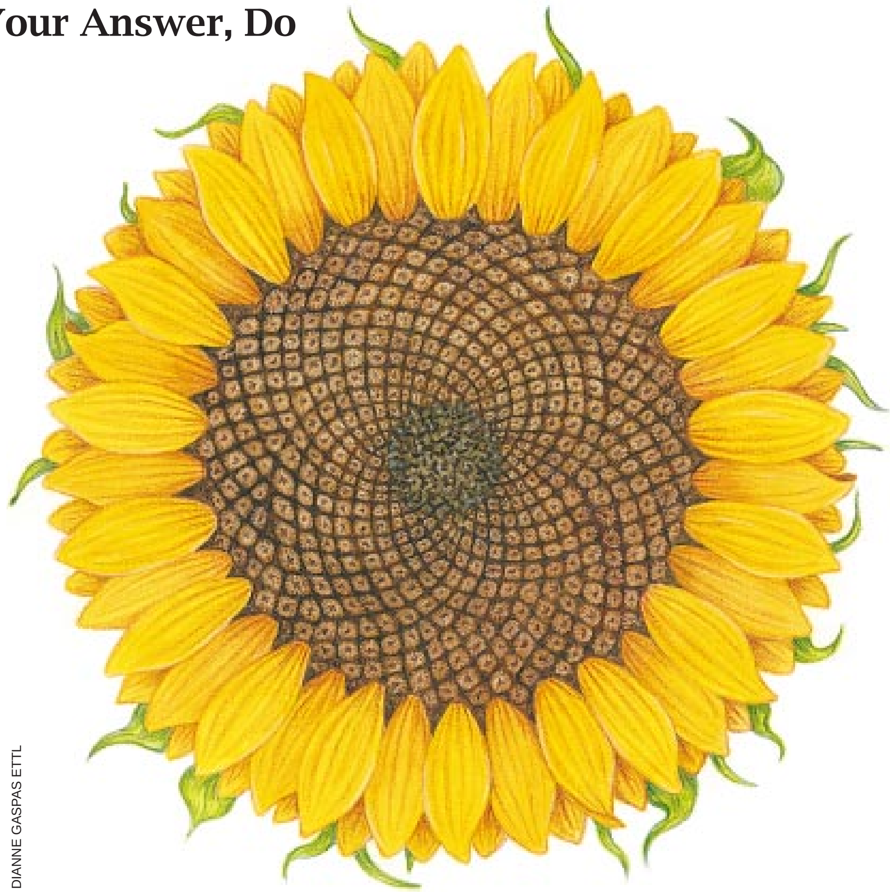
 by Ian Stewart

Daisy, Daisy, Give Me Your Answer, Do


DIANNE GASPAS ETTL

SUNFLOWER HEAD, like that of daisies and many other flowers, contains two families of interlaced spirals—one winding clockwise, the other counterclockwise. Models show that this regular pattern results from the dynamics of plant growth.

Bees buzzed drowsily, the sun beamed down and sunflowers waved in the breeze. Grimes the shepherd boy snored under a tree, while Bumps the goose girl made daisy chains. Suddenly she stopped.

"Grimes! I've just found a daisy with 31 petals. Usually this kind has 34."

Grimes sat up and stretched. "Really? It's curious that there's a specific number. Though I suppose that the flower's genes must specify—"

"I don't see that they must. I mean, genes tell plants how to make chlorophyll, but they don't tell them to make it green. That's chemistry, not genetics."

Grimes had been through this argument with her before. "Yeah, sure. Some features of the morphology of living creatures are genetic in origin, and others are a consequence of physics, chemistry and the dynamics of growth."

"Right," Bumps said. "Genetics can give rise to pretty much anything, whereas physics, chemistry and dynamics produce mathematical regularities."

"I wouldn't say that 34 is a very striking regularity," Grimes said.

