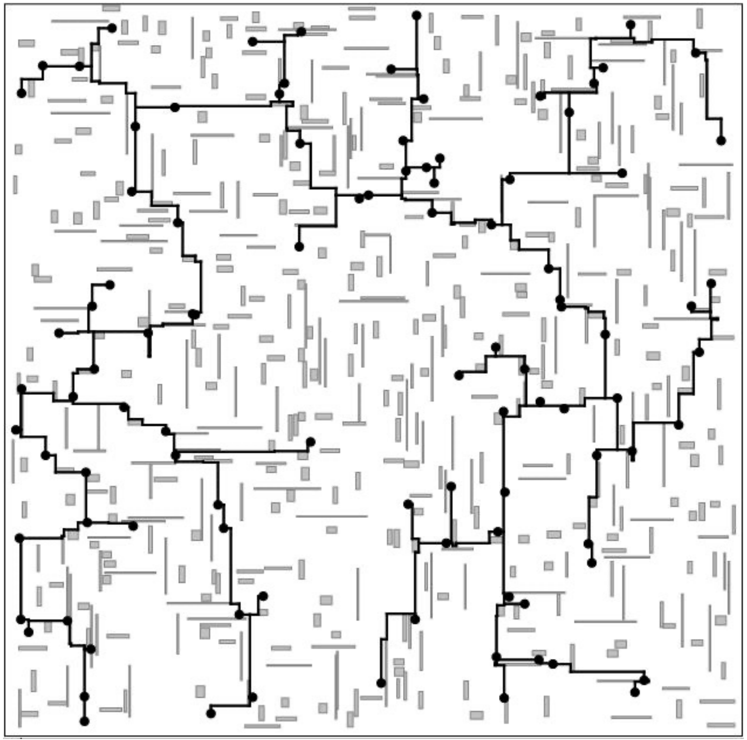
Appendix A:

Obstacle-Avoiding Rectilinear Steiner Tree Construction

Lin, Chen, Li, Chang, and Yang

ISPD-07



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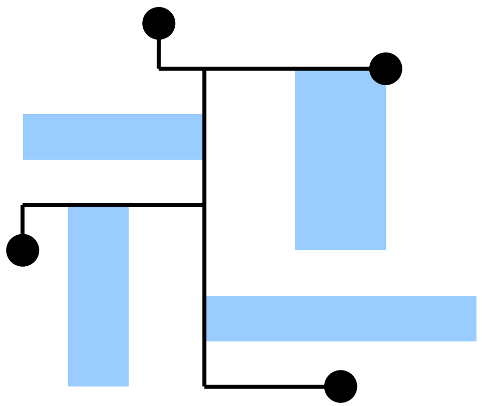
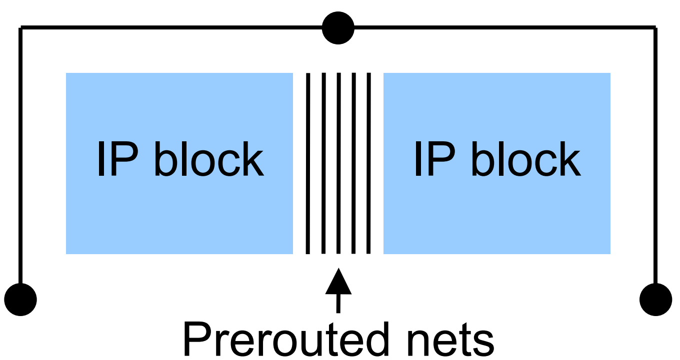
Introduction to OARSMT Problem

- Given a set of pins and a set of obstacles, an obstacle-avoiding rectilinear Steiner minimal tree (OARSMT)
  - Connect those pins, possibly through some Steiner points
  - Use only vertical and horizontal edges
  - Avoid running through any obstacle
  - Have a minimal total wirelength

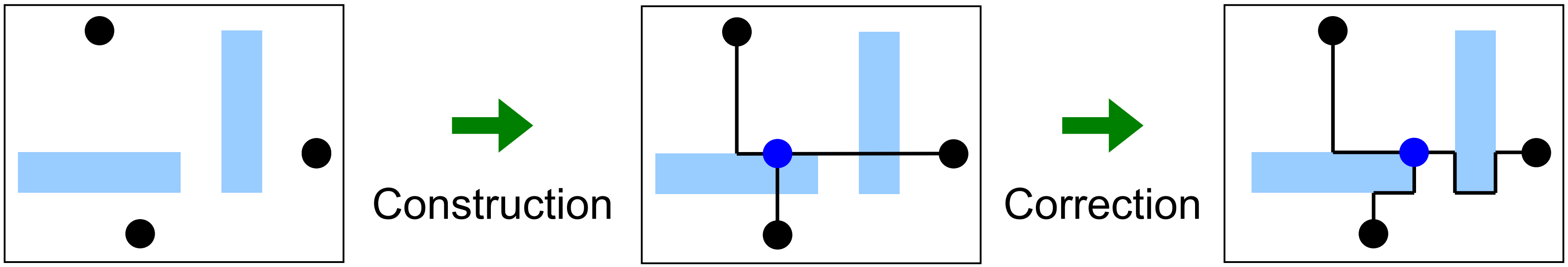


- It becomes more important than ever for modern nanometer IC designs.
  - The design needs to consider numerous routing obstacles incurred from
    - Prerouted nets
    - Large-scale power networks
    - IP blocks, etc



Construction-by-Correction Approach

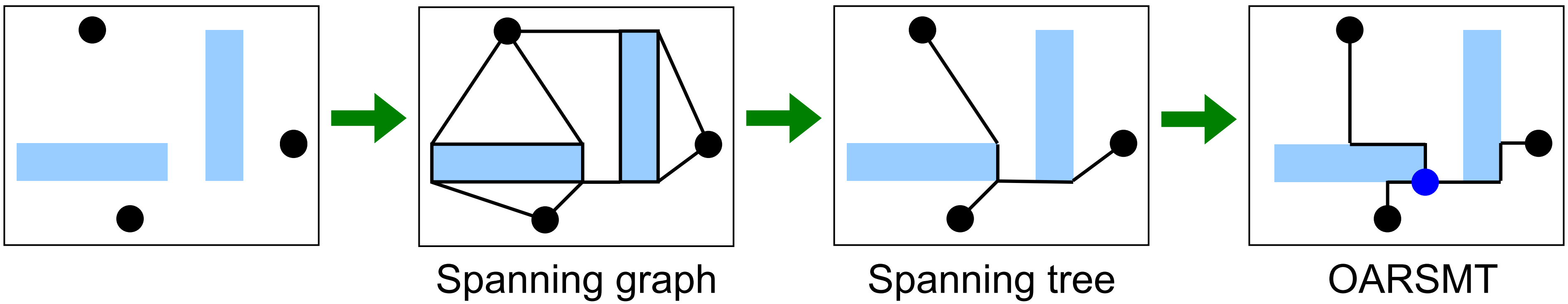
- Construct an initial tree without considering obstacles
- Correct edges overlapping obstacles



- Lack a global view of obstacles
- Have a limited solution quality
- Feng et al., ISPD-06
  - Construct OARSMT in the λ-geometry plane

Connection-Graph Based Approach

- Construct a connection graph in which there is a desired OARSMT
- Apply searching techniques to find the desired OARSMT

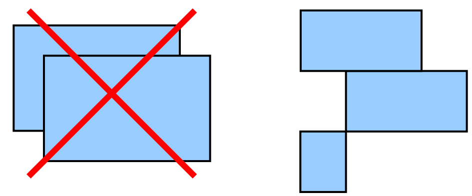


- Prune many redundant edges to reduce problem size
- Obtain much better solution quality
- Shen et al., ICCD-05
  - Achieve good results but can be further improved

Problem Formulation

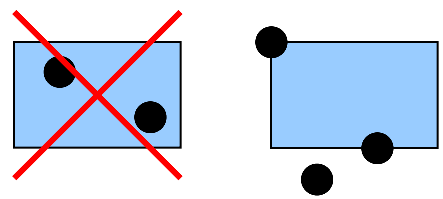
- Given *m* pins $\{p_1, p_2, \ldots, p_m\}$ and k obstacles $\{o_1, o_2, \ldots, o_k\}$, construct an OARSMT such that the total wirelength of the tree is minimized.

