A Comparison of The Discrete Fourier Transform and Two Discrete Wavelet Transforms (Haar and Daubechies 4)

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Implementations

Full Discrete Wavelet Transform

s - Dx1 input signal where D is of the form D=2^N for some +ve integer N

h - low pass filter of length L<=D

```
function y=dwt(s,h)
    D=length(s);
    L=length(h);
    y=zeros(D,1);

    % construct high pass filter g from h
    g=zeros(L,1);
    for i=1:2:L
        g(i)=h(L-i+1);
        g(i+1)=-h(L-i);
    end

    j=1;
    while L<=D
        W=zeros(D,1);

        D=D/2;
        lower=1;
        for i=1:D
            upper=lower+L-1;
            % concatenate entries from the beginning of
            % sequence if filter spans past the end of s
            if upper>2*D
                s_tmp=[s(lower:end);s(1:end-lower+1)];
            else
                s_tmp=s(lower:upper);
            end
            % apply filters g and h to ith set of L entries
            W(i)=dot(s_tmp,h); % H
            W(D+i)=dot(s_tmp,g); % G

            lower=lower+2; % shift filter by 2 positions
        end
        y(j:j+D-1)=W(D+1:end); % append G to output vector
        j=j+D;

        s=W(1:D); % set s to new approximation H for next level
    end
    y(j:end)=s; % append final s to output vector
end
```

Discrete Fourier Transform

s - Dx1 input signal

```
function s=dft(s)
    N=length(s);
    DFT=ones(N);

    for m=1:N-1
        for n=1:N-1
            DFT(m+1,n+1)=cos(-2*pi*m*n/N)+1j*sin(-2*pi*m*n/N);
        end
    end

    s=1/sqrt(N) * DFT*s;
end
```

Function for thresholding and calculating compression rate

s - Dx1 input signal

epsilon - threshold entries of s smaller than this

```
function [s,compression_rate]=threshold(s,epsilon)
    N=length(s);
    count=0;

    for i=1:N
        if abs(s(i))<epsilon
            s(i)=0;
            count=count+1;
        end
    end

    compression_rate=(N-count)/N;
end
```

Main

Constructing input signal s = {sin(2πn/512) : n = 0,...,511}

```
N=512;
s=zeros(N,1);

for n=0:N-1
    s(n+1)=sin(2*pi*n/N);
end
```

Haar low pass filter

```
h_haar=1/sqrt(2) * [1 1]';
```

Daubechies 4 low pass filter

```
c=2-sqrt(3);
h_db4=1/(sqrt(2)*(c^2+1)) * [1-c 1+c c*(c+1) c*(c-1)]';
```

Applying transforms to signal s

```
y_db4=dwt(s,h_db4);
y_haar=dwt(s,h_haar);
y_dft=dft(s);
```

Thresholding and calculating compression rates

```
epsilon=1/1000;
[y_db4_compressed,db4_compression_rate]=threshold(y_db4,epsilon);
[y_haar_compressed,haar_compresssion_rate]=threshold(y_haar,epsilon);
[~,dft_compression_rate]=threshold(y_dft,epsilon);
```

Reshaping resulting vectors for readability *(results in next page)*

```
res_db4=reshape(y_db4_compressed,8,64)';
res_haar=reshape(y_haar_compressed,8,64)';
```

db4_compression_rate = 0.2188
haar_compression_rate = 0.9551
dft_compression_rate = 0.003906

DFT < Daubechies 4 < Haar

The discrete fourier transform yields the best compression rate for this input sequence by a significant amount in comparison to the wavelet transforms. The Daubechies 4 wavelet transform does yield a relatively significant compression rate but the Haar transform does not.

