Automatic Curriculum Construction for Deep Models Using Active Learning

Ian McWilliam

Master of Science
Artificial Intelligence
School of Informatics
University of Edinburgh
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Abstract

Curriculum Learning refers to the study of methods for improving supervised learning performance by presenting training data to the model in a meaningful order. Motivated by the way in which animals learn, this is usually achieved by using 'easy' samples in the early stages of training, allowing the model to learn basic concepts that can be used as building blocks for learning more complex, 'hard' samples in later training epochs. While learning curricula have been applied succesfully in a variety of studies, one of the main difficulties is that few real world datasets have a clear delineation between 'easy' and 'hard' training samples, often making handcrafted learning curricula infeasible. There is growing interest therefore in implementing methods for automating the construction of learning curricula.

In this paper we use uncertainty methods from the field of active learning to infer training sample difficulty, enabling the automatic construction of a variety of curriculum methods for deep models, which we term *active curricula*. We introduce two broad categories of active curricula; *bootstrapped active curricula*, which use a pretrained baseline model to calculate sample difficulty, and *dynamic active curricula*, which estimates sample difficulty dynamically throughout training, producing an adaptive curriculum that evolves as the model is trained. We test our curriculum methods on the Geometric Shapes, MNIST and CIFAR 10 datasets, showing consistent test set outperformance of the different active curriculum methods relative to baseline models, providing support for the use of curriculum learning and introducing several flexible methods for automating curriculum construction

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Declaration

I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification except as specified.

(*Ian McWilliam*)

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Chapter 1

Introduction

1.1 Motivation

Supervised learning is the area of machine learning in which algorithms learns the relationships between a set of input features and corresponding 'ground truth' labels, the ultimate goal being to construct a predictive model of the relationships between the inputs and the labels in order to predict the labels of future, unseen input samples [54]. Deep learning models perform this task by building hierarchical representations of the input features throughout a multitude of layers, often using feature maps such as convolutions to construct complex representations of the inputs [32]. Supervised learning algorithms are fitted using a *training set* of example input-label pairs; in deep models this is usually achieved by sampling uniformly from the entire training set throughout training, adjusting the model parameters to better fit the training data [44]. *Curriculum Learning* [6] [35], is the field of study which proposes that model learning performance can be improved by fitting the model on training samples in a meaningful order, typically by using 'easy' samples in the early stages of training in order to allow the model to learn basic concepts that be used as building blocks for training on more difficult samples [12]. A related area of research is *active learning*, which is generally used when there is a prohibitive cost to obtaining training sample labels for supervised learning [43]. The goal of active learning is to identify which samples, usually from a pool of unlabeled candidate samples, would improve model performance the most if they were labeled and added to the training set [43]. A common active learning approach is that of *uncertainty sampling* [43], wherein the samples that the model is most uncertain about classifying are chosen to be included in the training data.

A substantial challenge with when implementing curriculum learning is that in

many domains however it is challenging to identify a clear delineation between 'easy' and 'hard' samples through which to implement curriculum learning [6] [19]. As interest in curriculum learning has grown, there have been an increasing number of studies into how the process of curriculum construction can be automated so as to remove the requirement of handcrafted curricula, for example [19] [27] [41] [53]. In this paper we propose that methodologies developed for active learning, in particular uncertainty sampling, are well suited to estimating the difficulty of training samples, allowing for the automatic construction of learning curricula that will improve the training of deep networks on a wide range of machine learning problems. Specifically, we can use the classification uncertainty of deep models to approximate the difficulty of training samples, which can then be used to divide the training data into tasks of ascending difficulty, which can then be used to automatically construct a learning curriculum.

1.2 Contribution

In this paper we introduce two categories of automated curriculum construction, which we broadly term *Active Curricula*. In Chapter 4 we implement the first approach, termed *Bootstrapped Active Curricula*, wherein we use the classification uncertainty of a pre-trained baseline line model to estimate the difficulty of the training data in the Geometric Shapes [7] dataset. We use the uncertainty scores to divide the training data into equally sized tasks, training an identical curriculum model on the tasks in increasing order of difficulty. We show that the curriculum model consistently outperformed the baseline model, supporting the hypothesis that training with a curriculum can improve model performance, and suggesting that using model uncertainty can be an effective way of inferring sample difficulty in order to automate the curriculum construction process.

In Chapter 5 we develop this idea further by introduced *Dynamic Active Curricula*, where, instead of using a baseline model to estimate sample difficulty, the classification uncertainty of the curriculum model itself is calculated throughout training, resulting in a dynamic learning curriculum that adapts throughout the training process. We test several dynamic curriculum methods on the Geometric Shapes dataset, as well as the MNIST handwritten digit recognition task [33] and the CIFAR 10 image recognition dataset [30]. We show that the dynamic curriculum methods lead to a further improvement in model performance over the boostrapped curriculum method for the Geometric Shapes dataset, as well as consistently outperforming baseline models in the MNIST

and CIFAR 10 datasets. As well as presenting the results of the experiments we analyse the way in which the various curricula affect the model training, showing how different uncertainty estimation approaches affect the training samples provided to the model at different stages of training.

1.3 Document Structure

The subsequent chapters of this thesis will be organised as follows:

- Background - Here we will introduce in more detail the topics introduced above, giving the reader the background required to understand and appreciate the rest of work. We will also develop some of the terminology and notation that will be used in the rest of the paper.
- Related Work - In this chapter we set our contributions in the context of related studies, discussing various other works which have tested approaches for automating curriculum construction to improving learning performance.
- Bootstrapped Active Curricula - In this chapter we introduce the bootstrapped active curriculum approach, explaining the curriculum construction, the methods used to test it, as well as the results of the experiments and subsequent discussion.
- Dynamic Active Curricula - Motivated by the previous chapter, we test methods for dynamically constructing and implementing learning curricula throughout training, again laying out the curriculum construction approach, the methods and experiments used to test its efficacy and a discussion of the results of the experiments.
- Conclusion and Further Work - Finally, we summarise the findings of the thesis and suggest directions in which future work can build on the experiments detailed in this paper.

Chapter 2

Background

2.1 Supervised Learning

Machine learning is the field of study concerned with building algorithmic systems that can automatically infer patterns and relationships in data. While machine learning has several main subfields, for example reinforcement [48] and unsupervised learning [21], the most commonly studied area of the field is arguably *supervised* learning. Supervised learning is characterised by learning a function that maps inputs to outputs (or 'labels'), using a *training set* of example input-output pairs [54], (supervised learning has previously been referred to as "learning from examples" [10]). The training set, which we will denote by $\mathcal{T}$, consists of a number of inputs-output pairs: $\mathcal{T} = \{(x_1, y_1), (x_2, y_2), ...(x_N, y_N)\}$, where N is the number of examples in the training set. We will refer to an input-output pair from the training set as a 'sample' or an 'example' interchangeably. Concretely, the goal of supervised learning is to find a function $f : X \to Y$, where X denotes the input space and Y the output space, so as to minimize the *generalization error* of the function [37]. The generalization error is defined as the expected error of the function averaged over future input-output samples [37], where the error is defined using a loss function $L : Y \times Y \to \mathbb{R}$, such that $L(f(x_i), y_i)$ gives the error of the function on the input-output pair $\{x_i, y_y\}$. Generalization error can therefore be defined as $\mathbb{E}_{X \times Y}(L(f(x), y))$; as finding a closed form solution for the generalization error is generally intractable, it is usually approximated using the empirical error on a held out *test set* of samples that were not used when fitting the function [37]. Using a test set of M input-output examples, we therefore approximate the generalization error as $\frac{1}{M} \sum_{i=1}^{M} L(f(x_i), y_i)$. A supervised learning algorithm is a method for fitting the function f using the training set $\mathcal{T}$, usually by

minimizing the loss of the function over the input-output samples in the training set [37], the process of fitting the function to the training data is referred to as *training*. In order to avoid *overfitting*, where the function has a low error on the training set but a high generalization error[1], the loss function often includes regularization terms that penalize function complexity, or the range of functions that can be fitted by the algorithm is constrained [46]. Furthermore, in addition to a training set and test set, a *validation set* of samples is often used to monitor the function error on samples outwith the training set during training, in order to measure whether or not the function is overfitting [54].

2.2 Deep Learning

Deep learning, as applied in the supervised learning setting, refers to algorithms which model the relationships in the training set using complex, hierarchical representations of the data, usually doing so using multiple 'deep' layers, as well as feature maps such as convolutions or recurrent layers [32]. Deep learning models fit functions by passing the input signals through several layers of transformations, multiplying the signal by parameterised weights and then passing the weighted signal through non-linear *activation functions* [54]. The combined model of weights and transformations is referred to as a 'deep network', or 'deep neural network', as a result of similarities to the functionalities of neurons in brain [23]. This approach allows the algorithm to transform the input space in order to make the network outputs closely match the output, with the weight parameters being tuned throughout training in order minimise the network loss on the training set. Deep learning has been the subject of much research in recent years, showing state of the art performance in a range of domains including image recognition and natural language processing [32] [5].

2.2.1 Feedforward Networks

Arguably the simplest example of a deep network is a *feedforward network*, usually consisting of an input layer, an output layer and several intermediate 'hidden' layers [54]. There are a variety of 'hyper-parameters' that must be set when implementing a feedforward network, or any deep model, for example the architecture (i.e. number of

[1] More specifically, overfitting refers to the situation where the function has a low error on the training set and a higher generalization error than a similar function with a higher error on the training set.

hidden layers and the number of nodes each layer contains), learning rate and chosen activation functions [54].

In classification problems, where the aim is to classify an input x into one of several possible classes, the output activation function used is the softmax function [46], which maps the output vector to a vector of probabilities, summing to one. Denoting by $\mathbf{h}$ the signal forward propagated through the network to the output layer, the final output vector of the network is then given by $softmax(\mathbf{h})$, where:

$$softmax(\mathbf{h})_i = \frac{e^{h_i}}{\sum_{c=1}^{C} e^{h_c}}, \tag{2.1}$$

where C is the number of possible class labels. The loss function typically used to train such networks is the *cross-entropy* loss function, defining the output vector of the network for input sample x by $f(x) = \tilde{\mathbf{y}}$, cross-entropy error is defined as follows:

$$crossentropy(\tilde{\mathbf{y}}, \mathbf{y}) = -\sum_{c=1}^{C} y_c log(\tilde{y}_c). \tag{2.2}$$

In several of the experiments in this report we will use feedforward networks for image classification, using a softmax output layer and cross-entropy error as the chosen loss function.

2.2.2 Convolutional Networks

The state of the art deep models for image recognition are *convolutional networks* [32] [5]. Convolutional layers pass filters of a predetermined size over an image, allowing for weight sharing between nearby pixels and the subsequent extraction of visual features [32]. Usually, multiple convolutional layers are used in order to extract a hierarchical representation of different features, before the signal is flattened to a vector output and passed through the softmax function for classification. Convolutional layers are often followed by *max pooling* layers [38], which downsample the the signal into a lower dimension, and also *batch normalisation* layers [25], which normalises the signal as it propagates through the network.

2.3 Stochastic Gradient Descent

The standard method for training deep models is *gradient descent (GD)* [54] [46], an optimization algorithm which varies the parameters in the model depending on the

gradient of the chosen error function with respect to the model parameters. Defining the current weight parameters of the network by θ_{old}, the updated weight after gradient descent as θ_{new} and the loss function of the model on the training set $\mathcal{T}$ by $L(\theta)$, we update the weight parameters by performing a gradient descent update as follows:

$$\theta_{new} = \theta_{old} + \Delta\theta, \tag{2.3}$$

where

$$\Delta\theta = -\alpha \frac{\partial L(\theta)}{\partial \theta}|_{\theta_{old}}. \tag{2.4}$$

α in this case is the pre-set *learning rate* [54], controlling the step-size of the gradient descent update. To implement GD it is necessary to calculate the gradient of the error function with respect to the parameters of the model, usually this is done layer by layer, starting with the output layer, in a method referred to as *backpropogation* [34].

One of the drawbacks of gradient descent is that calculating this gradient over the entire training set can be very computationally expensive, particularly when dealing with large training sets; to address this issue a variant of GD, *stochastic gradient descent (SGD)*, is often used [44]. With SGD, instead of calculating the error gradient over the entire training set, only one sample is used to calculate the gradient and update the model parameters. Alternatively a selection or 'mini-batch' of training samples may be used to calculate the gradient, in which case the optimization algorithm is referred to as *mini-batch stochastic gradient descent* [42]. To implement mini-batch SGD, batches of samples are selected uniformly from the training data, usually without replacement, and gradient descent updates are performed using the sample in the batch. This is then repeated until all of the training samples have been used in a batch; a complete pass through the training data is referred to as an *epoch*, and the number of training epochs is set as a parameter of the experiment [42].

It can be shown that that SGD and mini-batch SGD produce an unbiased estimate of the error gradient [44], with various convergence proofs showing that SGD will eventually converge to an optimal solution under certain conditions [44]. There are many adaptations to 'vanilla' SGD, for example momentum based methods [47] and other more sophisticated optimization algortihms which build on SGD to result in better and quicker fitting of the model parameters. In this paper we will employ the ADAM optimiser [28], the details of which are laid out in Algorithm 1 of [28].

2.4 Curriculum Learning

When training a deep model, or indeed any supervised learning algorithm, one usually trains the model over the entire training set throughout the entirety of training. When using stochastic gradient descent for example, all epochs would typically consist of a full pass through all available training samples. *Curriculum learning*, as introduced in [6], hypothesises that model performance can be improved by instead training on samples in a meaningful order, with the order defining a 'curriculum'. The motivation stems from the way in which humans and other animals learn, usually beginning with easy concepts before moving onto more complex facets of the area of study. The same principle can be applied to training deep models, and the authors of [6] suggest that, by initially training only on 'easy' samples, one can reduce overall generalization error.

While the term curriculum learning in the machine learning setting may be a relatively new one, the concept was arguably introduced with Elman's 1993 paper "Learning and development in neural networks: the importance of starting small" [12]. In this paper the author demonstrates in a language modeling task that inhibiting the memory of the network in the early stages of learning, so that it can only analyse a small subset of the training set, ultimately improves performance. The author motivates the method by analysing the learning dynamics of connectionist networks, specifically the sensitivity of the overall model to the early stages of training and the subsequent implications for what data should be used in these early epochs. Interestingly, the author suggests motivations for beginning training with either 'easy' or 'hard'/'noisy' samples. In the case of using easy samples, he argues that this will prevent the network from falling into "early commitment to erroneous hypotheses", instead enabling it to learn broader concepts at the key early stage of training that will act as "scaffolding" for more complex concepts. Conversely, he also suggests that using harder/noisier samples may also prevent the network from reaching such erroneous hypotheses, as the high variance in noisier samples will lead to less consistent gradient descent updates and prevent the networks weights from converging to an area of parameter space from which it will be difficult to exit in the later stages of training. This dynamic, wherein initialising learning with either easy and hard samples somewhat paradoxically seems to improve learning in both cases, is something which will be revisited throughout this report.

The authors of [6] also offer several theoretical justifications for curriculum learning, for example comparing the technique to *continuation methods* [1]. It is proposed that the easier samples represent a smoother, more convex version of the error space

of the overall problem, and that, by training on easier samples, the parameters of the model are effectively initialized into an area of parameter space closer to the global optimum. This argument is similar to that of unsupervised pre-training [13] which again has been shown to lead to better generalized models by initializing the parameters into parts of the error space closer to the global optimum [5]. Comparisons have also been drawn between curriculum learning and *transfer learning* [40], with the easier samples being seen as a separate task that the model is trained on, before using the weights for a different task (i.e. the harder samples) as in transfer learning [53].