Obstacles overlap each other



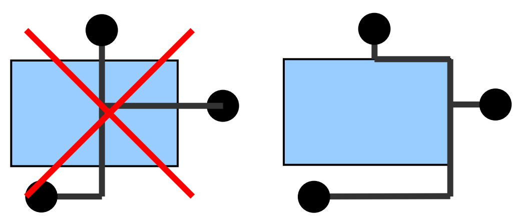
Obstacles are point-touched or line touched

A pin locates inside an obstacle



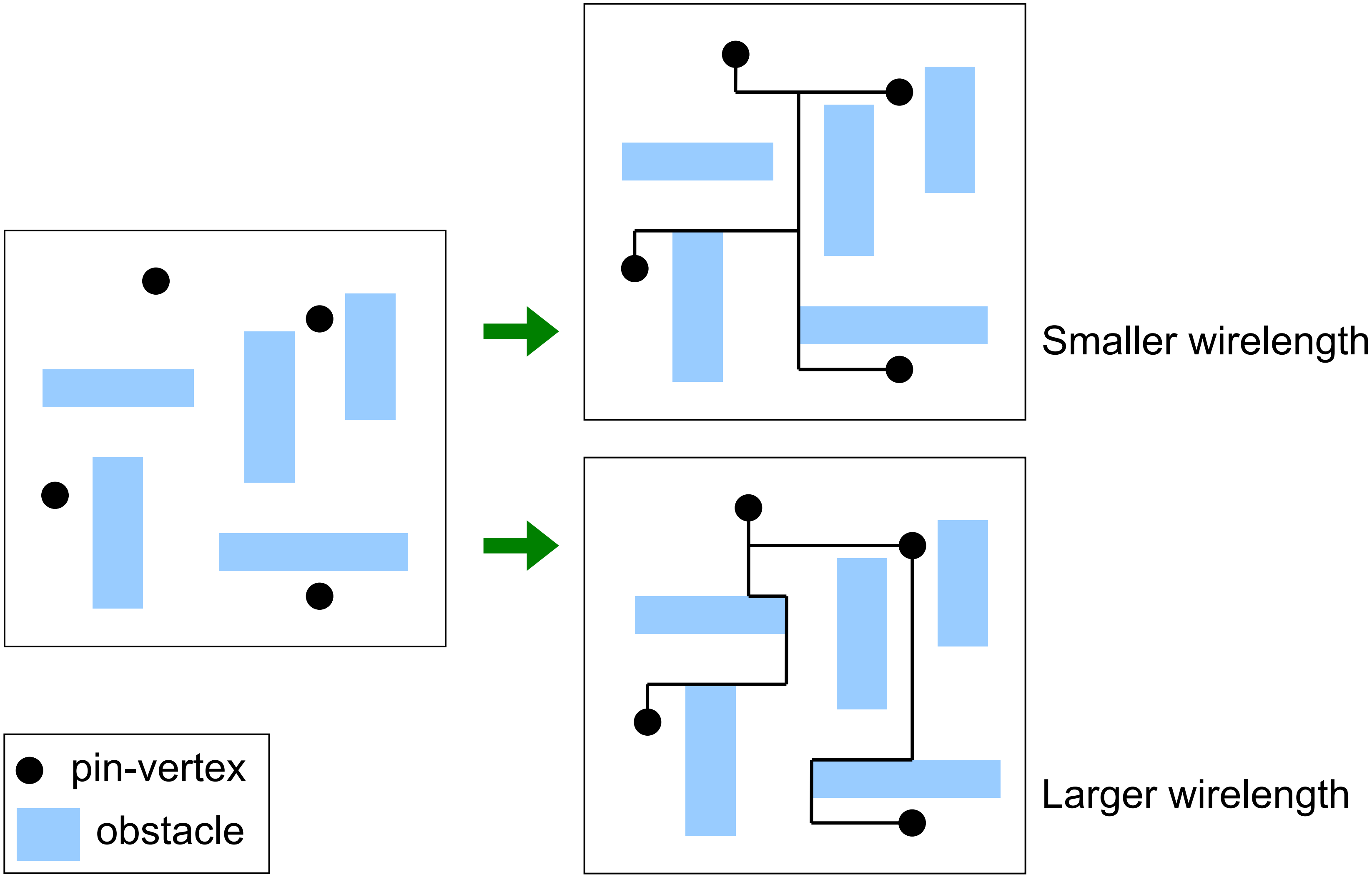
A pin is at the corner or on the boundary of an obstacle

An edge intersects an obstacle



An edge is point-touched or line-touched with an obstacle

An OARSMT Example



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Algorithm Flow

Obstacle-Avoiding Spanning Graph Construction

↓

Obstacle-Avoiding Spanning Tree Construction

↓

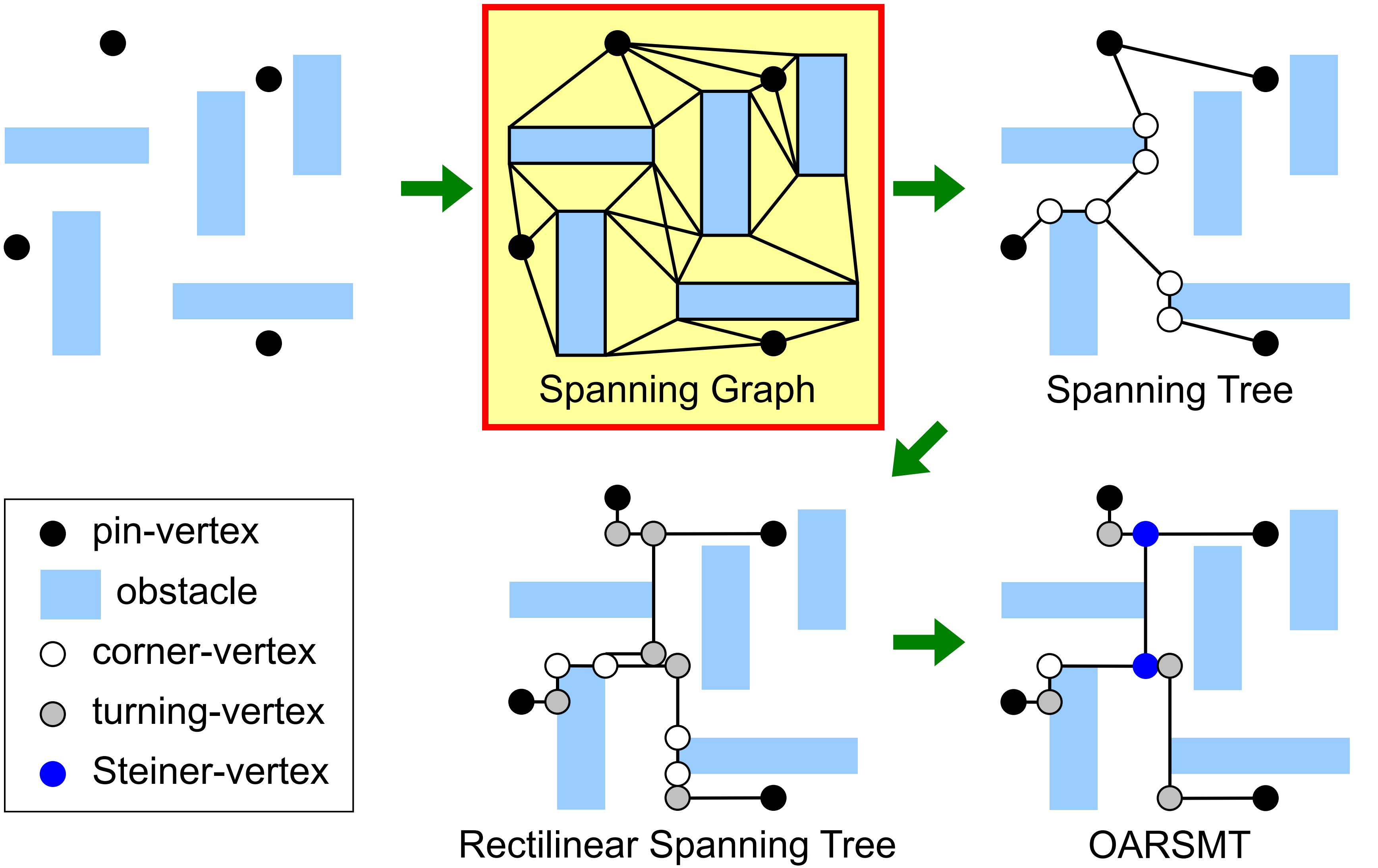
Obstacle-Avoiding Rectilinear Spanning Tree Construction

