ON EXPANSION OF LEBESGUE INTEGRABLE FUNCTIONS IN SERIES OF LEGENDRE FUNCTIONS

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The Legendre functions considered are certain solutions $y = P_\nu^\mu(x)$, on $-1 < x < 1$ of the following equation (see [1]):

$$\frac{d}{dx}\left((1-x^2)\frac{dy}{dx}\right) + \left(\nu(\nu+1) - \frac{\mu^2}{1-x^2}\right)y = 0. \tag{1}$$

In our report we discuss the possibility for an integrable function f to be expanded in series of Legendre functions. The results presented generalize those obtained in [2].

Theorem 1. *If* $(1-t^2)^{-1/4}f(t) \in L(-1,1)$ *and* f *satisfies the Dini condition at a certain* $a \in (-1,1)$ *(see e.g. [3]),* $|\mathrm{Re}\,\mu| < 1/2$, ν *is not a half of an odd integer, and*

$$a_n = (-1)^n \frac{\nu+n+\frac{1}{2}}{2\cos\nu\pi} \int\limits_{-1}^{1} f(t) P_{\nu+n}^{-\mu}(-t)\,dt,$$

then

$$f(x) = \sum_{-\infty}^{+\infty} a_n P_{\nu+n}^{\mu}(x),$$

where P_k^μ *are determined in* (1).

Sketch of the proof. ...

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References

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