CHAPTER 12

Fundamentals of RF Testing

In the first chapter of the RF portion of this textbook, we will review the physical basis of an RF signal so that we can better understand the RF test techniques introduced in the following chapter. We will start with the fundamentals of waves, which are the most efficient way of describing power transport within the RF frequency spectrum, followed by a vector description of an RF signal with scattering parameters.

12.1 INTRODUCTION TO RF TESTING

Radio transmission is over 100 years old, and the frequency bands for wireless transmission have been allocated and reallocated as technologies have progressed. In the beginning, there was only an AM modulated radio transmission, but in the last decades the technology has together with low cost manufacturing enabled a large variety of RF systems, like cellular telephones, pagers, satellite radio, or data services like WLAN, WiMAX, or UWB. Many of these applications had been previously served by wired application, but with the technology available, more and more of these services became wireless and commercially available in high volume. These and many other services use the electromagnetic spectrum depicted in Figure 12.1. A more detailed frequency allocation can be requested from national telecommunication organizations, like FCC in the US1 or ETSI2 in Europe, to name just a few.

With progressing technology, it was also commercially valid to build systems covering a wide power range for personal two-way radios. The low-power range had been accessible for a long time with radio receivers, while the high-power range had been used for transmitting radio stations. Modern personal two-way radio systems will continue to use the lower end of the power range, because the high end is limited due to (a) supply power restrictions and (b) as well as concerns over the health effects of high-power, high-frequency radio waves. The power range used for cellular phones or data services like WiMAX is in the range of a few watts.

There are multiple aspects the RF test engineer will have to consider when working on new devices. One is the extreme dynamic range covered with RF devices, ranging from noise measurement in the sub-attowatt range ($10^{-18}\,\mathrm{W}$) up to a few watts when testing commercial power

Figure 12.1. Frequency spectrum of some commercial RF devices.

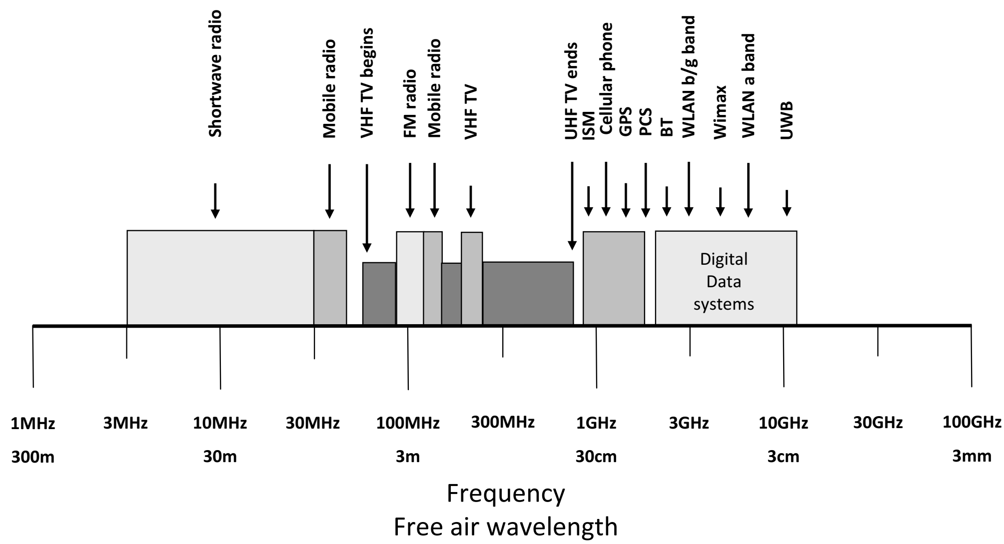
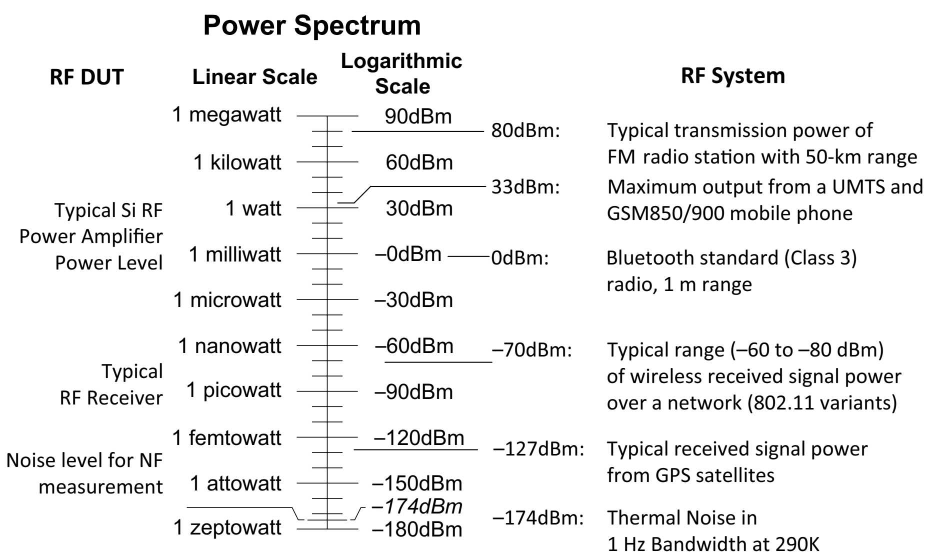


Figure 12.2. Power spectrum of consumer and commercial RF devices.



amplifiers, as depicted by the illustration in Figure 12.2. Commercial ATE systems typically cover a dynamic range as high as 170 dB without any external amplifier or attenuator.

A second RF differentiator to digital and mixed-signal testing is the huge bandwidth the test system needs to cover. Existing commercial ATE already cover the 12-GHz range for testing UWB devices. Recently, RF devices in the >50-GHz range have become commercially available for radar detection in automobiles. The authors assume that these frequency ranges will be around in the foreseeable time frame for data transmitting systems and other commercial products. In this chapter, we will limit our focus to test techniques in the sub-12-GHz range, however, the physics are still applicable to the high-gigahertz frequency range.

A third aspect that we will discuss in great depth is the physics of high frequency signals and its corresponding power. At a lower frequency, the power associated with an electrical signal can be expressed in terms of electron flow and electron position, while it will be more convenient at high frequency to describe the physical phenomena as waves. Later in this chapter, the scattering parameter will be explained based on the concept of waves.

12.2 SCALAR VERSUS VECTOR MEASURES

In RF test technology, we typically have two kinds of values: scalar and vector. Scalar values have by definition no direction and can be expressed by a single point in an n-dimensional space, while a vector has a magnitude and direction. Interesting enough, the magnitude of a vector can be described by a scalar value. In many cases in RF test technology, the magnitude of the wave is sufficient to describe the test metric, however, in some cases it will be necessary to take vector components into account.

Since time is a scalar (a nondirected value), we can assume that time-dependent performances, like magnitude over time as we have seen previously in Chapter 9, is a scalar measurement. However, it is best to describe multiple phenomena with waves (like the reflection we know from the propagation of light) in RF. These waves are not only time dependent, but also have a direction. Therefore we should treat electrical waves and their phenomena as vector values.

12.2.1 Wave Definition of Electrical Signals

Based on the definitions and the experimental results of the English chemist and physicist Michael Faraday, the Scottish mathematician and theoretical physicist James Clerk Maxwell published *A Treatise on Electricity and Magnetism* in 1873, in which he described all known dependencies of electricity. He also described the basics of the electromagnetic theory, which was proven experimentally in subsequent years. We will avoid a detailed description of Maxwell's equations here, and instead we jump to a specific result of these equations as it applies to plane waves (i.e., waves that have the same magnitude at any point in a plane perpendicular to the wave propagation).

Based on the Maxwell's equation, the equation for a plane wave with electric field E traveling in one direction in space in a homogeneous material can be expressed as

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \tag{12.1}$$

The same form applies for the magnetic field wave B. For a plane wave, the electric field vector and the magnetic field vector are in a plane perpendicular to their direction of travel and they are also perpendicular to each other in this plane. In this discussion they are assumed to be traveling in the x direction. The symbol c represents the speed of the electromagnetic wave.

A solution to Eq. (12.1) for the electrical field E can easily be shown to be

$$E = E_m \sin\left(kx - \omega t\right) \tag{12.2}$$

and, similarly, for the magnetic field it can be shown as

$$B = B_m \sin\left(kx - \omega t\right) \tag{12.3}$$

Figure 12.3 is an illustration of a transverse electrical and magnetic wave (TEM) traveling in the x-direction as described by Eqs. (12.2) and (12.3). It is interesting to note that any linear combination of any number of sinusoids will also satisfy Eq. (12.1), hence, we can imagine through the concept of a Fourier series that plane waves can take on arbitrary shape.

12.2.1 Measures of Electrical Waves

Accepting the premise thaat energy is transported in RF systems in the form of propagating waves as described by Maxwell equation; we will now consider several attributes and metrics associated with RF signals.

A sinusoidal wave at some arbitrary point in space can be expressed in the time domain as

$$v_n(t) = A_n \sin(\omega_n t + \Theta_n) \tag{12.4}$$

or with complex notational form as

$$v_n(t) = \Re e\left\{A_n e^{j(\omega_n t + \Theta_n)}\right\} \tag{12.5}$$

where A_n is the amplitude, $\omega_n = 2\pi f_n$ is the frequency and Θ_n is the phase lead relative to a defined reference. The wavelength λ of a propagating wave is defined as the distance in space over which the wave shape repeats as depicted in Fig. 12.4.

For the case of a sinusoidal wave of frequency f propagating in free space, the wavelength λ_0 is defined as

$$\lambda_0 = \frac{c_0}{f} = \frac{2.998 \times 10^8 \,[\text{m/s}]}{f\,[\text{Hz}]} \approx \frac{30\,[\text{cm/s}]}{f\,[\text{GHz}]} \tag{12.6}$$

Figure 12.3. RF energy transport through space by perpendicular electrical and magnetic fields.

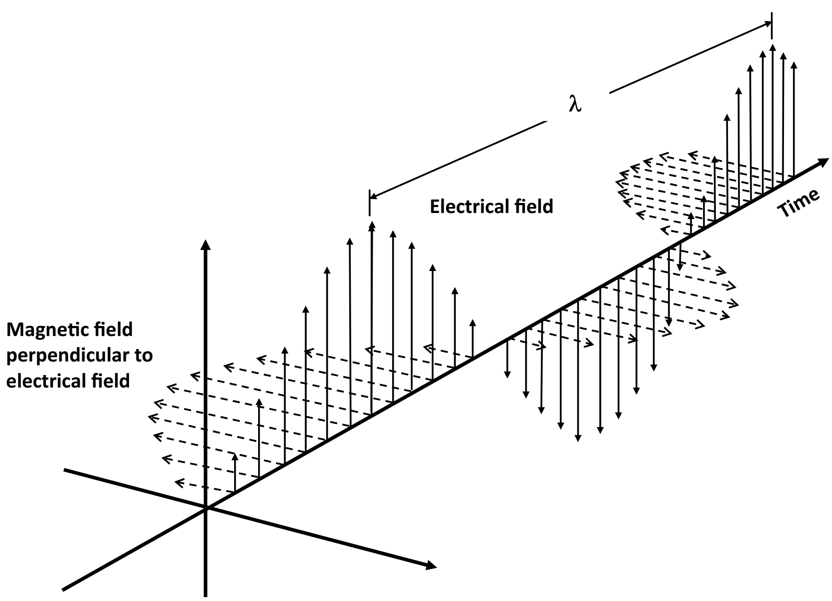
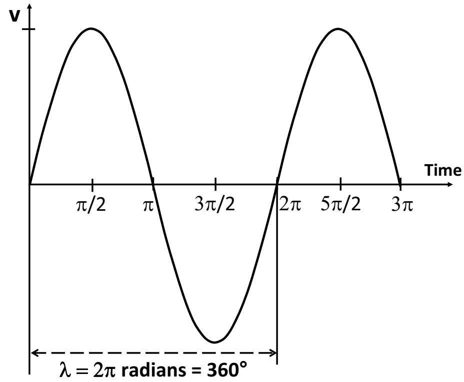


Figure 12.4. Wavelength λ is the distance in space between the start and the end of a complete sinusoidal wave.



when c_0 is the phase velocity of an electromagnetic wave in vacuum. Using Eq. (12.6), we see that the wavelength of a 6-GHz WLAN signal in vacuum is about 5 cm.

If the same electromagnetic wave is propagating in an arbitrary nonmagnetic, homogeneous lossless dielectric medium, the propagation speed, denoted by v_p, will decrease with increasing relative dielectric constant ε_r value, as described by

$$v_p = \frac{c_0}{\sqrt{\varepsilon_r}} \tag{12.7}$$

The dielectric constant ε_r is the ratio of the electric field storage capability of the nonmagnetic, homogeneous and lossless material compared to that of free space, and it is a dimensionless number greater than 1. For optical waves, this dielectric constant ε_r is the square of the refractive index n. At this point we are using the dielectric constant ε_r as a constant; however, this parameter is highly dependent on frequency. It is for this reason that RF capacitor values need to be carefully chosen based on their operating frequency.

For nonhomogeneous media, like a microstrip or coplanar RF line, we will use the effective dielectric constant ε_{eff} instead of the dielectric constant ε_r. With this effective dielectric constant ε_{eff}, the effective wavelength λ_{eff} becomes

$$\lambda_{eff} = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} \approx \frac{30\,[\text{cm/s}]}{f\,[\text{GHz}] \cdot \sqrt{\varepsilon_{eff}}} \tag{12.8}$$

An RF transmission line on FR4 board material might have an effective dielectric constant ε_{eff} of about 3.4. With this effective dielectric constant, the same WLAN 6-GHz signal wave, which had a wavelength of ~5 cm in free space, will have an effective wavelength of 2.7 cm (~1.06 in) on the board.

The fractional wavelength (FW) in percentage can be calculated using l, the physical wavelength of the line, and the effective wavelength λ_{eff} according to

$$\text{FW} = \frac{l}{\lambda_{eff}} 100\% \quad [\%] \tag{12.9}$$

or it might be calculated in terms of phase Θ, expressed in degrees, as

$$\Theta = \frac{l}{\lambda_{eff}} 360° \quad \text{[degrees]} \tag{12.10}$$

A common rule of thumb is that a RF transmission line can be treated as a connection of two nodes with the same voltage at all points at any given time if the fractional wavelength is < 5%. For a transmission line on FR4 material with the $\varepsilon_{eff} = 3.4$ mentioned above, this would be given for frequencies up to 813 MHz for 10 mm (~0.39 in) connecting lines. For our 6-GHz WLAN signal the 5% rule would be equal to a line length of

$$l_{5\%} = FW \cdot \lambda_{eff} = FW \cdot \frac{30\,[\text{cm/s}]}{f\,[\text{GHz}] \cdot \sqrt{\varepsilon_{eff}}} = 0.05 \cdot \frac{30\,[\text{cm/s}]}{6\,[\text{GHz}] \cdot \sqrt{3.4}} = 1.35\,\text{mm} \tag{12.11}$$

For connecting lines longer than 1.35 mm (~0.053 in), the connecting line should be treated as a set of distributed elements rather than an lumped LCR equivalent.

EXAMPLE 12.1

What is the free space wavelength of an electromagnetic wave of a GPS signal with a frequency of 1575.42 MHz?

Solution:
Using Eq. (12.6),

$$\lambda_0 = \frac{c_0}{f} = \frac{2.998 \times 10^8\,[\text{m/s}]}{1575.42 \times 10^6\,[\text{Hz}]} = 190.3 \text{ mm}$$

The wavelength of a GPS signal in free space is 190.3 mm.

EXAMPLE 12.2

An RF transmission line on FR4 board material is used to delay a signal with a frequency of 1575.42 MHz by 90 degrees. The effective dielectric constant for this material is $\varepsilon_{eff} = 3.4$. What line length should be used?

Solution:
First, using Eq. (12.8), we calculate the wavelength of the signal as

$$\lambda_{\text{eff}} = \frac{1}{\sqrt{3.4}} \cdot \frac{2.998 \times 10^8\,[\text{m/s}]}{1575.42 \times 10^6\,[\text{Hz}]} = 103.2 \text{ mm}$$

For a 90-degree phase shift we require one-quarter of this wavelength, resulting in

$$l_{90\,\text{deg}} = \frac{103.2 \text{ mm}}{4} = 25.8 \text{ mm}$$

The board trace needs to be made 25.8 mm long.

Exercises

12.1. What is the frequency of a wave that has a wavelength of 10 cm in free space? What is the corresponding frequency if the wave is traveling in water with a relative dielectric constant of 80.1 at 20 degrees centigrade?

ANS. 2.998 GHz; 2.998 GHz (no change in frequency, because speed and wavelength change in equal proportions).

12.2. An RF transmission line on Roger's printed circuit board material is used to delay a signal with the frequency of 6 GHz by 45 degrees. The effective dielectric constant for this material is $\varepsilon_{eff} = 2.2$. What line length is required?

ANS. 4.2 mm.

12.3. An RF transmission line constructed on an FR4 printed circuit board is 10 mm in length. If the RF line is excited by a signal with frequency of 1 GHz do we treat this line as a distributed set of elements or as a lumped LRC equivalent assuming the 5% line length rule.

ANS. $l_{5\%} = 8.1\text{ mm}$; because the line length of 10 mm is greater than $l_{5\%}$, the line must be treated as a distributed line.

12.2.2 Power Definition

In the previous subsection, we described several attributes of a traveling plane wave: wavelength, frequency, and amplitude. This subsection will describe the concept of scalar power and voltage. The goal here is to gain an understanding how the shape of the waveform impacts a power measurement. It is important to understand expressions for average and peak power, as well as understand the role that they have when conducting a power measurement with an ATE.

In general, in a conservative field, voltage is defined as the line integral over an electrical field of an electromagnetic field. Sparing the reader the mathematical details, a voltage wave has a form of expression similar to that of Eq. (12.1); that being, a sinusoidal function of both time and space.

At low frequencies, the well-known Ohm's law relating a current signal to a voltage signal, that is, $v(t) = R \cdot i(t)$, provides a convenient shorthand solution to Maxwell's electromagnetic field equations. Maxwell's equation shows that the voltage and current are actually waves dependent on the frequency and the medium in which they are propagating through. At these low frequencies, the definition of power is the product of the instantaneous current and the instantaneous voltage or, together with Ohm's law, can be written in the following two ways:

$$p(t) = i(t) \cdot v(t) = \frac{v^2(t)}{R} \tag{12.12}$$

Often in RF circuit literature, the resistance of interest involved in the power transfer process is denoted with symbol Z_0. Since it is customary to use the capital letter Z for impedance, it is assumed that this impedance is real-valued. Using this notation, the power expression often appears as

$$p(t) = \frac{v^2(t)}{Z_0} \tag{12.13}$$

Figure 12.5 illustrates the instantaneous voltage and current waveforms corresponding to a voltage-source–resistive-load configuration. Here the voltage and current waveforms are in-phase. In contrast, Figure 12.6 illustrates the instantaneous voltage and current waveforms corresponding to a voltage-source–arbitrary-load configuration (i.e., one with resistive, capacitance and inductive components). Here we see the voltage and current waveforms are out-of-phase by some angle ϕ. For both cases, the instantaneous power *p(t)* are seen superimposed on these two plots. In the case of the resistive load or in-phase situation, the instantaneous power is always positive in value, whereas, in the situation depicted in Figure 12.6 for the out-of-phase case, the instantaneous power goes negative at some points along the waveform. Positive instantaneous power refers to power being delivered by the source to the load, and negative instantaneous power refers to power being delivered to the source by the load. Since the load is passive, one would not expect a net flow of power to be delivered by the load to the source. A measure that describes this behavior is average power, P_{avg}.

Average power is defined as the average value of the instantaneous power waveform taken over some integration time, T_r. As the meaning of an average value depends on the integration time, it is important to standardize on this quantity. We encountered this same issue previously with DSP-based testing back in Chapter 9. There we spoke in terms of coherency; that is, input

Figure 12.5. Time dependent power as a function of in-phase voltage and current waves for a voltage source—resistive load configuration.

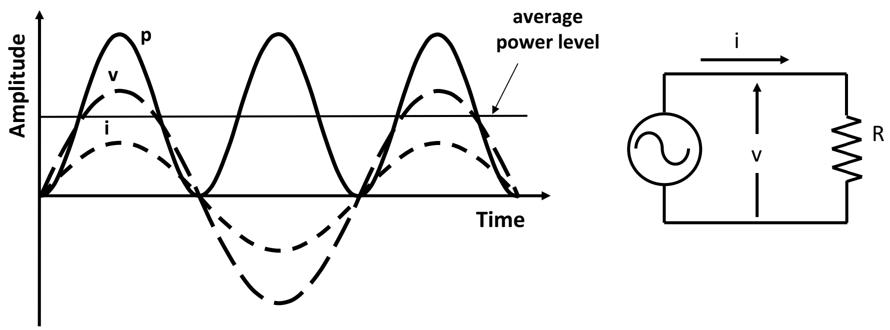
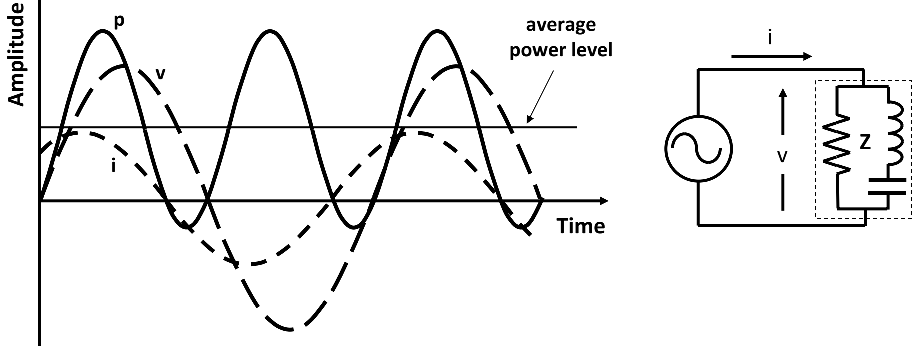


Figure 12.6. Time dependent power as a function of out of phase voltage and current waves for an arbitrary voltage source—arbitrary load configuration.



and output signals are coherent with respect to the unit test period (i.e., integration time). RF power measurements follow similar coherency constraints. For sinusoidal signals with period T_o, referred to as continuous wave (CW) sinusoidal signals in RF literature, as opposed to pulsed signals that also appear, we can state this same coherency condition by ensuring that the integration time is an integer multiple n of the period T_o. Assuming that the magnitude of the sinusoidal voltage wave is denoted by V_{peak} and magnitude of the sinusoidal current wave is described by I_{peak}, the average power is given by

$$P_{avg} = \frac{1}{nT_0} \int_0^{nTo} V_{peak} \sin\left(\frac{2\pi}{T_0} t \right) \cdot I_{peak} \sin\left(\frac{2\pi}{T_0} t + \phi \right) dt \tag{12.14}$$

where ϕ is the phase difference between the voltage and the current wave. Because n is an integer, the average power reduces to

$$P_{avg} = \frac{V_{peak} \cdot I_{peak}}{2} \cos(\phi) \tag{12.15}$$

Here we see that P_{avg} is not a function of frequency and is dependent only on the phase difference ϕ, the magnitude of the voltage wave V_{peak}, and current wave I_{peak}. The average power for the in-phase and out-of-phase cases depicted in Figure 12.5 and Figure 12.6 is superimposed on each plot. As expected, the average power delivered to the load is positive in both cases.