Bumps pulled petals from her daisy. "Agreed, but the numbers that arise in plants—not just for petals but for all sorts of other features—are normally very special. Lilies have three petals, buttercups have five, marigolds 13, asters 21, and most daisies have 34, 55 or 89. You don't find any other numbers very often. The main exceptions are when those same numbers occur doubled or when the so-called anomalous series appears—3, 4, 7, 11, 18 and so on."

Grimes scratched his head. "I've seen those numbers before."

"Yes. The numbers 3, 5, 8, 13, 21, 34, 55, 89 form the beginning of—"

"The Fibonacci series," Grimes said in triumph. "Each number is the sum of the two that precede it. Your 'anomalous series' has the same pattern."

"Right," Bumps replied. "Fibonacci was a medieval mathematician who invented the series to model the growth of rabbit populations. That didn't work very well. But his numbers turn up in many different places. Look at how a sunflower's florets are arranged."

Grimes eyed a nearby plant myopically. "Wow. Spirals—34 spirals wind clockwise, like the spokes of a wheel, but curved. Counterclockwise, there are 55."

"Consecutive Fibonacci numbers," Bumps added. "The precise numbers depend on the species of sunflower, but you get 34 and 55, or 55 and 89, or even 89 and 144. Daisies, too."

"Weird," Grimes said.

"Entirely," Bumps agreed. "If genetics can give a flower any number of petals it likes, why such a preponderance of Fibonacci numbers?"

Grimes snapped his fingers. "You're telling me that the numbers arise through some mathematical mechanism? Physics, or chemistry, or—"

"Dynamics," Bumps said firmly.

"Has somebody actually explained how plant growth might yield Fibonacci numbers?" Grimes asked.

"Well, lots of people have suggested many different kinds of answers. But for me, the most dramatic insight comes from Stéphane Douady and Yves Couder of the Laboratory of Statistical Physics in Paris. They recently showed that the dynamics of plant growth could account for the Fibonacci numbers—and much more.

"The basic idea is an old one," Bumps continued. "If you look at the tip of the shoot of a growing plant, you can detect the pieces from which all the main features of the plant—leaves, petals, sepals, florets or whatever—form. At the center of the tip is a circular region of tissue having no special features, called the apex. Around the apex, one by one, tiny lumps called primordia emerge. Each primordium migrates away from the apex and eventually develops into a leaf, petal or the like. So you must explain why you see spiral shapes and Fibonacci numbers in the primordia."

"How?" Grimes wondered.

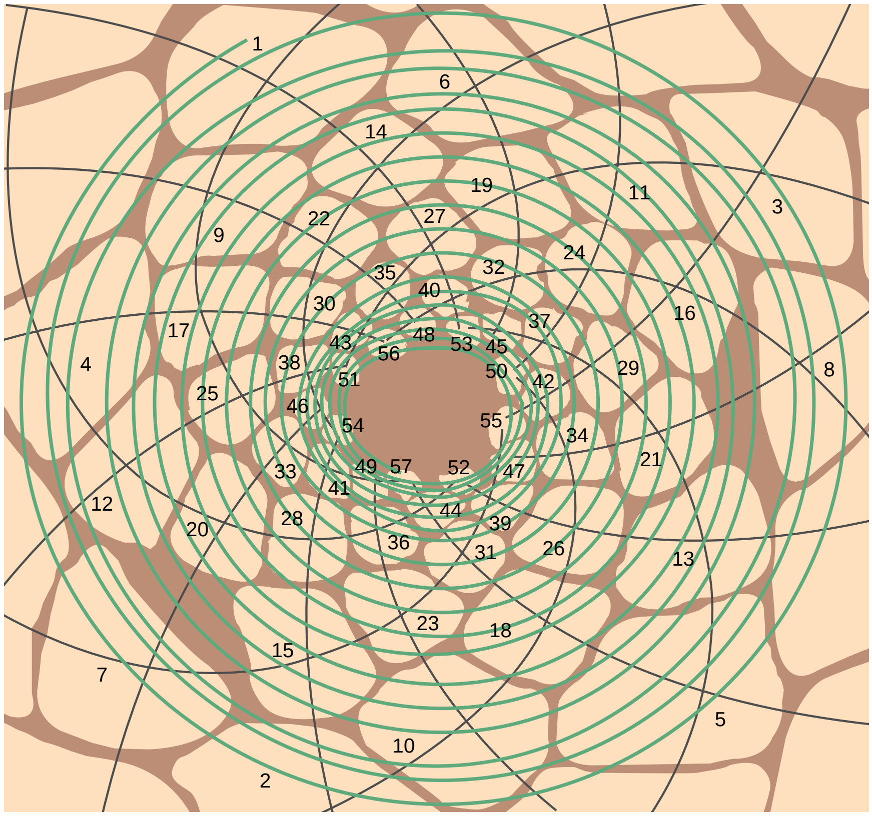
"The first step is to appreciate that the spirals most apparent to the human eye—the parastichies—are not fundamental. The most important spiral is formed by considering the primordia in their order of appearance [*see illustration at right*]. Primordia that appear earlier travel farther, so you can deduce their order based on their distance from the apex. You find that the primordia are spaced rather sparsely along a tightly wound spiral, called the generative spiral. With me so far, Grimes?"