The example given in [6] for curriculum learning is the 'Geometric Shapes' dataset [7], an image classification task where a network attempts to classify whether or a not an image shows a rectangle, ellipse or triangle. In this case there is a natural subset of 'easy' samples; specifically squares (i.e. regular rectangle), circles (regular ellipses) and equilateral triangles. The authors show that, by training initially on only the regular shapes, then transitioning to training on harder shapes, the test set performance is significantly improved compared to training only on the hard shapes for the entirety of training. One issue with this study is that it can be argued that the curriculum trained model has seen more samples overall than the baseline, as the curriculum model is trained on both an 'easy' training set and a 'hard' training set, whereas the baseline is trained only on the hard training set. A better baseline therefore is a model trained uniformly on the union of the easy and hard training sets. While the authors do comment on this issue, and claim that the curriculum method still outperforms uniform sampling from the combined training set, some of the results we will set out in this paper did not reach the same conclusions.

Curriculum learning is similar in concept to *self-paced learning* [31]. In a paper of the same title [31], the authors train a latent SSVM model [14] by adding a regularization term to the objective function that indicates whether or not a sample is 'easy' or note, depending on how well the current model parameters correctly classify the sample. Using this approach they limit the number of training samples used for parameter updates, as governed by a weight parameter that is annealed throughout training until all training samples are considered. They test their method on a variety of tasks including natural language processing and image classification. While the self-paced learning (SPL) method is tested for the SSVM model in [31], an exploration of SPL as applied to convolution networks is laid in [2]. The author applies SPL and a variant defined as 'Self-Paced Learning with Diversity (SPLD)' [26] which expands the SPL method to emphasise a diverse representation of samples.

The author of [2] tests various curriculum methods on the CIFAR 10 image classification dataset, interestingly, similar to the remarks mentioned above in [12], the author found that training on samples with *decreasing* difficulty outperformed the standard curriculum of training on samples with increasing difficulty. However, the author reports that little overall improvements were seen in any of the curriculum methods compared to standard training procedure using the entire training set. There are various examples of curriculum learning being applied in areas where the learning problem can be split into discrete 'tasks'. For example in [41], the authors use an animal image classification dataset that has been manually annotated with a perceived 'easiness' score. The authors divide the dataset into five equal tasks depending on difficulty score, demonstrating that their curriculum method outperforms a standard multi-task learning algorithm. The authors of [35] also find a natural curriculum in a handwritten text line recognition task, finding that by first training on short sequences of text, before progressing to to the longer sequences significantly accelerates the training time of of a recurrent neural network [36] model for text recognition. More recent applications of curriculum learning include [53], where the authors provide a theoretical study of how curriculum learning affects the convergence of stochastic gradient descent, as well as implementing a curriculum using transfer learning by using a model trained on a separate task to construct a curriculum for an image classification task, finding that the curriculum improves performance. The authors also test the effect of using an "anti-curriculum", where training progress from difficult to easy samples, as in [2], however they find this significantly underperformance the baseline, as illustrated in Figure 2.1, taken from Figure 3b of [53].

While the number of studies implementing curriculum learning methods is increasing, a key difficulty in constructing a curriculum is that it is often very difficult to delineate between 'easy' and 'difficult' samples [6], particularly for large datasets where manually annotating the difficulty of the each sample is infeasible. It is also an open question as to how best to construct a learning curriculum, given a measure of sample difficulty, in particular given the aforementioned questions around whether or not training should progress through increasingly difficult or easy samples. A key issue therefore is that of exploring methods for automating the construction of learning curricula, and it is towards this goal that this paper contributes; specifically investigating how active learning methods (introduced in the next section) can aid such curriculum construction. In Chapter 3, we will discuss a variety of other studies that have implemented methods for automated curriculum construction. We first however introduce

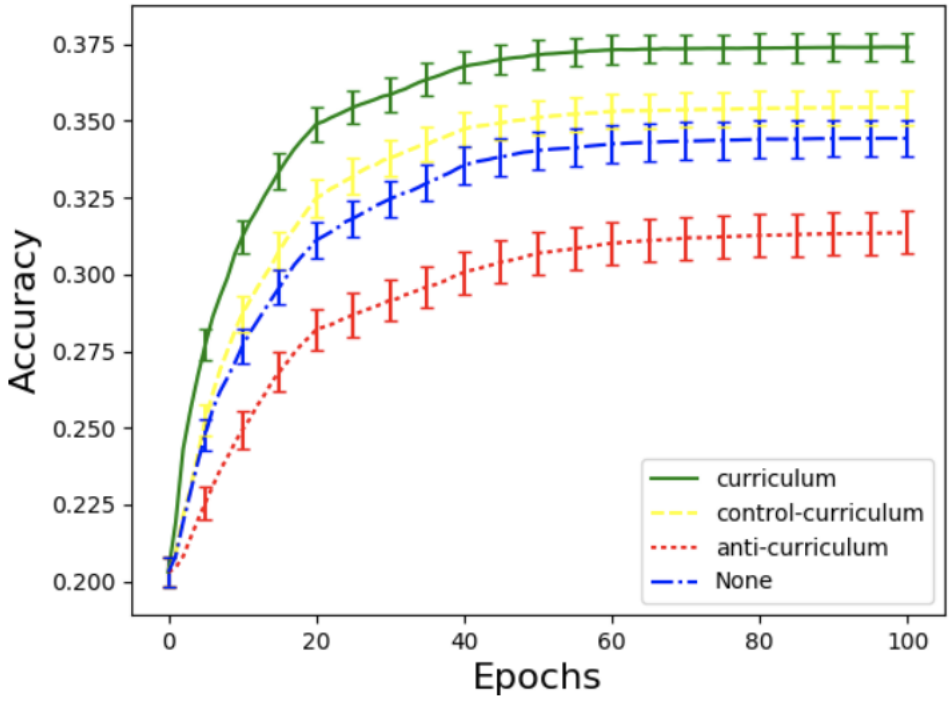


Figure 2.1: Figure 3b from [53], test classification accuracy of a convolutional network trained using various curriculum methods on the CIFAR 100 dataset [30]. The curriculum method trains on progressively more difficult samples, while the anti-curriculum method trains on progressively easier samples. The control-curriculum is trained using a random curriculum to control for differences in training between the curriculum methods and the baseline.

active learning, including some of the methods we will use to estimate sample difficulty and construct learning curricula in Chapters 4 and 5.

2.5 Active Learning

A key component in any supervised learning effort is labeled data; in many domains it is relatively easy and cheap to obtain a large number of training samples, however in others it can be far more costly, particularly acquiring accurate labels. In medical image analysis for example one may require a domain expert to spend significant time analysing each image before assigning label, or in document tagging it can take time to read a document and assign a topic label. It can therefore be very useful for a designer

to understand which samples they should go to the effort of acquiring, labeling and feeding into their chosen learning algorithm. A field of study that attempts to address this is *active learning* [43] [10] [11], which studies methods for choosing, often from a pool of unlabeled candidate samples, which sample should be labeled and used to train the model. Generally the efficacy of an active learning method is measured by how much the chosen samples improve the model performance, compared to if samples were instead selected randomly [43].

Figure 2.2, taken from Figure 1 of [43] shows a typical active learning cycle; a machine learning model is trained on the current set of labeled training data, $\mathcal{L}$, the model then "queries" an "oracle", in this case a human annotator, for the labels of one or more samples from a pool of unlabeled samples, denoted $\mathcal{U}$. The queried samples are then labeled and added to $\mathcal{L}$, at which point the model is retrained using the expanded training set and the cycle reiterates until training is stopped. Note that, as well as pool based query strategies, some active learning methods may enable the model to query samples from the entire input space, as opposed to being limited to a pool of unlabeled samples [43].

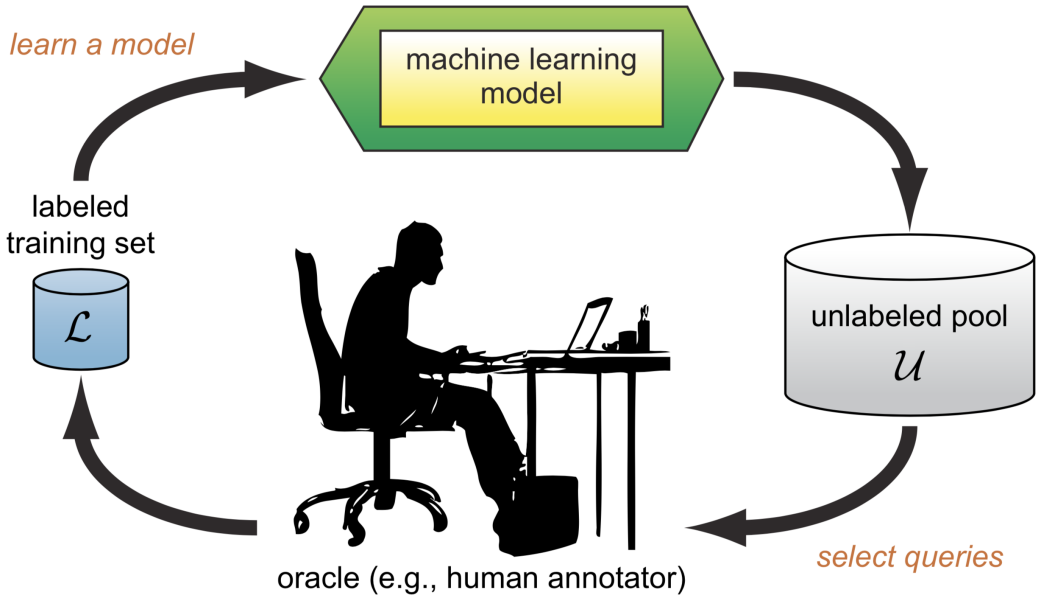


Figure 2.2: Figure 1 from [43], showing a typical "pool-based active learning cycle".

There are a variety of approaches to the active learning problem, however most involve the use of an *acquisition function*, which selects which sample from $\mathcal{U}$ (the set of candidate unlabeled examples) should be selected for labeling and training [43]. As the most appropriate training examples varies depending on the model itself, the

chosen sample is said to be 'queried' by the algorithm. One of the most common query methods is that of *uncertainty sampling* [43], wherein the samples that the learning algorithm is most uncertain about labeling are queried. We will suggest in this paper that algorithm uncertainty can provide an effective measure of sample difficulty which can be used to automate the construction of learning curricula.

A common way of estimating model uncertainty is by analysing the distance to classification threshold of the model outputs; for example one method is to select the sample about which the model is least confident in predicting (taken from [43]):

$$x^*_{LC} = \arg\max_x 1 - P_\theta(\hat{y}|x), \tag{2.5}$$

where

$$\hat{y} = \arg\max_{y_i} P_\theta(oftsy_i|x). \tag{2.6}$$

Where x^*_{LC} is the queried training sample and $P_\theta(y_i|x)$ is the model's predicted probability that sample x is of class y_i, given model parameters θ. Similarly, samples can be queried by their average distance to classification threshold or, similarly, the entropy of the algorithm prediction, again taken from [43]:

$$x^*_H = \arg\max_x -\sum_i P_\theta(y_i|x) \log P_\theta(y_i|x), \tag{2.7}$$

where the sum runs over the possible classes y_i. More sophisticated methods for estimating prediction uncertainty include the "Bayesian Active Learning by Disagreement (BALD)" [24] acquisition function, which is used for deep models by the authors of [17] and is introduced and used in Chapter 5 of this paper.

It is interesting to note the somewhat opposing assumptions underlying active and curriculum learning; in the active learning approach the *most* uncertain samples, which could arguably be seen as 'hard' samples from a curriculum point of view, are selected for training. In curriculum learning however, the focus is usually on selecting easy samples, particularly early on in training, to improve model performance. Again, this speaks to the dynamic discussed in [12], where the author lays out potential justifications for the learning benefits of emphasizing both easy and hard/noisy training samples. As we will see in the results sections of Chapter 5, our experiments support this idea, with both 'easy to hard' and 'hard to easy' curricula improving model performance in some cases

There are many examples of authors implementing active learning approaches for a variety of algorithms; [49] develops various methods to query unlabeled samples using

support vector machines to categorize news stories, while [55] uses active learning to reduce the burden of human effort in labeling video data. In a more recent study, the authors of [52] employ active learning with deep networks for image classification, successfully reducing the number of training samples required to achieve promising results on challenging image classification tasks such as face recognition. Several studies investigate theoretical justifications for the benefit of active learning and the situations in which it is beneficial to learning, for example [3] and [4]. However in most cases these studies are based on severely restricted hypothesis spaces and more general proofs are yet to be developed [43]. While the main goal of active learning is generally to reduce the number of samples required to achieve a certain level of test set performance [43], there are some studies that examine how active learning approaches can reduce overall generalization error, for example [10]. This motivates the idea underlying this paper, that active learning methods can be used in a an automated curriculum learning setting in order to improve the overall accuracy of deep models.

We have now introduced the main componentry that will be employed in the rest of the paper; specifically we will use methods motivated by uncertainty sampling in active learning to estimate training example difficulty scores which will be used to automatically construct learning curricula. In the following chapter we will outline several recent studies which have tested methods for automating the process of curriculum construction, before laying out the methods and experimental results carried out for this report in the subsequent two chapters.

Chapter 3

Related Work

In this chapter we discuss several works that have also implemented methods to automate the curriculum construction process, note that some of the content in this section is based on the author's 'Informatics Project Proposal', completed in preparation for this thesis.

3.1 Self-Paced Learning

The first area we discuss is *self-paced learning*, as defined in [31] and introduced briefly in Chapter 2. In the original paper the authors reference the original curriculum learning paper by Bengio et al [6], discussing curriculum learning and the difficulties posed by constructing hand crafted curricula and measuring training sample difficulty. Their solution is self-paced learning (SPL), an alternative which approaches sample difficulty in a different way. The authors note that "in self-paced learning, the characterization of what is easy applies not to individual samples, but to sets of samples; a set of samples is easy if it admits a good fit in the model space". To implement self-paced learning the authors introduce a regularization term into the objective function of a latent SSVM [14] model, using a vector indicating the difficulty of each training sample. The regularization vector is not constructed prior to training, rather it is fitted throughout training in conjunction with the latent variable model parameters, with the objective function being minimised when a suitably sparse selection of training samples with high likelihood are considered. The level of sparsity is controlled with a tuning parameter which is annealed throughout training until the entire training set is included in training. The authors of [26] build on the self-paced learning idea by adapting the method to prefer a diverse selection of samples, as opposed to simply

samples with a high likelihood. The authors note that SPL can suffer from a lack of diversity in the training samples used during training as often the samples with high likelihood will be clustered together; by introducing an additional term encouraging diversity in the objective function, the authors correct for this effect and show an improvement on the usual SPL method. Finally, in [27], the authors further develop the link between SPL and curriculum learning. In this paper the authors suggest that a disadvantage with SPL is the inability to introduce prior knowledge into into the learning process, unlike in curriculum learning where prior knowledge is often used to craft a curriculum. To implement their training method, which they term *self-paced curriculum learning*, the authors combine the SPL and curriculum learning approaches, using a predefined curriculum to order the inclusion of training samples, then also including the usual SPL regularization term in the objective function. The authors demonstrate that their method outperforms SPL without using a prior curriculum and curriculum learning without SPL, testing their method on a multimedia event detection task.