↓

OARSMT Construction

Courtesy of Prof. Y.-W. Chang

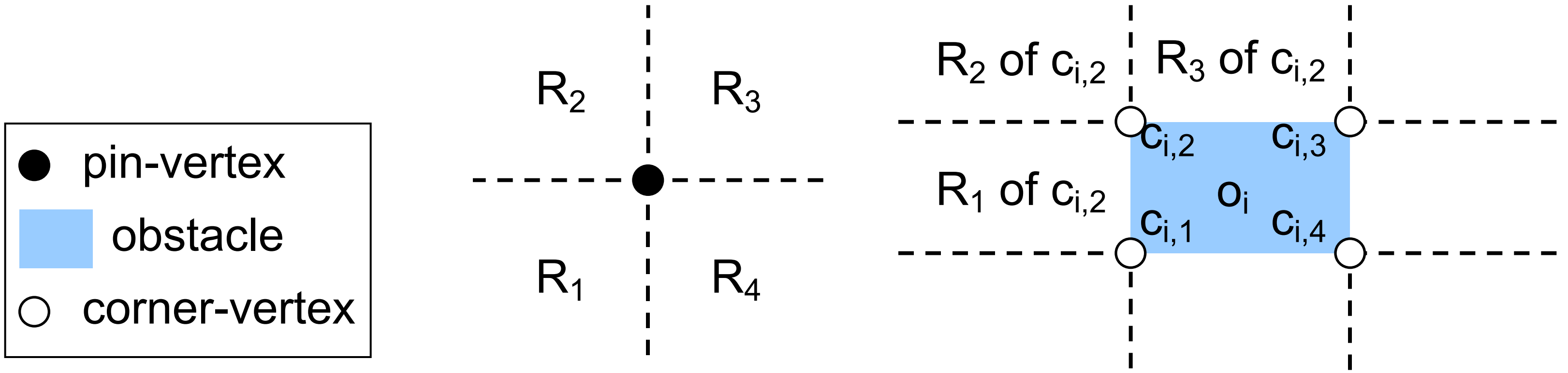
Algorithm Flow



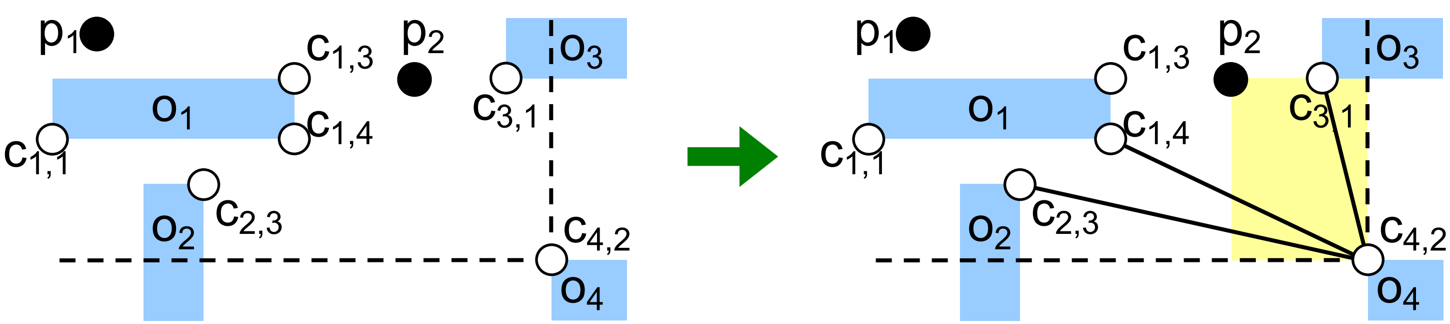
Courtesy of Prof. Y.-W. Chang

Spanning Graph Construction

- The plane is divided into four regions for each vertex.



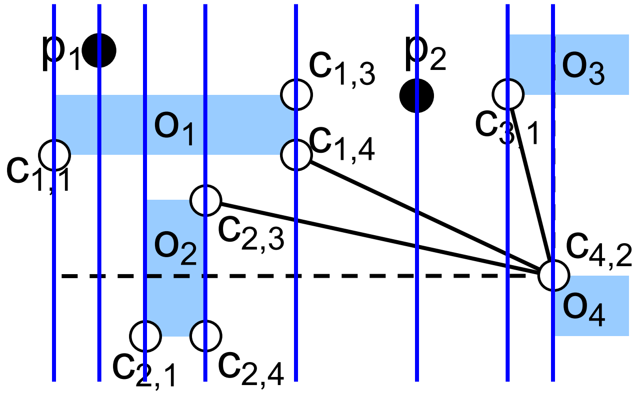
- v_1 and v_2 are connected if no other vertex or obstacle is inside or on the boundary of the bounding box of v_1 and v_2.



Spanning graph for R_2 of $c_{4,2}$

Spanning Graph Construction – Example

- Apply a sweeping line algorithm for each region
- Use blocking information to check a vertex is blocked or not