"No problem. But why do the primordia form spirals?"

"That comes a bit later. The pioneer crystallographer Auguste Bravais and his brother, Louis, observed one essential quantitative feature in 1837. They drew lines from the center of each primordium to the center of the apex and measured the angles between successive primordia, as seen from the center of the apex. Look at the angle between the primordia numbered 29 and 30, or 30 and 31. What do you notice?"

Grimes squinted. "They look the same."

"Precisely. The successive angles are pretty much equal; their common value is called the divergence angle. The primordia are equally spaced, in an angular sense, along the generative spiral. How big do you think the divergence angle is?"



JOHNNY JOHNSON

PRIMORDIA, numbered from 1 to 57, appear in sequence along a tight spiral **(green).** *Another 21 spirals, called parastichies, are more obvious. Eight parastichies curve clockwise, and 13 run counterclockwise* **(black).**

"Quite big, more than a right angle."

"Good. It's usually close to 137.5 degrees." Bumps looked smug, although Grimes didn't know why.

"Take consecutive numbers in the Fibonacci series," Bumps began to explain.

"Like 34 and 55?"

"Exactly. Now, form the fraction 34/55 and multiply by 360 degrees."

Grimes fished out the pocket calculator he normally used to keep track of feedstocks. "Um. It's 222.5 and a bit."

"You can measure angles externally or internally. Your answer is more than 180 degrees, so subtract it from 360."

"Right," Grimes said, punching the buttons. "That's 137.5 degrees."

"You got it. The ratio of successive Fibonacci numbers, as they get bigger, gets closer to 0.618034, which is $(\sqrt{5}-1)/2$, the so-called golden number, denoted by the Greek letter phi, φ."

"I thought the golden number was $(\sqrt{5}+1)/2$," Grimes queried.

"That's 1.618034. The golden number equals both $1+\varphi$ and $1/\varphi$. If you look at the ratio the other way up, say, $55/34 = 1.6176$, then the limit of increasingly larger Fibonacci ratios tends toward 1.618034 instead. At any rate, the key to the whole shebang is the 'golden angle,' which is $360(1-\varphi)$ degrees, or 137.50776 degrees. The Bravais brothers observed that the angle between successive primordia is very close to the golden angle."

"Gotcha."

"If you plot successive points on a tightly wound spiral at angles of 137.5 degrees, because of the way neighboring points align, you get two families of interpenetrating spirals. And because of the relation between Fibonacci numbers and the golden number, the numbers of spirals in the two families are consecutive Fibonacci numbers."

Grimes watched the butterflies flitting about the meadow for a moment. "So it all boils down to explaining why successive primordia are separated by the golden angle?"

"Yes. Everything else follows from that, provided you assume that successive primordia spring up around the edge of the apex—as suggested by Wilhelm Hofmeister in 1868—and that they move away in a radial direction."