3.2 Transfer Learning

In a recent paper, the authors of [53] implement an automated curriculum for image classification using transfer learning [40]. The authors use the CIFAR 100 dataset [30], passing the images through a large pre-trained network (pre-trained on a different image classification task, hence the connection with transfer learning), then using the embeddings of the penultimate layer of the pre-trained network as inputs to a shallow classifier, in this case a support vector machine (SVM) [22], which is then trained to classify the CIFAR 100 training images given the inputs from the pre-trained network. The distance to classification margin from the shallow classifier is then used as a measure of sample difficulty, sorting the samples according to the confidence of the SVM. This approach is similar to the method we lay out in Chapter 4, however, instead of using a pre-trained network, we 'bootstrap' a curriculum by training a baseline model on the target problem, then use the uncertainty of the baseline model to infer sample difficulty. The transfer learning approach removes the need to train a complex model, with the pre-trained network able to 'transfer' its knowledge of image data to help infer the difficulty of the training samples.

3.3 Reinforcement Learning

A different way of approaching the curriculum construction method is to take a 'meta-learning' [50] approach towards identifying the best order in which to include training samples throughout learning, with the best curriculum being a parameter that can be learned and optimised through a meta-learning approach. To this end, the authors of [19] lay out a method in which they use a reinforcement learning [48] approach to learn the best way to construct a curriculum for an n-gram language modeling problem [29]. In the paper, the authors divide the data into several 'tasks' of increasing complexity, then used a multi-armed bandit approach [48] to train a meta-learner to identify which order of tasks (i.e. a curriculum) should be used to train an LSTM model on the language modeling task. The 'arms' or 'actions' of the multi-armed bandit in this study therefore represent the choice to sample from the different tasks, with the reinforcement learning model being optimised to maximise learning performance of the LSTM model by sampling from the different tasks throughout training. Improvements in learning performance are measured in a variety of ways, but are broadly divided into "loss-driven progress", where the reinforcement algorithm is rewarded for reducing the loss of the LSTM algorithm after training on the samples selected by the reinforcement learning algorithm. Alternatively, "Complexity-driven progress" rewards the reinforcement algorithm for increasing the complexity of the LSTM algorithm; this is motivated on the concept of *minimum description length* [20], which suggests that the model complexity will "increase most in response to the training examples from which the network is best able to generalise". The metrics used to measure improvement in learning performance could be interesting from both a curriculum learning and an active learning perspective, offering novel ways to measure the potential benefit of training on a given sample. Interestingly, the authors show that, despite the fact they have not enforced any preference for beginning training with the easier tasks, the reinforcement learning algorithm autonomously learns to implement easy to hard learning curricula, with increasing task difficulty.

3.4 Active Bias

Finally we discuss a recent paper titled "Active Bias" [8], which inspired some of the sampling based methods used in the paper. While the authors do not implement a strict curriculum construction method, they investigate how varying the probability

with which training examples are sampled throughout the training of deep models affect learning performance. In Chapter 5, we will employ a similar approach, arguing that it can effectively act a form of stochastic curriculum, where instead of strictly limiting the samples used in different training phases, we instead bias the sampling probabilities towards easy/hard training examples. The authors of [8] test a variety of methods for biasing sampling probability throughout training, for example by recording how successful the model has been in classifying the training samples in prior epochs, and biasing the sampling probability either in favour of or against selecting the samples that the model has been most unsuccessful at classifying previously. This is similar to the dynamic sampling curriculum methods that we implement in Chapter 5, although we use the model's prediction uncertainty as opposed to recording previous classification outcomes. The authors test their results over a variety of deep architectures and datasets, including image classification and text analysis, concluding that many of their sampling methods outperform default uniform sampling. As we will develop later in this paper, the authors discover a relationship between task difficulty and the most effective sampling method, with difficult learning problems being most improved by emphasizing easy training samples, and vice versa, with more easier classification tasks being most improved by biasing learning to focus on difficult training samples. These results are supported by those laid out in the results section of Chapter 5 of this paper, with more difficult image classification tasks seeing greater learning improvement when using easy to hard learning curricula than easier problems.

The papers laid out in this section present a broad overview of the various methods that other studies have used to automate the process of constructing learning curricula, including papers that have inspired some of the techniques used in this thesis. The next two chapters will discuss the active curriculum methods we introduce and implement in this paper, outlining their construction methodology and the experiments carried out to test their effect on learning performance in deep models over various image classification tasks.

Chapter 4

Bootstrapped Active Curricula

The first curriculum construction approach we investigate is what we term *bootstrapped active curricula (BAC)*, the name is chosen as it involves 'bootstrapping' a curriculum from a separate model. The intuition behind this approach is that, while it may be difficult to ascertain a-priori which training samples are 'hard' or 'easy' (particularly for very large datasets where it is infeasible to analyse every sample), we can automatically infer which samples are difficult by first training a model on the problem and then using an active learning uncertainty metric to investigate which samples the model is uncertain about classifying. Using prediction uncertainty to approximate difficulty, we can then construct a learning curriculum which can be used to train a new model, for example by splitting the training samples into separate tasks of increasing difficulty, as detailed below.

4.1 Curriculum Construction

In the BAC approach we train two models; the first, which we term the 'baseline model' and denote $\theta_{baseline}$, is trained to convergence using a standard mini-batch gradient descent optimisation on the entire available training set $\mathcal{T}$. We then use $\theta_{baseline}$ in conjunction with the Absolute Average Distance to Threshold uncertainty function as defined below in Section 4.1.1, scoring each training sample in $\mathcal{T}$. This produces $\mathbf{s}^{\theta_{baseline}}$, a vector of N scores, where N is the number of samples in $\mathcal{T}$, such that the i^{th} element of $\mathbf{s}^{\theta_{baseline}}$ is the output of the AADT uncertainty function for the i^{th} training sample inputs:

$$s_i^{\theta_{baseline}} = AADT_{\theta_{baseline}}(x_i) \tag{4.1}$$

where x_i are the inputs of the i^{th} training sample.

We then sort the training samples according to their score, producing an ordered training set $\mathcal{T}_{\mathbf{S}^{\theta_{baseline}}}$, such that the first training sample in $\mathcal{T}_{\mathbf{S}^{\theta_{baseline}}}$ is the sample which produces the highest value from the AADT function. We then split $\mathcal{T}_{\mathbf{S}^{\theta_{baseline}}}$ into equally sized 'tasks', with the number of tasks being a hyperparameter of the curriculum construction. The learning curriculum is then constructed from these tasks; the chosen approach is to split training into discrete phases, with the number of phases being equal to the chosen number of tasks. The first phase of the curriculum consists of training only on the first task, denoted $\mathcal{T}^1_{\mathbf{S}^{\theta_{baseline}}}$, in the second phase, the second task is added to the training samples, training the model on $\mathcal{T}^1_{\mathbf{S}^{\theta_{baseline}}} \cup \mathcal{T}^2_{\mathbf{S}^{\theta_{baseline}}}$. In the third phase (if the number of tasks is greater than 2), the next task is added, and so on until all tasks have been added and the final phase consists of training on the entirety of the original training set $\mathcal{T}$. Having constructed the BAC learning curriculum, we train a new 'curriculum' model', which we denote by $\theta_{curriculum}$, using the curriculum. Pseudocode for the BAC method is given in Section 4.1.3.

4.1.1 Average Absolute Distance to Threshold (AADT)

A popular active learning uncertainty method is to examine the proximity of the model's outputs to the classification boundary. The assumption is that samples that the model is uncertain about classifying will produce probabilities close to the classification boundary; indeed, the authors of [8] show that prediction variance is inversely proportional to the distance to the boundary. From a curriculum perspective we can estimate a sample's difficulty by the algorithm's uncertainty in predicting the class label, with uncertain samples being seen as hard, and vice versa. We thus define the uncertainty function *AADT* which calculates the absolute distance to classification threshold of the model outputs for a given input sample, averaged across all possible classes. Note that the function is dependent on a the output of the model in question, which is denoted θ.

$$AADT_\theta(x_i) = \frac{\sum_{c=1}^{C} \left| P_\theta(y_c|x_i) - \frac{1}{C} \right|}{C}, \tag{4.2}$$

where $|.|$ represents the L1 norm/absolute value function. Here N is the number of training samples, C is the number of output classes and $P_\theta(y_c|x_i)$ is the output softmax probability for class y_c of the model θ, given input x_i. We also tested the average *squared* distance to threshold, as opposed to the absolute distance to threshold, as well as testing the entropy of the outputs as an uncertainty measure, however results were extremely similar in all cases.

4.1.2 Phase Training Epochs

A naive approach to the BAC approach would be to train the curriculum model for the same number of epochs as the baseline model, split equally across training phases. For example if the baseline model was trained for 100 epochs and we then constructed a two task curriculum, the curriculum model would be trained on the first task for the first 50 epochs, then on both tasks for the second 50 epochs. The issue with this method however is that, as during the first 50 epochs there are only half as many training samples, the curriculum model will not have as many parameter updates as the baseline model. To emphasize this, consider that if we were using stochastic gradient descent (i.e. a mini-batch size of 1) with a training set of 1000 samples, the baseline model in this instance would have (# epochs * # training samples) parameter updates, i.e. 100 * 1000 = 100,000 parameter updates. The curriculum model on the other hand would have (50*500) parameter updates during the first training phase and (50*1000) in the second, resulting in a total of (50*500) + (50*1000) = 75,000 parameter updates, 25% less updates than the baseline model. To address this, we increase the number of epochs in each phase by the ratio of the number of samples used in the phase to the size of the whole training set. Specifically, we set the number of training epochs in each phase as follows:

$$NumEpochs^t = \lfloor \frac{BaselineEpochs}{t} \rfloor \tag{4.3}$$

Where $BaselineEpochs$ is the number of epochs used to train the baseline model and $NumEpochs^t$ denotes the number of training epochs in the t^{th} training phase. For example, in the first phase, $NumEpochs^1 = \frac{BaselineEpochs}{1} = BaselineEpochs$. $\lfloor . \rfloor$ represents the floor function; as $\frac{BaselineEpochs}{i}$ will not always be integer, we round down the number of epochs in each phase. The curriculum model will therefore be trained for a higher number of epochs (precisely, $\sum_{t=1}^{NumTasks} \frac{1}{t}$ times more epochs), however the number of parameter updates will be equalised.

4.1.3 Pseudocode for BAC

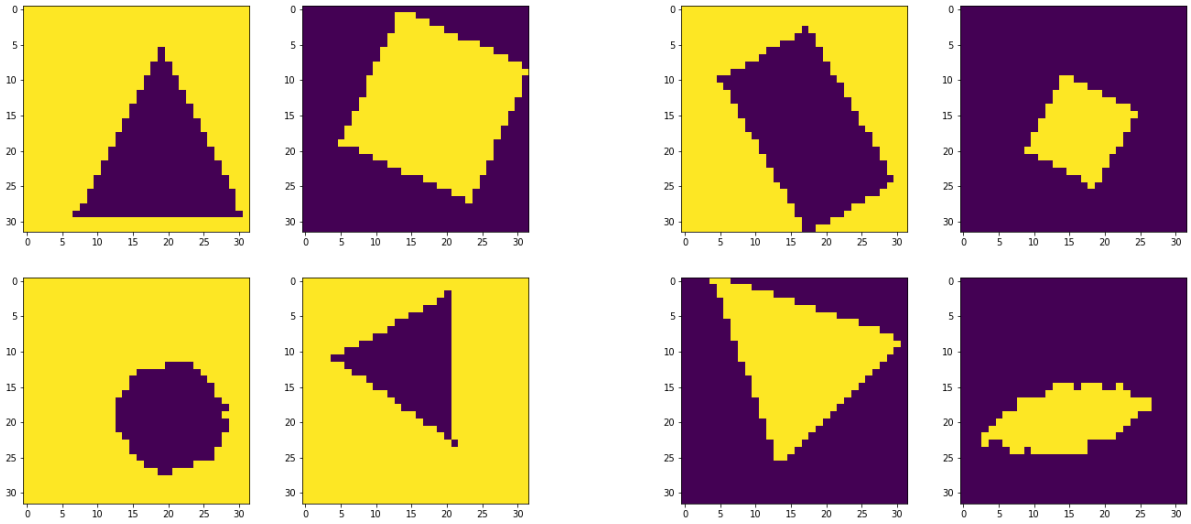
Pseudocode for the boostrapped active curriculum method (assuming the use of the AADT uncertainty function and an easy to hard curriculum). Note that we slightly abuse the definition of the AADT function from Section 4.1.1, which was defined only for individual input samples, setting $\mathbf{s}^{\theta_{baseline}} = AADT_{\theta_{baseline}}(\mathcal{T})$, i.e. signifying that $\mathbf{s}^{\theta_{baseline}}$ is the output of the AADT uncertainty function over the entire training set

$\mathcal{T}$:

Require: Num_Epochs {Number of epochs for baseline model}
Require: $\mathcal{T}$ {Training Set of input-label pairs}
Require: $\theta_{baseline}$ {Baseline model}
Require: $\theta_{curriculum}$ {Curriculum model, identical to Baseline model}
Require: Num_Tasks
  for Num_Epochs **do**
    train $\theta_{baseline}$ on training set $\mathcal{T}$
  end for
  $\mathbf{s}^{\theta_{baseline}} = AADT_{\theta_{baseline}}(\mathcal{T})$
  $\mathcal{T}_{\mathbf{s}^{\theta_{baseline}}} = \mathcal{T}$, sorted by $\mathbf{s}^{\theta_{baseline}}$ (descending order)
  $Num\_Samples = |\mathcal{T}_{\mathbf{s}^{\theta_{baseline}}}|$
  for $t = 0$ to Num_Tasks **do**
    $TaskStartIndex = Num\_Samples * \frac{t}{Num\_Tasks}$
    $TaskEndIndex = Num\_Samples * \frac{t+1}{Num\_Tasks}$
    $\mathcal{T}^{t}_{\mathbf{s}^{\theta_{baseline}}} = \mathcal{T}_{\mathbf{s}^{\theta_{baseline}}}[TaskStartIndex : TaskEndIndex, :]$
  end for
  for $t = 0$ to Num_Tasks **do**
    $Num\_Epochs\_Task = \lfloor \frac{Num\_Epochs}{t} \rfloor$
    for Num_Epochs_Task **do**
      train $\theta_{curriculum}$ on training set $\mathcal{T}^{0:t}_{\mathbf{s}^{\theta_{baseline}}}$
    end for
  end for

4.2 Geometric Shapes Dataset

To test the boostrapped active curriculum approach we use the 'Geometric Shapes' dataset [7], as used in [6]. This dataset consists of 32x32 pixel images of geometric shapes, specifically ellipses, rectangles and triangles; the class labels are one-hot vectors which indicate to which of the three classes each sample belongs. As well as varying the shapes shown in the image, the samples also vary in colour, orientation, size and position. This dataset is used in [6] as it is easy to construct a predefined curriculum for geometric shapes; circles, squares and equilateral triangles represent regular, 'easy' versions of the broader classes of ellipses, rectangles and triangles, respectively. The authors of [6] train a curriculum model by first training only on an easy



(a) Samples from ‘easy’ training set

(b) Samples from ‘hard’ training set

Figure 4.1: Sample figures from the Geometric Shapes dataset

training set consisting of circles, squares and equilateral triangles, before then training on a hard training set with the more general shapes. The easy and difficult training sets each consist of 10,000 training samples, giving a total of 20,000 training samples, while the test set, consisting only of hard samples, contains 5,000 images. The authors demonstrate that their approach outperforms an identical model trained only on the harder shapes; the curriculum model consistently achieves greater test accuracy, with the greatest improvement coming when the first half of the training epochs train on the easier shapes, and the second half the harder ones, as shown in Figure 3 of their paper. As discussed by the authors, one potential pitfall of their experiments is that the curriculum model has seen more samples than the benchmark model, as it has been trained on both the easy and the difficult training sets. To avoid this in our experiments, the training set we use is the union of both the easy and difficult samples. Our training set therefore consists of 20,000 geometric shape images, including both the easy, regular shapes and the more difficult shapes, the test set is unchanged however, consisting of the 5,000 difficult samples. Some example images for the Geometric Shapes dataset are given in Figure 4.1, with Figure 4.1a showing examples from the easy training set featuring circles, squares and equilateral triangles, while Figure 4.1b shows samples from the more difficult training set of general shapes.