As a matter of convenience, one often replaces the peak of a signal by its RMS value. In the case of the sinusoidal voltage and current waveforms describe here, we can write

$$V_{peak} = \sqrt{2}\, V_{RMS} \quad \text{and} \quad I_{peak} = \sqrt{2}\, I_{RMS} \tag{12.16}$$

Substituting the above two quantities into Eq. (12.15), we write the well-known average power expression as

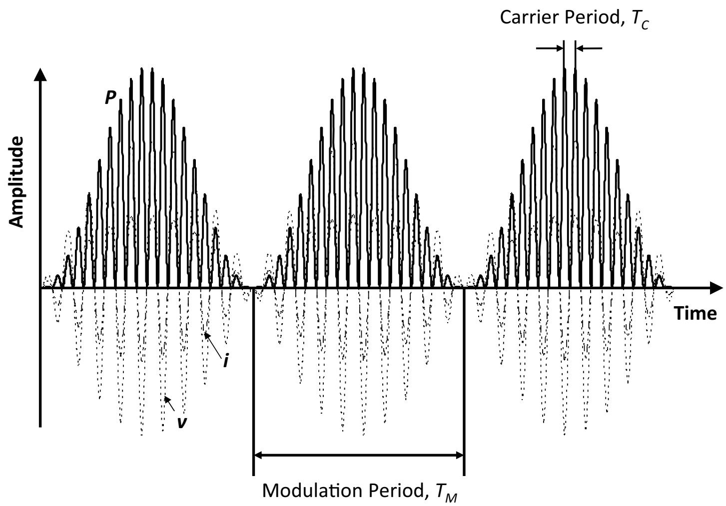
$$P_{avg} = V_{RMS} \cdot I_{RMS} \cos(\phi) \tag{12.17}$$

In RF testing, signals can be more complex than a single-frequency sinusoidal wave, which requires a more general definition of the integration time. It is obvious that in the case of a superposition of multiple sinusoidal waves, the average for a power measurement needs to be taken over multiple periods of the lowest frequency involved. For example, in the case of the amplitude-modulated signal shown in Figure 12.7, the integration time is defined as an integer multiple of the period of the modulation signal T_m, not the carrier period T_c, as shown in Figure 12.7. We can then write the average power as

$$P_{avg} = \frac{1}{nT_m} \int_0^{nT_m} v(t) \cdot i(t)\, dt \tag{12.18}$$

In modern digital RF transmission systems, the signals are often transmitted only in so called frames. In the time domain, the envelope of the signal power looks like a string of pulses as shown

Figure 12.7. Power of an amplitude-modulated signal.



in Figure 12.8. Here the pulsed signal has a duration of τ seconds and a repetition period of T_r. For the system performance itself, only the power within the frame is of interest. One performance measure is the pulse power P_{Pulse}. It is defined as the average power in the pulse period τ according to

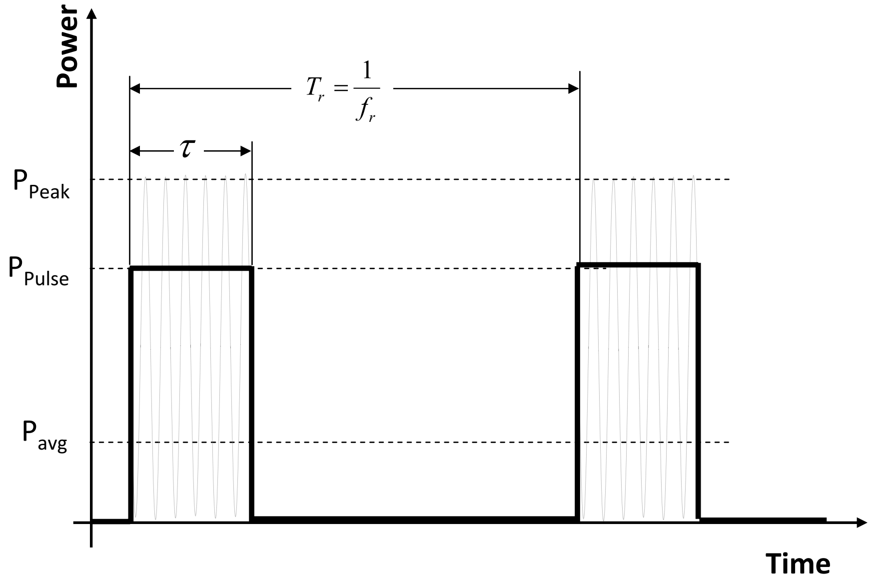
$$P_{Pulse} = \frac{1}{\tau} \int_0^{\tau} v(t) \cdot i(t)\, dt \tag{12.19}$$

In many cases, the pulse power can also be expressed as a function of the average power associated with the continuous wave and the duty cycle of the pulsing action; that is, consider rewriting Eq. (12.19) as

$$P_{Pulse} = \frac{T_r}{\tau} \frac{1}{T_r} \int_0^{\tau} v(t) \cdot i(t)\, dt = \frac{P_{avg}}{\tau / T_r} = \frac{P_{avg}}{DutyCycle} \tag{12.20}$$

As described above, it is important to consider the time dependence of the signal in order to be able to determine the correct power. It will often be beneficial to determine the signal in the time and frequency domains and then apply the correct power calculation or measurement method. For an appropriate ATE setting, it is important to understand the peak power P_{Peak} of the signal, even when only the average power is to be measured. This is especially important when there is a significant difference between the average and peak power, such as that which occurs with a pulse signal (see Figure 12.8). As we will discuss later in Chapter 13, it is critical to measure in the linear range of the ATE to guarantee that no stage of the measurement path enters the compression state at any time during the modulation cycles.

Figure 12.8. Power of a pulsed signal.



12.2.3 Crest Factor

In the previous section we have seen that the average power of a signal can have a complex relation to its peak power. For optimal RF system performance, like the test of an RF DUT with an ATE, it is essential to know not only the average power level but just as important the peak power used to source or measure the DUT, or even within the source and measurement path of the ATE. A description of the peak and average power used for complex signals is the signal dependent crest factor (CF). The CF is the ratio of the peak power P_{Peak} to the average power value P_{avg} of an instantaneous power waveform. CF can be written simply as

$$\mathrm{CF} = \frac{P_{Peak}}{P_{avg}} \tag{12.21}$$

Exercises

12.4. The instantaneous voltage across a pair of terminals in a circuit can be described as $v(t) = V_{peak}\cos(2\pi f_0 \cdot t)$. Similarly the current in and out of each of these terminals can be described as $i(t) = I_{peak}\cos(2\pi f_0 \cdot t + \phi)$. Compute the instantaneous power associated flowing across these two terminals?

ANS. $p(t) = V_{peak} I_{peak} \cos(2\pi f_0 \cdot t + \phi) \cos(2\pi f_0 \cdot t)$ W.

12.5. For the circuit situation described in Exercise 12.4 compute the average power.

ANS. $P_{avg} = f_0 \times \int_0^{1/f_0} p(t)dt = \frac{1}{2} V_{peak} I_{peak} \cos(\phi)\ W.$

12.6. Using the trigonometric identity, $\cos(\alpha)\cos(\beta)= 1/2\cos(\alpha-\beta)+\cos(\alpha+\beta)$, write the instantaneous power expression found in Exercise 12.4 in terms of the sum and difference phase terms.

ANS.
$$p(t)=\frac{V_{peak}I_{peak}}{2}\cos(\phi)+\frac{V_{peak}I_{peak}}{2}\cos(4\pi f_O\cdot t+\phi)$$

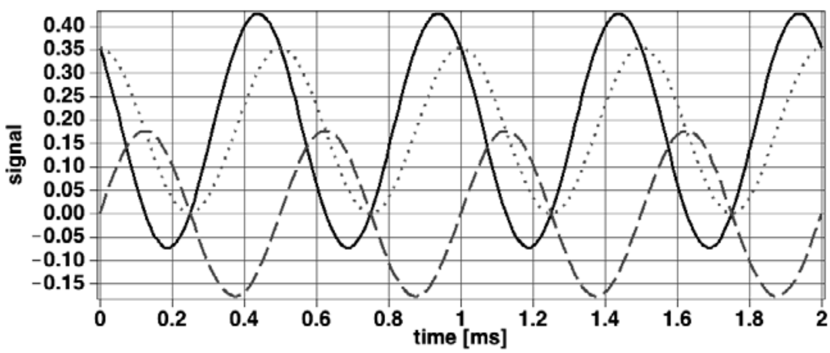
12.7. Using the trigonometric identity, $\cos(\alpha+\beta)= \cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\alpha)$, rewrite the instantaneous power expression found in Exercise 12.6.

ANS.
$$p(t)=\frac{V_{peak}I_{peak}}{2}\cos(\phi)+\frac{V_{peak}I_{peak}}{2}\cos(\phi)\cos(4\pi f_O\cdot t)-\frac{V_{peak}I_{peak}}{2}\sin(\phi)\sin(4\pi f_O\cdot t)$$

12.8. Plot the total instantaneous total power in terms of thereal and imaginary instantaneous power if $v(t)=1.0\cos\left(2\pi\times10^3\cdot t\right)$ and $i(t)=0.5\cos\left(2\pi\times10^3\cdot t+\pi/4\right)$.

Legend: solid = total power, dot = real power, dash = imaginary power.

ANS.



Often the crest factor is expressed in terms of decibels, where the conversion is done through the operation,

$$\text{CF}\big|_{\text{dB}}=10\log_{10}\text{CF}=10\log_{10}\left(\frac{P_{Peak}}{P_{avg}}\right) \tag{12.22}$$

Back in Chapter 8, Section 8.3.3, we described the idea of a crest factor of a multitone voltage signal in terms of the peak-to-RMS ratio. This definition is often used in mixed-signal testing. However, for RF testing, the idea of a crest factor is related to a power waveform, and it should not be confused with parameters extracted from a voltage or current signal.

Referring to the power waveforms shown in Figure 12.5 we see that the peak value of this waveform is equal to twice the average power level, resulting in a crest factor of 2 or, equivalently, 3 dB.

For modulated RF signals, the crest factor is referred to as the peak value of the instantaneous power modulation envelope, instead of the peak value of the instantaneous power of the RF carrier signal. A frequency-modulated (FM) signal has a constant envelope and thus a crest factor, CF = 1 or 0 dB.

For signals with numerous uncorrelated sinusoidal voltages, we can bound the peak value of the instantaneous power waveform as the sum of individual voltage amplitudes, all squared, normalized by the appropriate impedance level, Z; that is,

$$P_{Peak} = \frac{\left(V_{Peak,1} + V_{Peak,2} + V_{Peak,3} + \cdots + V_{Peak,n}\right)^2}{Z} \tag{12.23}$$

Correspondingly, the average power of a multitone signal is simply the sum of the power of each sinusoidal term written as follows

$$P_{avg} = \frac{1}{2}\frac{V_{Peak,1}^2}{Z} + \frac{1}{2}\frac{V_{Peak,2}^2}{Z} + \frac{1}{2}\frac{V_{Peak,3}^2}{Z} + \cdots + \frac{1}{2}\frac{V_{Peak,n}^2}{Z} \tag{12.24}$$

Using Eqs. (12.23) and (12.24), the crest factor of multiple uncorrelated sinusoidal signals can be estimated to be

$$\text{CF} = \frac{P_{Peak}}{P_{avg}} \approx 2 \times \frac{\left(V_{Peak,1} + V_{Peak,2} + V_{Peak,3} + \cdots + V_{Peak,n}\right)^2}{V_{Peak,1}^2 + V_{Peak,2}^2 + V_{Peak,2}^2 + \cdots + V_{Peak,n}^2} \tag{12.25}$$

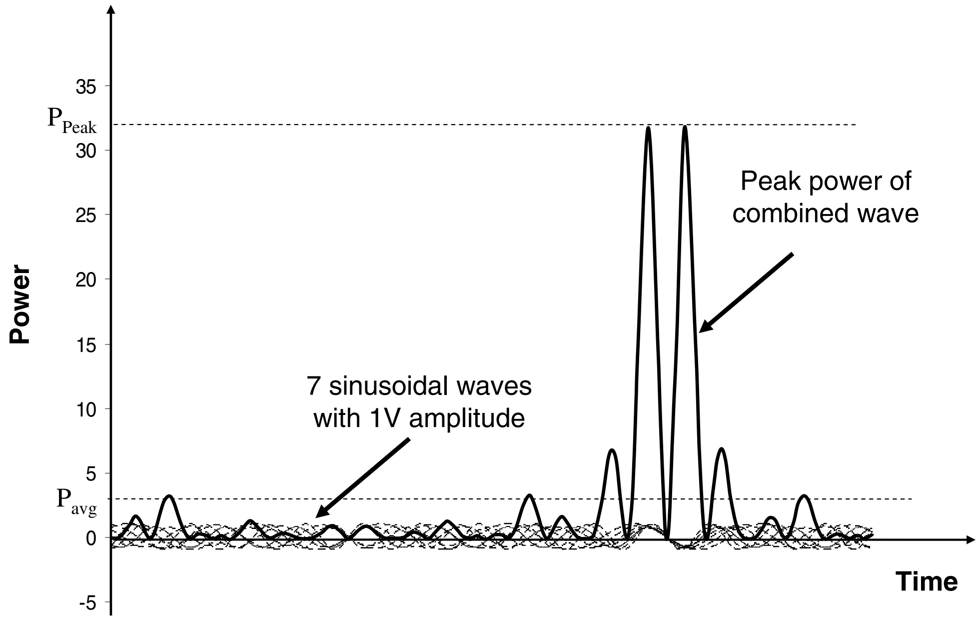
Figure 12.9 shows an example of the peak-to-average ratio for seven uncorrelated sinusoidal signals. Here we see the peak power exceeds the average signal power by a factor of about 11 times. If we assume the peak value each sine wave is 1 V, then according to Eq. (12.25) one would expect to see a CF of about $2 \times 7^2/7 = 14$. This is indeed what we see here.

The crest factor of other signal types are: noise has a crest factor of around 11 dB, an OFDM signal used in DAB, DVB-T and WLAN also have a crest factor of around 11 dB. The crest factor of CDMA2000 and UMTS mobile radio standards range up to 15 dB, but these could be reduced to 7–9 dB using special filtering techniques. Except during a burst, GSM signals have a constant envelope resulting from MSK modulation and thus have a crest factor of 0 dB. EDGE signals have a crest factor of 3.2 dB due to the filter function of the 8PSK modulation.

In the design of any RF system, the average power requirements are traded-off for lower peak power requirements. Crest factor is a parameter that relates these two quantities. Knowing any two parameters enables one to calculate the third. For instance, knowing the crest factor and the average power, the test engineer can calculate the expected peak power. This situation often arises in RF test when one is concerned about the maximum power a signal source can drive. The source will often go into compression at the peak power at any given time. In turn, source compression will set the maximum available power in a test system when using a specific modulation feature.

The complementary cumulative distribution function (CCDF) is the probability that a power is equal to or greater than a certain peak-to-average ratio as a function of such ratios. The higher the peak-to-average ratio, the lower the probability of reaching this point. The statistics of the signal determine the headroom required in the RF system. CCDF statistics are important in understanding digital modulated RF systems, because the statistics may vary. For instance, in CDMA systems, the statistics of the signal will vary with the number of codes that are used simultaneously.

Figure 12.9. Superposition of multiple sinusoidal waves resulting in high peak-power-to-average-power ratio defined as Crest Factor.



Since a couple of RF parameters like EVM and ACPR (to be introduced later in Chapter 13) are dependent on the peak-to-average ratio, it is important to understand their statistics to set up repeatable measurements.

Exercises

12.9.	The crest factor of a continuous waveform is 7 dB. If the average power is 13 W, what is the peak power associated with this waveform?	ANS. 65.2 W.
12.10.	A signal consisting of 4 uncorrelated sinusoids of 2 V. Estimate the crest factor associated with the composite waveform.	ANS. 9.03 dB.

12.2.4 Power in dBm

In actual RF systems, power is often defined for a (real) reference or characteristic impedance of Z_0. While any value can be selected for Z_0, most test equipment is built with a reference impedance Z_0 of 50 Ω. As such, a power measurement is equivalent to computing the square of the RMS value of the voltage and normalizing this value by 50 Ω, that is,

$$P_{avg} = \frac{V^2_{RMS}}{Z_0} = \frac{V^2_{RMS}}{50\,\Omega} \text{ W} \tag{12.26}$$

It is a common practice in RF measurement and test to use the unit dBm instead of watt for a measurement of power. The dBm value is calculated by dividing the power transferred to a 50-Ω load by 1 mW as shown below:

$$P\big|_{\mathrm{dBm}} = 10 \cdot \log\left(\frac{P_{avg}}{1\ \mathrm{mW}}\right) \tag{12.27}$$

A summary of some useful dBm values is given below:

13 dBm corresponds to 1-V RMS into 50 Ω
0 dBm corresponds to 0.224-V RMS into 50 Ω
–174 dBm is the thermal noise power in 1-Hz bandwidth at room temperature (293 K)

Likewise, some commonly used power ratios on a dB logarithmic scale is listed below:

3 dB is approximately the power ratio of 2
6 dB is approximately the voltage ratio of 2

12.2.5 Power Transfer

As seen previously, the flow of instantaneous power $p(t)$ for a given instantaneous voltage $v(t)$ and current $i(t)$ is given by

$$p(t) = v(t) \cdot i(t) \tag{12.28}$$

This instantaneous power represents the power that is dissipated, which is also known as real power, and the power that is bouncing back and forth between the reactive elements of the circuit. The latter power is known as the imaginary power. When the power is applied to an antenna, the real power is partially lost in dissipative elements of the antenna while the majority of it is transmitted (radiated) away from the antenna. The reactive power becomes the near-field reactive energy required to match boundary conditions of the antenna structure and, for the most part, is not radiated away from the antenna.

In the following, we will discuss the real power flow in an RF circuit. Here we will assume that the voltage and current signals are sinusoidal with some arbitrary phase angle difference Θ. If we assume that the voltage has the form $v(t) = V_{Peak}\sin(\omega t)$, then the current takes on the form $i(t) = I_{Peak}\sin(\omega t + \Theta)$. When working with RF circuits, it is preferable to make use of phasors, because it greatly simplifies their AC analysis. Consider representing the voltage and current signals using phasor notation. As mentioned previously, we write the voltage using complex notation as

$$v(t) = \Re e\left\{V_{Peak}e^{j\omega t}\right\} \tag{12.29}$$

The leading term inside the brackets is the voltage phasor $\mathscr{V}$, which we shall denote using complex notation as

$$\mathscr{V} = V_{Peak}e^{j0} \tag{12.30}$$

Similarly, we can write the current signal using complex notation as

$$i(t) = \Re e\left\{I_{Peak}e^{j(\omega t+\Theta)}\right\} = \Re e\left\{\left(I_{Peak}e^{j\Theta}\right)e^{j\omega t}\right\} \tag{12.31}$$

leading to the current phasor $\mathscr{I}$ as

$$V_L = I_{Peak}e^{j\Theta} \tag{12.32}$$

The average power dissipated is then found from the expression

$$P_{avg} = \frac{1}{2}\cdot\Re e\left(\mathscr{V}\cdot\mathscr{I}^*\right) = \frac{1}{2}\cdot V_{Peak}I_{Peak}\cos(\Theta) \tag{12.33}$$

We can easily write the average power in terms of the RMS value of the voltage and current waveforms and get the familiar expression

$$P_{avg} = V_{RMS}I_{RMS}\cos(\Theta) \tag{12.34}$$

The above expressions and their extension to the *E* and *B* fields are straightforward. Useful expressions for the power propagation in waveguides, transmission lines, and free space can be derived.

Maximum Power Transfer

When a voltage source is connected to a load, a flow of power from the source to the load results. As a matter of power transfer, the question related to the circuit conditions for maximum power transfer arises. Consider a voltage source with a real valued voltage level of $2V_A$ having a complex source impedance of $Z_S = R_S + jX_S$ connected to a load with impedance $Z_L = R_L + jX_L$. The corresponding circuit is shown in Figure 12.10. The voltage term V_A is called the *available voltage* from the source. This is not the maximum source voltage, as this would occur when the source is driven into an open circuit.

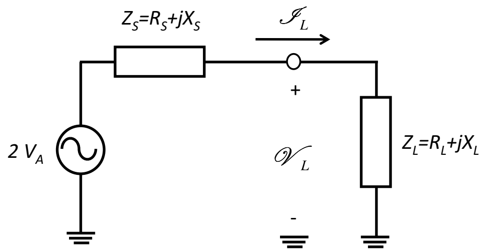
Through the applications of phasors, the average power delivered to the load is given by

$$P_{L,avg} = \frac{1}{2}\cdot\Re e\left(V_L\cdot I_L^*\right) \tag{12.35}$$

The load voltage and current phasors are found by straightforward circuit analysis resulting in

$$\begin{aligned}\mathscr{V}_L &= \frac{Z_L}{Z_S+Z_L}2V_A = \frac{R_L+jX_L}{R_S+R_L+j\left(X_S+X_L\right)}2V_A\\ \mathscr{I}_L &= \frac{2V_A}{Z_S+Z_L} = \frac{1}{R_S+R_L+j\left(X_S+X_L\right)}2V_A\end{aligned} \tag{12.36}$$

Figure 12.10. General network for power transfer calculations.



Substituting these two expressions back into Eq. (12.35), we obtain an expression for the average load power as

$$P_{L,avg} = 2|V_A|^2 \cdot \Re e\left(\frac{R_L + jX_L}{R_S + R_L + j(X_S + X_L)} \cdot \frac{1}{R_S + R_L - j(X_S + X_L)} \right) \tag{12.37}$$

or we can write it more simply as

$$P_{L,avg} = 2|V_A|^2 \cdot \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2} \tag{12.38}$$

Assuming a fixed source impedance, together with some calculus (not shown), the maximum power transfer from source to load occurs when the following two conditions are met:

$$\begin{aligned} R_L &= R_S \\ X_L &= -X_S \end{aligned} \tag{12.39}$$

Collectively, we can summarize the above two conditions and write $Z_L = Z_S^*$. This is known as the conjugate match condition. Under these matched conditions, the maximum power delivered to the load is referred to as the *maximum power available* from the source and is commonly denoted by the symbol P_A. Substituting the conditions listed in Eqs. (12.39) into (12.38), we find

$$P_A = \frac{1}{2}\frac{|V_A|^2}{R_S} \tag{12.40}$$

It is interesting to note that the source delivers twice this amount of power. However, the other half of this power is dissipated by the source resistance, R_S. Under general circuit conditions, the average power delivered by the source is given by

$$P_{S,avg} = 2|V_A|^2 \frac{(R_S + R_L)}{(R_S + R_L)^2 + (X_S + X_L)^2} \tag{12.41}$$

EXAMPLE 12.3

A voltage source with a voltage level described by phasor $100e^{j\pi/4}$ V and source impedance $Z_S = 54 + j6\ \Omega$ drives load impedance of $Z_L = 25 + j25\Omega$. What is the average power delivered to the load and how does it compare to the maximum power available from the source? What is the average power delivered by the source?

Solution:
From Eq. (12.38), together with the relationships $2V_A = 100e^{j\pi/4}$, $R_S = \Re e(Z_S) = 54\ \Omega$, $X_S = \mathrm{Im}(Z_S) = 6\ \Omega$, $R_L = \Re e(Z_L) = 25\ \Omega$, and $X_L = \mathrm{Im}(Z_L) = 25\ \Omega$, we solve for the average load power as

$$P_{L,avg} = 2\left|\frac{100e^{j\pi/4}}{2}\right|^2 \cdot \frac{25}{(54+25)^2 + (6+25)^2} = 17.3\ \mathrm{W}$$

The maximum available power can found from Eq. (12.40) as

$$P_A = \frac{1}{2}\left|\frac{100e^{j\pi/4}}{2}\right|^2 \frac{1}{54} = 23.1\ \mathrm{W}$$

Using Eq. (12.41), we solve for the average source power as

$$P_{S,avg} = 2\left|\frac{100e^{j\pi/4}}{2}\right|^2 \cdot \frac{(54+25)}{(54+25)^2 + (6+25)^2} = 54.8\ \mathrm{W}$$

EXAMPLE 12.4

Repeat Example 12.3 using a load impedance of $Z_L = 54 - j6\ \Omega$ (conjugate matched to the source impedance).