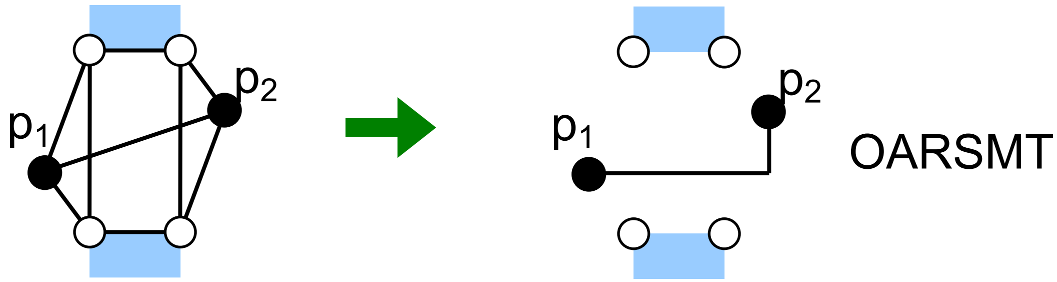


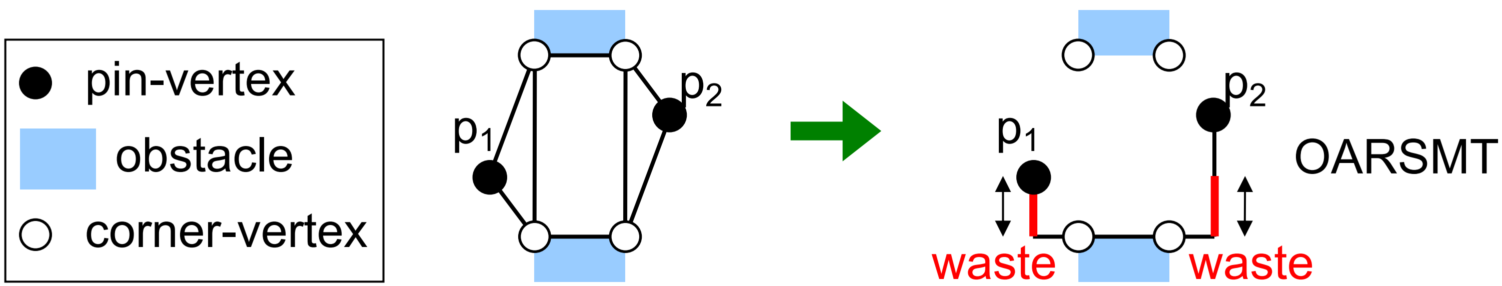
Candidate set	Blocking information
$c_{1,1}$	o_1
$c_{2,3}$	o_2
$c_{1,4}$	o_3
p_2	
$c_{3,1}$	

Spanning graph for R_2 of $c_{4,2}$

Properties of Our Spanning Graph (1/2)

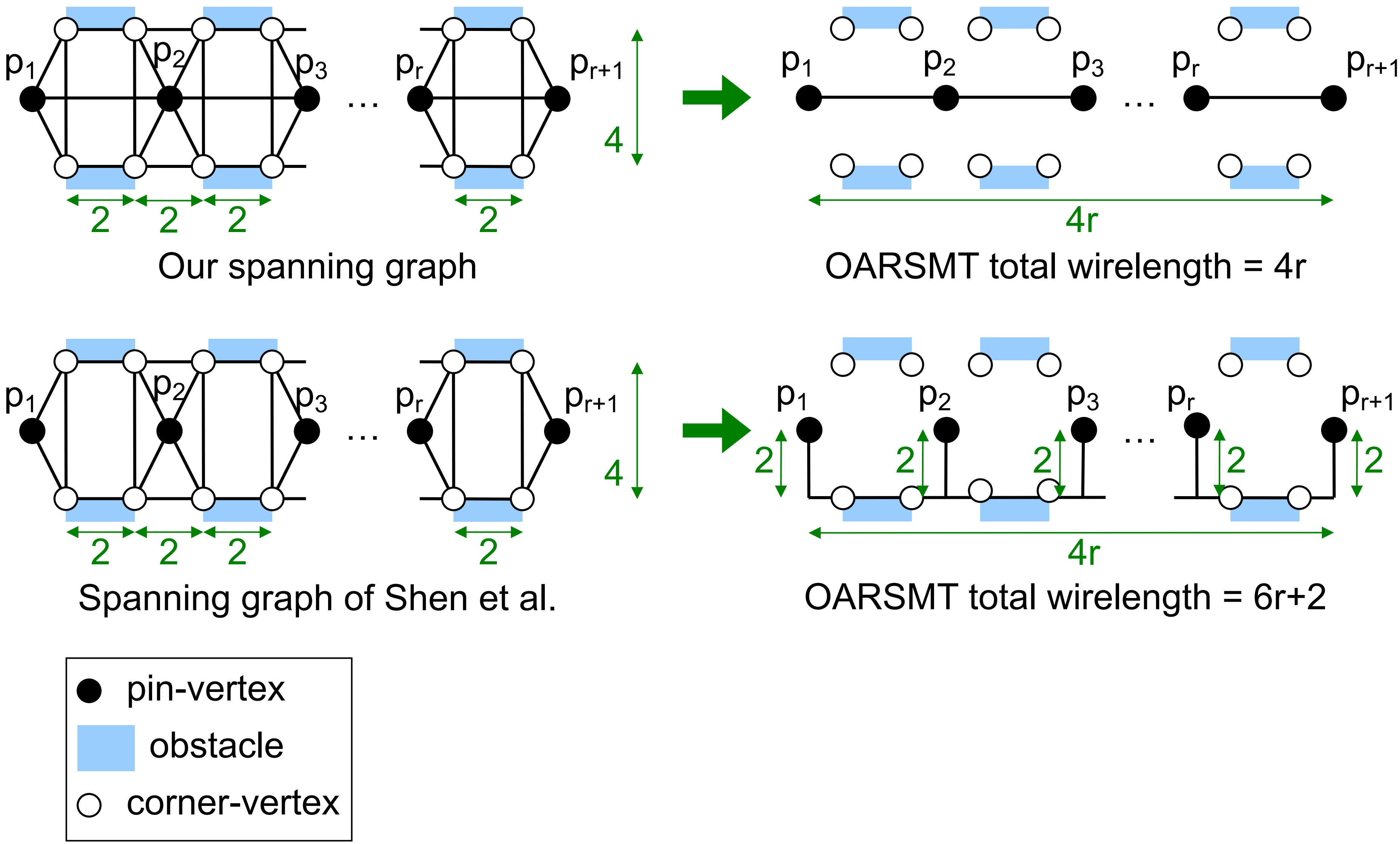
- Our spanning graph guarantees a shortest path of any two vertices.





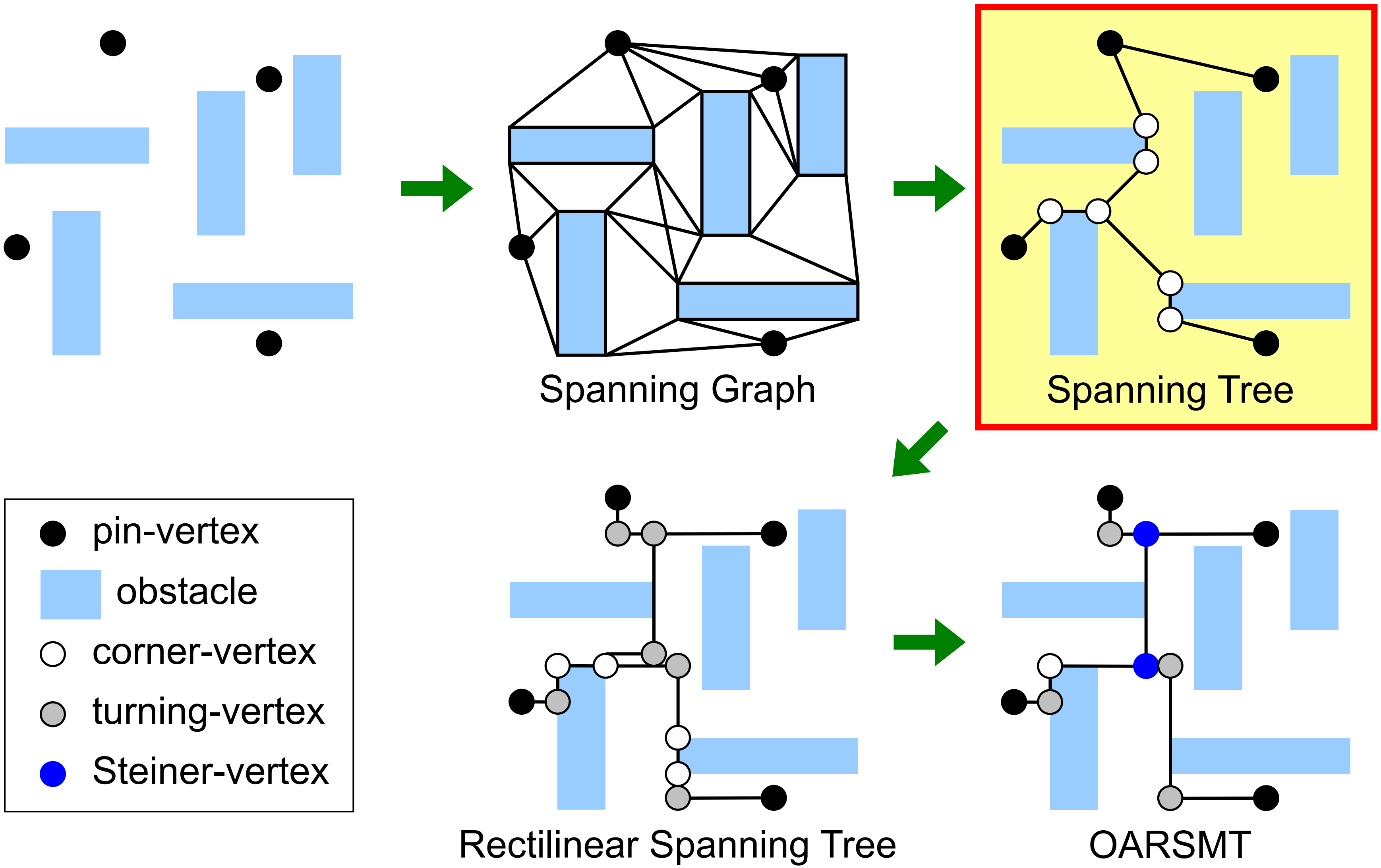
- Spanning graph of Shen et al. (ICCD-05) does not guarantee it.