In the results Section 4.4, we analyse how successful our tested curriculum method is at automatically identifying which samples come from the easy or hard training sets, investigating which training samples fall into the different tasks automatically constructed through the BAC curriculum method.

4.3 Experiments

In order to measure the effect of the curriculum on test performance, we set $\theta_{baseline}$ and $\theta_{curriculum}$ to have identical architectures, hyperparameters and initial weights, essentially minimizing any differences in training besides the learning curriculum. We use the Average Absolute Distance to Threshold (AADT) function, detailed in Section 4.1.1, to score the training samples using the trained baseline model in each experiment. Furthermore, as well as testing 'easy to hard' curricula, where the training phases progress from $\mathcal{T}^1_{\mathbf{s}^{\theta_{baseline}}}$ to the final task, we also test the opposite approach, with the first training phase using only the hardest task, then incorporating the other, easier tasks throughout training. We run the experiments using the Geometric Shapes dataset, as introduced in Section 4.2, allowing us to compare results with the predefined curriculum as in [6]. To do so, as well as training the baseline and BAC curriculum models, we also train a model using the predefined curriculum method laid out in [6], training on only the easy samples for the first half of the training epochs, and only the hard samples in the second half. In order to better compare the predefined curriculum method from [6], we also run an adjusted version of their method; unlike the BAC approach, where we correct for the difference in number of parameter updates between the curriculum model and the baseline model, the predefined curriculum model will end up undergoing significantly less parameter updates than our baseline (in fact it will have half as many updates). We therefore correct for this in two ways; in the first phase of training, in which the model is trained only on the easy images, lasts for twice as many epochs as in the unadjusted method (correcting for the fact that it consists of half as many samples as the full training set). The second phase of the adjusted model is then trained on the *full* training set, consisting of the union of the hard and easy samples. This should allow for a better comparison between the BAC models, the baseline model, and the predefined curriculum approach. We therefore train 5 separate models:

- Baseline Model - Trained on the full training set of easy and hard geometric shape images for all epochs, with standard mini-batch stochastic gradient descent.

- Easy to Hard Model - Trained with a boostrapped active curriculum construct from the above Baseline Model, beginning with the easiest/least uncertain task and including the other tasks throughout training.
- Hard to Easy Model - As above, but training begins with the hardest/most uncertain tasks before incorporating the easier tasks throughout training.
- Predefined Curriculum - As in [6], the first half of the training epochs use only the 10,000 easy training samples, while the second half train on only the 10,000 hard training samples.
- Adjusted Predefined Curriculum - As above, but the number of epochs in the first phase of training is doubled to account for the difference in parameter updates, and the second phase uses both easy and hard training samples for better comparison with the BAC models.

All models are tested on the same test set of 5000 images, all containing 'hard' geometric shapes; experiments are repeated multiple times with different weight initialisations. To analyse the effect of the curriculum on learning we report the accuracy and cross-entropy error of the different models over the held out test set. The exact model architecture uses three hidden layers, each consisting of 300 nodes using the Hyperbolic Tangent activation function [**?**] [1], and are followed by dropout layers for regularization. We also lay out the hyperparameters for the training procedures in table 4.1 below, experiments were carried out in Keras [9] with the Adam optimiser implemented with the default hyperparameters (except for the learning rate which is given in Table 4.1):

Table 4.1: Experiment Hyperparameters

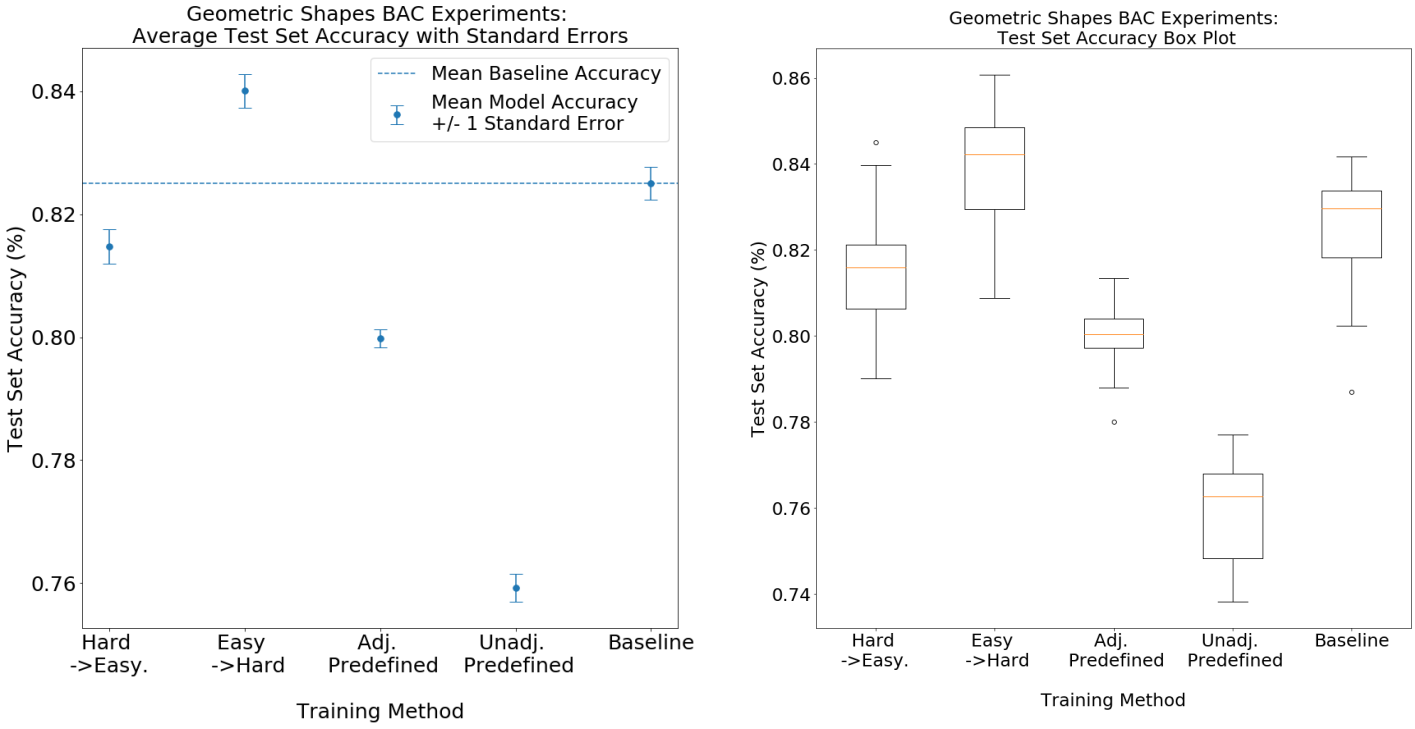
Experiment Hyperparameters						
	Epochs	Optimiser	Learning Rate	Dropout %	Batch Size	Num Tasks
GeoShapes	350	Adam	0.0001	0.25	32	2

Architectures and hyperparameters were chosen and tuned in order to deliver a good level of model convergence and validation performance; robustness tests were however carried out varying the model architectures and hyperparameters resulting in little change in the relative performance of the different methods.

[1] The Hyperbolic Tangent activation is chosen as this was this the activation used in [6].

4.4 Results and Discussion

The results of running the bootstrapped active curriculum experiments with the Geometric Shapes dataset are summarised Figure 4.2 which shows the test set accuracies, including standard errors, derived from 24 experiments with different initialisations, full results are given in Table A.1.



(a) Mean Test Set Accuracies

(b) Test Set Accuracy Boxplots

Figure 4.2: Geometric Shapes Bootstrapped Active Curriculum (BAC) Experiment Results

We see from the results that the Easy to Hard curriculum model significantly outperforms the baseline model, with an average test accuracy improvement of around 1.5%, over 5 standard errors, higher than the baseline test performance. Cross-entropy error is also reduced, with the Easy to Hard curriculum test error over 6 standard errors below that of the baseline model. These results suggest that, not only is there a benefit to training the model with a curriculum, the BAC model seems to be successful at identifying which samples should be used in the different curriculum learning phases. Conversely, the other curriculum methods all underperform the baseline model; the Hard to Easy curriculum in some ways acts as a control, showing that the improve-

ment in the Easy to Hard model is driven by the order in which the tasks are included in training, as opposed to being a consequence of another part of the BAC method. It is interesting to note that both of the predefined curriculum models underperform the baseline model; the lower accuracy of the unadjusted method would seem to support the authors' suggestion in [6] that the performance differences in their experiments may have been a result of their curriculum model having seen overall more training samples than their baseline. However, even our adjusted version of the predefined curriculum results in a significantly lower test accuracy than the baseline. This is interesting as this method is effectively trained in the same way as the BAC approach, however the tasks are predetermined by an intuitive sense of difficulty, as opposed to being bootstrapped from a baseline model. This would suggest that, even for datasets where there as a somewhat clear distinction between hard and easy samples, a bootstrapped curriculum may be more useful. Potentially, this may because the BAC method specifically calculates which samples the model itself is uncertain in classifying, as opposed to assuming that our intuitive sense of difficulty corresponds to the best curriculum for the model. We would note however, that given the relatively low resolution of the images (as illustrated in Figure 4.1), there is perhaps less observable difference between the easy and hard training sets than might be assumed, and that this may contribute to the poor performance of the predefined curriculum. Note that as we use a different baseline, these results are still consistent with those shown in [6]; the baseline used in the referenced paper is trained only on the hard training set, whereas our baseline is trained on both the easy and hard training sets. In fact, in early experiments we tested using the same baseline as in [6] and found results consistent with the paper, with what we have termed the 'predefined curriculum' outperforming this more simple baseline.

We further analyse the the Easy to Hard curriculum experiments in more depth, in order to ascertain which samples are typically included in the two automatically constructed curriculum tasks. Recall that in the BAC method we train a baseline model, then use the uncertainty in the model's output class probabilities for the training samples, calculated using the AADT function defined in Section 4.1.1, to score and rank the training samples. The training samples are then split into two tasks, the first consisting of the training samples that the baseline model is least uncertain in classifying, and the second task containing the samples about which the baseline model's outputs are most uncertain. We can analyse the composition of the two tasks to infer which samples the model is least/most uncertain about. Doing so for one of the experiments presents some interesting conclusions; first of all, there is a significant lack of balance

in the target classes in the two tasks. In task 1, the easier task, 43% of the images are of triangles, while 31% are of ellipses and 26% are of rectangles, implying that the baseline model is significantly more confident in classifying whether or not an image shows a triangle than it is in classifying the other shapes. The bottom right image in Figure 4.1b illustrates why this may be, with an example of an ellipse that looks quite similar to a mis-shapen rectangle, again this is probably the result of the low resolution of the images. We also observe that 64% of the samples in the first task are from the easy training set of squares, circles and equilateral triangles, showing that the AADT function is somewhat successful in separating the two training sets, however the first task still contains a substantial number of the harder samples.

A potential enhancement to the BAC method would be to control the balance of the classes in either task, however if one class is indeed easier than the others to classify then it may be better for it to be over-represented in the easier task(s). While we used two tasks for the Geometric Shapes dataset as this corresponded to the number of tasks of the predefined curriculum, we also ran some initial experiments with a larger number of tasks. We found some variation in results with a larger number of tasks, with some choices of task number underperforming the baseline model, however further tests could be carried out in future work to ascertain how robust these results are and investigate what is driving the different performances.

4.5 Summary

To summarize, these experiments seem to support the hypothesis that training a deep model with a curriculum can reduce generalization error, and furthermore that using an active learning approach to infer sample difficulty through model prediction uncertainty can be an effective way of automatically constructing such curricula. In particular such curriculum construction methods may even outperform a preconstructed curriculum using an intuitive sense of sample difficulty, as model uncertainty incorporates information about which samples the model itself finds difficult. One of the arguable drawbacks to the BAC method is that it requires that a baseline model is trained to convergence, in order to use its outputs to construct the curriculum to train the curriculum model. This effectively doubles the training time and, while this may not be too significant a computational burden in some problems, motivates the approach of the next chapter, which investigates whether or not a curriculum can be constructed dynamically throughout training, without first training a baseline model.

Chapter 5

Dynamic Active Curricula

The results laid out in Chapter 4 suggest that generalization performance can indeed be improved through the use of a learning curriculum, and that using active learning approaches to score which training samples are hard or difficult can be an effective way of automatically constructing such curricula without manually analysing the training samples. The disadvantage with the BAC method from Chapter 4 however is that it is necessary to first train a 'baseline' model that can be used to derive a curriculum based on which samples it is uncertain about classifying. While in some cases this may not be too significant a computational burden, it effectively doubles the overall training time, motivating the approach laid out in this section. Specifically, we wish to investigate methods that can dynamically construct a curriculum throughout training, without needed to reference another model. We do this by calculating the prrediction uncertainty of the curriculum model itself over the training samples throughout training, using the uncertainty scores to construct dynamic curricula which adapt to the changing parameterisation of the model. If successful, these methods should lead to model performance which beats a the benchmark test set performance of a baseline model trained on the entire training set without a curriuclum, and hopefully will achieve results similar to those achieved by the bootstrapped active curricula from Chapter 4.

5.1 Curriculum Construction

5.1.1 Dynamic Task Curricula (DTC)

The first dynamic curriculum method we test we term *dynamic task curricula*; this approach is very similar to that of the BAC method laid out in Chapter 4, however

instead of constructing tasks based on the uncertainty scores of a fully trained 'baseline' model, tasks are constructed dynamically using the uncertainty of the curriculum model itself throughout training. To do this, we calculate the model's uncertainty in predicting the classes of the training samples in the full training set $\mathcal{T}$ using the AADT function 4.1.1 (or another active learning uncertainty measure) at the beginning of every epoch[1], then ranking the training samples by the the model's uncertainty. We then split the training data into separate tasks, with the first task consisting of the first $\frac{1}{T}$ samples where T is the chosen number of tasks. At the start of each epoch, we will obtain a new set of ordered tasks, $\mathcal{T}_i^1, \mathcal{T}_i^2, ..., \mathcal{T}_i^T$, where i is the current epoch, and T is the chosen number of tasks. Training is divided into T phases, such that, during epoch i of phase t, the model is trained on a training set consisting of $\mathcal{T}_i^1 \cup \mathcal{T}_i^2 \cup ... \cup \mathcal{T}_j^t$, where t denotes the current training phase.The final training phase will therefore consist of training on the full training set, as $\mathcal{T} = \mathcal{T}_i^1 \cup \mathcal{T}_i^2 \cup ... \cup \mathcal{T}_i^T, \forall i$. As in the BAC curriculum method from Chapter 4, we normalise the number of parameter updates in each phase by scaling the number of epochs relative to the number of samples used to train in that phase. Similarly to Equation 4.3, the number of epochs in training phase t is set as follows:

$$NumEpochs^t = \lfloor \frac{TotalEpochs}{t} \rfloor \tag{5.1}$$

where $TotalEpochs$ is the number of epochs the model would need to be trained for on the whole training set in order to achieve the same number of parameter updates as the DTC curriculum method.