Solution:
For the revised load, we write $R_L = \Re e(Z_L) = 54\ \Omega$ and $X_L = \mathrm{Im}(Z_L) = -6\ \Omega$ and solve for the average load power as

$$P_{L,avg} = 2\left|\frac{100e^{j\pi/4}}{2}\right|^2 \cdot \frac{54}{(54+54)^2 + (6-6)^2} = 23.1\ \mathrm{W}$$

The maximum available power can found from Eq. (12.40) as

$$P_A = \frac{1}{2}\left|\frac{100e^{j\pi/4}}{2}\right|^2 \frac{1}{54} = 23.1\ \mathrm{W}$$

Using Eq. (12.41), we solve for the average source power as

$$P_{S,avg} = 2\left|\frac{100e^{j\pi/4}}{2}\right|^2 \cdot \frac{(54+54)}{(54+54)^2 + (6+-6)^2} = 46.2 \text{ W}$$

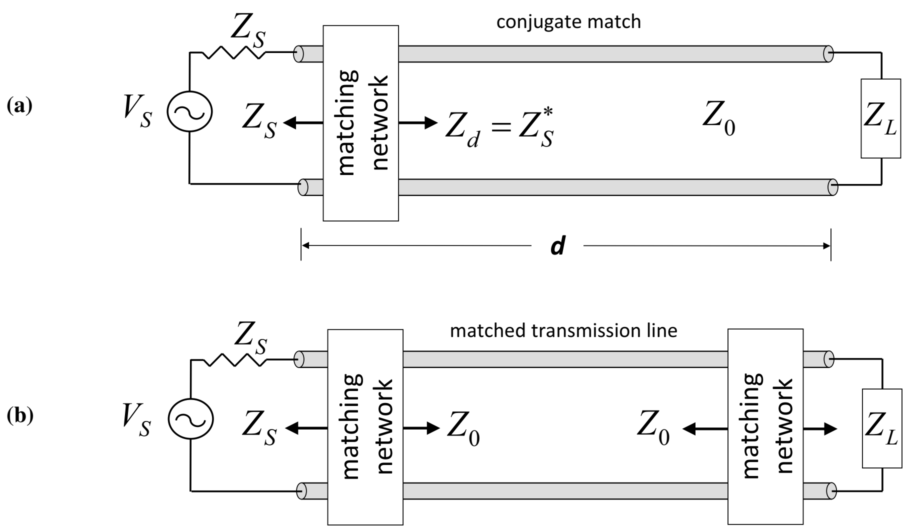
As the source and load average power are equal, the circuit is operating under maximum power transfer conditions. Nothing can be done to have the load receive more power than ½ the source power.

12.2.6 Conjugate and Reflectionless Matching

Having just learned that the maximum available power at the load is half the source power, we must condition this statement with the fact that this is a best-case situation and only occurs when the load impedance is matched to the complex conjugate of the source impedance. In reality the source and the load are connected through a transmission line, whereby the line alters the load impedance seen by the source. If we denote the impedance seen by the source looking into the transmission line as Z_{TL}, one can show for a transmission line of length d having characteristic impedance Z_0 that

$$Z_{TL} = Z_0 \times \frac{Z_L + jZ_0 \tan(\beta \cdot d)}{Z_0 + jZ_L \tan(\beta \cdot d)} \tag{12.42}$$

Figure 12.11. (a) Complex conjugate match transmission line, (b) reflectionless matched transmission line.



where $\beta = \dfrac{2\pi f}{v_p}$. As the impedance of Z_S and Z_L is generally targeted for 50 Ω, the maximum power transfer condition might not be met on account of the transmission line effect. To circumvent this situation, a matching network is inserted in the line at the source end as shown in Figure 12.11a. Here the matching network alters the impedance seen by the source and denoted as Z_d, such that the conjugate matched condition is met, that is, $Z_d = Z_S^*$. For a real-valued source impedance, that is, $Z_S = Z_S^*$, this condition ensures that no reflections occur at the source side of the transmission line. However, reflections will occur at the load side if $Z_L \neq Z_0^*$. In this case, an additional matching network is used to match the load to the transmission line as depicted in Figure 12.11b. Using two matching networks at each end is called reflectionless matching.

An interesting benefit arises with reflectionless matching. As the load and matching network combine to realize an impedance equal to the line impedance, Eq. (12.42) reveals that the impedance seen at the source end of the line is equal to Z_0 regardless of the line length or operating frequency *f*. This implies that the signaling will remain reflectionless with maximum power transfer. Moreover, this matched condition will be independent of line length and operating frequency. Section 5 of Chapter 15 will discuss in more detail the design of matching networks using Smith Charts.

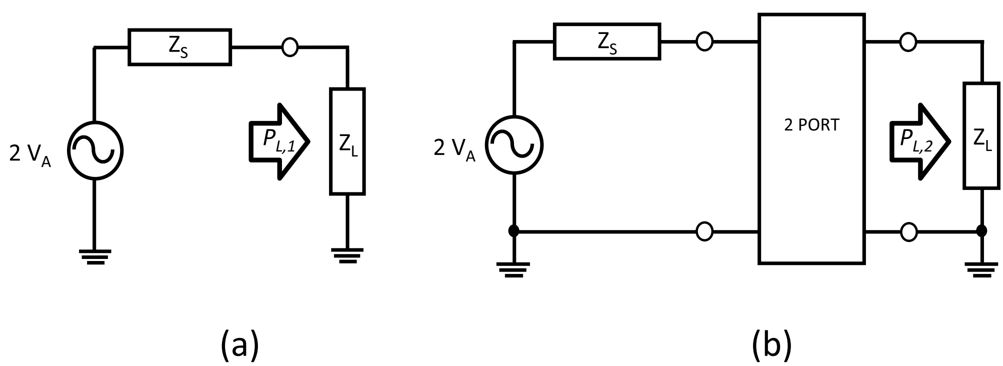
12.2.7 Power Loss Metrics

Power transfer between source and load experiences various losses. In the following we shall describe two commonly used measures of power loss: insertion and transducer loss.

Insertion Loss

Insertion loss is the loss of signal power that result from the insertion of a device between the source and load connections as illustrated in Figure 12.12. Here a two-port network is shown inserted between the source and load. A two-network is a general term for an electrical network with two ports—for example an input and an output port. Two terminals constitute a port together with the condition that the current that enters one terminal of the port must equal the current leaving the other terminal. An *n*-port network is a network with n ports. A typical example of a two-port network is an amplifier. An example of a three-port is a mixer with an input RF port, an output IF port and a local oscillator port.

Figure 12.12. Two-step method to measure insertion and transducer loss: (a) power delivered to a load directly from source, (b) power deliver to load via a two-port network.



A two-step procedure to obtain the insertion loss of a two-port network is as follows:

1. A load is connected directly to the source and the average load power P_{L1} is measured as shown in Figure 12.12a. The load impedance can be, for example, the input impedance of a power meter.
2. A two-port is placed between the source and the load, as shown in Figure 12.12b, and again the average load power P_{L2} is measured.

The insertion loss IL is then calculated according to the formula

$$\text{IL} = \frac{P_{L1}}{P_{L2}} \tag{12.43}$$

Insertion loss IL can also be expressed in dB as

$$\text{IL}\big|_{\text{dB}} = 10\log\left(\frac{P_{L1}}{P_{L2}}\right) \tag{12.44}$$

As can be seen from Figure 12.12, an insertion loss measurement is dependent on the specific values of Z_L and Z_S. Without knowing these values exactly, the effect of inserting a two-port cannot be repeated at different times or locations. In the next chapter, we will discuss the insertion loss using S-parameters. This will provide more information about the test situation and allow for repeatable insertion loss measurements. The interested reader is recommended to study more details found in, for example, reference [5] or [6].

Transducer Loss

Transducer loss (TL) is the ratio of the maximum average power available from a source to the average power that the transducer delivers to a load, that is,

$$\text{TL} = \frac{P_A}{P_L} \tag{12.45}$$

or expressed in decibels as

$$\text{TL}\big|_{\text{dB}} = 10\log\left(\frac{P_A}{P_L}\right) \tag{12.46}$$

The maximum available source power can be derived from the test setup of Figure 12.12a with the load impedance conjugate matched to the source impedance. Because this power is equal to the maximum available source power, we can write $P_A = P_{L1,match}$, where $P_{L1,match}$ signifies the conjugate matched test condition. Note that the condition of conjugate matching makes this test setup different from that used in the insertion loss test. Next, the load power is measured with the transducer inserted in the circuit. This is identical to the test setup of Figure 12.12b where the load power is equal to $P_{L,2}$. The transducer loss can then be expressed in terms of the two separate measurements as

$$\text{TL} = \frac{P_{L1,match}}{P_{L2}} \tag{12.47}$$

When the source and load impedances are complex conjugates of one other, insertion and transducer loss are equal. During a microwave test, the generator and load are commonly matched to a real impedance Z_0 known as the standard characteristic impedance (typical 50 Ω). By doing so, the conjugate match requirement will be satisfied.

When using this method to measure either insertion or transducer loss, it is important to remember that reflections should not compromise the test results. This can be achieved by adding attenuators to the source and load, which will minimize the impact of the reflected power. Another method would be with use of a network analyzer, which separates the incident and reflected wave with a directional coupler.

EXAMPLE 12.5

A 6-in. cable is used to connect a load to a source. A power measurement is made at the load and found to be equal to 1.2 W. It is assumed that this cable has negligible insertion loss. If a second cable is used instead with an insertion loss of 0.9 dB, what is the expected level of load power?

Solution:
Using Eq. (12.44), we write

$$IL\big|_{\text{dB}} = 10\log\left(\frac{1.2\text{ W}}{P_{L2}}\right) = 0.9\text{ dB}$$

Solving for the unknown power level, PL2, we write

$$P_{L2} = \frac{1.2\text{ W}}{10^{4.5/10}} = 0.97\text{ W}$$

Therefore the expected level of load power is 0.97 W.

12.3 NOISE

The signal-to-noise ratio (SNR) at the output of RF systems is a very important quantity, if not the most important parameter. The SNR often sets the dynamic range limit of system operation. Listening or watching an analog audio RF system, like analog radio or television, in the presence of noise will have an adverse effect on the experience. Noise figure and sensitivity are other important system parameters, and are in most, if not all, cases the key figures of merit. For digital RF systems, the reliability of an RF system is often stated in terms of bit error rate (BER). The BER is related to the noise figure and the SNR in a nonlinear manner. For signals close to the noise floor (often referred to as low SNR signals), the noise level will make it impossible to decode such signals leading to transmission errors.

Exercises

12.11. A 10-V source with source impedance $Z_S = 43 + j12\ \Omega$ drives a load impedance of $Z_L = 52 + j5\Omega$. What is the average power delivered to the load and how does it compare to the maximum power available from the source? What is the average power delivered by the source?

ANS. P_L = 0.279W, P_A = 0.291 W, P_S = 0.51 W.

12.12. A load is connected directly to a source where the load power was measured to be 2.4 W. The same load was then connected to the source through an RF connector and DIB trace. The load power was then measured to be 2.3 W. What is the insertion loss associated with the RF connector and DIB trace?

ANS. 0.185 dB.

12.13. The maximum available power from a source is 1 W. If a transducer with a 16-Ω load is connected to the source, what is the transducer loss if the load power is 0.9 W?

ANS. 0.46 dB.

A continuous sinusoidal wave (CW) can be described mathematically as

$$v(t) = A_0 \sin(\omega_0 t + \Theta_0) \tag{12.48}$$

where A_0 is the voltage amplitude of the sinusoidal wave, ω_0 is the angular frequency expressed in rad/s, and Θ_0 is the phase offset (also in radians). In practice, a CW signal will experience additive noise as it propagates through a channel. There are two types of noises that the wave will experience: amplitude and phase noise. We can model these two time-varying noise effects by including an additive amplitude noise term $a(t)$ and an additive phase noise term $\phi(t)$ in the expression above and write

$$v(t) = [A_0 + a(t)]\sin[\omega_0 t + \Theta_0 + \phi(t)] \tag{12.49}$$

Noise in RF test technology is a random variation of one or more characteristics of any entity such as voltage, current, phase, distribution, or spectral density. Noise effects are modeled in general as random processes. Typical measurements for RF noise effects are noise figure (NF) and phase noise ($\pounds(f)$). These measurements account for the random variations in magnitude and phase associated with the CW signal. Some system-like measurements such as EVM (error vector magnitude) and BER (bit error rate) also take into account noise effects while at the same time measuring system transmission capability.

In the following we will discuss the physics behind these amplitude and phase noise measurements.

12.3.1 Amplitude Noise

Generally, noise consists of spontaneous stochastically distributed fluctuations caused by ordinary phenomena in electronic circuits. These fluctuations can arise from different physical effects, but all lead to a random effect. The most important noise effects are due to:

- Thermal or Johnson–Nyquist noise
- Shot noise
- Flicker or $1/f$ noise

Thermal Noise

The thermal or Johnson–Nyquist noise arises from vibrations of the electrons and holes in conducting or semiconducting material due to their finite temperature. Some of the vibrations have a spectral content, but the magnitude of these variations for RF frequencies is nearly uniform. As such, we talk about the noise as being white, which is assumed to have a constant power as a function of the frequency. For a single resistor, the thermal noise power made available at its terminals is

$$P_N = kTB \tag{12.50}$$

where k is the Boltzmann constant ($k = 1.38\times10^{-23} JK^{-1}$), T is the temperature in kelvin, and B is the bandwidth expressed in hertz. It is important to note that the thermal noise power available is independent of the system impedance. Using Eq. (12.50) we can calculate the noise power available at room temperature (say 20°C) for a 1 Hz bandwidth as follows

$$P_{N,1\mathrm{Hz}} = 1.38\cdot10^{-23}\ \mathrm{WsK}^{-1}\cdot(273+20)\mathrm{K}\cdot1\,\mathrm{Hz} = 4.04\times10^{-21}\ \ \mathrm{W} \tag{12.51}$$

Assuming this noise power is constant over a large range of frequencies, we can speak in terms of the power per 1-Hz bandwidth, or power spectral density (PSD), denoted by S_N, and write

$$S_N \triangleq \frac{P_{N,1\mathrm{Hz}}}{1\,\mathrm{Hz}}\left[\frac{W}{\mathrm{Hz}}\right] \tag{12.52}$$

In the case described here, the PSD would be $S_N = 4.04\times10^{-21}$ W/Hz. We can convert the PSD to a dBm/Hz scale by referencing to a 1-mW power level according to

$$S_N\big|_{\mathrm{dBm/Hz}} = 10\log_{10}\left(\frac{S_N}{1\,\mathrm{mW}}\right) \tag{12.53}$$

For the running example given here, the PSD expressed in dBm/Hz is found according to

$$S_N\big|_{\mathrm{dBm/Hz}} = 10\log_{10}\left(\frac{4.04\times10^{-21}}{1\,\mathrm{mW}}\right) = -173.93\frac{\mathrm{dBm}}{\mathrm{Hz}} \tag{12.54}$$

If the noise power is measured with an instrument that has a resolution bandwidth other than 1 Hz, say instead equal to $\mathrm{BW_{RES}}$ in Hz, then the PSD will be increased by a factor equal to this $\mathrm{BW_{RES}}$ and falsely representing the PSD level. Therefore, to correct for this effect, we modify the PSD given in Eq. (12.52) according to

$$S_N \triangleq \frac{1}{\mathrm{BW}_{RES}} \frac{P_{N,BW_{RES}}}{1\ \mathrm{Hz}} \quad \left[\frac{W}{\mathrm{Hz}}\right] \tag{12.55}$$

or in dBm/Hz as

$$S_N\Big|_{\mathrm{dBm/Hz}} = 10\log_{10}\left(\frac{1}{\mathrm{BW}_{RES}} \frac{P_{N,BW_{RES}}}{1\,\mathrm{mW}}\right) \tag{12.56}$$

which can also be written as

$$S_N\Big|_{\mathrm{dBm/Hz}} = 10\log_{10}\left(\frac{P_{N,BW_{RES}}}{1\,\mathrm{mW}}\right) - 10\log_{10}\left(BW_{RES}\right) \tag{12.57}$$

Here the left-hand side term represents the actual PSD of the signal, the first term on the right-hand side is the actual power level captured by the instrument, and the second term on the right is related to the instrument bandwidth. If the PSD of an input signal is -173.93 dBm/Hz at room temperature and an instrument with a 100-kHz resolution bandwidth measures it, then the capture power would display a power level equal to

$$10\log_{10}\left(\frac{P_{N,BW_{RES}}}{1\ \mathrm{mW}}\right) = -173.93 + 10\log_{10}\left(100\times 10^3\right) = -173.93 + 50 \approx -124\ \mathrm{dBm}$$

This example shows the impact of the bandwidth and the minimum detectable power. In this case, a noise power level of -124 dBm is displayed. If we want to measure lower amplitude signals, we need to reduce the measurement or receiver bandwidth.

Shot Noise

Shot or Schottky noise in electronic devices is caused by the random fluctuations of the number of charge carriers (electrons and holes) that cross a region of a conductor in a given amount of time. Shot noise is most pronounced in *pn*-junction-associated semiconductors. It is less of a concern in metal wires, as correlations between individual electrons remove these random fluctuations. The spectrum of shot noise is broadband and flat for RF frequencies, that is, white noise.

Shot noise is to be distinguished from thermal or Johnson–Nyquist noise. Thermal noise occurs without any applied voltage and without any current flow. This is not the case for shot noise. The noise power of shot noise can be described in most practical applications by

$$P_N = 2qIBR \tag{12.58}$$

where q is the elementary charge, B is the bandwidth in hertz over which the noise is measured, and I is the average current through the effective resistance R of the signal path.

Flicker Noise

Flicker noise is a type of electronic noise with a *1/f* frequency behavior or pink spectrum. It is therefore often referred to as *1/f* or pink noise, though these terms have wider definitions. It occurs in almost all electronic devices and results from a variety of effects, including (a) impurities in a conductive channel and (b) generation and recombination effects associated with a semiconductor material. In electronic devices, it is a low-frequency phenomenon, as the higher frequencies are overshadowed by white noise from other sources. In oscillators, however, the low-frequency noise is mixed up to frequencies close to the carrier that results in oscillator phase noise.

Active devices like MOSFETs, JFETs, and BJTs all suffer from flicker noise. Flicker noise is often characterized by the corner frequency f_c, the frequency at which the flicker noise meets the thermal noise level. MOSFETs have a higher than JFETs or bipolar transistors, which is usually below 2 kHz.

12.3.2 Noise Figure

The basic definition of noise factor (F) was introduced in the 1940s by Harold Friis as the ratio of SNR at a system input to the SNR at the system output.[3] It is written as

$$F = \frac{\mathrm{SNR}_i}{\mathrm{SNR}_o} = \frac{S_i/N_i}{S_o/N_o} \tag{12.59}$$

It is common to express the noise factor on a logarithmic scale in decibels with a new name, the noise figure (NF), according to

$$NF\big|_{\mathrm{dB}} = 10\log_{10}(F) \tag{12.60}$$

A perfect system would add no noise to the signal, which would result in the output SNR being the same as that occurring at its input. In other words, the ideal system would have a noise factor or noise figure F of 1 or NF of 0 dB, respectively. Of course, real systems will add noise to the incoming signal (including the input noise) resulting in a reduction of the output SNR, in turn, leading to a noise factor F greater than one. Such a situation is illustrated in Figure 12.13 for an RF amplifier. Assuming that the amplifier corresponding to the data shown in Figure 12.13 is tested with a signal with a limited bandwidth, we can assume that we can read the signal power at the frequency of 2.45 GHz while the noise power can be read at, for example, 2.4 GHz. The error introduced is minimal since the noise power added to the signal at a frequency of 2.45 GHz is basically the same as the power around 2.4 GHz. Using Eqs. (12.59) and (12.60), we get

$$\mathrm{NF} = \mathrm{SNR}_i\big|_{\mathrm{dB}} - \mathrm{SNR}_o\big|_{\mathrm{dB}} = \left[(-69)-(-100)\right]-\left[(-43)-(-64)\right] = 10 \text{ dB}$$

Assuming an RF system with a power gain $G = P_o/P_i$, where P_i and P_o is the input and output power, respectively, the noise factor F can be expressed in terms of the input noise level N_i as well as the additive amplifier noise N_a as follows:

$$\begin{aligned} F &= \frac{S_i/N_i}{S_o/N_o} \\ &= \frac{S_i/N_i}{GS_i/(N_a+GN_i)} \\ &= \frac{N_a+GN_i}{GN_i} \end{aligned} \tag{12.61}$$

Equation (12.61) shows the dependency of the noise at the input. Often this noise can be assumed to be thermal noise.

For a system with multiple gain stages, the system noise factor can be calculated by the Friis equation given by

$$F_{sys} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_n - 1}{G_1 G_2 \ldots G_{n-1}} \tag{12.62}$$

where F_i and G_i are the noise factor and gain, respectively, of the ith amplifier stage expressed in linear magnitude form (rather than in dBm).

Figure 12.13. Typical signal and noise levels vs. frequency at an amplifier's input and at its output.

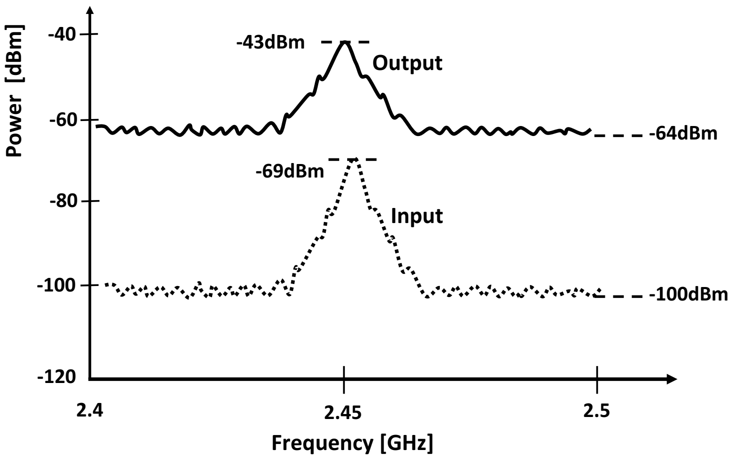
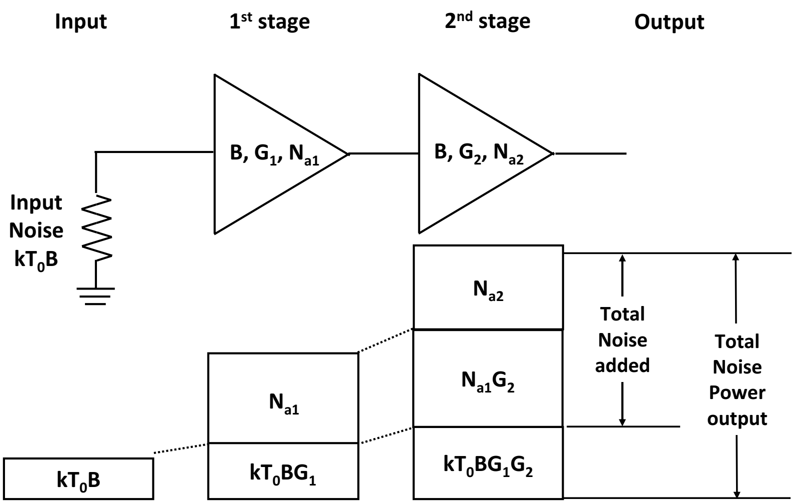


Figure 12.14. Illustrating the noise additions as a function of the number of amplifier stages.



The basis of the Friis general expression of Eq. (12.62) can be better understood from the two-stage amplifier example shown at the top of Figure 12.14. Here the amplifier consists of two stages with gains G_1 and G_2. Each stage is assumed to have the same bandwidth B. Assume that the input noise to the amplifier is thermal noise arising from a resistor operating at a temperature of T_o with bandwidth B. The first stage will amplify this noise component by the gain of the first stage, G_1, as well contribute an additional noise component of N_{a1}, resulting in the output noise from the first stage as $kT_oBG_1 + N_{a1}$. This is depicted in the middle block in the lower figure of Figure 12.14. Subsequently, this noise component appears as the input to the second stage, whereby it is amplified with gain G_2 and combined with the noise of the second stage. The net result is the noise at the amplifier output becomes $kT_oBG_1G_2 + N_{a1}G_2 + N_{a2}$. As depicted by the lower sketch of Figure 12.14, the amplifier has contributed a noise component of $N_{a1}G_2 + N_{a2}$ that is not present at the input. As this additive noise component is the sum of two terms, where N_{a1} and N_{a2} are generally similar in magnitude, the first noise term $N_{a1}G_2$ dominates the sum. It also suggests that the noise of the first stage is the most important component affecting the amplifier's noise factor.