Resulting Sequences (ordered row by row left to right)

From Daubechies 4 filter (showing the last 128 entries; all entries prior are 0)

	1	2	3	4	5	6	7	8
49	0.00E+00	0.00E+00	0.00E+00	1.16E-03	1.42E-03	1.67E-03	1.90E-03	2.11E-03
50	2.30E-03	2.47E-03	2.62E-03	2.74E-03	2.83E-03	2.90E-03	2.94E-03	2.95E-03
51	2.93E-03	2.89E-03	2.81E-03	2.71E-03	2.58E-03	2.43E-03	2.26E-03	2.06E-03
52	1.84E-03	1.61E-03	1.36E-03	1.09E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00
53	0.00E+00	0.00E+00	0.00E+00	-1.16E-03	-1.42E-03	-1.67E-03	-1.90E-03	-2.11E-03
54	-2.30E-03	-2.47E-03	-2.62E-03	-2.74E-03	-2.83E-03	-2.90E-03	-2.94E-03	-2.95E-03
55	-2.93E-03	-2.89E-03	-2.81E-03	-2.71E-03	-2.58E-03	-2.43E-03	-2.26E-03	-2.06E-03
56	-1.84E-03	-1.61E-03	-1.36E-03	-1.09E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00
57	3.80E-03	6.89E-03	9.72E-03	1.22E-02	1.42E-02	1.56E-02	1.64E-02	1.67E-02
58	1.62E-02	1.52E-02	1.35E-02	1.14E-02	8.79E-03	5.86E-03	2.70E-03	0.00E+00
59	-3.80E-03	-6.89E-03	-9.72E-03	-1.22E-02	-1.42E-02	-1.56E-02	-1.64E-02	-1.67E-02
60	-1.62E-02	-1.52E-02	-1.35E-02	-1.14E-02	-8.79E-03	-5.86E-03	-2.70E-03	0.00E+00
61	4.23E-02	7.12E-02	8.91E-02	9.36E-02	8.37E-02	6.12E-02	2.93E-02	-7.08E-03
62	-4.23E-02	-7.12E-02	-8.91E-02	-9.36E-02	-8.37E-02	-6.12E-02	-2.93E-02	7.08E-03
63	4.22E-01	5.14E-01	3.05E-01	-8.31E-02	-4.22E-01	-5.14E-01	-3.05E-01	8.31E-02
64	2.56E+00	-9.11E-01	-2.56E+00	9.11E-01	-7.81E+00	7.81E+00	7.68E+00	-7.68E+00

From DFT

```
All entries are 0 except
y_dft_compressed(2)=-11.314i and
y_dft_compressed(512)=+11.314i
```

From Haar filter

	1	2	3	4	5	6	7	8
1	-8.68E-03	-8.67E-03	-8.66E-03	-8.65E-03	-8.63E-03	-8.61E-03	-8.58E-03	-8.54E-03
2	-8.50E-03	-8.45E-03	-8.40E-03	-8.35E-03	-8.29E-03	-8.22E-03	-8.15E-03	-8.08E-03
3	-8.00E-03	-7.91E-03	-7.82E-03	-7.73E-03	-7.63E-03	-7.52E-03	-7.42E-03	-7.30E-03
4	-7.19E-03	-7.06E-03	-6.94E-03	-6.81E-03	-6.67E-03	-6.54E-03	-6.39E-03	-6.25E-03
5	-6.10E-03	-5.94E-03	-5.79E-03	-5.63E-03	-5.46E-03	-5.30E-03	-5.13E-03	-4.95E-03