Note that we can rank the samples both from low to high uncertainty or from high to low uncertainty; in the low to high case the first tasks will contain samples about which the model is confident in predicting, corresponding to dynamically constructing an 'easy to hard' curriculum. Alternatively, ranking the samples from high to low uncertainty will produce a curriculum more similar to an active learning approach, focusing on training the model on areas of the input space that it is more uncertain about classifying. While the latter method may contradict some of the usual motivations for curriculum learning [6], we would still argue that the hard to easy approach still qualifies as a curriculum method, and may be appropriate in instances where the model already has achieved a high level of expertise in the problem, and improvements are arguably more likely to be made by studying harder samples than easy ones. We therefore have two variations of the Dynamic Task Curriculum method; 'Easy to Hard'

[1]With the exception of the first training epoch which uses the entire training set in order to allow the model to begin to calibrate to the problem.

DTC and 'Hard to Easy' DTC. This approach is similar to that of *Self-Paced Learning* [31], where they apply a similar approach with a latent SSVM model [14], however they implement their method by introducing a regularization term reflecting the difficulty of each sample into the objective function of the model, as opposed to employing an actual curriculum. As the model parameters are initialised randomly, it would be inappropriate to infer model uncertainty before the first epoch, as such, the first epoch is trained using the whole training set, and the use of the curriculum method begins starting with the second epoch. Pseudocode for the DTC method is given in section 5.1.1.1.

5.1.1.1 Pseudocode for the DTC curriculum

Pseudocode for the dynamic task curriculum method (assuming the use of the AADT uncertainty function and an easy to hard curriculum):

Require: Num_Epochs
Require: Num_Tasks
Require: $\mathcal{T}$ {Full training set on input-label pairs}
Require: $\theta_{curriculum}$ {Model to be trained with the curriculum}
  for $i = 1$ to Num_Epochs **do**
    $TrainingPhase = \lfloor \frac{i * Num\_Tasks}{Num\_Epochs} \rfloor$
    $PhaseEpochScale = \frac{Num\_Tasks}{TrainingPhase}$
    for $PhaseEpochScale$ **do**
      $\mathbf{s}^{\theta_{curriculum}} = AADT_{\theta_{curriculum}}(\mathcal{T})$
      $\mathcal{T}_{\mathbf{s}^{\theta_{curriculum}}} = \mathcal{T}$, sorted by $\mathbf{s}^{\theta_{curriculum}}$ (descending order)
      $Num\_Samples = |\mathcal{T}_{\mathbf{s}^{\theta_{curriculum}}}|$
      for $t = 0$ to Num_Tasks **do**
        $TaskStartIndex = Num\_Samples * \frac{t}{Num\_Tasks}$
        $TaskEndIndex = Num\_Samples * \frac{t+1}{Num\_Tasks}$
        $\mathcal{T}^{t}_{\mathbf{s}^{\theta_{curriculum}}} = \mathcal{T}_{\mathbf{s}^{\theta_{curriculum}}}[TaskStartIndex : TaskEndIndex, :]$
      end for
      train $\theta_{curriculum}$ on training set $\mathcal{T}^{0:TrainingPhase}_{\mathbf{s}^{\theta_{curriculum}}}$
    end for
  end for

5.1.2 Dynamic Sampling Curricula (DSC)

The second dynamic curriculum construction method we test is *Dynamic Sampling Curricula (DSC)*; one of the potential problems with some curriculum and active learning methods is that of *diversity* [26]. Diversity refers to how well the whole training set is represented in the samples used to train the model; if a sample selection method is extremely un-diverse then it may end up selecting only a very small selection of the training samples, leading to the model not training on a suitably broad set of training samples, resulting in high generalization error [26]. One way to approach this problem is to avoid deterministic methods of selecting training examples, and instead sample from the training data using some probability distribution. Mini-batch stochastic gradient descent is one such method, sampling batches of examples from the training data using a uniform probability distribution, however by varying the sampling distribution away from a uniform distribution we can bias training towards different samples, resulting in them being selected more or less often than others.

We implement the DSC curriculum method by biasing the sampling probability proportionally to the model uncertainty in predicting the labels of the sample. In particular, at the start of every epoch we use an uncertainty function (for example the AADT function introduced in 4.1.1) to infer the model's current uncertainty in predicting the labels of the training data, giving $\mathbf{s}^{\theta_{curriculum}}$, a vector of N scores as in Equation 4.1, where N is the number of samples in the full training set $\mathcal{T}$ and the i^{th} element of $\mathbf{s}$ corresponds to the output of the AADT function for the i^{th} training input. We then pass the vector of scores through the softmax function [46], effectively transforming the scores into probabilities that can be used as the sampling probability distribution. For example, assuming the uncertainty function used is the AADT function, we can set:

$$\tilde{p_j}(x_i|\theta_j) = \frac{exp(\frac{AADT_{\theta_j}(x_i)}{\tau})}{\sum_k^N exp(\frac{AADT_{\theta_j}(x_k)}{\tau})} \tag{5.2}$$

where θ_j is the model being trained with the curriculum after $j-1$ epochs, $\tilde{p_j}(x_i|\theta)$ is the sampling probability of training sample i in the j^{th} training epoch, and τ is the softmax temperature parameter that can be set as a hyperparameter or tuned each epoch to achieve a target diversity level in the sampling probability. The sampling probability is updated at the start of each epoch and training proceeds as with normal mini-batch stochastic gradient descent, using the sampling probabilities calculated in Equation 5.3 to sample from the entire training set. As we wish to bias training towards

certain certain/uncertain samples, we also do not follow the typical SGD practice of sampling without replacement, instead sampling with replacement. As sampling with replacement implies that some samples will probably not be used in every epoch, this in effect introduces a stochastic curriculum where, instead of removing samples entirely from certain training phases, we instead decrease their probability of being sampled during an epoch. Unlike the DTC curriculum method in Section 5.1.1, the DSC method does not have distinct training phases; the sampling probability is biased throughout the entirety of training. As we are no longer using training phases with smaller task training sets we do not need to modify the number of training epochs as we did with the BAC method in Chapter 4 and the DTC method. As with the DTC method in the previous section, we train the model using the whole training set, with uniform mini-batch SGD, for one epoch, before we begin estimating model uncertainty and training with the DSC curriculum. Pseudocode for the DSC curriculum method is given in Section 5.1.2.1.

5.1.2.1 Pseudocode for the DSC curriculum

Pseudocode for the dynamic sampling curriculum method, assuming the use of the AADT uncertainty function and an easy to hard curriculum (exlcuding for the softmax temperature control method, given in 5.1.5):

Require: Num_Epochs

Require: $\mathcal{T}$ {Full training set on input-label pairs}

Require: $\theta_{curriculum}$ {Model to be trained with the curriculum}

$N = |\mathcal{T}|$

for $j = 1$ to Num_Epochs **do**

$\forall x_i \in \mathcal{T}, \tilde{p}_j(x_i|\theta_j) = \frac{exp(\frac{AADT_{\theta_j}(x_i)}{\tau})}{\sum_k^N exp(\frac{AADT_{\theta_j}(x_k)}{\tau})}$

train $\theta_{curriculum}$ on training set $\mathcal{T}$, sampled with probabilities $\tilde{p}(.|\theta_j)$

end for

5.1.3 Dynamic Sampled Task Curricula (DSTC)

The final dynamic curriculum method implemented combines the the DTC and DSC methodologies, into an approach we term *dynamic sampled task curricula (DSTC)*. As in the dynamic task curricula method, at the start of every epoch we calculate the model uncertainty on the samples in the training set $\mathcal{T}$, dividing the training set into several

equally sized tasks of increasing or decreasing uncertainty. As in the DTC approach, the model is trained in several phases; in the first phase only the first task is using to train the model, then in the second phase the first two tasks are used, and so on until the entirety of $\mathcal{T}$ is being used. In the DSTC approach however, instead of sampling uniformly from training samples used in each phase, we bias the sampling probability as in the DSC model. We therefore calculate the sampling probability for sample x_i in epoch j as follows, assuming j falls in phase t, assuming we are using the AADT uncertainty function to estimate model uncertainty:

$$\tilde{p}_j(x_i|\theta_j) = \begin{cases} \frac{exp(\frac{AADT_{\theta_j}(x_i)}{\tau})}{\sum_k^K exp(\frac{AADT_{\theta_j}(x_k)}{\tau})} & \text{if } x_i \in \mathcal{T}_j^1 \cup \mathcal{T}_j^2 \cup ... \cup \mathcal{T}_j^t \\ 0 & \text{if } x_i \notin \mathcal{T}_j^1 \cup \mathcal{T}_j^2 \cup ... \cup \mathcal{T}_j^t \end{cases} \tag{5.3}$$

where K is the number of training samples in the phase training set $\mathcal{T}_j^1 \cup \mathcal{T}_j^2 \cup ... \cup \mathcal{T}_j^t$.

The DSTC method therefore combines the DTC and DSC methods, dynamically splitting the training data into uncertainty based tasks, and then sampling from the tasks proportionally to the uncertainty of the samples relative to the rest of the task. DSTC could therefore be considered a more general version of the DSC method, where the DSC method is simply a special case of DSTC when the number of tasks is equal to one. Pseudocode for the DSTC method is given in the following section.

5.1.3.1 Pseudocode for the DSTC curriculum

Require: Num_Epochs
Require: Num_Tasks
Require: $\mathcal{T}$ {Full training set on input-label pairs}
Require: $\theta_{curriculum}$ {Model to be trained with the curriculum}
  for $i = 1$ to Num_Epochs **do**
    $TrainingPhase = \lfloor \frac{i * Num\_Tasks}{Num\_Epochs} \rfloor$
    $PhaseEpochScale = \frac{Num\_Tasks}{TrainingPhase}$
    for $PhaseEpochScale$ **do**
      $\mathbf{s}^{\theta_{curriculum}} = AADT_{\theta_{curriculum}}(\mathcal{T})$
      $\mathcal{T}_{\mathbf{s}^{\theta_{curriculum}}} = \mathcal{T}$, sorted by $\mathbf{s}^{\theta_{curriculum}}$ (descending order)
      $Num\_Samples = |\mathcal{T}_{\mathbf{s}^{\theta_{curriculum}}}|$
      for $t = 0$ to Num_Tasks **do**
        $TaskStartIndex = Num\_Samples * \frac{t}{Num\_Tasks}$
        $TaskEndIndex = Num\_Samples * \frac{t+1}{Num\_Tasks}$

$\mathcal{T}^{t}_{\mathbf{s}^{\theta_{curriculum}}} = \mathcal{T}_{\mathbf{s}^{\theta_{curriculum}}}[TaskStartIndex : TaskEndIndex, :]$

end for

$\forall x_i \in \mathcal{T}^{0:TrainingPhase}, \tilde{p}_j(x_i|\theta_j) = \frac{exp(\frac{AADT_{\theta_j}(x_i)}{\tau})}{\sum_k^N exp(\frac{AADT_{\theta_j}(x_k)}{\tau})}$

train $\theta_{curriculum}$ on training set $\mathcal{T}^{0:TrainingPhase}_{\mathbf{s}^{\theta_{curriculum}}}$, sampled with probabilities $\tilde{p}(.|\theta_j)$

end for

end for

5.1.4 BALD Active Score Function

As well as using the AADT uncertainty function, defined in Section 4.1.1, we will also test the use of the BALD acquisition function[24] [17]. Recent advances in Bayesian neural networks and variational inference motivate an alternative approach to measuring uncertainty [15]; while the distance to classification threshold may encapsulate classification uncertainty for samples close to the boundary, it does not consider the uncertainty associated with analysing samples from parts of the feature space that is not represented in the training data [15]. One approach to this problem is given by [16], where they motivate using the "*Monte Carlo dropout*" method as a way of approximating variational inference in neural networks. As with the usual dropout procedure [45], weights are randomly set to zero throughout the training phase, however, unlike the usual approach, dropout is maintained at the test stage, and a number of forward passes are carried out, resulting in a distribution of outputs. The resultant distribution can subsequently be analysed to infer which test samples the model is more or less confident in predicting, for example by comparing the variance of the distribution of outputs. In [17], the authors use the MC dropout method to implement the BALD acquisition function for deep models, querying points which "maximise the mutual information between predictions and model posterior", identifying samples that have a high probability of being placed into different classes in the different stochastic forward passes. One interpretation of the BALD method is that it is similar to the 'Query by Committee' active learning methods [43], with the different forward passes representing different models' votes.

We calculate the BALD active score function as follows, as in [17]:

$$BALD_\theta(x_i) = -\sum_c^C \bar{P}_\theta(y_c|x_i)\log(\bar{P}_\theta(y_c|x_i)) + \frac{1}{M}\sum_m^M(\sum_c^C P_\theta^m(y_c|x)\log(P_\theta^m(y_c|x))), \quad (5.4)$$

and

$$\bar{P}_\theta(y_c|x_i) = \frac{\sum_m^M P_\theta^m(y_c|x)}{M}. \quad (5.5)$$

Here M is the number of stochastic forward passes carried out and $P_\theta^m(y_c|x)$ is the softmax probability of class c from the m^{th} forward pass. The score can therefore be interpreted as the difference between the entropy of the average softmax output and the average entropy of the output of each forward pass [17].