"Does the speed at which they move matter?" Grimes asked.

"Definitely. Because of the way plants grow, migrating primordia actually speed up as the radius increases—keeping a velocity proportional to the radius."

"And that's where this theory of Douady and Couder comes in?"

"That's right," Bumps said. "They built their ideas on an earlier insight. If you model plant seeds as circular disks having a fixed radius and pack them together as closely as possible, while retaining a constant divergence angle of 137.5 degrees, then the nth seed (counting from the newest to the oldest) must be placed at a distance proportional to the square root of n. So the golden angle allows the seeds to pack most efficiently."

"Say again?"

"Well, suppose you did something stupid and used a divergence angle of 180 degrees, which divides 360 degrees exactly. Then successive primordia would be arranged along two opposite radial lines. In fact, if you use any rational multiple of 360 degrees—an angle that can be expressed as $360p/q$ for whole numbers p and q—you get q radial lines and big gaps between them."

Grimes nodded sagely. "So the seeds don't pack efficiently."

"Precisely. To do so requires a divergence angle that is an irrational multiple of 360 degrees—the more irrational, the more efficient. Number theorists have long known that the most irrational number is the golden number."

Grimes looked baffled. "What do you mean, 'most irrational'? Numbers are either irrational or not, right?"

"Yes, but some are more irrational than others. Remember that the ratios of successive Fibonacci numbers tend toward the golden number φ. So φ is

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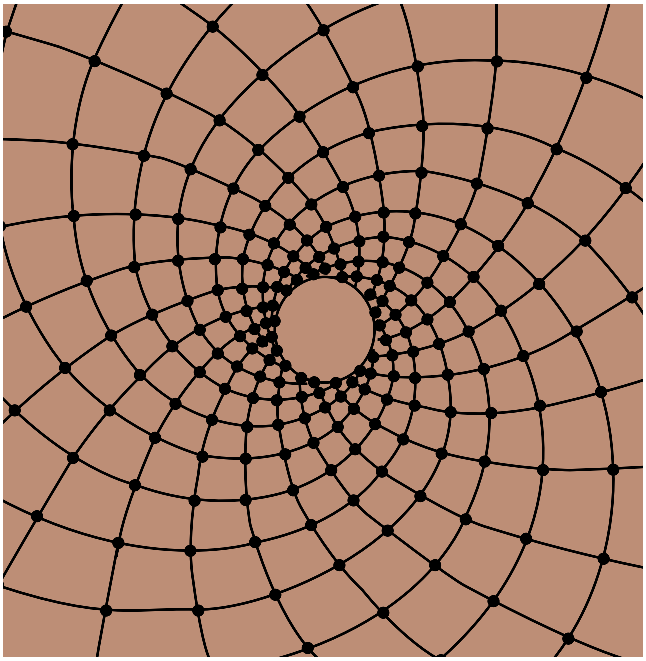

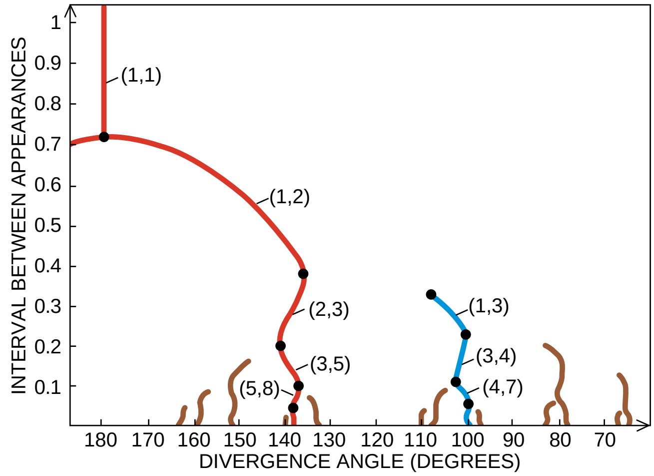

***ARRANGEMENT of primordia shown here is produced by separating points by the golden angle along a tightly wound spiral* (left). *The number of resultant parastichies depends on the time elapsed between the appearance of successive primordia* (right). *The main curve corresponds to pairs of consecutive Fibonacci numbers* (red). *The secondary curve gives the anomalous series* (blue).**