EXAMPLE 12.6

What is the combined noise figure of a two-stage amplifier if the first stage has a noise figure of 2 dB with a gain of 7 dB, and the second stage has a noise figure of 5 dB with a gain of 20 dB? What would be the overall noise figure if the amplifier cascade is rearranged so that the second stage becomes the first stage?

Solution:
Our first step is to convert all given terms into linear magnitude form, that is,

$$G_1 = 10^{G_1/10} = 10^{7/10} = 5.012,\ \ G_2 = 10^{G_2/10} = 10^{20/10} = 100,$$
$$F_1 = 10^{NF_1/10} = 1.585, \qquad F_2 = 10^{NF_2/10} = 3.162$$

Next, using Friis general noise equation (i.e., Eq. (12.62)), we calculate

$$F_{sys} = F_1 + \frac{F_2 - 1}{G_1} = 1.585 + \frac{3.162 - 1}{5.012} = 2.016$$

Finally, the overall system noise figure in decibels is

$$NF = 10\log_{10}(2.016) = 3.046 \text{ dB}$$

Now, if the second stage is placed in front of the first stage, the noise factor becomes

$$F_{sys} = F_2 + \frac{F_1 - 1}{G_1} = 3.162 + \frac{1.585 - 1}{5.012} = 3.168$$

and is expressed in decibels as

$$NF = 10\log_{10}(3.168) = 5.008 \text{ dB}$$

Clearly, the arrangement of the first stage gain of 7 dB followed by the second stage gain of 20 dB results in the lower noise figure of 3.046 dB.

12.3.3 Phase Noise

One of the most significant parameters impacting the overall performance of an RF system is phase noise. Phase noise often becomes the limiting factor with respect to system sensitivity, maximum data rate, and bit error rate, to name just a few. This subsection will outline the fundamentals of phase noise, its sources, and how jitter in digital systems is related to phase noise such as that described in reference [10].

Exercises

12.14.	A 1-kΩ resistor is connected across a 1-μF capacitor, what is the available noise power due to thermal noise at the terminals of this resistor at 300 K?	ANS. −151.8 dBm.
12.15.	If a spectral analyzer reveals a power level of -150 dBm at 500 MHz using a resolution bandwidth of 50 kHz, what is the actual PSD appearing at the input to this instrument?	ANS. −147.0 dBm/Hz.
12.16.	If a given amplifier has a 4 dB noise figure at 300 kelvin, a noise bandwidth of 500 kHz and an input resistance of 50 Ω, what is the output signal-to-noise ratio if the input signal RMS level is 1 mV.	ANS. 65.9 dB.
12.17.	What is the combined noise figure of three-stage amplifier? The first stage has a noise figure of 2.1 dB and a gain of 8 dB, the second stage has a noise figure of 3 dB and a gain of 10 dB, and the third stage has a noise figure of 5 dB and a gain of 14 dB.	ANS. 2.59 dB.

Fundamentals

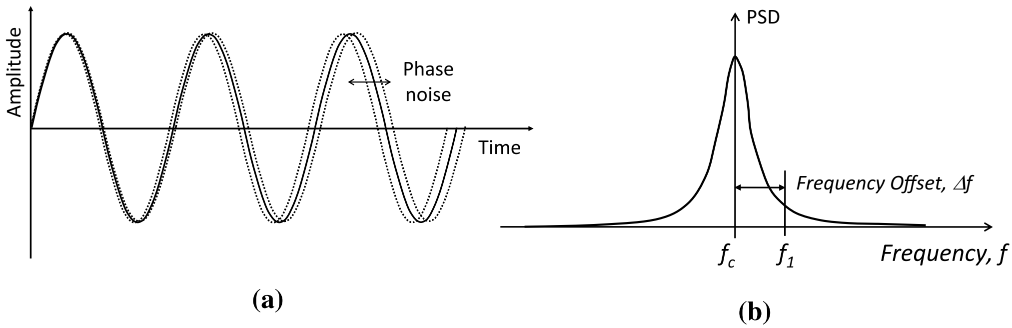
Phase noise, which falls into the wider category of frequency stability, is the degree to which an oscillating source produces the same single frequency value throughout a given time. This definition suggests that the frequency stability of an oscillator that produces a signal other than a perfect sinusoidal will be less than perfect.

A continuous sinusoidal wave subject to phase noise can be described as

$$v(t) = A_c \sin\left[\omega_c t + \Theta_0 + \phi(t)\right] \tag{12.63}$$

where A_c is the voltage amplitude of the sinusoidal wave, ω_c is the angular frequency expressed in rad/s, and Θ_0 is the phase offset (in radians) and $\phi(t)$ is a noise signal that modulates the phase of the carrier, also expressed in radians. Figure 12.15a illustrates the behavior of a sinusoidal signal whose phase is modulated by a noise signal in the time domain. One effect of the noise signal is to alter the zero-crossing values of the sinusoidal wave. Because noise affects the zero crossing values in a random manner, it is customary to model the attributes of this noise signal as a random process with specific types of statistics (e.g., Gaussian statistics). Alternatively, these phase fluctuations will also appear as changes in the period of the signal and thus show up as variations in the frequency of the signal. One can describe these frequency variations by a power spectral density (PSD) function as illustrated in Figure 12.15b.

Figure 12.15. (a) Impact of phase noise on sinusoidal signal in time domain. (b) PSD of sinusoidal signal in the frequency domain.



Spectral Density of Phase Fluctuations

As just mentioned, phase noise can be described in the time or frequency domain, depending on the application. For digital signals, a time domain description is used whereby a statistical model of the jitter introduced. For RF systems, a frequency domain description is more commonly used, because most measurement techniques are frequency-domain based.

The double-sided power spectral density (PSD) of a sinusoidal signal with phase noise can be described in the frequency domain along the positive frequency axis as having a frequency component concentrated at the carrier frequency f_c combined with a noise component that generally falls off on each side of the carrier at a rate of $\sim 1/(f - f_c)^2$ as illustrated in Figure 12.15b. Mathematically, one can describe the positive components of the frequency distribution in general terms as

$$S_v(f) = a \cdot \delta(f - f_c) + \frac{a \cdot b}{\pi^2 b^2 + (f - f_c)^2} \frac{\mathrm{V}^2}{\mathrm{Hz}} \tag{12.64}$$

where a and b are arbitrary positive constants. Defining $\Delta f = f - f_c$ as the frequency offset from the carrier, the general form of the PSD for $v(t)$can be described as

$$S_v(f_c + \Delta f) = a \cdot \delta(\Delta f) + \frac{a \cdot b}{\pi^2 b^2 + \Delta f^2} \quad \frac{\mathrm{V}^2}{\mathrm{Hz}} \tag{12.65}$$

The rightmost term in the above PSD is known as a Lorentzian distribution and often appears in oscillator studies.

While the above PSD was for the power associated with the signal $v(t)$, one can extract a single-sided PSD that reflects the RMS variations in the instantaneous phase difference φ between the carrier signal located at f_c and a signal occupying some bandwidth BW offset from the carrier by some frequency Δf. Assuming the RMS value of the phase signal extracted at each Δf is Φ_{RMS}, the PSD of the phase difference can be defined as

$$S_\phi(\Delta f) = \frac{\Phi_{RMS}^2}{\mathrm{BW}} \frac{\mathrm{rad}^2}{\mathrm{Hz}} \tag{12.66}$$

Such a measurement would be carried out using a phase demodulator technique rather than a spectrum analyzer (see Chapter 13, Section 13.4).

Through some rigor, this phase PSD can be approximated in terms of the original waveform PSD according to

$$S_\phi(\Delta f) \approx 2\frac{S_v(f_c+\Delta f)}{S_v(f_c)} \quad \frac{\text{rad}^2}{\text{Hz}} \tag{12.67}$$

where $S_v(f_c+\Delta f)$ represents the PSD of the signal $v(t)$ at a distance Δf from the carrier frequency f_c and $S_v(f_c)$ represents the PSD at the carrier frequency (a power term). In terms of the PSD defined by Eq. (12.65), $S_\phi(\Delta f)$ would be written as

$$S_\phi(\Delta f) = \frac{2b}{\pi^2 b^2 + \Delta f^2} \frac{\text{rad}^2}{\text{Hz}} \tag{12.68}$$

Often $S_\phi(\Delta f)$ is expressed in logarithmic terms as

$$S_\phi(\Delta f)\Big|_{\text{dBm/Hz}} = 10\log_{10}\left[S_\phi(\Delta f)\right] = 10\log_{10}(2b) - 10\log_{10}\left(\pi^2 b^2 + \Delta f^2\right) \ \frac{\text{dBm}}{\text{Hz}} \tag{12.69}$$

or it can be rewritten in a piecewise linear fashion as

$$S_\phi(\Delta f)\Big|_{\text{dBm/Hz}} \approx \begin{cases} 10\log_{10}\left(2/\pi^2 b\right), & \Delta f < \pi b \\ 10\log_{10}(2b) - 20\log_{10}\Delta f, & \Delta f > \pi b \end{cases} \tag{12.70}$$

As is evident from the above expression, for $\Delta f > \pi b$ the $S_\phi(\Delta f)$ will roll off at a rate of -20 dBm/Hz per decade or -6 dBm/Hz per octave.

It is important to note at this point in the discussion that not all oscillators behave according to a Lorentzian distribution. Often their behaviors can deviate quite significantly from this simple first-order model.

In the RF literature one often speaks about the quantity called *phase noise*. While multiple definitions of phase noise exist, here we make use of the definition provided in the 1139 Institute of Electrical and Electronic Engineers (IEEE) standard where the single sideband phase noise with symbol $\pounds(\Delta f)$ is defined as

$$\pounds(\Delta f) \stackrel{\Delta}{=} \frac{1}{2} S_\phi(\Delta f) \ \frac{\text{rad}^2}{\text{Hz}} \tag{12.71}$$

Combining Eq. (12.67) with (12.71), we can write the phase noise as

$$\pounds(\Delta f) = \frac{S_v(f_c+\Delta f)}{S_v(f_c)} \quad \frac{\text{rad}^2}{\text{Hz}} \tag{12.72}$$

Taking the logarithm of each side of Eq. (12.72) and multiplying by 10, we can write the above phase noise expression as

$$10\log_{10}\left[\pounds(\Delta f)\right] = 10\log_{10}\left[S_v(f_c+\Delta f)\right] - 10\log_{10}\left[S_v(f_c)\right] \tag{12.73}$$

While the units of the two right-hand terms are well-defined—that is, the term 10 $\log_{10}[S_v(f_c + \Delta f)]$ is expressed in dBm/Hz and the term $10\log_{10}[S_v(f_c)]$ is expressed in dBm—the left-hand side term is less clear. The units of the phase noise $10\log_{10}[£(\Delta f)]$ is simply $10\log_{10}(\text{rad}^2/\text{Hz})$, which is conveniently written as dBc/Hz. The dBc is stated as "decibels with respect to carrier."

Often the phase noise is measured with an instrument that has a bandwidth other than 1 Hz, say, a spectrum analyzer with resolution bandwidth BW. As such, the PSD displayed must be normalized by the instrument bandwidth. The phase noise would then be computed according to

$$£\big|_{\text{dBc/Hz}}(\Delta f) = P_{SSB}\big|_{\text{dBm}} - P_{carrier}\big|_{\text{dBm}} \tag{12.74}$$

where we define

$$\begin{aligned} £\big|_{\text{dBc/Hz}}(\Delta f) &\triangleq 10\log_{10}\left[£(\Delta f)\right] \ \frac{\text{dBc}}{\text{Hz}} \\ P_{carrier}\big|_{\text{dBm}} &\triangleq 10\log_{10}\left[S_v(f_c)\right] \ \text{dBm} \\ P_{SSB}\big|_{\text{dBm}} &\triangleq 10\log_{10}\left[S_v(f_c+\Delta f)\right] - 10\log_{10}(\text{BW}) \ \text{dBm} \end{aligned} \tag{12.75}$$

Figure 12.16. Illustrating the definition of phase noise in terms of the carrier power and single sideband power at an offset of Δf.

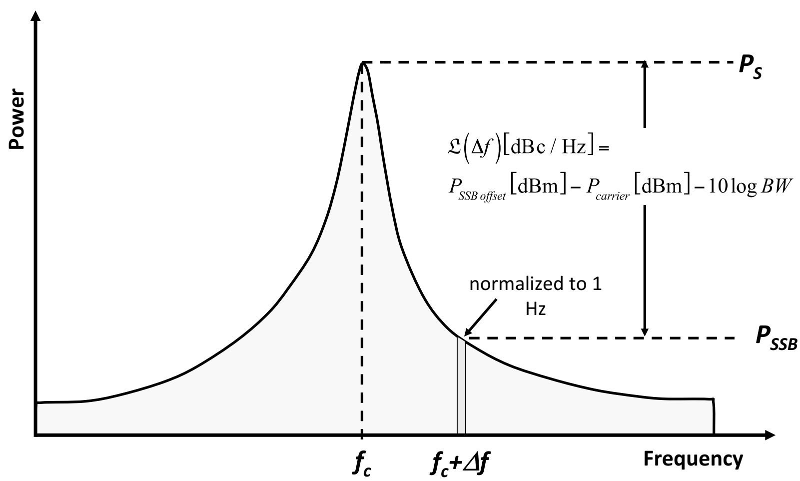


Figure 12.17. Single-sided phase noise $\pounds(f)$ shown as function of frequency offset.

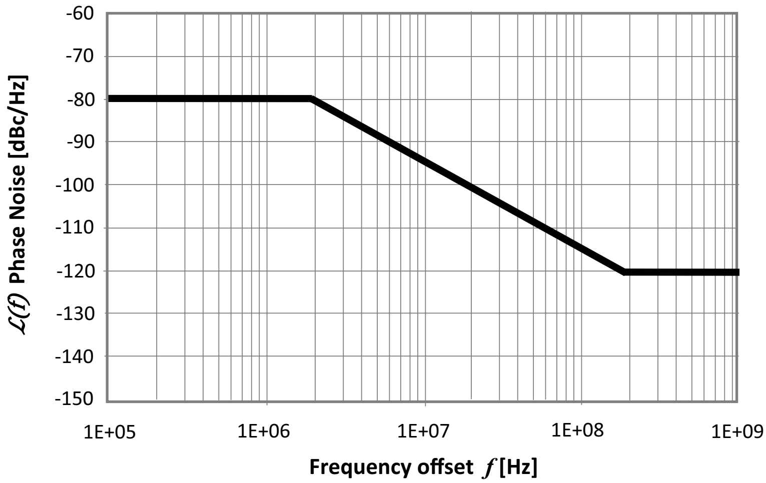


Figure 12.16 illustrates the relationship between the phase noise definition and power attributes of PSD of the original sinusoidal waveform defined in Eq. (12.63). If the PSD of a signal has been captured on a spectrum analyzer, then the phase noise is simply the difference in power of the carrier $P_{carrier}$ and the power at the some offset frequency Δf from the carrier frequency f_c normalized over 1 Hz bandwidth, denoted P_{SSB}.

While in this subsection we define the frequency offset with the explicit frequency term Δf, one commonly sees phase noise written without the delta symbol as $\pounds(f)$. It should be clear to the reader when they see this notation that the frequency f refers to a frequency offset from the carrier frequency.

In practice, rather than display the spectrum as shown in Figure 12.16, it is more common in RF discussions to display the phase noise $\pounds(f)$ as a single-sided spectrum dependent on the frequency offset from the carrier as shown in Figure 12.17. Both the vertical and horizontal axes are plotted on a log scale, that is, dBc/Hz versus Hz.

Spectral Density of Frequency Fluctuations

Another PSD that one may encounter with a phase noise description is the spectral density of frequency fluctuations, denoted $S_f(\Delta f)$, that results when measuring phase noise with a frequency discriminator. Here $S_f(\Delta f)$ represents the total RMS frequency variation Ψ_{RMS} in a given bandwidth, BW, defined as

$$S_f(\Delta f) \triangleq \frac{\Psi^2{}_{RMS}}{\mathrm{BW}} \quad \frac{\mathrm{Hz}^2}{\mathrm{Hz}} \tag{12.76}$$

Because instantaneous frequency is the derivative of the instantaneous phase of a signal, that is, $f = d\phi/dt$ the PSD of the frequency fluctuations is simply related to the PSD of the phase according to

$$S_f(\Delta f)^2 = (\Delta f)^2 S_\phi(\Delta f) \tag{12.77}$$

Integrated Frequency and Phase Noise

In modern communication systems with signals using multiple carriers, it is of interest to have a measure of short-term instabilities integrated over the channel bandwidth or the integrated noise power in the communication channel. For example, in an FM radio, the integrated frequency noise (measured in RMS-Hz) over the channel bandwidth is important. In digital communication systems, the integrated phase fluctuations in RMS-radians or RMS-degrees can be useful for analyzing the system performance.

Integrated noise over any bandwidth of interest is easily determined from using spectral density functions. Integrated frequency noise, commonly called residual FM, can be calculated by integrating the spectral density function of frequency fluctuations $S_f(\Delta f)$ over the frequency band $\mathrm{BW} = f_2 - f_1$ to arrive at

$$\text{Residual FM} = \Psi_{RMS} = \sqrt{\int_{f_1}^{f_2} S_f(\Delta f)\, d\Delta f} \quad [\text{RMS-Hz}] \tag{12.78}$$

The integrated phase noise (referred to here as the residual PM) can also be calculated by integrating the spectral density function of phase noise fluctuations $S_\phi(\Delta f)$ over a similar bandwidth according to

$$\text{Residual PM} = \Phi_{RMS} = \sqrt{\int_{f_1}^{f_2} S_\phi(\Delta f)\, d\Delta f} \quad [\text{RMS-radians}] \tag{12.79}$$

or in rms-degrees as

$$\text{Residual PM} = \frac{180}{\pi} \times \sqrt{\int_{f_1}^{f_2} S_\phi(\Delta f)\, d\Delta f} \quad [\text{RMS-degrees}] \tag{12.80}$$

Jitter and Integrated Phase Noise

Jitter is a common term in digital testing, as will be explained in greater detail in Chapter 13 as well as in Chapter 14. Jitter is the measure of the variation of the instantaneous frequency of an oscillator, which is not constant but varies slightly around an average value, thus creating an uncertainty in the frequency at any given point in time. This frequency change can be viewed as a change in the time of the waveform edge from the nominal frequency edge. We often speak about variation in the zero-crossing point as defined by the waveform edge.

Jitter can be measured in the time domain as a statistical variation in the timing of the edges. For instance, one statistical measure is to compute the RMS value of the signal period (denoted by T_n for the nth period) and expressed in radians according to

$$\sigma_\phi\Big|_{\text{rad}} = \sqrt{\lim_{N\to\infty}\left(\frac{1}{N}\sum_{n=1}^{N}\left(T_n - \mu_T\right)^2\right)} \tag{12.81}$$

where μ_T is the average period given by

$$\mu_T = \lim_{N\to\infty}\left(\frac{1}{N}\sum_{n=1}^{N} T_n\right) \tag{12.82}$$

We can also relate the integrated phase noise quantity to this RMS quantity (in radians) through the following expression, by integrating the area underneath the phase noise curve over some frequency range, f_1 and f_2, according to

$$\sigma_\phi\Big|_{\text{rad}} = \sqrt{\text{Residual PM}} = \sqrt{\int_{f_1}^{f_2} S_\phi(\Delta f)\, d\Delta f} \tag{12.83}$$

which is sometimes more conveniently expressed in terms of the phase noise as

$$\sigma_\phi\Big|_{\text{rad}} = \sqrt{2\int_{f_1}^{f_2} \pounds(\Delta f)\, d\Delta f} \tag{12.84}$$

The integrated phase noise can also be expressed in degrees according to

$$\sigma_\phi\Big|_{\deg} = \frac{180}{\pi}\sigma_\phi\Big|_{\text{rad}} \tag{12.85}$$

Likewise, we can express this RMS quantity in seconds by multiplying the average signal period, μ_T, normalized by 2π, according to

$$\sigma_\phi\Big|_{\sec} = \frac{\mu_T}{2\pi}\sigma_\phi\Big|_{\text{rad}} \tag{12.86}$$

The above four equations were derived assuming that there is no $1/f$ noise or burst noise present with the signal.

EXAMPLE 12.7

The phase noise of a VCO with an oscillation frequency of 1 MHz has the phase noise spectrum shown in Figure 12.18. What is the RMS value of the phase jitter over a frequency range of 100 kHz to 10 MHz? Express your answer in degrees, radians, and seconds.

Solution:

As is evident from the phase noise $\pounds(\Delta f)$ plot, the phase noise is equal to -80 dBc/Hz from 100 kHz to 10 MHz. This is equivalent to 10^{-8} rad^2/Hz over the same frequency range. Using Eq. (12.84), the integrated phase noise is computed according to

$$\sigma_\phi\Big|_{\text{rad}} = \sqrt{2\times\int_{10^5}^{10^7} 10^{-8} d\Delta f} = \sqrt{2\times\left(10^7 - 10^5\right)} = 0.447\text{rad}$$

We can convert this to degrees simply by multiplying $\sigma_\phi\Big|_{\text{rad}}$ by $180/\pi$ and get

$$\sigma_\phi\Big|_{\deg} = \frac{180}{\pi}\times 0.447 = 25.5 \text{ degrees}$$

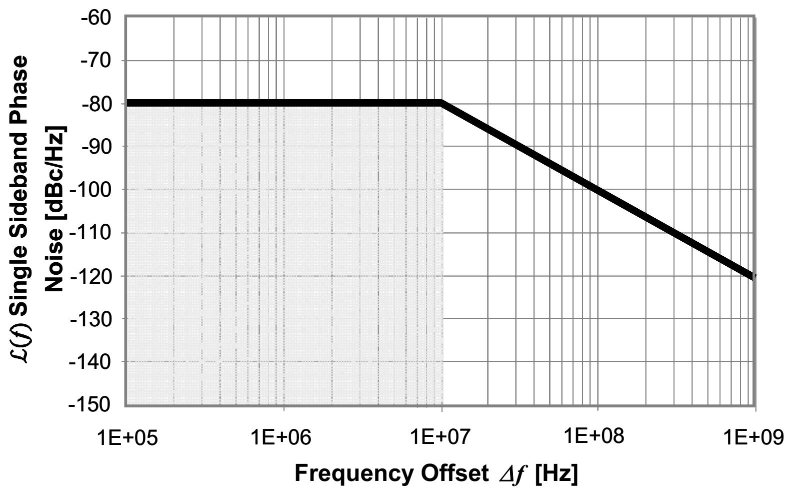
Similarly, we can express the RMS value in seconds through the application of Eq. (12.86) where the average period of the oscillator is

$$\mu_T = \frac{1}{f_{VCO}} = \frac{1}{10^6} = 10^{-6}\ \text{s}$$

leading to

$$\sigma_\phi\Big|_{\text{sec}} = \frac{10^{-6}}{2\pi} \times 0.447 = 71.1\,\text{ns}$$

Figure 12.18. Phase noise plot for Example 12.7. Area under curve from 100 kHz to 10 MHz represents integrated phase noise.