5.1.5 Softmax temperature

In order to homogenize the outputs of the different active score functions, we pass the the scores through a softmax function, resulting in an output of softmax probabilities summing to one. Using the softmax function also allows us to use the softmax temperature in order to control the diversity of the sampling probabilities. A common issue with active learning is that the acquisition functions can end up sampling from an unrepresentative subset of the input space, resulting in significant bias in the training of the model [43]. We control this effect by using the softmax temperature to target a preset *maximum probability ratio*, defined as follows:

$$MaxProbRatio = \frac{\max_i \tilde{p}_\theta(x_i)}{\min_i \tilde{p}_\theta(x_i)}. \tag{5.6}$$

To do this, we begin with a softmax temperature of 1, then calculate the current maximum probability ratio and increment the temperature until the target ratio is achieved. Pseudocode is given below:

Require: $TargetMaxProbRatio$
Require: $SoftmaxTemperature$
Require: Training Set $\mathcal{T}$
Require: Sampling Probabilities $\tilde{p}_\theta(x_i) \forall x_i \in \mathcal{T}$
  $MaxProb = \max_{x_i \in \mathcal{T}}(\tilde{p}_\theta(x_i))$
  $MinProb = \min_{x_i \in \mathcal{T}}(\tilde{p}_\theta(x_i))$
  $MaxProbRatio = \frac{MaxProb}{MinProb}$
  if $MaxProbRatio > TargetMaxProbRatio$ **then**
    while $MaxProbRatio > TargetMaxProbRatio$ **do**
      $SoftmaxTemperature \leftarrow SoftmaxTemperature * 1.01$
      Recalculate $\tilde{p}_\theta(x_i) \forall x_i \in \mathcal{T}$
      $MaxProb = \max_{x_i \in \mathcal{T}}(\tilde{p}_\theta(x_i))$
      $MinProb = \min_{x_i \in \mathcal{T}}(\tilde{p}_\theta(x_i))$
      $MaxProbRatio = \frac{MaxProb}{MinProb}$
    end while

else

while $MaxProbRatio < TargetMaxProbRatio$ **do**

$Softmax Temperature \leftarrow Softmax Temperature * 0.99$

Recalculate $\tilde{p}_\theta(x_i) \forall x_i \in \mathcal{T}$

$MaxProb = \max_{x_i \in \mathcal{T}}(\tilde{p}_\theta(x_i))$

$MinProb = \min_{x_i \in \mathcal{T}}(\tilde{p}_\theta(x_i))$

$MaxProbRatio = \frac{MaxProb}{MinProb}$

end while

end if

return $\tilde{p}_\theta(x_i) \forall x_i \in \mathcal{T}$

The downside to this method is it could be skewed by outlier probabilities; if there were one sample with an extremely large sampling probability this method would still achieve a target maximum probability ratio, without achieving sufficient diversity in the sampling probabilities. We investigated using outlier removal techniques such as winsorization [18] prior to tuning the temperature parameter, however, as the uncertainty functions we use generally did not result in outliers, this did not affect results. While we do not lay out a thorough analysis of this in this paper, we did found that controlling the maximum probability ratio throughout training helped to stabilise model performance in the DSC and DSTC curriculum methods. Without this softmax temperature control approach, for example by simply setting the softmax temperature to a constant value of 1, there were times when the sampling probabilities in the DSC and DSTC methods were such that either the maximum probability ratio grew extremely large, resulting in a lack of diversity in the sampling, or the ratio was so small as to render the biased sampling method negligible. In the experiments laid out below, we use a target maximum probability ratio of 10, which we generally found led to a good level of diversity without limiting the impact of the sampling bias.

5.2 Datasets

We test the dynamic active curricula on the Geometric Shapes data, introduced in Section 4.2, comparing the curriculum model test performances to baseline models trained on the entire training set. In order to broaden out the scope of the experiments, we also test the curricula on the MNIST [33] and CIFAR 10 [30] datasets, detailed below:

5.2.1 MNIST

The MNIST dataset [33] contains images of handwritten digits, consisting of a training set of 60,000 training images and 10,000 test images (in our experiments we further split the training set into a training set and a validation split using 75% training, 25% validation split). The input data are 28x28 greyscale pixel values, while the target labels are one-hot vectors indicating which digit is written in the image, from 0 to 9. MNIST is a popular image classification task due to its relative simplicity, with state of the art architectures achieving test accuracies well in excess of 99% [51]. Several examples images from the MNIST dataset are shown in Figure 5.1.

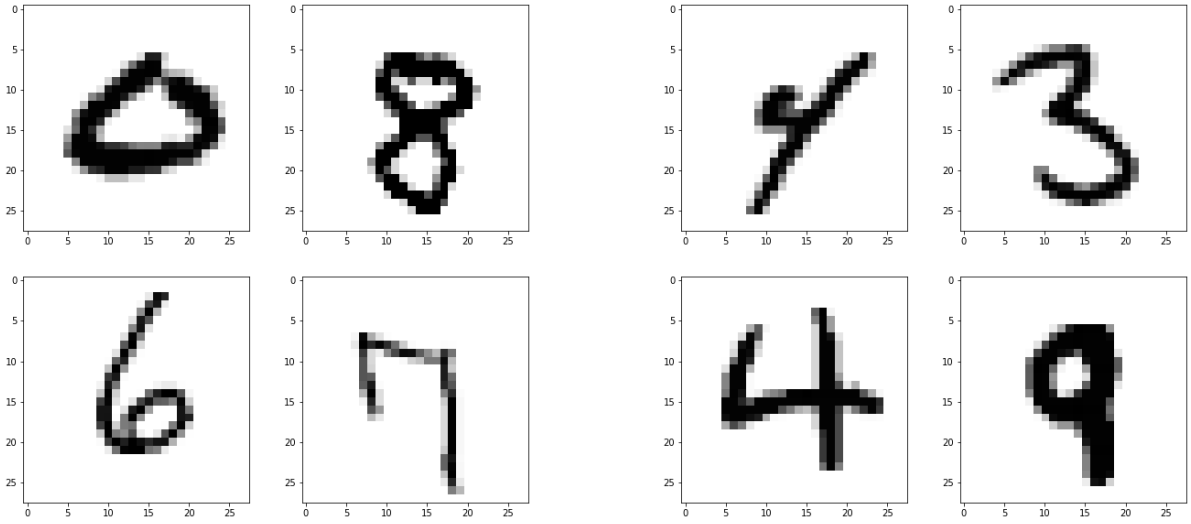


Figure 5.1: Sample figures from the MNIST dataset

5.2.2 CIFAR 10

The CIFAR 10 dataset [30] is another image classification dataset; the inputs for each samples are 32x32x3 RGB colour pixel values (3 separate channels for the Red, Green and Blue colour value), while the labels specify the class of each image. The possible classes are ‘airplane’, ‘automobile’, ‘bird’, ‘cat’, ‘deer’, ‘dog’ ,’frog’, ‘horse’, ‘ship’ and ‘truck’. As can be seen in Figure 5.2, the images are significantly downsampled, resulting in images that can be difficult even for a human observer to classify, resulting in a significantly more difficult task than either MNIST or the Geometric Shapes

database. For this dataset we use convolutional networks, as introduced in Section 2.2.2, this will both improve the results of the experiments with CIFAR 10, but will also test the performance of the curriculum methods with a different type of network.

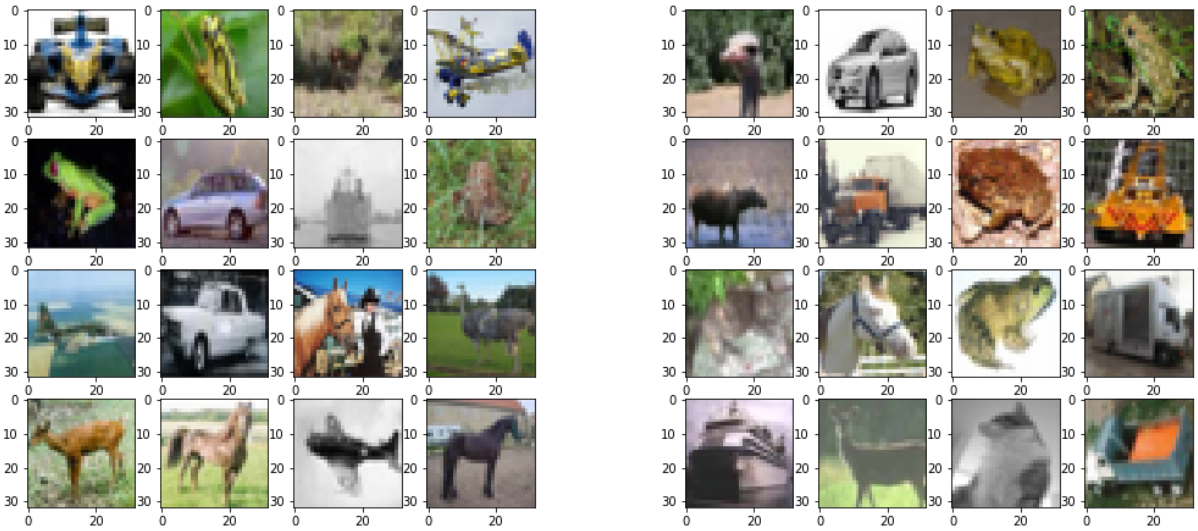


Figure 5.2: Sample figures from the CIFAR 10 dataset

5.3 Experiments

We test the different dynamic curriculum methods introduced above on the Geometric Shapes, MNIST and CIFAR 10 image classification datasets, as introduced in Sections 4.2, 5.2.1 and 5.2.2, respectively. For each dataset we used a validation set to construct an architecture and hyperparameters which achieved reasonable accuracies, then used those setting in the experiments for the different curriculum methods, the architectures for each dataset are laid out in Tables 5.1 to 5.3. Note that in the case of CIFAR, the convolutional layer (denoted "conv") are following by a max pooling layer [38] and a batch normalization layer [25], and ReLU denotes the Rectified Linear Unit activation function[39].

We run experiments across all three datasets using the curriculum methods laid out above, testing both 'easy to hard' curricula and 'hard to easy' curricula, as well as tested using different uncertainty functions. In each case we also train a baseline model trained without a curriculum to provide a comparison for the curriculum model test

Table 5.1: Geometric Shapes Dataset Model Architecture

Geometric Shapes Dataset Model Architecture							
	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5	Layer 6	Layer 7
Layer Type	FC	Dropout	FC	Dropout	FC	Dropout	FC
Units	300	NA	300	NA	300	NA	3
Activation	Tanh	NA	Tanh	NA	Tanh	NA	Softmax

Table 5.2: MNIST Dataset Model Architecture

MNIST Dataset Model Architecture							
	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5	Layer 6	Layer 7
Layer Type	FC	Dropout	FC	Dropout	FC	Dropout	FC
Units	300	NA	300	NA	300	NA	3
Activation	ReLU	NA	ReLU	NA	ReLU	NA	Softmax

Table 5.3: CIFAR Dataset Model Architecture

CIFAR Dataset Model Architecture							
	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5	Layer 6	Layer 7
Layer Type	Conv	Conv	Flatten	Dropout	FC	Dropout	FC
Units	50	50	NA	NA	100	NA	10
Activation	ReLU	ReLU	NA	NA	ReLU	NA	Softmax
Kernel Size	3x3	3x3	NA	NA	NA	NA	NA

performances. We did not test all permutations of curricula and uncertainty functions as it became clear in some experiments that there was little difference from the baseline performance, the experiments for which we performed repeated tests and present results are as follows:

- Geometric Shapes, using the AADT uncertainty function:
  - Dynamic Task Curriculum, Easy to Hard
  - Dynamic Task Curriculum, Hard to Easy
  - Dynamic Sampling Curriculum, Hard to Easy
  - Dynamic Sampled Task Curriculum, Hard to Easy
  - Uniform Sampling Baseline

- MNIST, using the AADT uncertainty function:
  - Dynamic Task Curriculum, Easy to Hard
  - Dynamic Sampling Curriculum, Easy to Hard
  - Dynamic Sampled Task Curriculum, Easy to Hard
  - Dynamic Task Curriculum, Hard to Easy
  - Dynamic Sampling Curriculum, Hard to Easy
  - Dynamic Sampled Task Curriculum, Hard to Easy
  - Uniform Sampling Baseline
- CIFAR 10, using the BALD uncertainty function[2]:
  - Dynamic Task Curriculum, Easy to Hard
  - Dynamic Sampled Task Curriculum, Easy to Hard
  - Dynamic Task Curriculum, Hard to Easy
  - Dynamic Sampled Task Curriculum, Hard to Easy
  - Uniform Sampling Baseline

Table 5.4: Experiment Hyperparameters

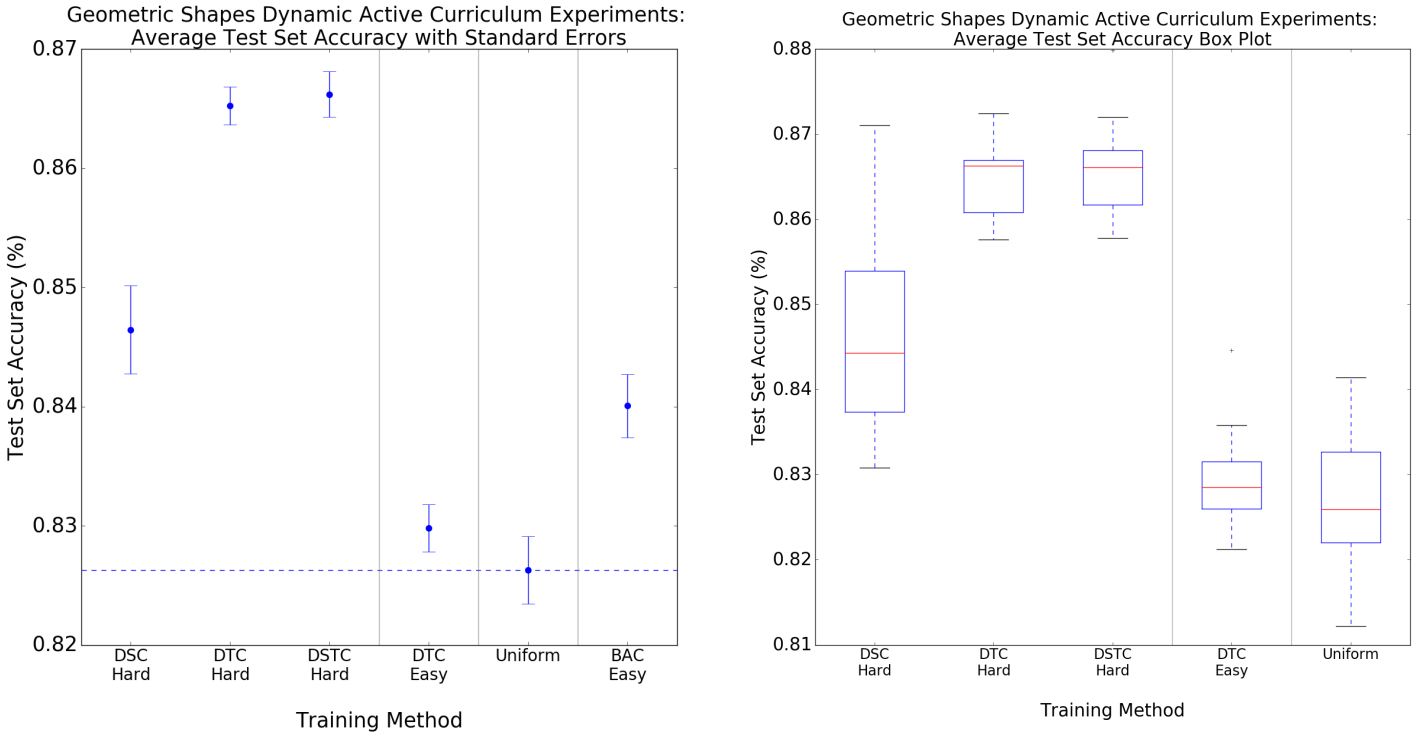
Experiment Hyperparameters						
	Epochs	Optimiser	Learning Rate	Dropout %	Batch Size	Num Tasks
Geometric Shapes	350	Adam	0.0001	0.25	32	2
MNIST	50	Adam	0.0001	0.25	50	2
CIFAR 10	50	Adam	0.0001	0.25	100	2

We run the experiments 10 times with different random initialistions, reporting average results and standard errors. In Table 5.4 we lay out the other hyperparameters for the experiments. Note that in each case we keep the number of tasks at 2; results were robust to differing the number of tasks however as we did not have time to do a full analysis we decided to keep the experimental setups relatively homogeneous in order to avoid tinkering with hyperparameters to achieve positive results. As in Chapter 4, experiments were carried out in Keras [9] and the Adam optimiser is implemented with the default hyperparameters (except for the learning rate which is given in Table 5.4).

[2]Tests for CIFAR 10 with the AADT function resulted in little difference with from baseline, however using the BALD uncertainty function improved performance significantly.