6	-4.78E-03	-4.60E-03	-4.42E-03	-4.23E-03	-4.04E-03	-3.85E-03	-3.66E-03	-3.47E-03
7	-3.27E-03	-3.07E-03	-2.87E-03	-2.67E-03	-2.47E-03	-2.26E-03	-2.06E-03	-1.85E-03
8	-1.64E-03	-1.43E-03	-1.22E-03	-1.01E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00
9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.12E-03	1.33E-03	1.54E-03
10	1.75E-03	1.95E-03	2.16E-03	2.37E-03	2.57E-03	2.77E-03	2.97E-03	3.17E-03
11	3.37E-03	3.57E-03	3.76E-03	3.95E-03	4.14E-03	4.32E-03	4.51E-03	4.69E-03
12	4.87E-03	5.04E-03	5.21E-03	5.38E-03	5.55E-03	5.71E-03	5.87E-03	6.02E-03
13	6.17E-03	6.32E-03	6.47E-03	6.61E-03	6.74E-03	6.87E-03	7.00E-03	7.13E-03
14	7.24E-03	7.36E-03	7.47E-03	7.58E-03	7.68E-03	7.77E-03	7.87E-03	7.95E-03
15	8.04E-03	8.12E-03	8.19E-03	8.26E-03	8.32E-03	8.38E-03	8.43E-03	8.48E-03
16	8.52E-03	8.56E-03	8.59E-03	8.62E-03	8.64E-03	8.66E-03	8.67E-03	8.68E-03
17	8.68E-03	8.67E-03	8.66E-03	8.65E-03	8.63E-03	8.61E-03	8.58E-03	8.54E-03
18	8.50E-03	8.45E-03	8.40E-03	8.35E-03	8.29E-03	8.22E-03	8.15E-03	8.08E-03
19	8.00E-03	7.91E-03	7.82E-03	7.73E-03	7.63E-03	7.52E-03	7.42E-03	7.30E-03
20	7.19E-03	7.06E-03	6.94E-03	6.81E-03	6.67E-03	6.54E-03	6.39E-03	6.25E-03
21	6.10E-03	5.94E-03	5.79E-03	5.63E-03	5.46E-03	5.30E-03	5.13E-03	4.95E-03
22	4.78E-03	4.60E-03	4.42E-03	4.23E-03	4.04E-03	3.85E-03	3.66E-03	3.47E-03
23	3.27E-03	3.07E-03	2.87E-03	2.67E-03	2.47E-03	2.26E-03	2.06E-03	1.85E-03
24	1.64E-03	1.43E-03	1.22E-03	1.01E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00
25	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	-1.12E-03	-1.33E-03	-1.54E-03
26	-1.75E-03	-1.95E-03	-2.16E-03	-2.37E-03	-2.57E-03	-2.77E-03	-2.97E-03	-3.17E-03
27	-3.37E-03	-3.57E-03	-3.76E-03	-3.95E-03	-4.14E-03	-4.32E-03	-4.51E-03	-4.69E-03
28	-4.87E-03	-5.04E-03	-5.21E-03	-5.38E-03	-5.55E-03	-5.71E-03	-5.87E-03	-6.02E-03
29	-6.17E-03	-6.32E-03	-6.47E-03	-6.61E-03	-6.74E-03	-6.87E-03	-7.00E-03	-7.13E-03
30	-7.24E-03	-7.36E-03	-7.47E-03	-7.58E-03	-7.68E-03	-7.77E-03	-7.87E-03	-7.95E-03
31	-8.04E-03	-8.12E-03	-8.19E-03	-8.26E-03	-8.32E-03	-8.38E-03	-8.43E-03	-8.48E-03
32	-8.52E-03	-8.56E-03	-8.59E-03	-8.62E-03	-8.64E-03	-8.66E-03	-8.67E-03	-8.68E-03
33	-2.45E-02	-2.45E-02	-2.44E-02	-2.42E-02	-2.40E-02	-2.37E-02	-2.34E-02	-2.30E-02
34	-2.25E-02	-2.20E-02	-2.14E-02	-2.08E-02	-2.02E-02	-1.94E-02	-1.87E-02	-1.79E-02
35	-1.70E-02	-1.61E-02	-1.52E-02	-1.43E-02	-1.33E-02	-1.22E-02	-1.12E-02	-1.01E-02
36	-8.97E-03	-7.84E-03	-6.69E-03	-5.52E-03	-4.34E-03	-3.15E-03	-1.96E-03	0.00E+00
37	0.00E+00	1.66E-03	2.85E-03	4.05E-03	5.23E-03	6.40E-03	7.56E-03	8.69E-03
38	9.81E-03	1.09E-02	1.20E-02	1.30E-02	1.40E-02	1.50E-02	1.59E-02	1.68E-02
39	1.77E-02	1.85E-02	1.93E-02	2.00E-02	2.07E-02	2.13E-02	2.19E-02	2.24E-02
40	2.28E-02	2.33E-02	2.36E-02	2.39E-02	2.42E-02	2.43E-02	2.45E-02	2.45E-02
41	2.45E-02	2.45E-02	2.44E-02	2.42E-02	2.40E-02	2.37E-02	2.34E-02	2.30E-02
42	2.25E-02	2.20E-02	2.14E-02	2.08E-02	2.02E-02	1.94E-02	1.87E-02	1.79E-02