Properties of Our Spanning Graph (2/2)



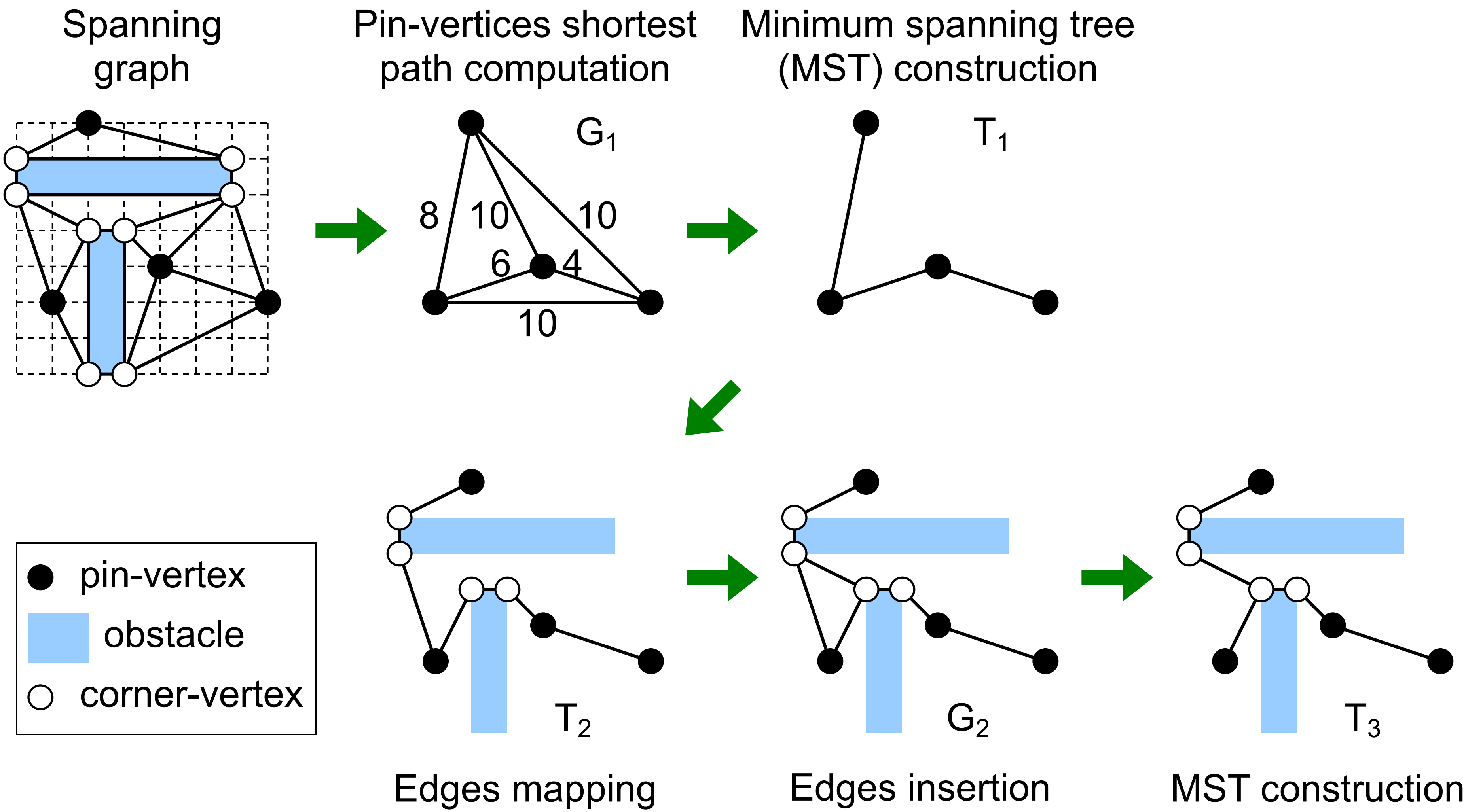
Courtesy of Prof. Y.-W. Chang

Algorithm Flow



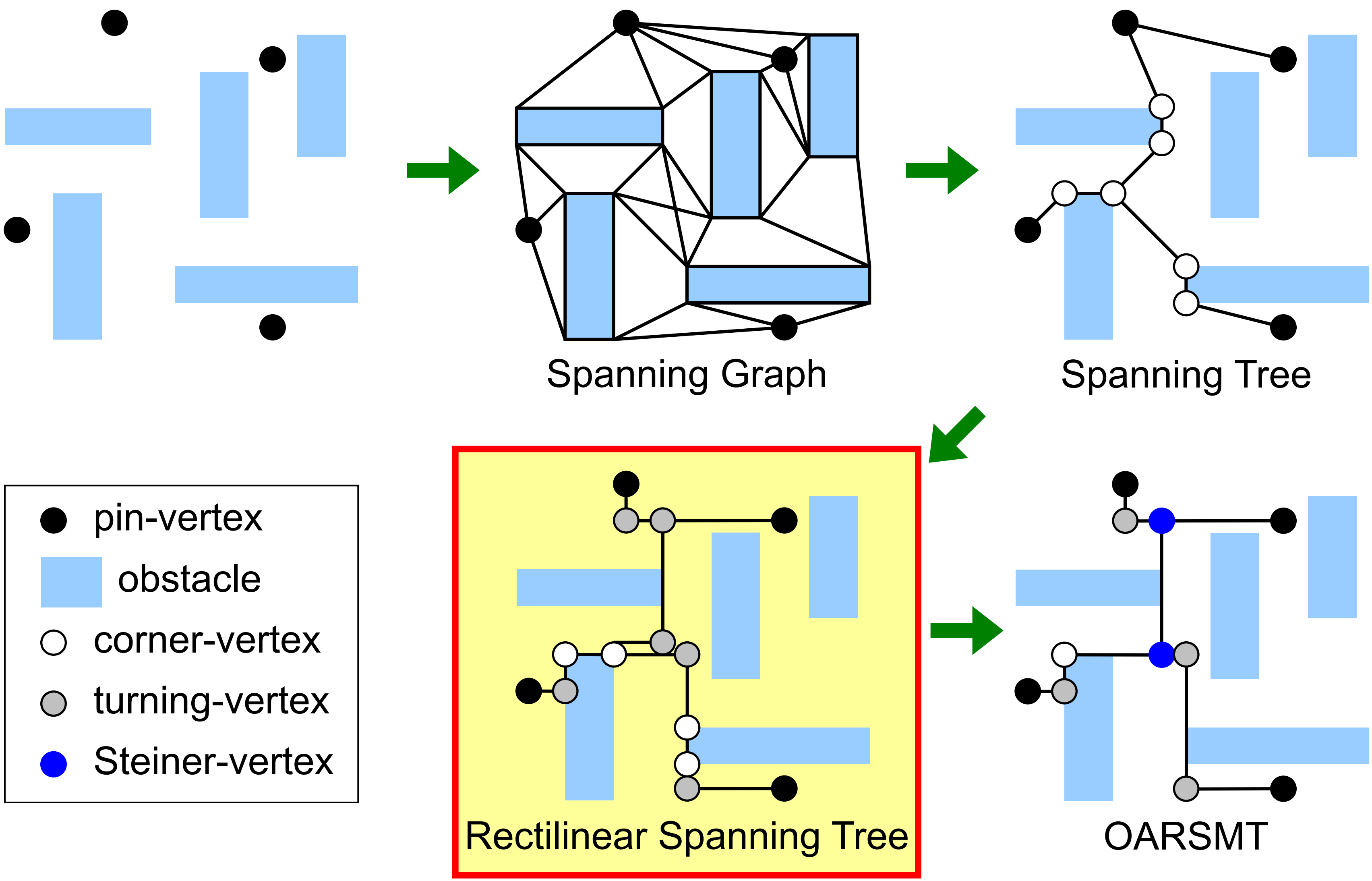
Courtesy of Prof. Y.-W. Chang

Spanning Tree Construction



Courtesy of Prof. Y.-W. Chang

Algorithm Flow

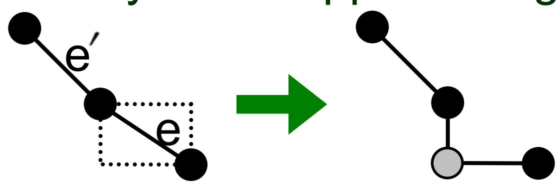


Courtesy of Prof. Y.-W. Chang

Rectilinear Spanning Tree Construction

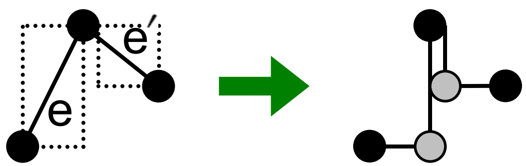
- Transform slant edges into vertical and horizontal edges
  - Longer edges are transformed first.
  - Three cases for a slant edge e and its neighboring edge e':