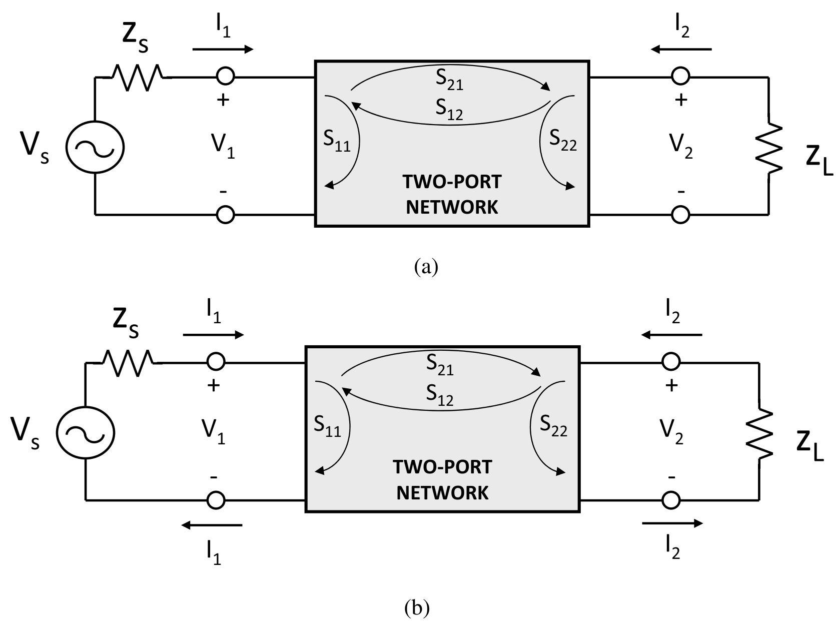
12.4 *S*-PARAMETERS

The performance and functionality of any RF circuit can be described by the signals going in and coming out of the circuit. In this case, the circuit can be viewed as an *n*-port. These *n*-ports are representatives for linear or nonlinear networks of the circuit. For signals sufficiently small to cause only a linear response, an *n*-port can be described via a set of parameters measured at the network ports. Once these parameters have been determined, the behavior in any external environment can be predicted without any detailed knowledge of the content of the *n*-port network. One set of parameters named *S*-parameters, also referred to as scattering parameters, has found widespread application for describing *n*-port networks. This was first published[4] by K. Kurokawa in 1965 and is used extensively in most RF work. In this section we will learn how *S*-parameters can help us better understand RF circuits.

12.4.1 Principles of *S*-Parameters of a Two-Port Network

S-parameters are commonly used to design and describe microwave circuits. They are easy to measure and are better suited to high-frequency applications unlike conventional two-port parameters

Figure 12.19. Two-port network with incident and reflected port waves: (a) waves associated with each port. (b) voltages and currents associated with each port.



like Y-, T- or h-parameters, which one may have encountered in an introductory circuit analysis course. They are conceptually simple and give deep insight into RF circuit behavior and their properties. Here the fundamentals of S-parameters will be discussed.

Although an arbitrary network might have any number of ports, we will begin with a two-port network like one shown in Figure 12.19a. We begin by identifying the incident and reflected waves a_i and b_i at each port of the network. These waves are normalized by the square root of either the source or load impedance, depending on which branch the wave is associated with.

The general form of the incident voltage wave a_i can be expressed as

$$a_i = \frac{V_i - Z_i^* I_i}{2\sqrt{\left|\Re e\left\{Z_i\right\}\right|}} \tag{12.87}$$

and the reflected wave, b_i as

$$b_i = \frac{V_i - Z_i^* I_i}{2\sqrt{\left|\Re e\left\{Z_i\right\}\right|}} \tag{12.88}$$

In both cases, V_i is the terminal voltage at the i^{th} port of the n-port network with terminal current I_i and complex impedance Z_i as highlighted in Figure 12.19b. The asterisk in Eq. (12.88) denotes the complex conjugate.

Assuming all impedances z_i are positive and real with value Z_0, Eqs. (12.87) and (12.88) can be expressed for a two-port network as

$$a_1 = \frac{V_1 + Z_0 I_1}{2\sqrt{Z_0}} = \frac{\text{voltage wave incident on port 1}}{\sqrt{Z_0}} \tag{12.89}$$

$$a_2 = \frac{V_2 + Z_0 I_2}{2\sqrt{Z_0}} = \frac{\text{voltage wave incident on port 2}}{\sqrt{Z_0}} \tag{12.90}$$

$$b_1 = \frac{V_1 - Z_0 I_1}{2\sqrt{Z_0}} = \frac{\text{voltage wave reflected from port 1}}{\sqrt{Z_0}} \tag{12.91}$$

$$b_2 = \frac{V_2 - Z_0 I_2}{2\sqrt{Z_0}} = \frac{\text{voltage wave reflected from port 2}}{\sqrt{Z_0}} \tag{12.92}$$

The following set of linear equations are used to describe the interrelationship of all the waves associated with the two-port network, in much the same way that nodal voltages and branch current interrelate in any electrical circuit:

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{12.93}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \tag{12.94}$$

The coefficients, S_{11}, S_{12}, S_{21}, and S_{22}, are referred to as the S-parameters of the two-port network. Sometimes, the above equations are lumped together into a matrix formulation as follows:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \tag{12.95}$$

Based of the above formulation, we can identify each S-parameter according to the following:

S_{11}: The input reflection coefficient with the output port terminated by a matched load ($Z_L = Z_0$) resulting in $a_2 = 0$;

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \tag{12.96}$$

S_{22}: The output reflection coefficient with the input port terminated by a matched load ($Z_S = Z_0$ and $V_S = 0$) resulting in $a_1 = 0$;

$$S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0} \tag{12.97}$$

S_{21}: The forward transmission (insertion) gain with the output port terminated by a matched load ($Z_L = Z_0$) resulting in $a_2 = 0$;

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \tag{12.98}$$

S_{12}: The reverse transmission (isolation) gain with the input port terminated by a matched ($Z_S = Z_0$ and $V_S = 0$) resulting in $a_1 = 0$;

$$S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \tag{12.99}$$

These S-parameters can be used to determine the percentage of the power going into a device and the amount reflected as a function of the impedance of the two-port network and the impedance of the termination. This termination might be the impedance of the source supplying an RF signal or the impedance of the test equipment measuring the RF power.

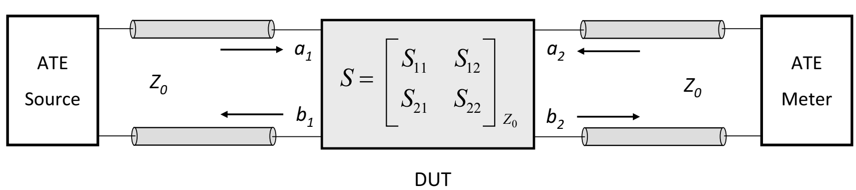
Equations (12.89)–(12.92) describe the incident voltage waves a_i and the reflected voltage waves b_i as effective (RMS) values and not peak values. Since the two-port network described with the *S*-parameter matrix,

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}_{Z_0} \tag{12.100}$$

is embedded in a environment with a characteristic impedance of Z_0, these waves can be interpreted in terms of normalized voltage or current waves. This relation is explained below. In most test situations, a real characteristic impedance $Z_0 = (50 + j0)\Omega$ is selected. For such cases, the RF measurement equipment including all cables will be designed for 50 Ω. As such, the environment will be operating under reflectionless and conjugate matched conditions. Natural reflections within the measurement environment will not exist. Nonetheless, the test engineer must concern himself or herself with matching the DUT to the test environment. The general situation is depicted in Figure 12.20 where the middle block consisting of the *S*-matrix represents the DUT.

Let us look at one example involving S_{11}. From Eq. (12.96), we recognize that S_{11} relates the reflected wave to the incident wave at port 1. It is therefore referred to as the reflection coefficient for port 1. Of course S_{22} has the same meaning but applies to port 2. Let us consider some further

Figure 12.20. Two-port network embedded into test environment with characteristic impedance Z_0.



meaning for S_{11}. Assuming matched terminations, according to Eq. (12.96), together with Eqs. (12.89) and (12.91), we can express S_{11} as

$$S_{11} = \frac{b_1}{a_1} = \frac{\frac{V_1}{I_1} - Z_0}{\frac{V_1}{I_1} + Z_0} \tag{12.101}$$

If we define the input impedance looking into port 1 as Z_1, we can write

$$Z_1 = \frac{V_1}{I_1} \tag{12.102}$$

and then we can write the reflection coefficient as

$$S_{11} = \frac{Z_1 - Z_0}{Z_1 + Z_0} \tag{12.103}$$

Clearly, if $Z_1 = Z_0$, then the reflection coefficient is zero and no reflected wave results. If $Z_1 = 0$ then the reflection coefficient is -1 and the reflected wave is equal but opposite to the incident wave.

Conversely, we can also rearrange this expression and write the input impedance of a two-port network as a function of the reflection coefficient S_{11} as

$$Z_1 = Z_0 \frac{1 + S_{11}}{1 - S_{11}} \tag{12.104}$$

Of course, we could develop similar looking expressions for the reflection coefficient S_{22} at port 2 in terms of the impedance $Z_2 = V_2/I_2$ and find

$$S_{22} = \frac{Z_2 - Z_0}{Z_2 + Z_0} \tag{12.105}$$

This relationship between the reflection coefficient and the port impedances is the basis of the Smith Chart transmission line technique. Consequently, the reflection coefficients S_{11} and S_{22} can be plotted on a Smith Chart (see Section 15.5). They can be converted directly into impedances; they can also be easily manipulated to determine a matching network to transform the port impedance so that the circuit operates more efficiently.

The remaining two S-parameters S_{12} and S_{21} are not reflection coefficients; rather they behave more as transmission coefficients from one port to the other. For instance, S_{21} can be shown to be equal to

$$S_{21} = \frac{I_2}{I_1}\left(\frac{Z_2 - Z_0}{Z_1 + Z_0}\right) \tag{12.106}$$

and S_{12} as

$$S_{12} = \frac{I_1}{I_2}\left(\frac{Z_1 - Z_0}{Z_2 + Z_0}\right) \tag{12.107}$$

The magnitude of the S-parameters are expressed in one of two ways, linear magnitude or in decibels. To convert between the two, we simply use the following:

$$\begin{aligned} S_{11}\big|_{\mathrm{dB}} &= 20\log_{10}\left|S_{11}\right|, \quad S_{12}\big|_{\mathrm{dB}} = 20\log_{10}\left|S_{12}\right| \\ S_{21}\big|_{\mathrm{dB}} &= 20\log_{10}\left|S_{21}\right|, \quad S_{22}\big|_{\mathrm{dB}} = 20\log_{10}\left|S_{22}\right| \end{aligned} \tag{12.108}$$

12.4.2 Scalar Representation of *S*-Parameters

It is a common practice to use scalar representatives for some of the S-parameters. The return loss, mismatch loss, VSWR, and mismatch uncertainty are scalar measures of different power parameters, which can be used to describe some RF circuit behavior, especially when formulating a test environment with a signal source, DUT, and measurement path. These scalar parameters are calculated using the magnitude of the complex value of the reflection coefficients.

Voltage Standing Wave Ratio

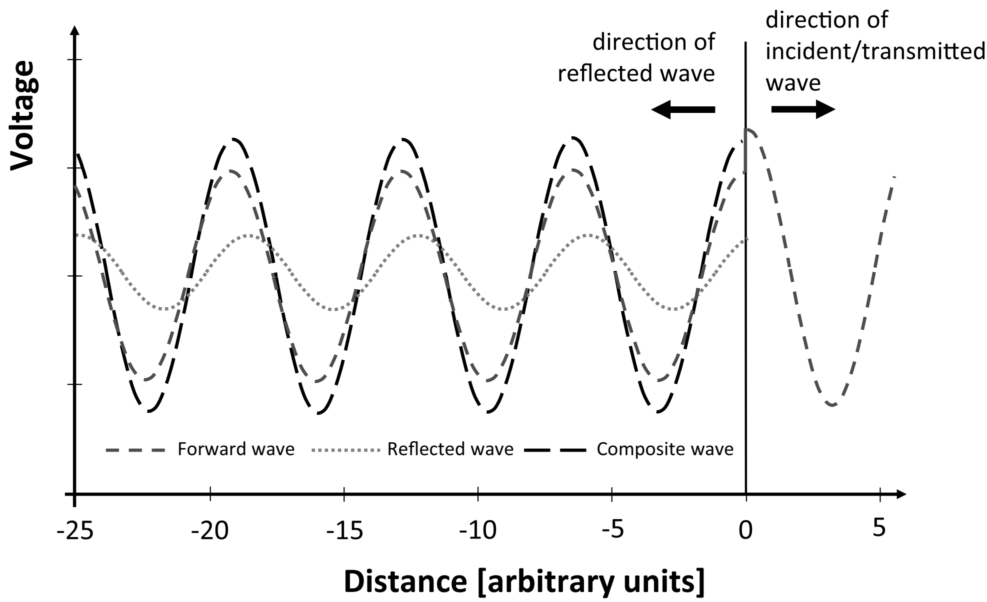
The concept of reflections can be best visualized by describing the reaction of an open-circuited lossless transmission line when a voltage pulse propagates along the transmission line and hits the open end. This is illustrated in Figure 12.21. The pulse will propagate as depicted in sketches 1 to 4, its voltage wave undiminished with the distance when the transmission line is assumed to be lossless (nondissipative and no radiative losses). When the pulse reaches the open end of the transmission line, all of the incident power must be reflected, since it can neither be radiated nor stored there. In practice some of the energy will radiate like an antenna. Since all the power needs to be reflected at the open end of the transmission line, the pulse will bounce back. For an open circuit, the current will need to be zero at the termination to satisfy this condition and the reflected current wave must be directed opposite to the incident current. This requires that the associated reflected voltage have the same polarity as the incident voltage, which results in a doubling of the voltage at the end of the line, as shown in Figure 12.21, sketches 5 to 7. The total voltage on the line is equal to the sum of the incident and reflected pulses. The current pulse at the end of the line is the sum of the incident and reflected pulse, resulting in a net zero. After the pulse is reflected at the end, it will propagate back in the direction of the source as shown in sketches 8 to 10, where the incident and reflected pulse are again separated in time. If the line impedance is different from the source impedance, the pulse will again reflect at the source end and this progress repeats itself until the energy of the pulse is all used up.

This pulse example is useful to demonstrate and visualize the effects of reflection. For one, it shows that the total voltage can vary on a transmission line due to reflections; on the other side it shows that a complex description is required to describe the effect of reflection. In the example above, we have seen that an open circuit, which requires having the condition of a zero current, causes the resulting pulse with twice the voltage amplitude to be reflected back. We can also consider the case of a short-circuited end as the other extreme. In this case, the voltage wave must be zero, and the current wave will be doubled, which will lead to an extinction of the voltage wave at the shorted end.

Just as pulses on a transmission line that is open-circuited produce a doubling of the incident voltage, so too do continuous sinusoidal waves (CW) propagating on a transmission line with

Figure 12.21. Pulse distribution on a loss-less transmission line and its reflections caused by the open circuit termination.

Figure 12.22. Sinusoidal CW incident, reflected and composite waves on a transmission line as a result of a mismatch in the line impedance at the far right hand side at distance 0.



similar termination conditions. This situation is illustrated in Figure 12.22, where the impedance mismatch appears of the far right-hand side where the distance is 0. The reflected voltage wave will interfere constructively with the incident wave on the transmission line, generating voltage maxima and minima along the length of the transmission line. To physically separate these two waves, a device referred to as a directional coupler is used.

In the early years of network analyzers for microwave frequencies, high-performance directional couplers were unavailable; hence slotted transmission lines were used for return loss measurements. A slotted transmission line is a carefully fabricated line element with a longitudinal slot cut in its outer conducting material and fitted with an exterior carriage. Within the carriage, a small wire probe is mounted and connected to a rectifying semiconductor crystal. The crystal's rectified voltage output is connected to a voltage measurement unit, like an oscilloscope. With this test, the minimum and maximum voltage across the line could be measured. Based on this measurement, the voltage standing wave ratio (VSWR) can be calculated; and with the assumption of a loss-free line, the maxima and minima voltages along the transmission line due to constructive interference vary according to

$$\mathrm{VSWR} = \frac{V_{\max}}{V_{\min}} = \frac{V_{incident} + V_{reflected}}{V_{incident} - V_{reflected}} \tag{12.109}$$

The VSWR is a scalar measure since V_{max} and V_{min} are themselves scalar quantities and VSWR is bounded between 1 and infinity.

Reflection Coefficient

For an expression of the fraction of the voltage wave reflected at the ith port of an n-port network, the complex representation of the reflection coefficient Γ_i can be introduced. It is defined as

$$\Gamma_i = \frac{\text{reflected voltage wave}}{\text{incident voltage wave}} = \frac{b_i}{a_i} = |\Gamma_i| e^{-j\Theta_i} \tag{12.110}$$

We saw a similar ratio in the context of the S-parameter definitions of Section 12.4.1, where the S_{ii} parameter was defined in terms of the ratio of the reflected-to-incident wave at the *i*th port. However, in the context of an *S*-parameter definition of an *n*-port, the incident wave on the other ports is assumed to be zero, that is, matched condition. In many test applications, a matched condition will be approached during test; if not exactly zero, it should be very close to zero. As a result, the reflection coefficient at the *i*th port can be used for S_{ii}. The reflection coefficient Γ_i can be expressed in polar coordinates with the phase angle Θ_i and the magnitude ρ_i as follows

$$\Gamma_i = \rho_i e^{-j\Theta_i} \tag{12.111}$$

This is a matter of convenience that is used extensively in the RF and microwave literature.

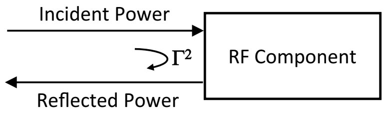
For the ith port, Γ_i is assumed to be equal to S-parameter S_{ii} (here we assume that the incident wave a_i on the other ports is zero). In the same manner as Eq. (12.103), we can then express the reflection coefficient in terms of the system impedances as

$$\Gamma_i = \frac{Z_i - Z_0}{Z_i + Z_0} \tag{12.112}$$

where Z_i is the impedance seen looking into the ith port and Z_0 is the impedance of the transmission line.

Often it is sufficient to represent an arbitrary reflection coefficient r with only its magnitude term ρ. The value of the reflection coefficient for a passive network is $0 \le \rho \le 1$. The value of $\rho = 1$ means that all power of the voltage wave at the port is reflected and the incident power is equal to the reflected power, while $\rho = 0$ means that no power is being reflected.

Figure 12.23. Definition of the refection coefficient in terms of the incident and reflective power.



When working exclusively with power quantities, the magnitude of the reflection coefficient can be calculated as the ratio of the reflected to incident power as

$$\rho^2 = \frac{\text{reflected power}, P_r}{\text{incident power}, P_i} \tag{12.113}$$

Figure 12.23 depicts the relationship between the incident and reflect power at the interface of some arbitrary RF component. Here it is important to recognize that power (average power to be exact) is a scalar quantity and has no phase quantity associated with it. An alternative form for the magnitude of the reflection coefficient can be written in terms of the VSWR quantity as

$$\rho = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} \tag{12.114}$$

On a lossless line, the respective magnitudes of the incident and reflected voltage waves do not change, thus the magnitude of the reflection coefficient $|\Gamma|$ does not change either, only the phase angle Θ changes as the two waves travel through each other in opposite directions. The phase angle Θ depends upon the relative phases of the incident and reflected voltage waves at the load, as well as the electrical distance from the load.

Return Loss

Return loss RL is defined as the ratio of the power incident to a discontinuity to that which is reflected. The return loss RL (often referred to as 'insertion return loss') for an RF port is equal to the reciprocal of the reflection coefficient according to

$$\text{RL} \triangleq \frac{\text{incident power}, P_i}{\text{reflected power}, P_r} = \frac{1}{\rho^2} \tag{12.115}$$

or expressed in decibels as

$$\text{RL}\big|_{\text{dB}} = 10\log_{10} RL = -20\log_{10}\rho \tag{12.116}$$

The return loss describes the difference between the incident wave and the reflected wave in decibels. The return loss itself is a scalar value and has no phase information. It is often used in production testing as a pass/fail criterion. A large RL value indicates little reflection; a small RL value means large levels of reflection. *Although RL is described as a loss quantity, its use is largely one to describe how well two devices match. The higher the value, the better the match.* Some literatures define return loss as the inverse of that which is given in Eq. (12.115). This typically results in a negative value for the RL in decibels. In this textbook, we make use of the positive value convention for return loss.

Passive circuits always have a return loss somewhere between 0 dB and infinity, that is, $0 \text{ dB} \leq RL \leq \infty$, while active circuits might also have a return gain, meaning that the reflected wave is greater than the incident wave, and as such, RL has a negative value.

Within the industry, VSWR and RL are used as alternative means for specifying match even though the slotted line is rarely used for measurements anymore. A perfect match of the load to the transmission line occurs when $Z_L = Z_0$. This condition results in all power being delivered to the load, and no reflected wave is produced. For this case, we get $\rho = 0$, VSWR = 1, and RL = ∞.

Mismatch Loss

Mismatch loss, ML, describes the amount of power loss at an impedance discontinuity due to reflections as shown in Figure 12.24. The path of the two-port is assumed lossless for the calculation of the mismatch loss. This condition can be expressed with *S*-parameters as $S_{21} = S_{12} = 1$. Physically, this describes a path without any dissipative loss or all power supplied into port one will exit the network at port two.

The mismatch loss *ML* is a computed parameter that describes the power attenuated (wasted) due to mismatch between two interconnects. It is a useful parameter that describes how much gain improvement could be achieved by an optimized matching.

For a two-port network, such as that shown in Figure 12.24 with a nonzero input reflection coefficient ($\Gamma \neq 0$), the normalized power delivered to the load as transmitted power is the difference between the available power from the source and the reflected power, that is, $1-|\Gamma|^2$. The reciprocal of this amount is the fractional increase of power delivered to the load without reflection and is known as the mismatch loss, ML. It is defined in terms of the reflection coefficient Γ as follows

$$\text{ML} \triangleq \frac{\text{incident power}, P_i}{\text{transmitted power}, P_t} = \frac{1}{1-\rho^2} \tag{12.117}$$

Figure 12.24. Illustrating the power flow associated with mismatch loss.

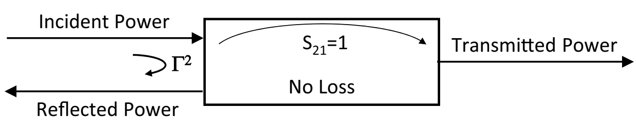
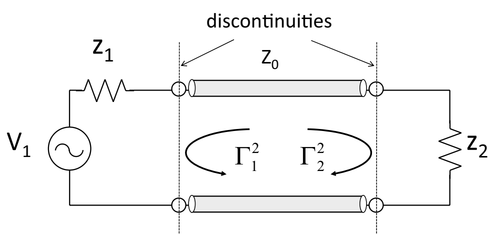


Figure 12.25. Illustrating the interfaces associated with power flow from source to load and their corresponding reflection coefficients Γ_1 and Γ_2.



or expressed in decibels as

$$\text{ML}\big|_{\text{dB}} = 10\log_{10} ML = -10\log_{10}\left(1-\rho^2\right) \tag{12.118}$$

For passive circuits, such as the lossless network used here, ML expressed in decibels is a positive number (i.e., transmitted power is less than or equal to the incident power). If we extend the definition of ML to active circuits, then it is possible for ML expressed in decibels to be a negative quantity, that is, the transmit power is larger than the incident power. In such cases, one must be careful with the sign.

In the case of two discontinuities associated with signal transmission (such as the two that arise at the interface between each end of a transmission line with characteristic impedance Z_0 as depicted in Figure 12.25), the total mismatch loss can be shown to be described by

$$\text{ML} = \frac{\left|1-\Gamma_1\cdot\Gamma_2\right|^2}{\left(1-\left|\Gamma_1\right|^2\right)\left(1-\left|\Gamma_2\right|^2\right)} \tag{12.119}$$

or, when expressed in decibels, as

$$\text{ML}\big|_{\text{dB}} = 10\log_{10}\text{ML} \tag{12.120}$$

Here the reflection coefficients are described as

$$\Gamma_1 = \frac{Z_1-Z_0}{Z_1+Z_0} \text{ and } \Gamma_2 = \frac{Z_2-Z_0}{Z_2+Z_0} \tag{12.121}$$

The mismatch loss predicted by either Eq. (12.119) or Eq. (12.120) describes the power that is bouncing back and forth between the two unmatched ports of the network. As seen from the numerator of Eq. (12.119), both the magnitude and phase of each reflection coefficients Γ_1 and Γ_2 is required for this calculation.