5.4 Results and Discussion

The results for the experiments are illustrated in Figures 5.3 and 5.5[3], full results are detailed in Tables B.1 to B.3. Note that the denotation 'easy' implies that the curriculum was trained on an easy to hard curriculum, with increasingly difficult tasks in the DTC case and by biasing sampling towards easy samples in the DSC method, and similarly for the DSTC method. Conversely, 'Hard' denotes that the curriculum used was a hard to easy curriculum.



(a) Mean Test Set Accuracies

(b) Test Set Accuracy Boxplots

Figure 5.3: Geometric Shapes Dynamic Active Curriculum (DAC) Experiment Results

The Geometric Shapes results shows a significant increase in test performance, particularly for the 'hard' curriculum models, where the model is trained on increasingly easy samples, contrary to the usual curriculum learning approach and more in line with an active learning philosophy on sample selection. All three 'hard' curriculum construction methods outperformed the benchmark, and, as shown in Figure 5.3a, also significantly outperformed the bootstrapped active curriculum approach from Chapter 4. Nevertheless, the more traditional 'easy' curriculum method also outperformed the

[3]Due to the experimental setup for the CIFAR 10 experiments, only test accuracies were recorded.

benchmark in the DTC case, although it underperformed the BAC test performance, and the 'easy' sampling based dynamic curriculum methods did not lead to significantly different results to the uniform baseline model (not shown here).

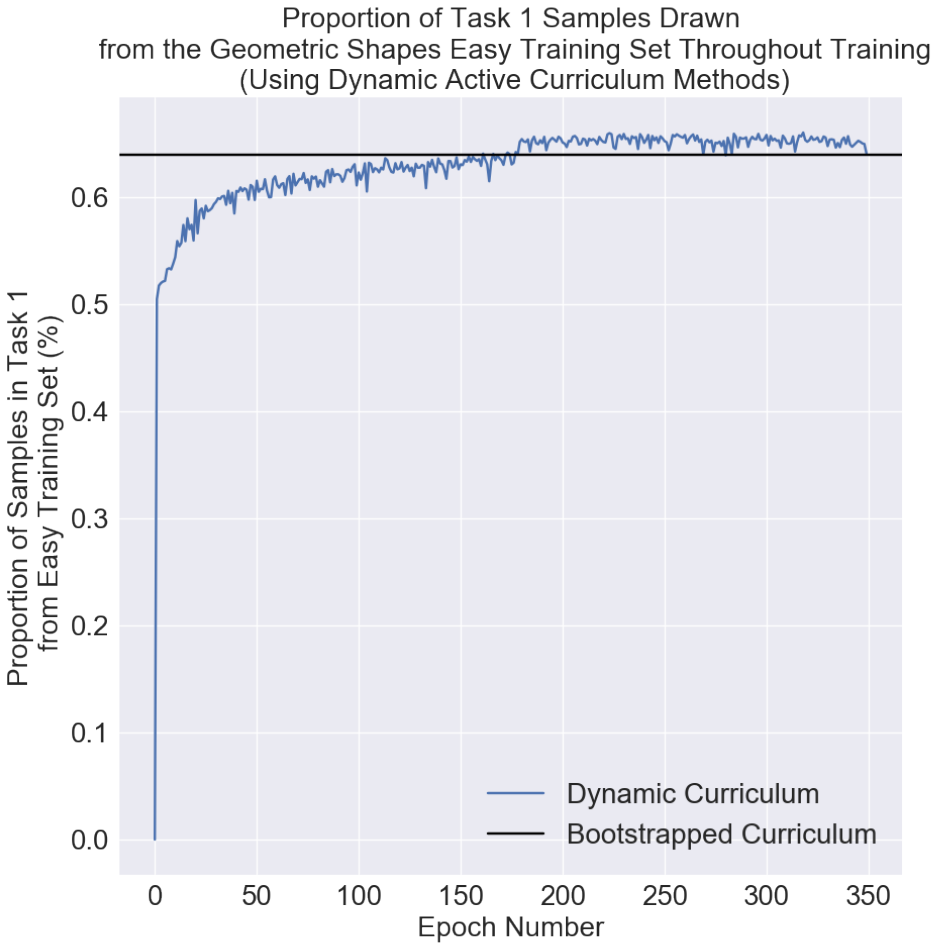
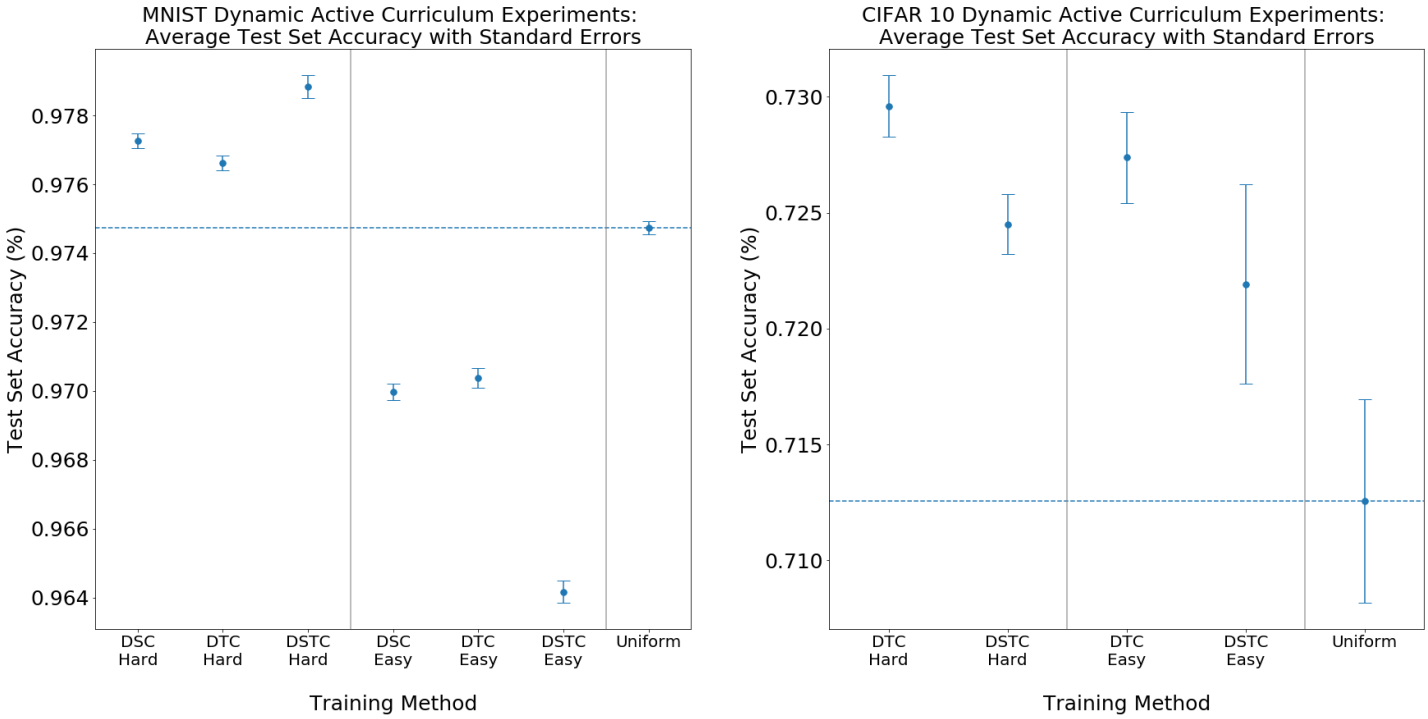


Figure 5.4: Analysis of the proportion of samples from the construct 'easy' first tasks of the dynamic active curricula taken from the easy Geometric Shapes training data.

Analysing the curriculum methods throughout training provides some insights into how the dynamic curricula choose training samples. As discussed in Section 4.2, the Geometric Shapes training data is made of up an 'easy' training set of regular shapes and a 'hard' training set of more general shapes. We can see from monitoring the output of the uncertainty function throughout training that the model can quickly starts to discriminate between the hard and easy training set, with the first task constructed at the start of each epoch of the easy dynamic task curriculum quickly becoming biased towards samples from the easy training set, suggesting that one does not need to fully train a baseline model, as in the bootstrapped curriculum method, to be able

to approximate the relative difficulty of the training samples. Indeed, the proportion of the first, easier task coming from the easy training set throughout training quickly converges and ultimately exceeds the proportion in the easy task of the boostrapped curriculum method, where the model used to measure uncertainty is fully trained, as illustrated in Figure 5.4. This result is slightly confounding however, as it does not explain why the hard to easy versions of the dynamic active curricula achieve much higher test performances than the easy to hard methods, given that the dynamic curricula seem to ranking the samples in similar ways, yet the hard to easy boostrapped curriculum method underperformed even the baseline model.



(a) Mean Test Set Accuracies

(b) Test Set Accuracy Boxplots

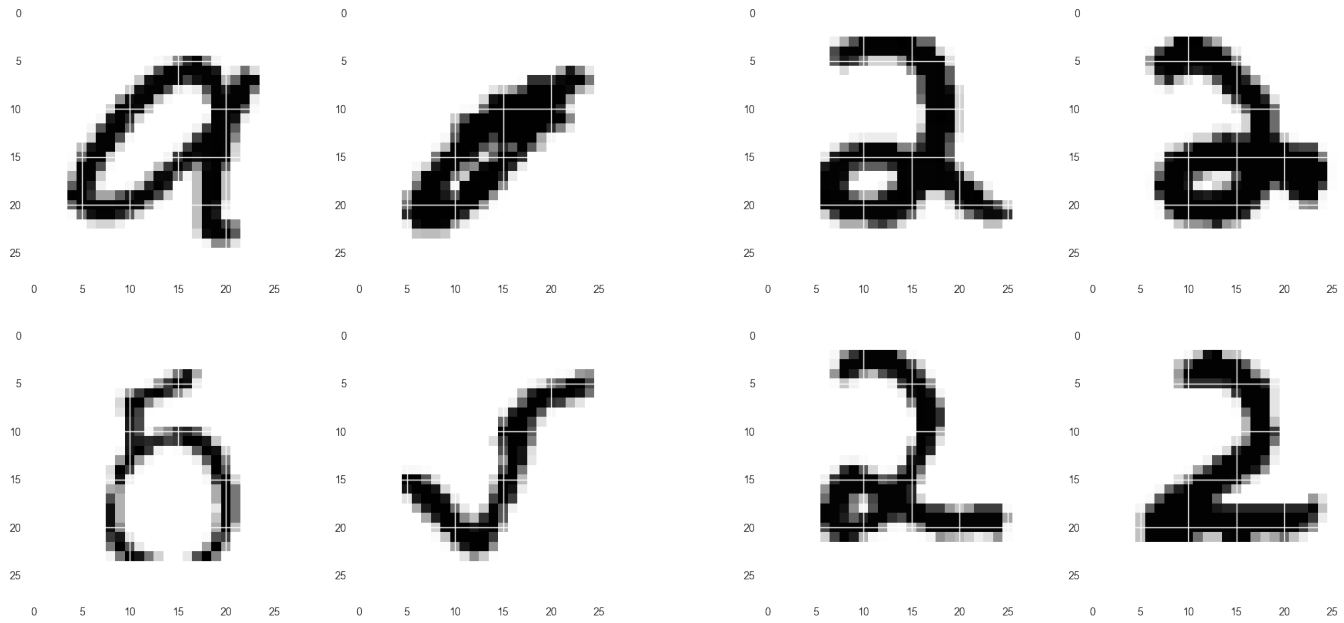
Figure 5.5: MNIST and CIFAR 10 Dynamic Active Curriculum (DAC) Experiment Results

Figure 5.5a shows the performance of the curriculum methods for the MNIST dataset; as in the case of Geometric Shapes, the best performing curriculum is the hard to easy dynamic sampled task curriculum, combining the dynamic task based approach with the biased sampling methodology. While the outperformance in the case of MNIST is fairly small on an absolute basis (the best performing model achieving an average test accuracy only 0.4% higher than the baseline model), given the rela-

tively high overall accuracy achievable on the MNIST dataset this might be considered significant, particularly given the consistency in the results illustrated by the standard error bars. Unlike in the Geometric Shapes results however, the MNIST test accuracies for the easy to hard curricula are significantly lower than the baseline. One hypothesis for explaining this is that emphasising easy samples may be most effective in the early stages of training when the model is still poorly calibrated on the task. In the case of MNIST however, validation accuracy can exceed 90% after only a single training epoch, suggesting that it is such a simple task that an easy to hard curriculum would not be beneficial, and that it would be better to focus training on difficult samples instead, explaining the performance of the hard to easy curriculum methods. This dynamic is also suggested in [8] and [53], where it is suggested that the efficacy of curriculum learning may be related to the difficulty of the task, relative to the capacity of the network. We attempted to test this hypothesis by repeating the experiments with a 'noisy' version of the MNIST dataset, where the input images are corrupted with increasing levels of random noise; while we did not find any variation in the relative order of test accuracies shown in Figure 5.5a, we did notice that the model was still able to achieve high levels of accuracy on the validation set very early in training, suggesting that the noise adding procedure was ineffective and that perhaps different data augmentation techniques would be better suited to testing this hypothesis.

While there is not a clear delineation of difficulty in the MNIST dataset, examining the model uncertainty for the training set reveals that the method does appear to be effective at identifying samples that correspond to an intuitive sense of difficulty. Figure 5.6a shows the samples that one of the curriculum models was most uncertain about classifying (at the end of training), while Figure 5.6b shows the samples the model was most certain about classifying. Figure 5.6b clearly shows that the model is most confident in classifying the number 2, while 5.6a illustrates the irregularity of the samples that the model is uncertain in classifying, with some samples not even appearing to be numbers. This analysis would suggest that using an active learning inspired uncertainty metric is an effective way of automatically inferring sample difficulty, even when done dynamically whilst training the model. Furthermore the outperformance of the curriculum methods would suggest that these uncertainty measures can effectively be used to construct a learning curriculum.