1. They are in opposite regions.



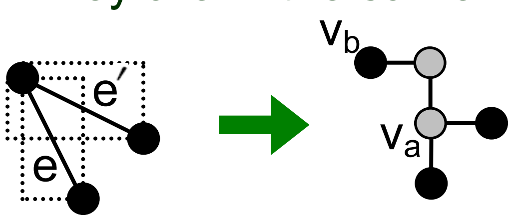
e is transformed randomly

2. They are in neighboring regions.



e and e' are transformed with edge overlap

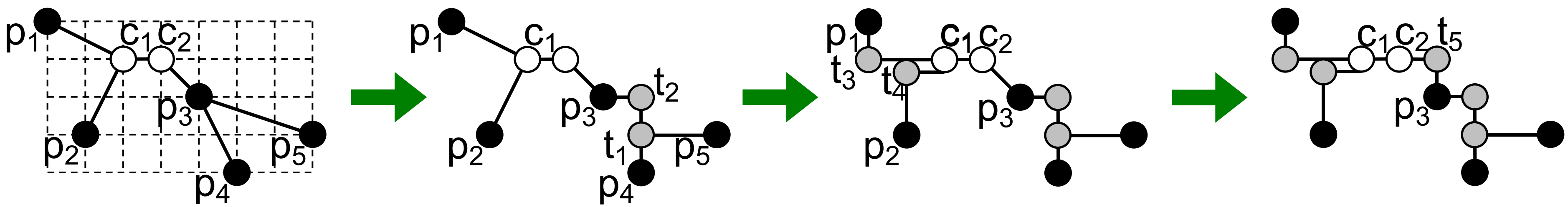
3. They are in the same region.



e and e' are transformed with edge overlap
(v_a, v_b) is transformed randomly

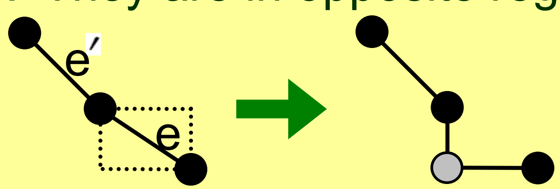
Courtesy of Prof. Y.-W. Chang

Rectilinear Spanning Tree Construction – Example



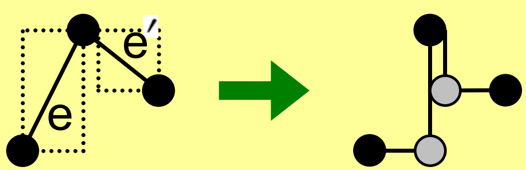
● pin-vertex ○ corner-vertex ◉ turning-vertex