Return loss and mismatch loss are frequently used to describe an RF circuit but often interpreted incorrectly. The return loss RL represents the ratio of the reflected power to incident power. This is sometimes stated as the difference in decibels between the reflected and the incident power. In general, it is desired to have a small positive return loss (large decibels value), indicating that the match at the interface is good. In contrast, mismatch loss represents the ratio of the power transmitted across an interface to the incident power, or the difference in decibels between the transmitted power and the power that is incident. This difference in decibels should be small. The following example will help to illustrate these differences.

Mismatch Uncertainty

In many cases, only the magnitude of the reflection coefficient is known, which may be due to the limitations of the measurement equipment used, like a power meter or a scalar network analyzer, or by simply calculating the reflection coefficient based on given VSWR values. As such, the precise mismatch loss cannot be calculated, however, upper and lower bounds can be established. The ratio of the upper to the lower bound of the mismatch loss is called mismatch uncertainty, *MU*.

EXAMPLE 12.8

Several power measurements were made on a two-port network. The incident power was found to be 1 W, the reflected power was 0.1 W and the power transmitted by the two-port was 0.9 W. Additional measurements indicate the two-port was lossless. The measurements are summarized in Figure 12.26. Compute the reflection coefficient, return loss, and mismatch loss associated with this two-port network.

Solution:
According to Eq. (12.113), we find the magnitude of the reflection coefficient as

$$\rho = \sqrt{\frac{P_r}{P_i}} = \sqrt{\frac{0.1\ \mathrm{W}}{1.0\ \mathrm{W}}} = \sqrt{0.1} = 0.316$$

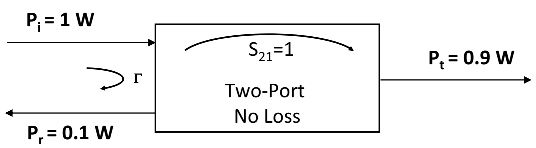
Next, we find the return loss, RL, from Eqs. (12.115) and (12.116),as follows

$$\mathrm{RL}\big|_{\mathrm{dB}} = -20\log_{10}\rho = -20\log_{10}(0.316) = 10\ \mathrm{dB}$$

Finally, we compute the mismatch loss ML for a single discontinuity using Eqs. (12.117) and (12.118), because the two-port network is assumed loss-less, resulting in

$$\mathrm{ML}\big|_{\mathrm{dB}} = -10\log_{10}\left(1-\rho^2\right) = 10\log_{10}\left(1-0.316^2\right) = 0.46\ \mathrm{dB}$$

Figure 12.26. Power measurements associated with a loss-less two-port network.



EXAMPLE 12.9

The reflection coefficients for a source and DUT were found to be 0.1778 and 0.3162, respectively. What is their corresponding VSWR?

Solution:
Rearranging Eq. (12.114), we can write

$$\mathrm{VSWR} = \frac{1+\rho}{1-\rho}$$

leading to

$$\text{VSWR}_{SMA} = \frac{1+0.1778}{1-0.1778} = 1.432 \quad \text{and} \quad \text{VSWR}_{DUT} = \frac{1+0.3162}{1-0.3162} = 1.924$$

Exercises

12.18.	The voltage wave incident at the interface of an RF component was measured with a network analyzer to be $0.9e^{j\pi/4}$ V, and the reflected wave was found to be $0.05e^{-j\pi/8}$ V. What is the reflection coefficient? What is the return and mismatch loss associated with this RF component?	ANS. $\Gamma = 0.0213 - j0.0513 = 0.0556e^{-j1.1781}$, RL = 25.1 dB, ML = 0.0134 dB.
12.19.	An RF connector has a VSWR of 2.0, what is the reflection coefficient and return loss associated with this connector.	ANS. $\rho = 1/3$, RL = 9.54 dB.

The mismatch uncertainty will give the range of the power uncertainty at the input of the DUT. This power can be higher or lower than the source power. The mismatch uncertainty is in most cases the most significant contributor to the total error related to an RF measurement.

Consider the following situation where the magnitude of the reflection coefficients for two interfaces was found to be ρ_1 and ρ_2. Since the phase of each reflection coefficients is unknown, we can only assume that each reflection coefficient is either positive or negative in value, that is, $\Gamma_1 = \rho_1$ or $\Gamma_1 = -\rho_1$. Similarly for the other reflection coefficient, $\Gamma_2 = \rho_2$ or $\Gamma_2 = -\rho_2$. Under such uncertainty, we can bound the mismatch loss by substituting each of these reflection coefficients into Eq. (12.119) and identify the range of possible ML value as

$$\frac{(1-\rho_1\cdot\rho_2)^2}{(1-\rho_1^2)\cdot(1-\rho_2^2)} \le \text{ML} \le \frac{(1+\rho_1\cdot\rho_2)^2}{(1-\rho_1^2)\cdot(1-\rho_2^2)} \tag{12.122}$$

Expressing the mismatch loss in decibels, we can write Eq. (12.122) as

$$10\log_{10}\left[\frac{(1-\rho_1\cdot\rho_2)^2}{(1-\rho_1^2)\cdot(1-\rho_2^2)}\right] \le \text{ML}\big|_{\text{dB}} \le 10\log_{10}\left[\frac{(1+\rho_1\cdot\rho_2)^2}{(1-\rho_1^2)\cdot(1-\rho_2^2)}\right] \tag{12.123}$$

The difference between the limits of this bound is called mismatch uncertainty MU and is expressed mathematically as

$$\text{MU}\big|_{\text{dB}} \triangleq 10\log_{10}\left[\frac{(1+\rho_1\cdot\rho_2)^2}{(1-\rho_1^2)\cdot(1-\rho_2^2)}\right] - 10\log_{10}\left[\frac{(1-\rho_1\cdot\rho_2)^2}{(1-\rho_1^2)\cdot(1-\rho_2^2)}\right] \tag{12.124}$$

Since the denominator is the same for each term on the right hand side, the above expression reduces to

$$\mathrm{MU}\big|_{\mathrm{dB}} = 20\log_{10}\left(1+\rho_1\cdot\rho_2\right) - 20\log_{10}\left(1-\rho_1\cdot\rho_2\right) \tag{12.125}$$

If VSWR measurements are made instead of reflection coefficients measurements, Eq. (12.124) can be rewritten using the relationship between reflection coefficient and VSWR provided through Eq. (12.110). For instance, representing the VSWR of a RF source by VSWR_{SRC} and that of the DUT by VSWR_{DUT}, the mismatch uncertainty can then be found from

$$\mathrm{MU}\big|_{\mathrm{dB}} = 20\log\left(1+\frac{\mathrm{VSWR}_{SRC}-1}{\mathrm{VSWR}_{SRC}+1}\cdot\frac{\mathrm{VSWR}_{DUT}-1}{\mathrm{VSWR}_{DUT}+1}\right) - 20\log\left(1-\frac{\mathrm{VSWR}_{SRC}-1}{\mathrm{VSWR}_{SRC}+1}\cdot\frac{\mathrm{VSWR}_{DUT}-1}{\mathrm{VSWR}_{DUT}+1}\right) \tag{12.126}$$

The following example commonly found in RF testing applications involving an SMA connector, RF transmission line and a DUT will help to illustrate the application of the above theory.

EXAMPLE 12.10

A typical RF test setup involving a DIB is shown in Figure 12.27. Here a source is to be connected to a DUT through an SMA connector and a 50-Ω RF line fabricated on a PCB. If the SMA connector and DUT have return losses of 15 dB and 10 dB, respectively, what is the expected level of mismatch uncertainty associated with this test setup? Assume that the RF transmission line is perfect and has no loss.

Solution:
According to Eq. (12.116), we find the magnitude of the reflection coefficients in terms of the return loss as

$$\rho = |\Gamma| = 10^{-\mathrm{RL[dB]}/20}$$

Correspondingly, the reflection coefficient for the SMA connector and DUT are found from above to be

$$\rho_{SMA} = 10^{-15/20} = 0.1778$$

and

$$\rho_{DUT} = 10^{-10/20} = 0.3162$$

According to Eq. (12.122), together with the knowledge of the magnitude of each reflection coefficient, we can bound the mismatch loss ML as

$$\frac{\left(1-\rho_{SMA}\cdot\rho_{DUT}\right)^2}{\left(1-\rho_{SMA}^2\right)\cdot\left(1-\rho_{DUT}^2\right)} \le \mathrm{ML} \le \frac{\left(1+\rho_{SMA}\cdot\rho_{DUT}\right)^2}{\left(1-\rho_{SMA}^2\right)\cdot\left(1-\rho_{DUT}^2\right)}$$

leading to

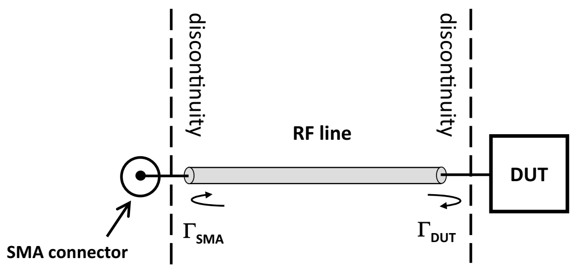
$$1.022 \le ML \le 1.280$$

or in dB as

$$0.094\ \text{dB} \le ML|_{dB} \le 1.07\ \text{dB}$$

This suggest that the mismatch loss can range from 0.094 dB to 1.07 dB; a 0.978-dB variation. This, of course, is the mismatch uncertainty $MU|_{dB} = 0.978\ \text{dB}$ (see Eq. (12.124)).

Figure 12.27. RF DUT connected with a RF transmission line to SMA connector.

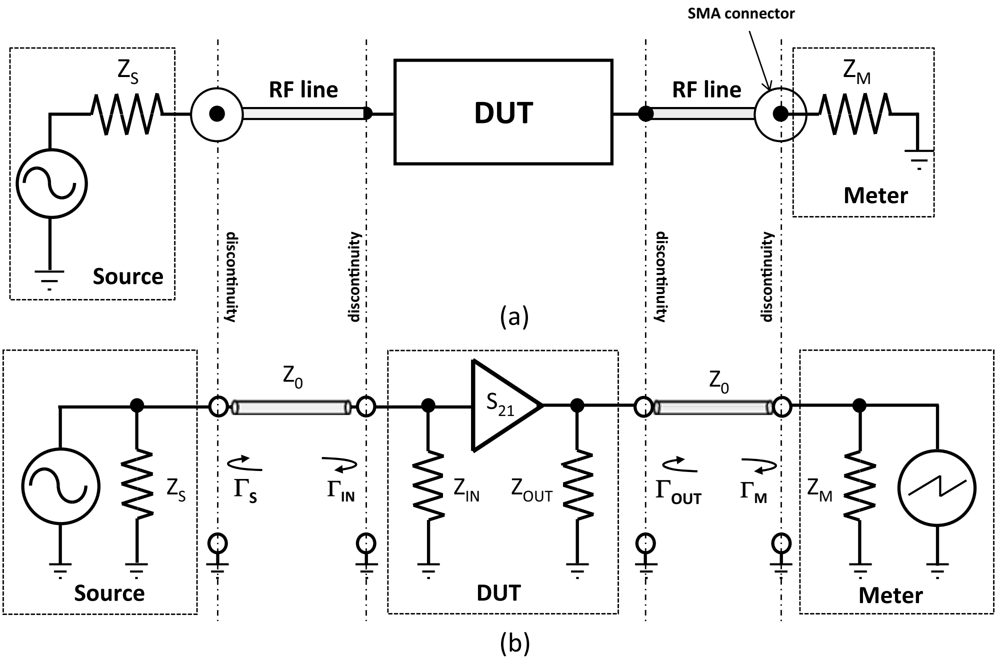


EXAMPLE 12.11

An RF source of an ATE system is coupled on a DIB to an amplifier (DUT) through an SMA connector with a return loss of 15 dB. The output of the amplifier is also coupled to a power meter in the ATE through another SMA connector with a return loss of RL_M = 15 dB. The DUT is known to have an input impedance of $Z_{IN} = 30\ \Omega$ and an output return loss RL_{OUT} of 10 dB. The reverse isolation of the amplifier S_{12} is assumed to be high, so this DUT can be treated as a unilateral device. All RF lines are assumed to have a characteristic impedance of 50 Ω and are loss-free. An illustration of the physical configuration is shown in Figure 12.28a, and its corresponding electrical schematic is given in Figure 12.28b. Compute the return loss, mismatch loss, and mismatch uncertainty of the amplifier gain test.

Solution:
To start, we have to convert the input impedance using Eq. (12.101) into the reflection coefficient ρ. For a unilateral system, the reflection coefficient is equal to the input reflection coefficient S_{11}. Using Eq. (12.101), we get

Figure 12.28. DUT connected to the source and measurement path of an ATE. (a) Physical representation of the test setup. (b) Equivalent electrical wave representation.



$$\rho_{IN} = |\Gamma_{IN}| = \left|\frac{Z_{IN} - Z_0}{Z_{IN} + Z_0}\right| = \left|\frac{30\ \Omega - 50\ \Omega}{30\ \Omega + 50\ \Omega}\right| = 0.250$$

and the return loss in decibels is

$$\mathrm{RL}_{IN}\big|_{\mathrm{dB}} = -20\log_{10}(\rho_{IN}) = -20\log_{10}(0.250) = 12.04\,\mathrm{dB}$$

The source on the test board has a return loss RL_S of 15 dB. This is equal to a reflection coefficient Γ_S of

$$\rho_S = |\Gamma_S| = 10^{-RL_S/20} = 10^{-15/20} = 0.178$$

These two reflections on the single RF line leading into the DUT causes a mismatch loss ML that can be bounded between the upper and lower mismatch loss as

$$\frac{(1-\rho_S\cdot\rho_{IN})^2}{(1-\rho_S^2)\cdot(1-\rho_{IN}^2)} \le \mathrm{ML}_{S-IN} \le \frac{(1+\rho_S\cdot\rho_{IN})^2}{(1-\rho_S^2)\cdot(1-\rho_{IN}^2)}$$

Substituting the appropriate values, we write

$$\frac{(1-0.250\cdot 0.178)^2}{\left(1-0.250^2\right)\cdot\left(1-0.178^2\right)} \le \mathrm{ML}_{S-IN} \le \frac{(1+0.250\cdot 0.178)^2}{\left(1-0.250^2\right)\cdot\left(1-0.178^2\right)}$$

which after some algebra reduces to

$$1.005 \le \mathrm{ML}_{S-IN} \le 1.202$$

We can express this mismatch loss in dB and write

$$0.025\ \mathrm{dB} \le \mathrm{ML}_{S-IN}\big|_{\mathrm{dB}} \le 0.798\ \mathrm{dB}$$

Here we see the mismatch loss varies between 0.025 dB to 0.798 dB. This is a 0.773 dB possible variation due to the standing wave on the front-end line on the DIB board. Hence, the mismatch uncertainty is

$$\mathrm{MU}_{S-IN}\big|_{\mathrm{dB}} = 0.773\ \mathrm{dB}$$

For the output side of the DUT similar calculations can be performed. First, we compute the reflection coefficient related to the DUT output, i.e.,

$$\left|\Gamma_{OUT}\right| = \rho_{OUT} = 10^{-RL_{OUT}/20} = 10^{-10/20} = 0.316$$

and the one related to the power meter is found according to

$$\left|\Gamma_{M}\right| = \rho_{M} = 10^{-RL_{M}/20} = 10^{-15/20} = 0.178$$

Using the magnitude of these two reflection coefficients, we can bound the mismatch loss ML and write

$$\frac{(1-\rho_{OUT}\cdot\rho_M)^2}{\left(1-\rho_{OUT}^2\right)\cdot\left(1-\rho_M^2\right)} \le \mathrm{ML}_{OUT-M} \le \frac{(1+\rho_{OUT}\cdot\rho_M)^2}{\left(1-\rho_{OUT}^2\right)\cdot\left(1-\rho_M^2\right)}$$

Substituting the appropriate values, wewrite

$$\frac{(1-0.316\cdot 0.178)^2}{\left(1-0.316^2\right)\cdot\left(1-0.178^2\right)} \le \mathrm{ML}_{OUT-M} \le \frac{(1+0.316\cdot 0.178)^2}{\left(1-0.316^2\right)\cdot\left(1-0.178^2\right)}$$

leading to

$$1.022 \le ML_{OUT-M} \le 1.280$$

We can express this mismatch loss in decibels and write

$$0.094\ \text{dB} \le ML_{OUT-M}\big|_{\text{dB}} \le 1.072\ \text{dB}$$

The uncertainty of the mismatch loss is then

$$MU_{OUT-M}\big|_{\text{dB}} = 1.072 - 0.094 = 0.978\ \text{dB}$$

The output side of the ATE will be measured with an uncertainty of 0.978 dB. When combining the results from the input and output side of the device, that is,

$$MU_{ALL}\big|_{\text{dB}} = MU_{S-IN}\big|_{\text{dB}} + MU_{OUT-M}\big|_{\text{dB}}$$

in decibels, we get a mismatch uncertainty of ~1.75 dB. Again, this level of uncertainty is the result of the lack of phase information associated with each reflection coefficient. In the next subsection, we shall consider a technique that can be used to reduce this uncertainty while working directly with scalar quantities for each reflection coefficient.

Exercises

12.20. The reflection coefficients between a source and DUT were found to be 0.21 and 0.25, respectively. What is the bound on the mismatch loss and the overall mismatch loss uncertainty?

ANS. $0.008\ \text{dB} \le ML \le 0.921\ \text{dB}$; $MU = 0.913\ \text{dB}$.

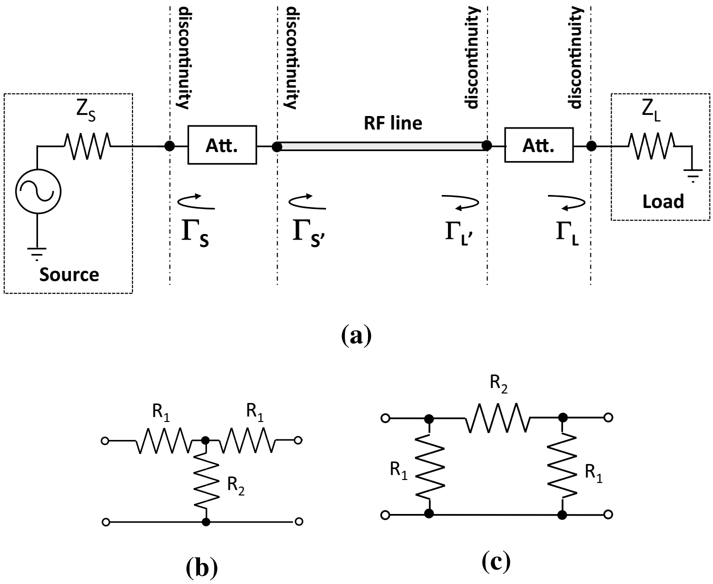
12.21. The mismatch uncertainty associated with the input path to a source-DUT connection is 1 dB. The uncertainty associated with the output path is 0.5 dB. What is the total uncertainty associated with this measurement?

ANS. MU = 1.5 dB.

Reducing Measurement Uncertainty

To make reliable RF measurements, it is often necessary to reduce the measurement uncertainty caused by multiple discontinuities. This is often done in two steps. In the first step, the impedances of the source and the DUT are transformed with impedance transformers (also known as matching networks) to the line impedance. Techniques for this matching are provided in Section 15.5 of Chapter 15. In the second step, the impact of any residual mismatch can be further reduced

Figure 12.29. (a) Attenuators inserted at source and load side of test setup. (b) Resistive attenuator in T configuration. (c) Resistive attenuator in Π configuration.



with the addition of an attenuator network or pad as depicted Figure 12.29a. In some cases the addition of this attenuator alone may be sufficient, especially when the impedance mismatch is not very large. Sometimes the attenuator network is combined with the matching network into a single network.

One simple realization of an attenuator is a T- or Π–shaped resistive network as shown in Figure 12.29b and 12.29c. Resistive attenuators typically have a wide bandwidth with a constant VSWR, as well as being easy to build using three resistors. The attenuator works on the principle of reducing the reflected wave back into the transmission line by twice its attenuation. To understand this, consider the following paragraph.

Let us assume that the reflection coefficient at an interface between the load and RF line is Γ_L. Recall that this reflection coefficients refers to the ratio of the reflected to incident power, that is, $P_L^- = \rho_L{}^2 P_L^+$. Now if an attenuator is inserted before the load interface, then the power incident at the interface is reduced, that is, $P_L^+ = A^2{}_{att} P_{att}^+$, where P_{att}^+ is the power incident on the input side of the attenuator and A_{att} is the attenuation factor. The reflected wave at the load interface can then be described as $P_L^- = \rho_L{}^2 \cdot P_L^+ = \rho_L{}^2 \cdot A^2{}_{att} \cdot P_{att}^+$. As this reflected wave passes back through the attenuator and gets attenuated by the same factor A_{att}, the reflected wave at the attenuator input is $P_{att}^- = A^4{}_{att} \cdot \rho_L{}^2 \cdot P_{att}^+$. We can then define the reflection coefficient at the input of the attenuator as

$$\rho_{L'}^2 = \frac{P_{att}^-}{P_{att}^+} = A_{att}^4 \cdot \rho_L^2 \qquad \textbf{(12.127)}$$

Because A_{att} is less than one (i.e., gain loss), we see that the reflection coefficient at the input side of the attenuator has been reduced in magnitude by this factor A_{att}.

Following a similar argument for the attenuator placed right after the source interface, we can express the modified reflection coefficient at the source side as

$$\rho_{S'}^2 = \frac{P_{att}^-}{P_{att}^+} = A_{att}^4 \cdot \rho_S^2 \tag{12.128}$$

While the reflection coefficient is improved with the addition of an attenuator, it comes at a reduction in the dynamic range of the measurement.

If we consider two attenuators are inserted into the signal path, such because that shown in Figure 12.29, then using the results of Eqs. (12.127) and (12.128), together with the expression for mismatch uncertainty given in Eq. (12.125), we write

$$\mathrm{MU}\big|_{\mathrm{dB}} = 20\log_{10}\left(1 + A_{att}^4 \cdot \rho_S \cdot \rho_L\right) - 20\log_{10}\left(1 - A_{att}^4 \cdot \rho_S \cdot \rho_L\right) \tag{12.129}$$

Adding attenuators in the signal path is common practice in RF test engineering because it makes the test results much more reliable and predictable. The drawback of this additional attenuation is that the signal strength will be reduced by the attenuation of the pad, which can lead to a reduced signal-to-noise ratio.

EXAMPLE 12.12

The reflection coefficients for a source and DUT were found to be 0.1778 and 0.3162, respectively. If a 3-dB attenuator is placed at the source and DUT side in the signal path, what are the new reflection coefficients? What is the corresponding return loss with the attenuators inserted in the line and how much improvement was realized over the original arrangement? What impact does the attenuators have on the mismatch uncertainty of the measurement?

Solution:
A 3-dB attenuator has an linear magnitude gain given by

$$A_{att} = 10^{-3/20} = 0.707$$

Using Eqs. (12.127) and (12.128), we find the revised reflection coefficients as

$$\rho_{S'} = A^2{}_{att}\rho_S = 0.707^2 \times 0.1778 = 0.0891$$
$$\rho_{DUT'} = A^2{}_{att}\rho_{DUT} = 0.707^2 \times 0.3162 = 0.1585$$

Using Eq. (12.116), we find the return loss at the source side with and without the attenuators in the signal path as follows:

$$RL_S\big|_{\mathrm{dB}} = -20\log_{10}\rho_S = -20\log_{10}(0.1778) = 15.0 \text{ dB}$$
$$RL_{S'}\big|_{\mathrm{dB}} = -20\log_{10}\rho_{S'} = -20\log_{10}(0.0891) = 21.0 \text{ dB}$$

Similarly at the load side, we find

$$\begin{aligned} RL_{DUT}\big|_{\text{dB}} &= -20\log_{10}\rho_{DUT} = -20\log_{10}(0.3162) = 10.0\text{ dB} \\ RL_{DUT'}\big|_{\text{dB}} &= -20\log_{10}\rho_{DUT'} = -20\log_{10}(0.1585) = 16.0\text{ dB} \end{aligned}$$

Here we see that the return loss has improved by twice the attenuator gain factor in decibels; that is, a 3-dB attenuator provides 6 dB improvement in return loss.