Figure 5.5 shows the results for the CIFAR 10 dataset, using the BALD activation function detailed in Section 5.1.4. Interestingly, both the easy to hard and hard to easy dynamic curriculum methods consistently exceeded the benchmark test accuracy, with

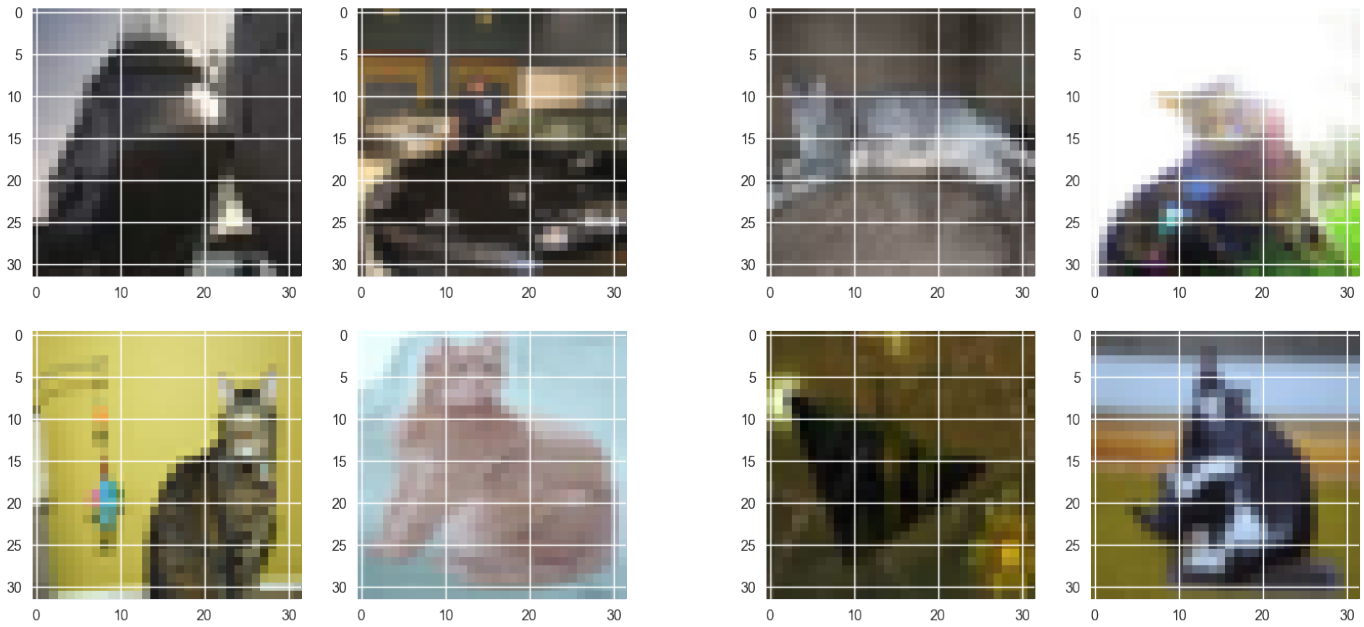


(a) 4 Most Uncertain MNIST Samples

(b) 4 Least Uncertain MNIST Samples

Figure 5.6: Top 4 most/least uncertain training samples from the MNIST dataset using a dynamic curriculum model.

the best performing curriculum method being the hard to easy DTC method which improved on the test accuracy by an average of 1.7%. Unlike with MNIST and Geometric Shapes however, in both cases adding the sampling component and sampling from the tasks proportionally to uncertainty inhibited performance, with the best performance in both the hard to easy and easy to hard case coming from the purely task based dynamic curricula. The fact that both the easy to hard and hard to easy curricula outperformed significantly in the case of CIFAR 10 is an interesting result, supporting the idea that beginning training with easy samples is most beneficial for 'difficult' problems. How one defines whether or not an entire dataset is difficult however, and is of course relative to the capacity of the learning algorithm. Nevertheless, simply based on test accuracy and training progression, CIFAR 10 would appear to be a more difficult task than learning on either the MNIST or Geometric Shapes datasets, supporting the suggestion in [8] and [53] that the effectiveness of curriculum methods is affected by the difficulty of the tasks. As with with the MNIST dataset there is no predefined curriculum for the CIFAR 10 dataset, however we can again analyse one of the trained

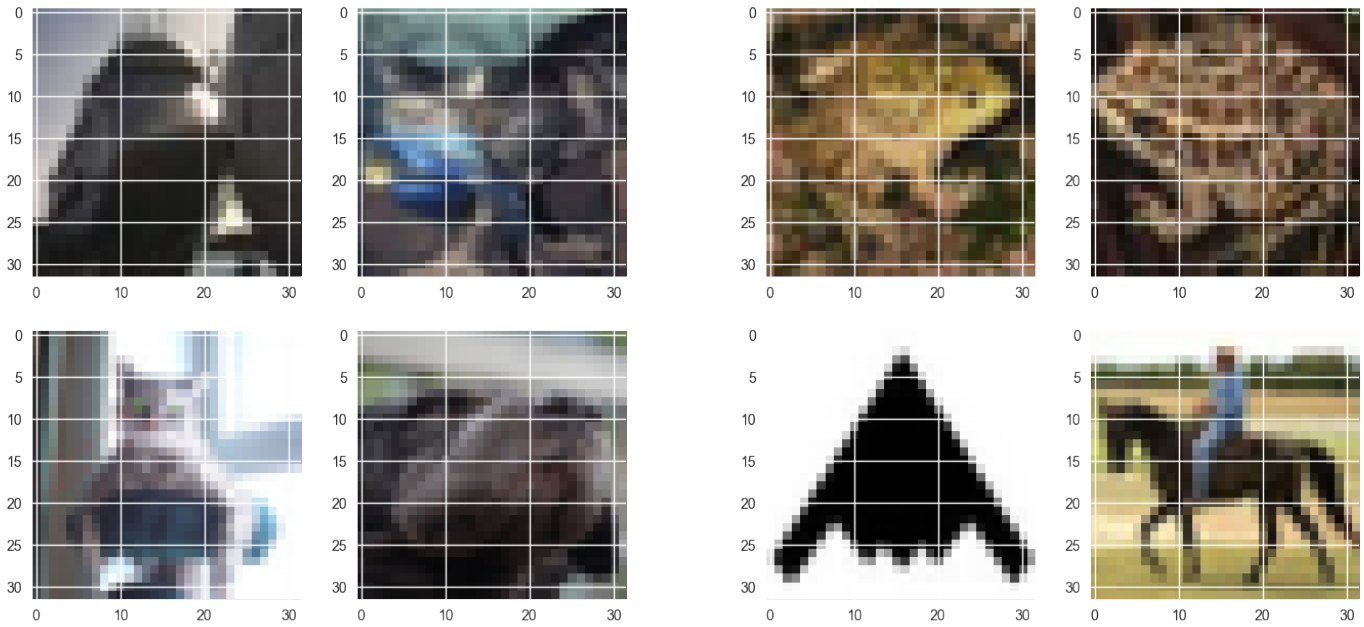


(a) 4 Most Uncertain CIFAR Samples (BALD)

(b) 4 Least Uncertain CIFAR Samples (BALD)

Figure 5.7: Top 4 most/least uncertain training samples from the CIFAR dataset using a dynamic curriculum model (BALD uncertainty function).

models' classification uncertainty over the training set. Figure 5.7 illustrates the four most and least uncertain samples at the end of one of the dynamic curriculum training runs, using the BALD uncertainty function. Unlike in the Geometric Shapes and MNIST datasets, the results are less intuitive; while the top two samples of Figure 5.7a do appear to be difficult to classify visually, the bottom two samples are clearly of the class 'cat'. Similarly while some of the least uncertain samples shown in Figure 5.7b appear to be 'easy' samples, insofar as the class labels seem obvious, the bottom two images are less clearly easy. This may be the result of using the BALD uncertainty function, which gives an arguably more abstract definition of uncertainty than the distance to classification threshold method. Indeed, repeating the exercise with the AADT uncertainty function (4.1.1), gives a slightly more intuitive result, as shown in Figure 5.8. Interestingly, the most uncertain image (the top left image in Figures 5.7a and 5.8a are the same, suggesting some overlap between the uncertainty measures. However the least uncertain images using the AADT function seem to correspond much more to an intuitive sense of difficulty compared to the BALD function, with the bot-



(a) 4 Most Uncertain CIFAR Samples (AADT)

(b) 4 Least Uncertain CIFAR Samples (AADT)

Figure 5.8: Top 4 most/least uncertain training samples from the CIFAR dataset using a dynamic curriculum model (AADT uncertainty function).

tom two images of Figure 5.8b clearly belonging to the 'Airplane' and 'Horse' classes. Nevertheless, the experiments with the AADT uncertainty with the CIFAR 10 dataset did not produce results significantly different to the baseline model, whereas using the BALD method led to consistently higher test accuracy. This might be seen to support the findings in Chapter 4 that even if we had a predefined curriculum according to a manual interpretation of sample difficulty, it may still be preferable to use an automated curriculum method that uses information from the model being trained to infer sample difficulty, such as those laid out in this paper.

To conclude, the experiments carried out show promising results that the dynamic approach of curriculum construction can lead to improved model generalization, outperforming the more costly 'boostrapped' curriculum approach laid out in Chapter 4. Moreover, the dynamic active curriculum approach offers a novel and relatively simple way of potentially improving model performance, requiring changes to the standard deep model learning procedure that can be implemented with most deep learning software. More research is needed into which curriculum method is most appropriate for

a given dataset, for example questions remain over which uncertainty function leads to the best results and whether or not an easy to hard or hard to easy curriculum is most appropriate. We develop these questions, as well as other suggestions for future research directions in the next chapter.

Chapter 6

Conclusion and Future Research Directions

6.1 Concluding Analysis

In this thesis we have introduced the concepts of curriculum and active learning, and motivated the use of active learning uncertainty methods as a way of approximating sample uncertainty in order to automate the process of constructing learning curricula for deep models. In Chapter ch:BootstrappedActiveCurricula we implemented the first curriculum construction approach, 'Bootstrapped Active Curricula (BAC)'. In this approach, we estimated training sample uncertainty from a fully trained model, using the uncertainty scores to construct a task based curriculum and effectively 'bootstrapping' a curriculum from a baseline model. We tested the method using the Geometric Shapes dataset, and the results laid out in Section 4.4 show that the boostrapped active curriculum method consistently outperforms a baseline model trained uniformly on the entire training set, as well as the predefined curriculum approach used in [6], a seminal paper in the curriculum learning literature. Analysing how the automated curriculum selects samples to use in the different training phases supports the use of uncertainty metrics as a measure of sample uncertainty, with the chosen samples corresponding well with an intuitive sense of difficulty in the data. An interesting result however was that the automated curriculum actually outperformed using a predefined curriculum, suggesting that inferring sample difficulty directly from the model being trained may lead to better results than using manually assigned difficulty ratings. We concluded this chapter by discussing some of the computational drawbacks of the boostrapped active curriculum approach, for example the need to first train a baseline model to implement

the curriculum, before proposing the use of dynamic active curriculum methods.

In Chapter 5, we implement several 'Dynamic Active Curricula (DAC)' methods; in contrast to the bootstrapped active curriculum approach sample difficulty is estimated throughout training using the curriculum model itself, as opposed to using a pre-trained baseline model. We tested three different variations of this method; the first, 'Dynamic Task Curricula', followed the BAC approach by repeatedly splitting the training set into separate tasks at the start of every epochs and training on increasingly difficult tasks throughout training. The 'Dynamic Sampled Curricula' method instead altered the usual mini-batch stochastic gradient descent optimisation algorithm by sampling from the training samples proportionally to the samples' relative classification uncertainty, as opposed to using a uniform sampling probability. Finally, the 'Dynamic Sampled Task Curricula' combined the two other dynamic curriculum approaches, diving the training set into tasks and then sampled mini-batches from the task proportionally to the uncertainty scores within the task. We again tested the different approaches on the Geometric Shapes dataset, as well as the MNIST handwritten digits recognition task and the CIFAR 10 image classification dataset. Again we found that most of the experiments consistently resulted in a higher test accuracy than a baseline model trained uniformly on the entire training set. As well as testing curricula where training progressed through increasingly difficult samples, as per the usual curriculum approach, we also tested the opposite, where training progresses through increasingly easy samples. Interestingly, in the dynamic curriculum methods we consistently found that using a 'hard to easy' curriculum led to superior results than the usual 'easy to hard' approach. We discussed why this may be the case, supporting ideas put forward in works such as [8] and [53] which suggest that the effect of curriculum methods may be affected by the difficulty of the learning task, relative to the capacity of the network being trained. We further analysed the experiments, investigating how the different uncertainty functions affected which samples are classified as easy or difficult, finding intuitive results in the case of the MNIST dataset, with the most uncertain samples corresponding to handwritten images that are indeed difficult to classify, even for a human observer. In the CIFAR 10 dataset the effect was less clear, particularly using the BALD [24] uncertainty function, however the curriculum nevertheless led to superior test performance than the baseline model, again suggesting that using the model to infer sample difficulty for curriculum construction may lead to even better results than using a handcrafted curriculum.

Overall we feel the results of this study support both the hypothesis that curriculum

learning can effectively be used to improve the generalization performance of deep models, and that using active learning approaches such as uncertainty estimation can be an effective way to construct such curricula in the absence of, and potentially even instead of, handcrafted curricula. The curriculum approaches laid out in this paper offer easily implementable methods to train deep models that can potentially be used with any network architecture, without significant computational cost.

6.2 Suggested Future Research Directions

In this paper we tested various automated curriculum construction techniques, using active learning uncertainty methods to infer sample difficulty. There are several directions we would suggest for developing the work further; in the first instance we would broaden out some of the methods used in this paper, for example by testing different uncertainty functions or even other active learning methods such as query by committee or expected model change [43]. Further work could also be done to analyse sensitivities to changing the parameters of the experiments, particularly those relevant to the curriculum construction, for example the chosen number of tasks. An open question as a result of this work is under what conditions do different curriculum methods best improve learning; we saw particularly in Chapter 5 that the easy to hard curricula performed best on the most difficult task, CIFAR 10. One could argue that, intuitively, a curriculum emphasising difficult samples may be best suited to easier tasks, in the same way that an expert in a field will learn most from studying tmore complex concepts, whereas a novice will benefit from beginning with learning more basic, foundational concepts. Future experiments could test for this effect further with different datasets and architectures, potentially formalising the link between curriculum design and problem complexity. Another limitation with this work is that it had been constrained to image classification, future work could test these methods as applied to other machine learning tasks such as natural language processing, or, more broadly, regression tasks.

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Appendix A

Bootstrapped Active Curriculum Results

Table A.1: Geometric Shapes BAC Results

GeoShapes BAC Results		
	Test Accuracy (%)	Test Cross-Entropy Error
Uniform Baseline	0.825 ± 0.00269	0.440 ± 0.00614
Easy to Hard Curriculum	0.840 ± 0.00267	0.398 ± 0.00670
Hard to Easy Curriculum	0.815 ± 0.00278	0.467 ± 0.00699
Adjusted Predefined Curriculum	0.800 ± 0.00143	0.486 ± 0.00409
Predefined Curriculum	0.757 ± 0.00279	0.576 ± 0.00489

Appendix B

Dynamic Active Curriculum Results

Table B.1: Geometric Shapes Dynamic Active Curricula Results

GeoShapes Dynamic Active Curricula Results		
	Test Accuracy (%)	Test Cross-Entropy Error
Uniform Baseline	0.826 ± 0.00281	0.437 ± 0.00696
DSC - Hard	0.846± 0.00369	0.391 ± 0.00827
DTC - Hard	0.865 ± 0.00157	0.362 ± 0.00350
DSTC - Hard	0.866 ± 0.00192	0.355 ± 0.00502
DTC - Easy	0.830 ± 0.00198	0.420 ± 0.00469

Table B.2: MNIST Dynamic Active Curricula Results

MNIST Dynamic Active Curricula Results		
	Test Accuracy (%)	Test Cross-Entropy Error
Uniform Baseline	0.975 ± 0.000188	0.229 ± 0.000562
DSC - Hard	0.977± 0.000216	0.250 ± 0.000497
DTC - Hard	0.977 ± 0.000216	0.227 ± 0.000611
DSTC - Hard	0.979 ± 0.000327	0.251 ± 0.000736
DSC - Easy	0.970± 0.000240	0.227 ± 0.00102
DTC - Easy	0.970 ± 0.000276	0.224 ± 0.000696
DSTC - Easy	0.964 ± 0.000314	0.228± 0.000595

Table B.3: CIFAR 10 Dynamic Active Curricula Results

CIFAR 10 Dynamic Active Curricula Results	
	Test Accuracy (%)
Uniform Baseline	0.713 ± 0.00439
DTC - Hard	0.730 ± 0.00132
DSTC - Hard	0.725 ± 0.00129
DTC - Easy	0.727 ± 0.00197
DSTC - Easy	0.722 ± 0.00429