1. They are in opposite regions.



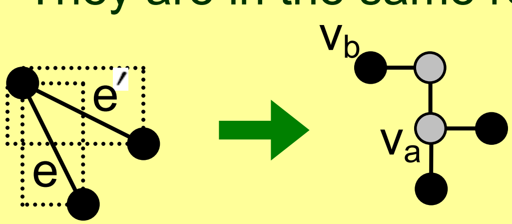
e is transformed randomly

2. They are in neighboring regions.



e and e' are transformed with edge overlap

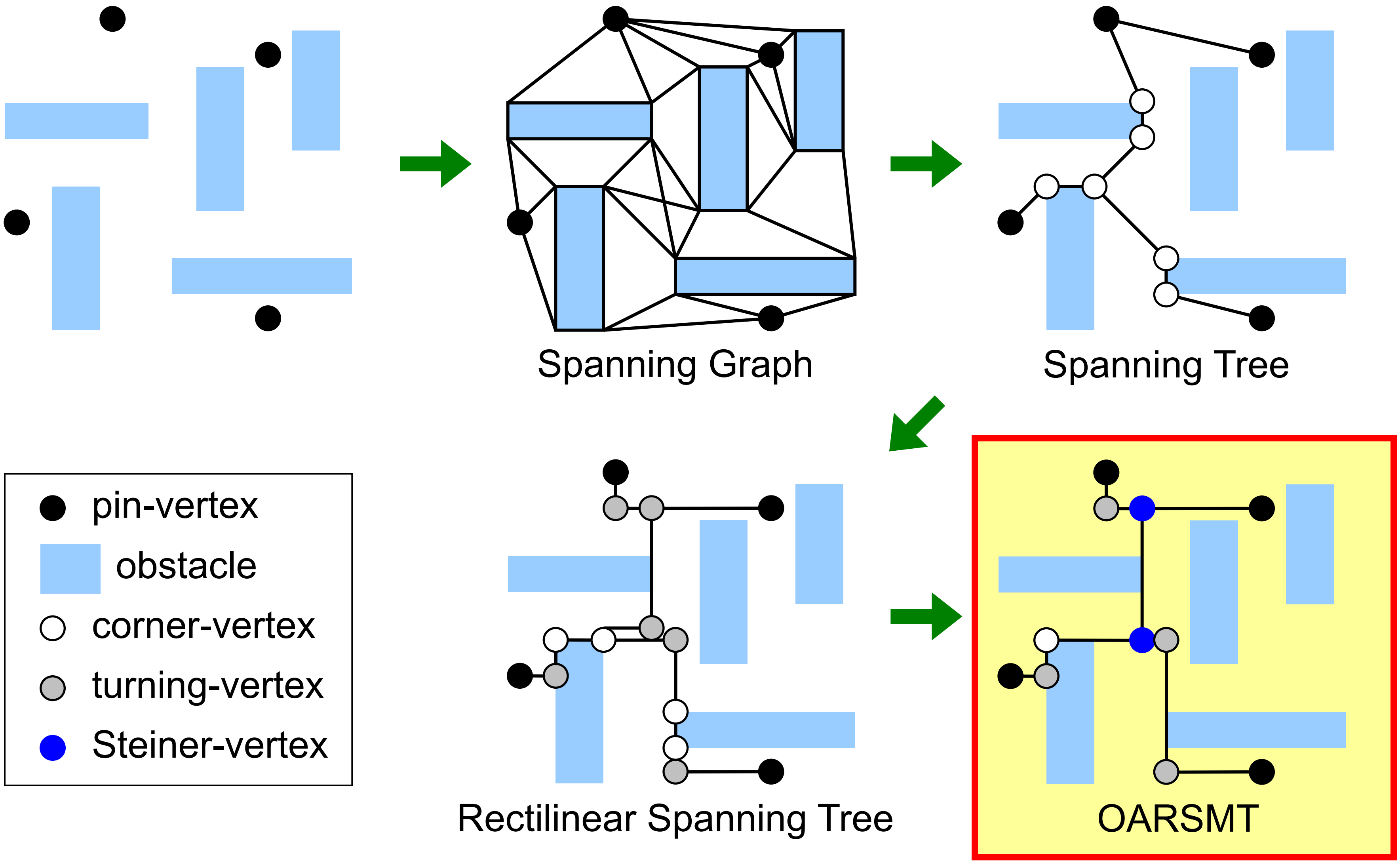
3. They are in the same region.



e and e' are transformed with edge overlap

(v_a, v_b) is transformed randomly

Algorithm Flow



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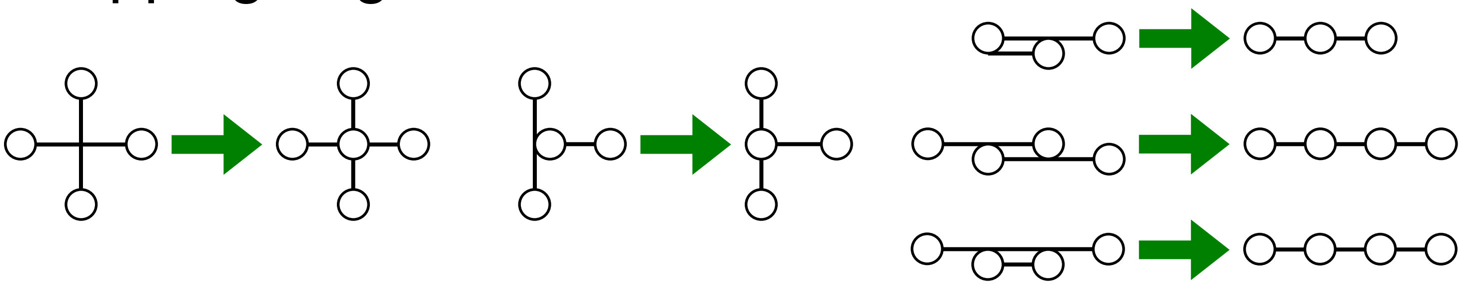
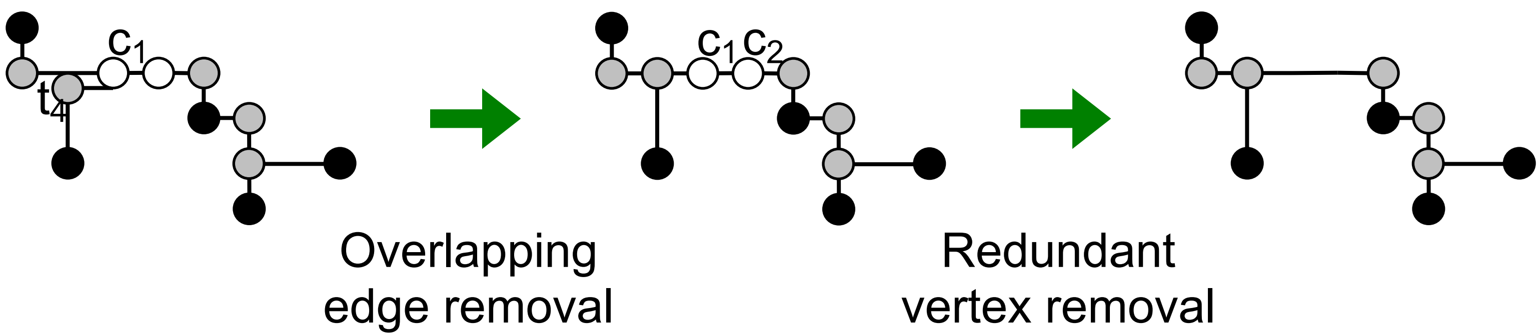
OARSMT Construction (1/2)

- Overlapping edge removal



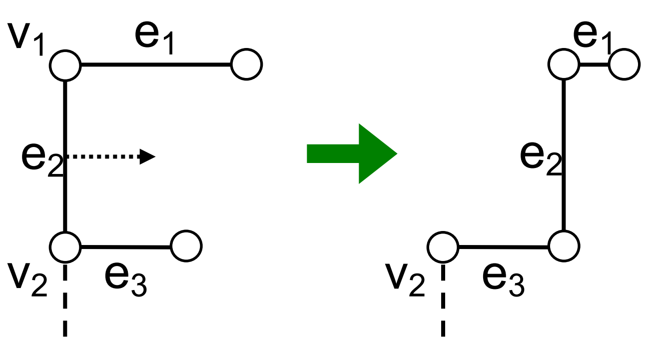
- Redundant vertex removal
  - A redundant-vertex is a non-pin-vertex with the degree of 2, and the two edges connecting to it are parallel.



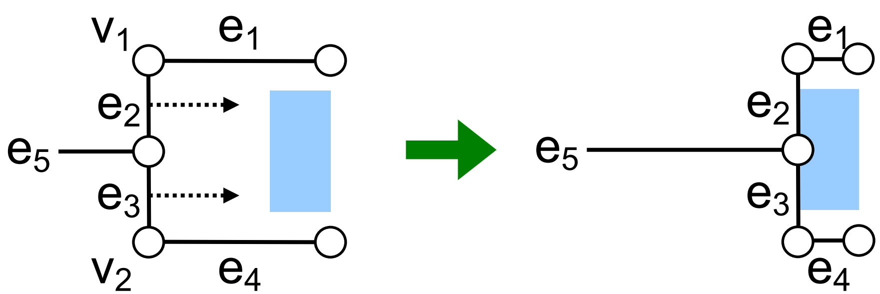
Courtesy of Prof. Y.-W. Chang

OARSMT Construction (2/2)

- U-shaped pattern refinement
  - A vertex satisfies the "U-shaped pattern refinement rules" if it is not a pin-vertex, and its degree is 2.
  - Two cases for the U-shaped pattern refinement:



One of the vertices v_1 and v_2 must satisfy the refinement rule.