According to Eq. (12.125), we find the mismatch uncertainty without the attenuators present in the line as

$$\begin{aligned} \text{MU}\big|_{\text{dB}} &= 20\log_{10}(1+\rho_S \cdot \rho_{DUT}) - 20\log_{10}(1-\rho_S \cdot \rho_{DUT}) \\ &= 20\log_{10}(1+0.1778\cdot 0.3162) - 20\log_{10}(1-0.1778\cdot 0.3162) \\ &= 0.978\text{ dB} \end{aligned}$$

Using the attenuators, we find the mismatch uncertainty as

$$\begin{aligned} \text{MU}\big|_{\text{dB}} &= 20\log_{10}(1+\rho_{S'} \cdot \rho_{DUT'}) - 20\log_{10}(1-\rho_{S'} \cdot \rho_{DUT'}) \\ &= 20\log_{10}(1+0.0891\times 0.1585) - 20\log_{10}(1-0.0891\times 0.1585) \\ &= 0.245\text{ dB} \end{aligned}$$

The mismatch uncertainty has been reduced to 0.245 dB from 0.978 dB.

Exercises

12.22. The reflection coefficient for at a load is 0.3. How much attenuation is required to reduce the reflection coefficient to 0.1?

ANS. $A_{att} = 0.577, -4.8$ dB.

12.23. A 9-dB resistive pad is inserted in series with a source with a reflection coefficient of 0.4. What is the effective reflection coefficient of the source?

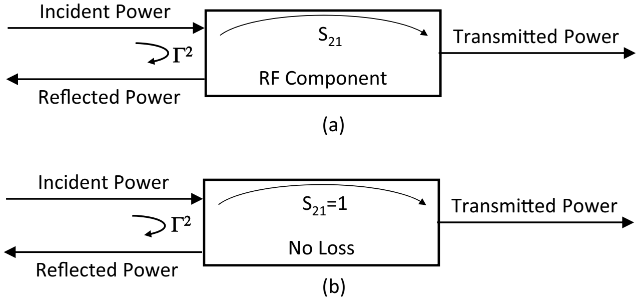
ANS. $\rho = 0.05$.

Insertion Loss

The reader first encountered insertion loss IL back in Section 12.2.7. There insertion losswas defined as the power loss associated with a two-port network inserted between the source and load. An alternative approach is one that involves the *S*-parameters of the two-port network. Assuming that the two-port network has been fully characterized, the insertion loss IL can be described as

$$IL\big|_{\text{dB}} = 10\log_{10}\left(\frac{|S_{21}|^2}{1-|S_{11}|^2}\right) \tag{12.130}$$

Figure 12.30. Highlighting the two-port transmission coefficient S_{21} for(a) insertion loss test setup, (b) mismatch loss test setup.



Here we adopt insertion loss as a loss instead of a gain (i.e., ratio of incident power to transmit power). We learned previously that $S_{11} = \Gamma$, so we can rewrite Eq. (12.130) as

$$IL\big|_{\text{dB}} = 10\log_{10}\left(\frac{|S_{21}|^2}{1-|\Gamma|^2}\right) \tag{12.131}$$

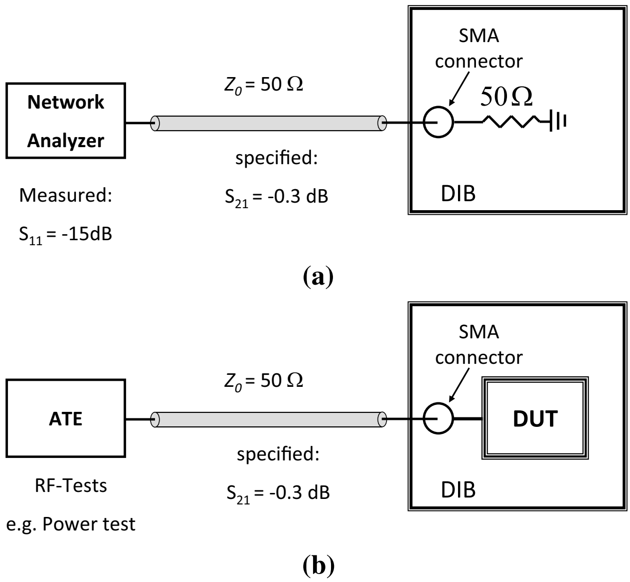
The reader may notice that the expression given here for insertion loss is very similar to that given in Eq. (12.117) for mismatch loss. The only difference is that insertion loss includes the term S_{21}. If one recalls the development of mismatch loss, the condition $S_{21} = 1$ was imposed on its definition.

While insertion loss and mismatch loss are very similar, they differ in terms of the transmission nature of the two-port network. To see this, compare the insertion loss expression given in Eq. (12.131) to the mismatch loss definition given in Eq. (12.117). On doing so, we see they are identical if and only if S_{21} is equal to unity. This leads us to conclude that insertion loss is a more general definition than mismatch loss; it is not restricted to the two-port condition of $S_{21} = 1$. To contrast these two loss definitions, Figure 12.30 summarizes their power flow and two-port conditions.

EXAMPLE 12.13

For a production test setup, an RF-DIB is connected to the ATE with an SMA RF cable. To characterize the losses, a network analyzer is first connected to the RF cable and the RF DIB as shown in Figure 12.31a. The RF cable is specified to have a forward transmission coefficient S_{21} equal to −0.3 dB. The DUT is replaced with a known 50-Ω load. The network analyzer measures S_{11} to be −15 dB. In production the network analyzer is replaced as shown in Figure 12.31b. What is the reflection coefficient Γ, VSWR, return loss, and the insertion loss of the RF cable-DIB combination? Also, what is the impedance seen by the network analyzer or ATE looking into the cable?

Figure 12.31. (a) A bench test setup for characterizing the interconnect associated with the RF line and DIB board. (b) Typical ATE production test setup for an RF DUT.



Solution:
The first step in solving this problem is to convert the two given *S*-parameters in decibels into voltage magnitude terms:

$$|S_{11}| = 10^{-15/20} = 0.1778 \text{ V}$$
$$|S_{21}| = 10^{-0.3/20} = 0.9661 \text{ V}$$

The magnitude of the reflection coefficient ρ associated with the RF line-DIB combination can be derived directly from the S_{11} measurement according to Eq. (12.96), leading to

$$\rho = |S_{11}| = 0.1778$$

Next, the VSWR is found from Eq. (12.113),

$$\text{VSWR} = \frac{1+0.1778}{1-0.1778} = 1.433$$

Similarly, the return loss is found from Eq. (12.116) to be

$$\text{RL} = -20\log_{10}(0.1778) = 15 \text{ dB}$$

The insertion loss of the DIB with the RF cable is then found from Eq. (12.131) with $\rho = |\Gamma|$ to be

$$\text{IL} = 10\log_{10}\left(\frac{|S_{21}|^2}{1-\rho^2}\right) = 10\log_{10}\left[\frac{0.9661^2}{1-0.1778^2}\right] = -0.161 \text{ dB}$$

Finally, the impedance seen by the network analyzer is found using Eq. (12.104) to be

$$Z_{IN} = Z_0 \frac{1+S_{11}}{1-S_{11}} = (50\ \Omega)\left(\frac{1+0.177e^{j0.125}}{1-0.177e^{j0.125}}\right) = 71.2 + j3.24\ \Omega$$

These numbers are showing a minimal impact on the test results while having a return loss of 15 dB. In Chapter 13 we will learn about a method in which to calibrate for this loss.

Exercises

12.24. An amplifier was characterized with the following S-parameters:

$$S_{11} = -0.5638 - j0.2052,\quad S_{12} = 0.0433 + j0.0124$$
$$S_{21} = 2.165 + j1.250,\qquad S_{22} = -j0.5$$

What is the insertion loss of this amplifier? What are the magnitudes of the reflection coefficients of the input and output ports of this amplifier? What is the input and output impedance of the amplifier, assuming that the *S*-parameters were obtained with ideal transmission lines with characteristic impedance of 50 Ω?

ANS.
IL = 9.9 dB, $\rho_{IN} = 0.6$, $\rho_{OUT} = 0.5$,
$Z_{IN} = 12.8 - j8.25\ \Omega$,
$Z_{OUT} = 30 - j40\ \Omega$

12.5 MODULATION

Modulation in mixed-signal or RF technology represents the process of varying a periodic waveform to transport a deterministic message. Typically, a sinusoidal waveform is used as a carrier signal for the modulation. The key parameters of a sinusoidal wave that can be modulated with a lower-frequency signal are the amplitude, the phase, and the frequency of the carrier signal. In the following sections, we will give a brief description of the fundamental modulations schemes.

12.5.1 Analog Modulation

Analog modulation is the transfer of an analog low-frequency signal—for example an audio signal for television over a high-frequency sinusoidal signal. In general, we can modulate the amplitude, frequency, and phase continuously, which results in amplitude modulation, frequency, modulation, and phase modulation, respectively. Below we will discuss the basics of these three analog modulation schemes.

Amplitude Modulation

Amplitude modulation (AM) had been the first technique used in telecommunication to transmit an audio message over a telephone line in the mid-1870s. Beginning with the Canadian inventor Reginald A. Fessenden in 1906, it was also the original method used for audio radio transmission and is still one of the most commonly used modulation schemes in modern telecommunication.

The general amplitude modulation formula is given by

$$s(t)=\lfloor 1+m\cdot s_m(t)\rfloor\cdot A_c\sin(2\pi f_c t+\phi_c) \tag{12.132}$$

where A_c, f_c, and ϕ_c are the parameters of the sinusoidal carrier wave, m is the modulation index, and $s_m(t)$ is the modulation signal. For a sinusoidal modulation signal,

$$s_m(t)=A_m\sin(2\pi f_m t+\phi_m) \tag{12.133}$$

with $\phi_c=\phi_m=0$, Eq. (12.132) reduces to

$$\begin{aligned} s(t) &= \left[1+m\cdot A_m\sin(2\pi f_m t)\right]\cdot A_c\sin(2\pi f_c t) \\ &= A_c\sin(2\pi f_c t)+m\cdot A_m\sin(2\pi f_m t)\cdot A_c\sin(2\pi f_c t) \end{aligned} \tag{12.134}$$

If we assign the leftmost term on the right-hand side as the carrier signal, i.e., $s_c(t)=A_c\sin(2\pi f_c t)$ and the rightmost term as the product of the carrier signal and the modulation signal, denoted by, $s_{mc}(t)=m\cdot A_m\sin(2\pi f_m t)\cdot A_c\sin(2\pi f_c t)$ then we can rewrite Eq. (12.134) as $s(t)=s_c(t)+s_{mc}(t)$. Using the trigonometric identity,

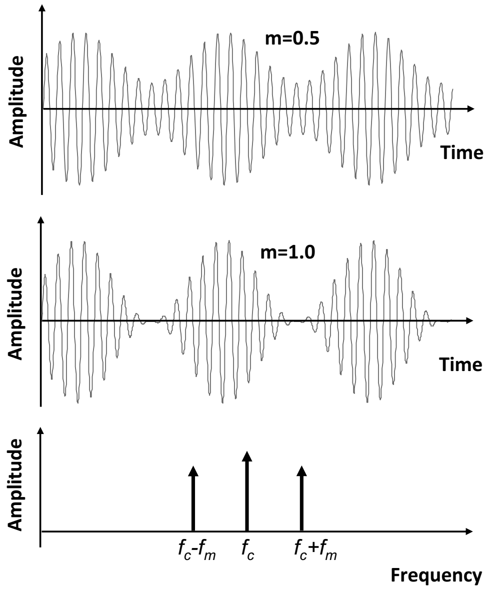
$$\sin\theta\sin\psi=\frac{1}{2}\cos(\theta-\psi)-\frac{1}{2}\cos(\theta+\psi) \tag{12.135}$$

the signal $S_{mc}(t)$ can be rewritten as

$$s_{mc}(t)=\frac{m\cdot A_m\cdot A_c}{2}\cos\left[2\pi(f_m-f_c)t\right]-\frac{m\cdot A_m\cdot A_c}{2}\cos\left[2\pi(f_m+f_c)t\right] \tag{12.136}$$

Equations (12.132) and (12.136) describe a general representation of an amplitude-modulated signal for the case in which the modulation signal is a sinusoidal signal. It consists of the term $s_c(t)$, which is independent of the modulated signal, and a signal on both sides of the carrier representing the modulated signal at the sum and difference frequencies. Figure 12.32 illustrates the time-domain representation of an amplitude modulated signal for two cases, m = 0.5 and m = 1.0. Here A_m is set equal to one. Also shown in this figure is the frequency domain representation of this signal consisting of three impulses in frequency; one at the carrier frequency f_c, and two sidebands at $f_c\pm f_m$. On account of the two sidebands, this type of modulation is called double-sideband amplitude modulation (DSB-AM). However, it should be noted that each sideband carries the same information; hence DSB-AM is inefficient in terms of power usage. At least two-thirds of the total RF power is used for the carrier and the second sideband, which carries no additional information beyond the fact that a signal is present. To optimize the power efficiency, the carrier will often be suppressed. This amplitude modulation is called a *double-sideband suppressed carrier* (DSBSC), but for demodulation, the carrier needs to be regenerated in the demodulator circuit for conventional demodulator techniques.

Figure 12.32. Time and frequency domains of an amplitude modulated sinusoidal wave.



Frequency Modulation

Frequency modulation (FM) is well known as the broadcast signal format for FM radio. In contrast to AM, the modulation is achieved through the variance of the frequency of the carrier signal. This makes the modulation scheme more robust against amplitude noise.

Frequency modulation involves altering the frequency of the carrier according to some modulating signal. One might ask what is the frequency of a waveform. For a simple wave like a sine wave, the answer appears quite obvious, we can define the wave using an expression like

$$s(t) = A_c \cos\left[2\pi f_c t + \phi\right] \tag{12.137}$$

and identify the f_c as the wave's frequency. An alternative way is to represent such a wave as

$$s(t) = A_c \cos\left\lfloor \theta_i(t) \right\rfloor \tag{12.138}$$

where

$$\theta_i(t) = 2\pi f_c t + \phi \tag{12.139}$$

This term represents the instantaneous phase of the wave at time instant, *t*. For a simple sine wave, its frequency is constant and $\theta_i(t)$ increases steadily with time at the rate

$$\frac{d\theta_i(t)}{dt} = 2\pi f_c \tag{12.140}$$

We can define the FM wave, as one that results from the modulation of a carrier $s_c(t) = A_c \sin(2\pi f_c \cdot t)$ with signal $s_m(t)$ to be

$$s(t) = A_c \cos\left\lfloor 2\pi f_i(t) \cdot t + \phi \right\rfloor \quad \textbf{(12.141)}$$

where

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + k_f s_m(t) \quad \textbf{(12.142)}$$

The term $f_i(t)$ represents the instantaneous frequency of the wave at the time instant, t. It is equal to the rate of change of the instantaneous phase $\theta_i(t)$ normalized by 2π. The term k_f is a constant whose value depends on the modulating system. It typically has units of hertz per volts.

The FM wave can now be used to convey information about the modulating signal in a manner very similar to that developed for AM modulation. Consider integrating Eq. (12.142) to get the instantaneous phase in terms of the modulating signal $S_m(t)$ as

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int s_m(t)\, dt \quad \textbf{(12.143)}$$

We can define the FM signal in terms of an arbitrary modulating signal by substituting Eq. (12.143) into Eq. (12.138) to obtain

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int s_m(t) dt\right] \quad \textbf{(12.144)}$$

To further our understanding of FM, consider the modulation signal as a simple sinusoidal signal as $s_m(t) = A_m \sin(2\pi f_m t)$. Substituting this signal into Eq. (12.142), we observe that the instantaneous FM signal frequency has the general form

$$f_i(t) f_c + k_f A_m \sin(2\pi f_m t) \quad \textbf{(12.145)}$$

This expression indicates that the instantaneous frequency swings up and down around the carrier frequency f_c with amplitude $k_f A_m$. The range of this frequency change is commonly referred to as the peak frequency deviation value, denoted as

$$\Delta f = k_f A_m \quad \textbf{(12.146)}$$

Combining the modulation signal $s_m(t) = A_m \sin(2\pi f_m t)$ together with Eq. (12.146) and Eq. (12.143), we can write the instantaneous phase as

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \quad \textbf{(12.147)}$$

It is conventional to define a quantity called modulation index,

$$\beta = \frac{\Delta f}{f_m} \tag{12.148}$$

where we can write the FM wave in the form of

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin\left(2\pi f_m t\right)\right] \tag{12.149}$$

This expression provides us with information about how the modulated signal varies with time. However, we often need to know the spectrum of the FM signal, as, for example, to determine the bandwidth of any filters, amplifiers, and so on, that the system requires.

Consider rewriting Eq. (12.149) using complex notation as follows

$$s(t) = A_c \Re e\left\{ e^{j\left[\omega_c t + \beta \sin(\omega_m t)\right]} \right\} \tag{12.150}$$

which, after several algebraic steps, leads to

$$s(t) = A_c \Re e\left\{ e^{j\left[w_c t + \beta \sin(\omega_m t)\right]} \right\} = A_c \Re e\left\{ e^{j\omega_c t} e^{j\beta \sin(\omega_m t)} \right\} \tag{12.151}$$

Using Bessel functional notation, we can write the above expression as

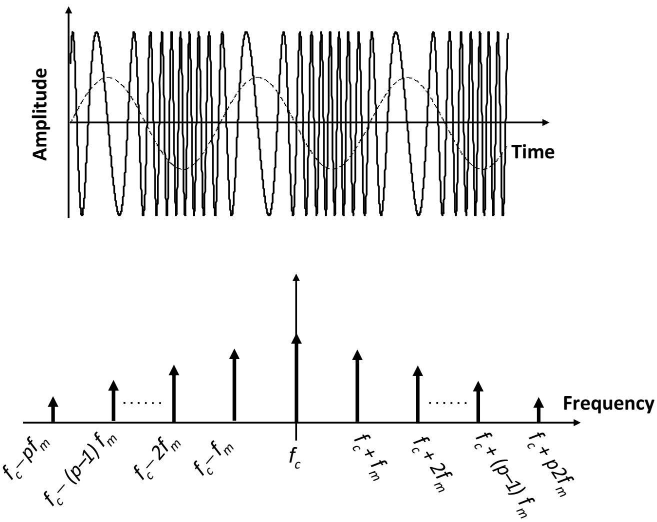
$$s(t) = A_c \Re e\left\{ e^{j\omega_c t} \sum_{k=-\infty}^{\infty} J_k(\beta) e^{j\beta k \omega_m t} \right\} = A_c \Re e\left\{ \sum_{k=-\infty}^{\infty} J_k(\beta) e^{j(\omega_c + k\omega_m)t} \right\} \tag{12.152}$$

where $J_k(\beta)$ is known as the Bessel function of the first kind of order k and argument β. The function $J_k(\beta)$ is a real number for real values of β (and any integer value for k). The reason for the introduction of the Bessel function representation is that Eq. (12.152) can be rewritten as a series of sinusoids (in much the same we do for a Fourier series) as follows

$$s(t) = A_c \sum_{k=-\infty}^{\infty} J_k(\beta) \cos\left[(\omega_c + k\omega_m)t\right] \tag{12.153}$$

Here we see an FM signal consists of an infinite number of sinusoidal components all positioned with respect to the carrier frequency ω_c provide $\beta \neq 0$. The side bands occur at multiples of the modulating frequency ω_m relative to the carrier frequency ω_c. As the series extends from minus infinity to plus infinity, FM signals have infinite bandwidth; albeit, the magnitude of these sidebands decrease with increasing index, k. This is a property of Bessel functions. For real systems, this requires limiting the bandwidth of the FM signal with a filter so that the signal stays within the channel bandwidth. An example of an FM signal is shown in Figure 12.33. The top plot illustrates the time domain FM signal for a sinusoidal modulating signal. The lower plot illustrates the multiple sidebands about the carrier signal.

Figure 12.33. Frequency modulated sinusoidal wave in the frequency and time domains.



An FM signal created with a low modulation index (i.e., where $\Delta f \ll f_m$) is called a narrowband FM signal. Such an FM signal can be approximated as

$$s_{NB}(t) = A_c J_0(\beta)\cos(2\pi f_c t) + A_c J_1(\beta)\cos\left[2\pi(f_m - f_c)t\right] - A_c J_1(\beta)\cos\left[2\pi(f_m + f_c)t\right]$$

(12.154)

This narrowband FM signal is very similar to AM in that it has sideband components at $(f_c \pm f_m)$; hence narrowband FM signals only need a bandwidth of $2f_m$. It should be noted that the spectrum of a narrowband FM signal differs from AM in two ways. Firstly, the amplitude of an FM signal remains constant and secondly the phase of the sideband components are 180 degrees out of phase with respect to one another.

In the case of a high modulation index, an FM signal can be viewed as a sinusoid whose frequency is varied over a relatively large range. This implies that the bandwidth required for transmission is at least equal to the peak-to-peak frequency deviation or $2\Delta f$. A more refined analysis reveals the minimum bandwidth *B* required for FM signals is

$$B = 2(f_m + \Delta f) \qquad \textbf{(12.155)}$$

This is known as Carson's rule for FM signals.

Phase Modulation

Phase modulation (PM) is very similar to frequency modulation, because any phase deviation can be interpreted as an instantaneous frequency variation. PM and FM are sometimes referred to as angular modulation. Consider a PM signal having the general form

$$s(t) = A_c \cos\left[\theta_i(t)\right] \qquad \textbf{(12.156)}$$

where

$$\theta_i(t) = 2\pi f_c t + k_p s_m(t) \tag{12.157}$$

Here the term k_p is a constant whose value depends on the system. Combining Eqs. (12.156) and (12.157), we observe that the general form of a PM-modulated signal is

$$s(t) = A_c \cos\left[2\pi f_c t + k_p s_m(t)\right] \tag{12.158}$$

In the case of a sinusoidal modulating signal, that is, $s_m(t) = A_m \sin(2\pi f_m t)$, we can write the PM signal as

$$s(t) = A_c \cos\left[2\pi f_c t + k_p A_m \sin(2\pi f_m t)\right] \tag{12.159}$$

Although the above expression appears quite similar to the one derived for FM, it is important recognize the difference. To see this, consider that the time derivative of the instantaneous phase given in Eq. (12.157) is equal to the instantaneous angular frequency, such that we can write

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} \\ &= f_c + \frac{k_p A_m}{2\pi}\frac{d\sin(2\pi f_m t)}{dt} \\ &= f_c + f_m k_p A_m \cos(2\pi f_m t) \end{aligned} \tag{12.160}$$

In comparison to the FM signal modulated by a similar signal, here we see the peak frequency deviation value is

$$\Delta f = f_m k_p A_m \tag{12.161}$$

which is dependent on both the amplitude and frequency of the modulating signal. This is quite different for the FM signal where we found $\Delta f = k_f A_m$ (see Eq. (12.146)). In terms of the modulation index, we find

$$\beta = \frac{\Delta f}{f_m} = k_p A_m \tag{12.162}$$

Substituting this result into Eq. (12.159), we write the PM signal subject to a sinusoidal modulating signal as

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin(2\pi f_m t)\right] \tag{12.163}$$

We immediately recognize that this form of the PM signal is identical to that for the FM signal (i.e., see Eq. (12.149)). Hence, the frequency domain conclusions drawn previously for an FM signal applies equally to the PM signal (albeit under a sinusoidal modulation condition).

Even though PM is very similar to FM for a sinusoidal modulating signal, there is one important difference. In FM, the maximum frequency deviation Δf is proportional to the amplitude of the modulating signal $s_m(t)$ (see Eq. (12.146)). In PM, the maximum frequency deviation Δf depends on the amplitude A_m of the modulating signal $s_m(t)$ as well as its frequency f_m (Eq. (12.161)). Therefore, modulating signals with the same amplitude but different frequencies gives different values of maximum frequency deviation. The next example will illustrate this effect. In practice, PM is rarely used for analog modulation on account of this dependence.