the limit of the sequence $^2/_3$, $^3/_5$, $^5/_8$ and so on. Those are rational approximations that get closer to, but never equal, φ. We can measure how 'irrational' φ is by seeing how quickly the differences between these fractions and φ shrink toward zero. In fact, they shrink more slowly for φ than they do for any other irrational number."

"So the golden number is distinct from every other number based on this simple mathematical property."

"Good point," Bumps said. "Well, Douady and Couder explained the golden angle as a consequence of dynamics, rather than postulate it directly on the grounds of efficient packing. They assumed that successive elements, representing primordia, form at equal intervals of time on the rim of a small circle, or the apex. These elements then migrate radially at some initial velocity and repel one another—a condition that ensures continuous motion and that each new element appears as far as possible from its predecessors."

"You mean it pops up in the biggest gap?" Grimes clarified.

"Grimes, you have a wonderful way with the English language. It's a good bet that such a system will pack efficiently, and so you would expect the golden angle to arise of its own accord. And it does—though with some interesting frills."

"Such as?"

"There are two ways to work out what happens. One is to perform an experiment, as did Douady and Couder. Instead of using plants, though, they filled a circular dish with silicone oil and placed it in a vertical magnetic field. Next they periodically dropped small amounts of magnetic fluid into the center of the dish. The magnetic field polarized the drops, which then repelled one another. To send the drops in a radial direction, they increased the magnetic field at the edge of the dish. The patterns that appeared depended on how much time passed between successive drops. But very often the drops lay on a spiral separated by divergence angles very close to 137.5 degrees."

"The golden angle!" Grimes exclaimed suddenly.

"Douady and Couder produced similar results from computer calculations. In detail, they found that the divergence angle depends on the time elapsed between drops, according to a complicated branching pattern of wiggly curves [*see right illustration above*]. Each section of a curve between adjacent wiggles corresponds to a particular pair of numbers of spiral families. The main branch runs close to a divergence angle of 137.5 degrees, and along it you find all possible pairs of consecutive Fibonacci numbers, in numerical sequence. The gaps between branches represent 'bifurcations' where the dynamics undergo significant changes."

Grimes thought for a few moments. "But there are branches that aren't close to 137.5 degrees, too."

"Yes. The main one corresponds to the anomalous series. The appropriate timing in this same model produces the most common exceptions to the Fibonacci rule and the Fibonacci rule itself—which shows why the exceptions occur while making it clear that they aren't really exceptions at all. But of course nobody is suggesting that botany is quite as perfect as this model. In many plants the rate of the appearance of primordia can speed up or slow down. In fact, whether a primordium becomes a leaf or a petal often accompanies such variations."

"So maybe a plant's genes affect the timing of the appearance of the primordia?" Grimes asked.

"Exactly. But genes need not tell them how to space the primordia. That's done by dynamics. It's a partnership of physics and genetics."

Grimes waved a particularly well-developed sunflower in the air. "You think maybe if I ate these seeds they'd improve my mathematical ability?"

"Try it," Bumps said. Grimes started to nibble at the seeds. "On the other hand—"

He stopped. "What?"

"Don't forget why Fibonacci invented his numbers. Maybe you'll turn into a rabbit."

FURTHER READING

The Algorithmic Beauty of Plants. Przemyslaw Prusinkiewicz and Aristid Lindenmayer. Springer-Verlag, 1990.

Phyllotaxis as a Self-Organized Growth Process. Stéphane Douady and Yves Couder in *Growth Patterns in Physical Sciences and Biology.* Edited by Juan M. Garcia-Ruiz et al. Plenum Press, 1993.

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