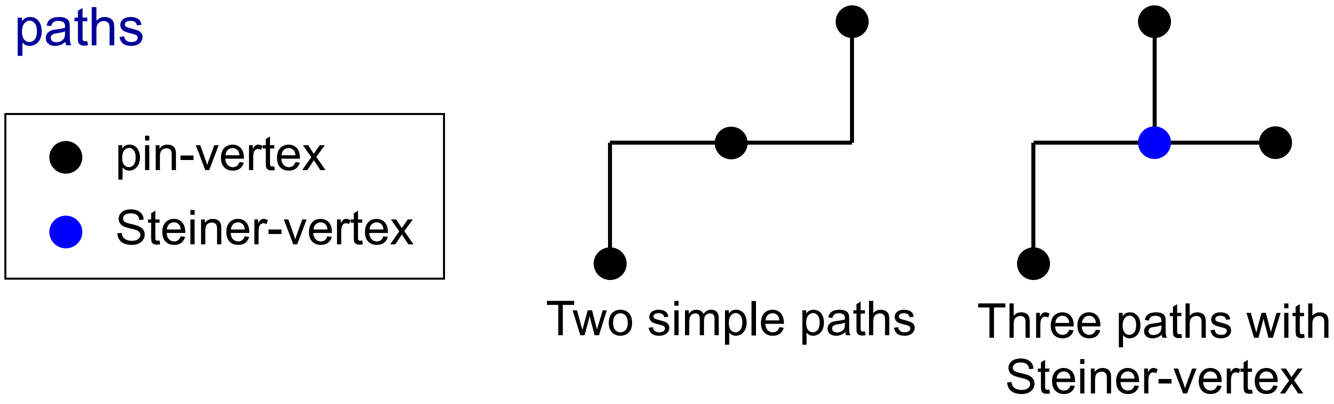


Both vertices v_1 and v_2 must satisfy the refinement rules.

Courtesy of Prof. Y.-W. Chang

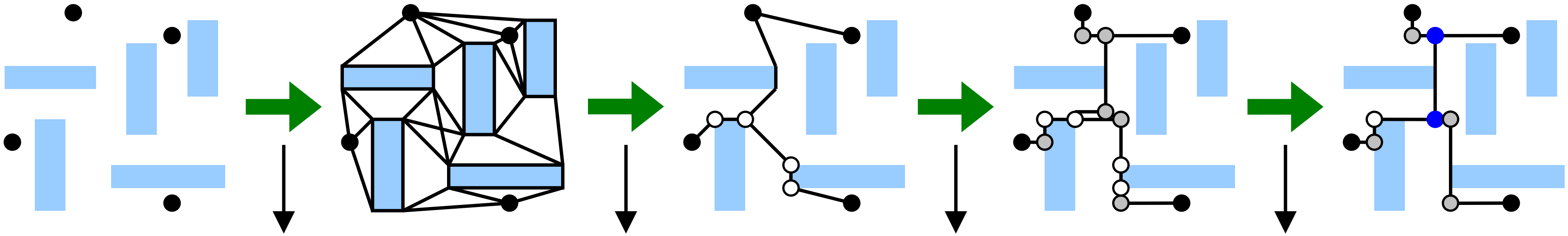
Properties of Our OARSMT

- Our OARSMT is an optimal solution for:
  - Any 2-pin net
  - Any 3-pin net without obstacles
  - Any net whose topology of an optimal solution contains only simple paths



- They are not guaranteed by any previous work.
- They give the sufficient but not necessary conditions for an optimal solution.
  - More optimal solutions may still be generated in other cases.

Complexity

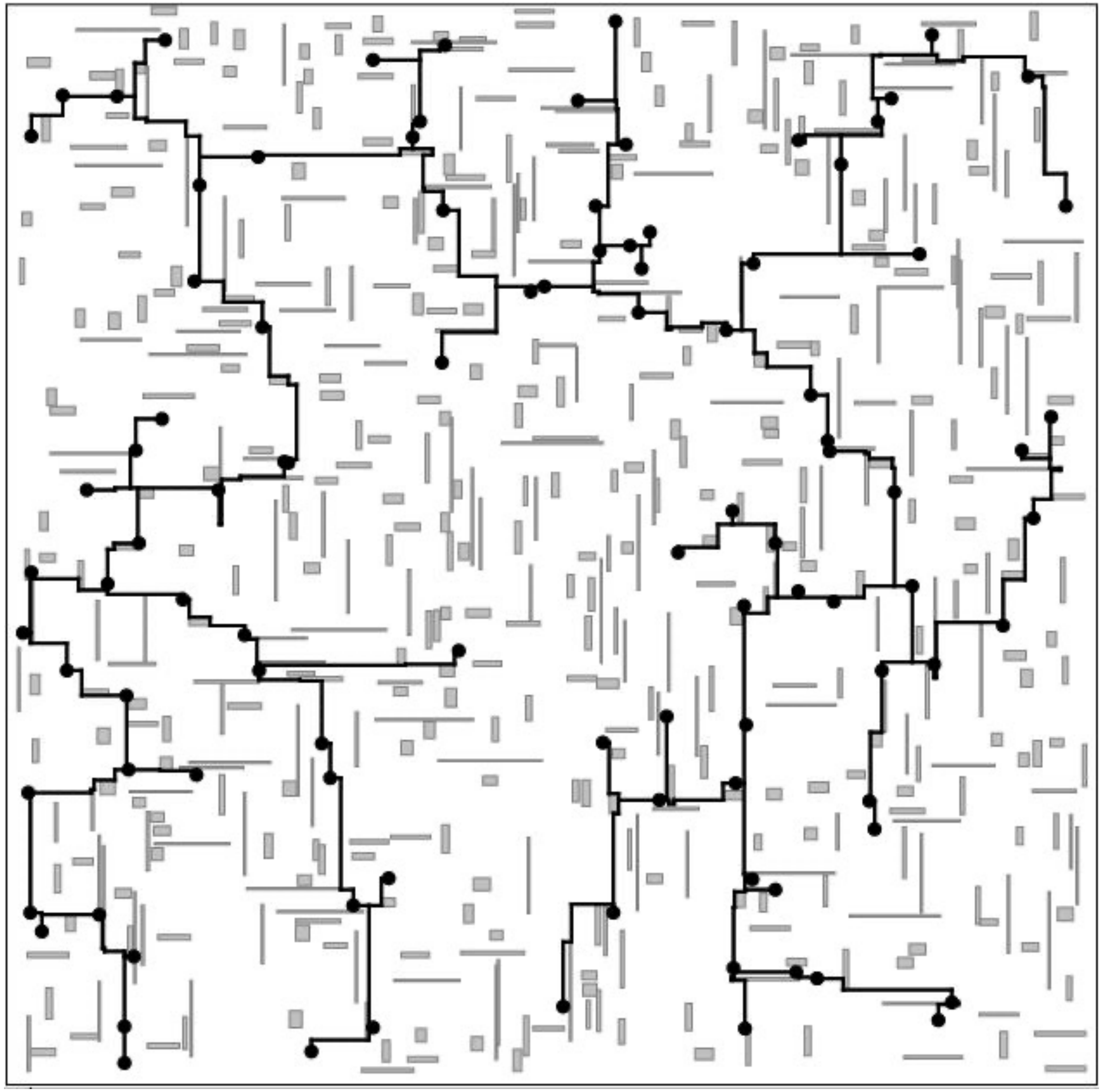


Spanning Graph Construction	Spanning Tree Construction	Rectilinear Spanning Tree Construction	OARSMT Construction
Worst Case			
$O(n^2 \lg n)$	$O(n^3)$	$O(n \lg n)$	$O(n^2)$
Practical Applications (# of edges in spanning graph is O(n).)			
$O(n \lg n)$	$O(n^2 \lg n)$	$O(n \lg n)$	$O(n^2)$

- Overall time complexity
  - $O(n^3)$ in the worst case
  - $O(n^2 \lg n)$ for practical applications
  - n: # of pin-vertices and corner-vertices

Courtesy of Prof. Y.-W. Chang

Experimental Results – Routing Result of rt3



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