EXAMPLE 12.14

A 100-mV, 1-kHz sinusoidal signal is used to modulate an FM and PM system with coefficients $k_f = 1$ kHz/V and $k_p = 1$ krad/V, respectively. The carrier frequency of both systems is 900 MHz. Compare peak-to-peak frequency deviation of these two systems. Repeat for a 100-mV, 10-kHz modulating signal.

Solution:

Using Eq. (12.146), at 1-kHz modulating frequency, we calculate the peak-to-peak-frequency deviation for the FM system as

$$\Delta f_{pp} = 2k_f A_m = 2\times 1000\ \frac{\text{Hz}}{\text{V}}\times 0.1\,\text{V} = 200\ \text{Hz}$$

Correspondingly, using Eq. (12.161), the peak-to-peak deviation of the PM system is

$$\Delta f_{pp} = 2f_m k_p A_m = 2\times 1000\ \text{Hz}\times 1000\ \frac{\text{rad}}{\text{V}}\times 0.1\,\text{V} = 200\ \text{kHz}\cdot\text{rad} = 200\ \text{kHz}$$

Repeating for a 10 kHz modulating signal, we find for the FM system remains the same as before, Δf_{pp}=200 Hz , but the PM system changes according to

$$\Delta f_{pp} = 2f_m k_p A_m = 2\times 1000\ \text{Hz}\times 1000\ \frac{\text{rad}}{\text{V}}\times 0.1\,\text{V} = 2\ \text{MHz}$$

This example illustrates the subtle difference between FM and PM systems.

12.5.2 Digital Modulation

In contrast to the analog modulation, a digital signal with only a discrete number of valid states can be transmitted with a digital modulation scheme. Since the sinusoidal carrier signal is not modulated with an analog signal, but is shifted between discrete values, we refer to it as "shift keying." Like analog modulation, digital modulation schemes also vary in amplitude, phase, and frequency.

Figure 12.34. OOK (ASK) modulated signal.

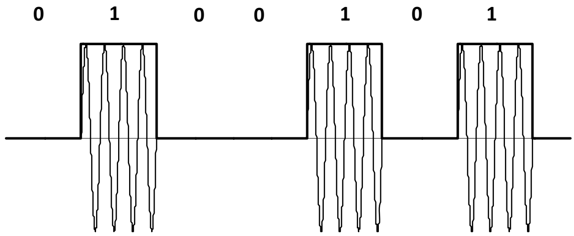
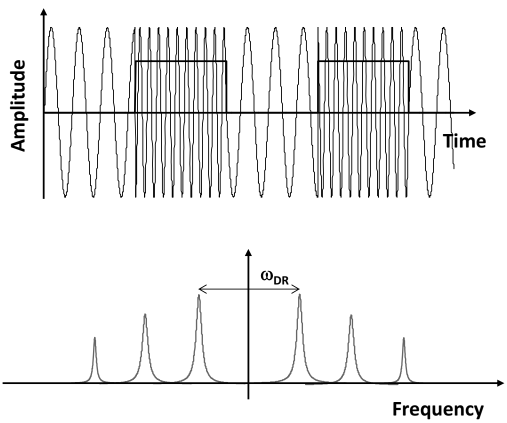


Figure 12.35. FSK modulation in the time and frequency domains.



Amplitude Shift Keying

Amplitude shift keying (ASK) represents a simple form of digital modulation. The amplitude of the carrier $s_c(t) = A_c \cos(2\pi f_c t)$ varies in accordance with the bit stream, which keeps the frequency and phase constant. The level of the amplitude can be used to represent the binary values of 0s and 1s, by assigning different amplitude to these binary states. A special form of ASK is on–off keying (OOK) modulation where a zero amplitude sinewave is assigned to one of the binary states. In Figure 12.34 we illustrate OOK with logic 0 being assigned the zero amplitude sinewave.

Frequency Shift Keying

Frequency shift keying (FSK) is a modulation scheme similar to frequency modulation (FM). Instead of an analog modulating signal, a digital bit pattern is used to modulate a carrier wave. The simplest FSK is the binary FSK (often refer to as BFSK). BFSK transmits binary 0s and 1s information via the allocation of a single frequency per bit. This is captured in the time-frequency diagram shown in Figure 12.35. As the bits change, the frequency of the signal is shifts from one value to the opposite one.

The minimum frequency shift keying, or minimum shift keying (MSK), is a FSK modulation where the difference between the higher and the lower frequency is identical to half of the data (bit) rate. A variant of the MSK is the Gaussian MSK, or GMSK, which is used for cellular handsets using the global system for mobile communications (GSM) standard.

Figure 12.36. Phase shift modulated bit stream. The binary "1" signal has a phase offset of π radians with respect to the binary "0" logic value.

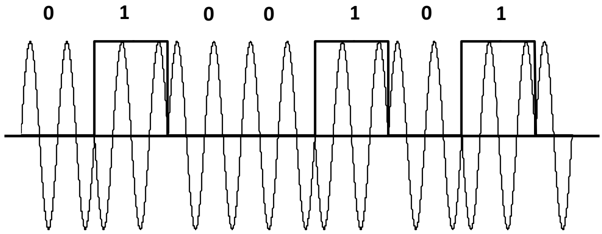
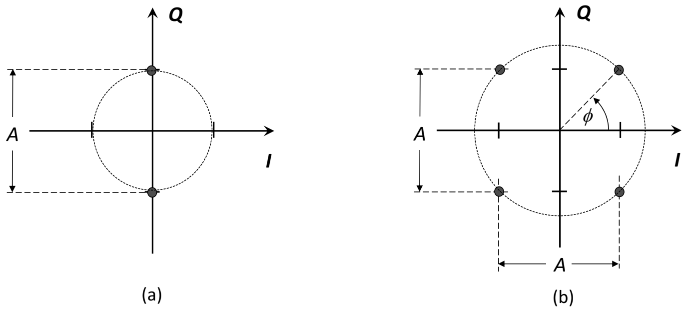


Figure 12.37. Constellation diagram for: (a) BPSK and (b) QPSK.



Phase Shift Keying

Phase shift keying (PSK) is a digital modulation scheme that conveys data by changing the phase of a carrier signal. The basic modulation scheme is the binary phase shift keying (BPSK), with exactly two defined phase states e.g.

$$s_0(t) = A\cos(2\pi f_c t) \text{ representing the binary "1"}$$

$$s_1(t) = A\cos(2\pi f_c t + \pi) \text{ representing the binary "0"}$$

A binary bit stream modulates the carrier waveform with constant amplitude, but alters the phase as a function of the binary signal to be transmitted, as seen at the beginning of every transmitted bit in Figure 12.36. A more general version of phase shift keying is the use of an array of $M = 2^n$ valid different phases, which represent multiple binary bits at one time.

Constellation Diagram

A convenient way of representing digital modulation schemes like PSK and ASK is on a constellation diagram. In a constellation diagram, the states of the modulation are shown in the in-phase-quadrature or IQ plane as shown in Figure 12.37 for two different signaling types: BPSK and QPSK. Here *I* represents the in-phase signal component and *Q* represents the quadrature or orthogonal signal component. In essence, the *IQ* plane is the familiar complex plane arising from complex number theory and is used to represent a transmitted signal using a complex number in rectangular form.

The constellation points in PSK are chosen such that the points are positioned with uniform angular spacing around a circle as shown in Figure 12.37. This gives maximum phase separation between adjacent points and thus gives the best immunity to a disturbance. As they are positioned around a circle, all points have the same amplitude, resulting in equal transmitted energy. For the BPSK signal shown in Figure 12.37a, two phases 180 degrees out of phase are used to represent the transmitted signal. In the case of a quadrature phase shift keying (QPSK) signal, each signal is placed 90 degrees out of phase of one another (i.e., occupies a quadrant of the complex plane).

12.5.3 Quadrature Amplitude Modulation

Quadrature amplitude modulation (QAM) is a modulation format that makes use of both PSK and ASK. The amplitude and phase of *M* sinusoidal waves are used to encode the in-coming digital data. The amplitude of each sine wave is uniformly distributed over some voltage range; also, the phase is evenly distributed over 180 degrees. For instance, for 2^M-QAM, where *M* is an integer, *M* sine waves having *M*/2 amplitudes and 180/*M* degrees out of phase with respect to one another are used to modulate *M*-bits of digital data. Commonly used signaling schemes are 16QAM and 64QAM. Figure 12.38 illustrates a 16QAM example. Here 4 bits of digital data are transmitted using two different amplitude sine waves with 90 degrees of phase separation.

12.5.4 Orthogonal Frequency Division Multiplexing

Orthogonal frequency division multiplexing (OFDM) is a frequently used modulation scheme for wideband digital communication. Wireless data communication systems like WLAN in the

Figure 12.38. 16QAM constellation diagram with 4 bit per symbol.

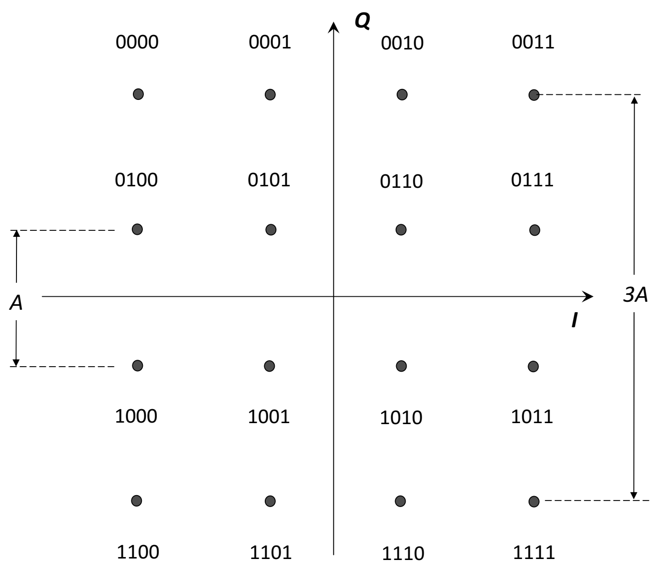


Figure 12.39. Orthogonal subcarrier frequency spectrum of an OFDM signal.

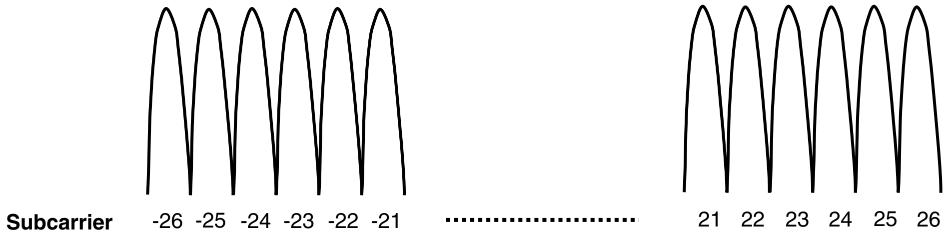
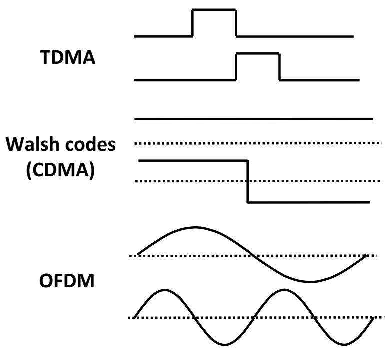


Figure 12.40. Orthogonal signals for different modulation schemes within one time interval. The integral of the product of such signals over a time interval results in a value of 0.



802.11a/b/g/n standard, or the WiMAX 802.16 standard are taking advantage on the high data rate combined with the robustness of data transmission. The high channel efficiency with OFDM modulated signals is utilized for digital radio systems like DAB, HD Radio or TV standards like DVB-T, DVB-H, as well as on wired standard for ADSL and VDSL broadband access and power line communication (PLC).

OFDM is a frequency division multiplexing (FDM) scheme, which utilizes a digital multi-carrier modulation method. A large number of closely spaced subcarriers are used to carry the data as shown in Figure 12.39. The data are divided into several parallel data streams, or channels, one for each subcarrier. Each subcarrier is modulated with a conventional modulation scheme, such as the QAM, and carries the data at a low symbol rate. The frequencies of these subcarriers are chosen so that the subcarriers are orthogonal to each other. Due to this orthogonally, the critical crosstalk between the subcarriers (subchannels) is eliminated, and inter-carrier guard bands are not required.

OFDM is addresses the problem with single carrier modulation (SCM) schemes. In a given environment, the symbol interval of a SCM becomes shorter than the delay spread associated with transmission. This effect becomes more acute with increased data rate. Multicarrier modulation (MCM) formats solve this problem by decreasing the symbol rate by increasing the number of carriers transmitted in parallel. The lower data rate within a single carrier of the MCM reduces the intersymbol interference (ISI) effect and reduces any multipath effects which might also occur. OFDM is one of the MCM formats and has a flexible modulation format, which can be scaled to meet the needs of multiple applications.

Another important consideration of OFDM is orthogonality (see Figure 12.40). Two signals are orthogonal in a given time interval if, when multiplied together and then integrated over that time interval, the result is zero. TDMA (time-domain multiple access) is not considered an orthogonal coding scheme. Over some time interval, the other signal is zero, so that the product of the two will be zero. Welch codes used in CDMA systems, like IS-95, are orthogonal and are probably the most common form of orthogonal signaling. OFDM uses Walsh codes instead of sinusoids. In a given period, the sinusoids will be orthogonally modulated over an integer number of cycles. Using multiple sinusoids instead of Walsh codes produces a spectrum, in which it is possible to assign a carrier frequency to a code channel. In other frequency division multiplexing systems (FDMA), the channel spacing is typically greater than the symbol rate to avoid an overlapping of the spectrum. In OFDM, the carriers are orthogonal and can overlap without interfering with one other.

Advantages of OFDM

- Increased efficiency due to reduced carrier spacing.
- Resistance to fading.
- Flexible data rate that can be adapted to conditions.
- Flexible frequency assignment up to single frequency for broadcast applications.

Disadvantages of OFDM

- The high peak-to-average power ratio requires highly linear transmit channels, especially a highly linear PA.
- The criticality of phase noise to the orthogonal condition require a tight phase noise and noise system.

12.6 SUMMARY

This chapter gave a brief introduction to the basics of RF theory as it applies to RF test. With the progression of semiconductor technology to higher bandwidths and higher operating frequencies to large volume applications, the need for the test engineer to expand his or her skill set with RF test has become a necessity.

This chapter began by introducing the reader to the concept of a propagating wave and the various means by which a wave is quantified (e.g., wavelength, velocity, etc.). Such metrics are important to the RF test engineer. For instance, understanding the wavelength of an RF signal is helpful when designing a device interface board and placing critical components on the test board. Knowledge of the wavelength will allow the engineer to find the best compromise between signal integrity, component size selection and placement, and position of the DUT.

Power measurements are the most common measurement in the RF test. Understanding its definition, as well as its related measures such as average power, peak power, power for framed signals, and so on, is critical to the power measurement itself. For instance, knowing the ratio of peak to average power commonly referred to as the crest factor enables the setting of the ATE to be optimized for maximum measurement accuracy.

A major portion of this chapter was dedicated to the theory of noise and phase noise. Like all electrical systems, noise is one of the limiting factors of system performance; the same holds true for RF systems. A brief description of various noise sources was introduced, as well as the concept of a system noise figure. Because most RF systems have a frequency translation device, such as a mixer and VCO, noise has a specific impact on their operation, which is quantified by a phase noise metric. The phase noise of the ATE reference source is often the limiting factor of any phase noise measurement, and it needs to be considered carefully in a production test.

A section of this chapter was dedicated to the concept of *S*-parameters because it applies to an *n*-port network, such as a two-port network. *S*-parameters are used to describe the small-signal performance of the network as seen from one port to another. *S*-parameters are useful descriptions of the how waves propagate and reflect at each interface of the network. Measures like reflection coefficient, mismatch loss, insertion and transducer loss, and various power gains can easily be defined in terms of these *S*-parameters. Most importantly, the idea of a mismatch uncertainty and its impact on a power measurement was introduced. Mismatch uncertainty is often the most significant contributor to measurement error. When estimating the quality of a test result, a careful discussion of the mismatch uncertainty will allow the engineer to estimate the tolerance in a measurement.

The final section of this chapter described several forms of modulation. We began with a set of traditional analog modulation schemes such as AM and FM used in early radio and television systems, followed with several digital modulation schemes used primarily for digital communication systems. This section concluded with a brief overview of an OFDM modulation as this scheme is used in many commercial RF communication systems.

Now that the reader has been introduced to the basics of RF theory they can move on to the next chapter where the specifics of various RF tests are described.

PROBLEMS

12.1. List four different RF applications and the corresponding frequency bands that they use.

12.2. List three major differences between mixed signal and RF test requirements.

12.3. Name the typical power level for noise, radio receive signals, data communication system transmit signals, and radio station transmit signals.

12.4. What is the difference between a scalar and a vector measure?

12.5. An RF transmission line on GETEK board material is used to delay a signal with a frequency of 2.4 GHz by 180 degrees. If the effective dielectric constant for this material is $\varepsilon_{eff} = 3.9$, what line length should be used?

12.6. An RF transmission line constructed on a Teflon printed circuit board is 10 mm in length. The effective dielectric constant for Teflon is $\varepsilon_{eff} = 2.2$. If the RF line is excited by a signal with frequency of 900 MHz do we treat this line as a distributed set of elements or as a lumped LRC equivalent assuming the 5% line length rule.

12.7. How is the wavelength changed when an electromagnetic wave is transferred from free space transmission to that guided by a microstripline?

12.8. Explain the superimposition principle.

12.9. Which circuits show power and voltage out of phase?

12.10. What is a universal definition of electrical power?

12.11. What is thereal and imaginary instantaneous power associated with a circuit if $v(t) = 0.25\cos\left(2\pi \times 10^4 \cdot t\right)$ and $i(t) = 0.25\cos\left(2\pi \times 10^4 \cdot t - \pi/5\right)$? *Hint*: See Exercise 12.7.

12.12. What is the average power associated with a circuit if $v(t) = 0.25\cos\left(2\pi \times 10^4 \cdot t\right)$ and $i(t) = 0.25\cos\left(2\pi \times 10^4 \cdot t - \pi/5\right)$?

12.13. What is the expected level of power associated with a pulsed signal if the continuous wave signal has an average power of 5 W and the pulsing action has a 20% duty cycle? Express your answer in terms of watts and in decibels.

12.14. Why is it important to take the crest factor of a signal into account when defining an RF system?

12.15. The crest factor of a continuous waveform is 12 dB. If the average power is 0.4 W, what is the peak power associated with this waveform?

12.16. A signal consisting of 16 uncorrelated sinusoids of 1.5 V. Estimate the crest factor associated with the composite waveform.

12.17. List the crest factors of three signals.

12.18. What is the formula to calculate power in dBm from the linear scale of Watt?

12.19. What is the ratio in decibels for doubling the power and doubling the voltage?

12.20. What load condition is required to transfer the maximum power to an electrical load?

12.21. What load condition is required to have no reflections?

12.22. A 1-V source with source impedance $Z_S = 100 + j21\ \Omega$ drives a load impedance of $Z_L = 50 + j10\Omega$. What is the average power delivered to the load and how does it compare to the maximum power available from the source? What is the average power delivered by the source?

12.23. A 1-V source with source impedance $Z_S = 50 + j10\Omega$ drives a load impedance of $Z_L = 100 + j21\ \Omega$. What is the average power delivered to the load and how does it compare to the maximum power available from the source? What is the average power delivered by the source?

12.24. Describe in your own words: (a) insertion loss, (b) transducer loss.

12.25. A source is terminated in a load where the load power was found to be 0.52 W. The same load was then connected to the end of a trace on a DIB board connected to the source through an SMA connector. The load power was then measured to be 0.42 dB. What is the insertion loss associated with the RF connector and DIB trace?

12.26. The maximum available power from a source is 5.0 W. If a transducer with a 50 Ω load is connected to the source, what is the transducer loss is the load power is 4.2 W?

12.27. What is the difference between a dissipative and reflection loss?

12.28. If a spectral analyzer reveals a power level of -92 dBm at 900 MHz using a resolution bandwidth of 80 kHz, what is the actual PSD appearing at the input to this instrument?

12.29. What method can be used to measure a signal below -174 dBm?

12.30. Why is it important to have a gain stage with a low noise figure as the first stage in an RF system?

12.31. If a given amplifier has a 3 dB noise figure at 300 K, a noise bandwidth of 10 MHz, and an input resistance of 50 Ω, what is the output signal-to-noise ratio if the input signal RMS level is 100 μV.

12.32. What is the combined noise figure of three-stage amplifier? The first stage has a noise figure of 5.2 dB and a gain of 10 dB, the second stage has a noise figure of 4 dB and a gain of 10 dB, and the third stage has a noise figure of 5 dB and a gain of 10 dB.

12.33. What is the principal definition of phase noise and how could it be measured?

12.34. What are the paramters S_{11}, S_{21}, S_{12}, and S_{22} describing in a two-port network?

12.35. For which case are the parameters S_{21} and S_{12} identical?

12.36. Describe in your own words VSWR, reflection coefficient, return loss, insertion loss, and mismatch loss.

12.37. The voltage wave incident at the interface of an RF component was measured with a network analyzer to be $3.5e^{-j\pi/16}$ V and the reflected wave was found to be $0.03e^{j\pi/4}$ V. What is the reflection coefficient? What is the return and mismatch loss associated with this RF component?

12.38. An RF connector has a VSWR of 1.45, what is the reflection coefficient and return loss associated with this connector.

12.39. What is the cause of mismatch uncertainty and what is the impact of mismatch uncertainty on a test result?

12.40. The reflection coefficients between a DUT and load were found to be 0.09 and 0.15, respectively. What is the bound on the mismatch loss and the overall mismatch loss uncertainty?

12.41. The mismatch uncertainty associated with the input path to a source-DUT connection is 0.43 dB. The uncertainty associated with the output path is 0.24 dB. What is the total uncertainty associated with this measurement?

12.42. A source is to be connected to a DUT through an SMA connector and a 50–Ω RF line fabricated on a PCB. If the SMA connector and DUT have return losses of 12 dB and 25 dB, respectively, what is the expected level of mismatch uncertainty associated with this test setup? Assume the RF transmission line is perfect and has no loss.

12.43. An RF source of an ATE system is coupled on a DIB to an amplifier (DUT) through an SMA connector with a return loss of 18 dB. The output of the amplifier is also coupled to a power meter in the ATE through another SMA connector with a return loss of $RL_M = 21$ dB. The DUT is known to have an input impedance of $Z_{IN} = 42\ \Omega$ and an output return loss RL_{OUT} of 18 dB. The reverse isolation of the amplifier S_{12} is assumed to be high, so this DUT can be treated as a unilateral device. All RF lines are assumed to have a characteristic impedance of 50 Ω and are loss free. Compute the return loss, mismatch loss, and mismatch uncertainty of the amplifier gain test.

12.44. The reflection coefficient for at a load is 0.2. How much attenuation is required to reduce the reflection coefficient to 0.1?

12.45. If a 6-dB attenuation pad is inserted in series with a source with a reflection coefficient of 0.5. What is the effective reflection coefficient of the source?

12.46. An amplifier was characterized with the following S-parameters:

$$S_{11} = 0.3e^{-j2.7925}, \quad S_{12} = 0.1e^{j0.2793}$$
$$S_{21} = 1.5e^{j0.5236}, \quad S_{22} = 0.9e^{-j1.5708}$$

What is the insertion loss of this amplifier? What are the magnitudes of the reflection coefficients of the input and output ports of this amplifier? What is the input and output impedance of the amplifier, assuming that the S-parameters were obtained with ideal transmission lines with characteristic impedance of 50 Ω?

12.47. Why is the frequency modulation more robust against noise compared to amplitude modulation?

12.48. How can the power efficiency improved for AM and FM signals?

12.49. What is the difference between GFSK and FSK and what is the advantage of GFSK?

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