43	1.70E-02	1.61E-02	1.52E-02	1.43E-02	1.33E-02	1.22E-02	1.12E-02	1.01E-02
44	8.97E-03	7.84E-03	6.69E-03	5.52E-03	4.34E-03	3.15E-03	1.96E-03	0.00E+00
45	0.00E+00	-1.66E-03	-2.85E-03	-4.05E-03	-5.23E-03	-6.40E-03	-7.56E-03	-8.69E-03
46	-9.81E-03	-1.09E-02	-1.20E-02	-1.30E-02	-1.40E-02	-1.50E-02	-1.59E-02	-1.68E-02
47	-1.77E-02	-1.85E-02	-1.93E-02	-2.00E-02	-2.07E-02	-2.13E-02	-2.19E-02	-2.24E-02
48	-2.28E-02	-2.33E-02	-2.36E-02	-2.39E-02	-2.42E-02	-2.43E-02	-2.45E-02	-2.45E-02
49	-6.93E-02	-6.87E-02	-6.74E-02	-6.55E-02	-6.29E-02	-5.97E-02	-5.60E-02	-5.17E-02
50	-4.69E-02	-4.17E-02	-3.60E-02	-3.01E-02	-2.38E-02	-1.73E-02	-1.06E-02	-3.83E-03
51	2.98E-03	9.76E-03	1.65E-02	2.30E-02	2.93E-02	3.53E-02	4.10E-02	4.63E-02
52	5.11E-02	5.55E-02	5.93E-02	6.26E-02	6.52E-02	6.72E-02	6.86E-02	6.93E-02
53	6.93E-02	6.87E-02	6.74E-02	6.55E-02	6.29E-02	5.97E-02	5.60E-02	5.17E-02
54	4.69E-02	4.17E-02	3.60E-02	3.01E-02	2.38E-02	1.73E-02	1.06E-02	3.83E-03
55	-2.98E-03	-9.76E-03	-1.65E-02	-2.30E-02	-2.93E-02	-3.53E-02	-4.10E-02	-4.63E-02
56	-5.11E-02	-5.55E-02	-5.93E-02	-6.26E-02	-6.52E-02	-6.72E-02	-6.86E-02	-6.93E-02
57	-1.95E-01	-1.88E-01	-1.74E-01	-1.52E-01	-1.25E-01	-9.35E-02	-5.81E-02	-2.04E-02
58	1.80E-02	5.58E-02	9.14E-02	1.24E-01	1.51E-01	1.72E-01	1.87E-01	1.95E-01
59	1.95E-01	1.88E-01	1.74E-01	1.52E-01	1.25E-01	9.35E-02	5.81E-02	2.04E-02
60	-1.80E-02	-5.58E-02	-9.14E-02	-1.24E-01	-1.51E-01	-1.72E-01	-1.87E-01	-1.95E-01
61	-5.44E-01	-4.62E-01	-3.10E-01	-1.11E-01	1.05E-01	3.05E-01	4.58E-01	5.42E-01
62	5.44E-01	4.62E-01	3.10E-01	1.11E-01	-1.05E-01	-3.05E-01	-4.58E-01	-5.42E-01
63	-1.44E+00	-6.02E-01	5.85E-01	1.43E+00	1.44E+00	6.02E-01	-5.85E-01	-1.43E+00
64	-3.00E+00	2.97E+00	3.00E+00	-2.97E+00	-6.25E-02	6.25E-02	1.44E+01	0.00E+00