LECTURE6 – PARALLEL COMPUTATION PATTERNS (SCAN)

- Prefix Sum
- A Work-inefficient Scan Kernel
- A Work-Efficient Parallel Scan Kernel
- More on Parallel Scan

Objective

- To master parallel scan (prefix sum) algorithms
  - Frequently used for parallel work assignment and resource allocation
  - **A key primitive in many parallel algorithms to convert serial computation into parallel computation**
  - A foundational parallel computation pattern
  - Work efficiency in parallel code/algorithms

- Reading –Mark Harris, Parallel Prefix Sum with CUDA
  - http://http.developer.nvidia.com/GPUGems3/gpugems3_ch39.html

Inclusive Scan (Prefix-Sum) Definition

Definition: *The* **scan** *operation takes a binary associative operator* $\oplus$ (pronounced as circle plus), *and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}],$$

and returns the array

$$[x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})].$$

Example: If $\oplus$ is addition, then scan operation on the array would return

[3 1 7 0 4 1 6 3],
[3 4 11 11 15 16 22 25].

An Inclusive Scan Application Example

- Assume that we have a 100-inch sandwich to feed 10 people
- We know how much each person wants in inches
  - [3 5 2 7 28 4 3 0 8 1]
- How do we cut the sandwich quickly?
- How much will be left?

- Method 1: cut the sections sequentially: 3 inches first, 5 inches second, 2 inches third, etc.

- Method 2: calculate prefix sum:
  - [3, 8, 10, 17, 45, 49, 52, 52, 60, 61] (39 inches left)

Typical Applications of Scan

- Scan is a simple and useful parallel building block
  - Convert recurrences from sequential:

```
for(j=1;j<n;j++)
  out[j] = out[j-1] + f(j);
```

  - Into parallel:

```
forall(j) { temp[j] = f(j) };
 scan(out, temp);
```


Other Applications

- Assigning camping spots
- Assigning Farmer's Market spaces
- Allocating memory to parallel threads
- Allocating memory buffer space for communication channels
- …

An Inclusive Sequential Addition Scan

Given a sequence $[x_0, x_1, x_2, \ldots]$
Calculate output $[y_0, y_1, y_2, \ldots]$

Such that $y_0 = x_0$

$y_1 = x_0 + x_1$

$y_2 = x_0 + x_1 + x_2$

...

Using a recursive definition

$$y_i = y_{i-1} + x_i$$

A Work Efficient C Implementation

```
y[0] = x[0];
for (i = 1; i < Max_i; i++) y[i] = y [i-1] + x[i];
```

Computationally efficient:

N additions needed for N elements - O(N) !
Only slightly more expensive than sequential reduction.

A Naïve Inclusive Parallel Scan

- Assign one thread to calculate each y element
- Have every thread to add up all x elements needed for the y element

$y_0 = x_0$

$y_1 = x_0 + x_1$

$y_2 = x_0 + x_1 + x_2$

"Parallel programming is easy as long as you do not care about performance."

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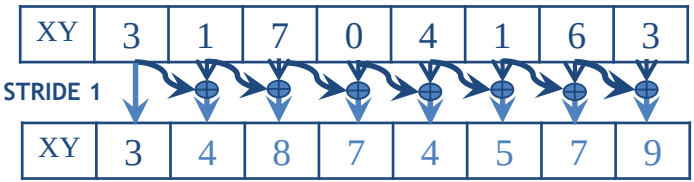
Objective

- To learn to write and analyze a high-performance scan kernel
  - Interleaved reduction trees
  - Thread index to data mapping
  - Barrier Synchronization
  - Work efficiency analysis

A Better Parallel Scan Algorithm

1. Read input from device global memory to shared memory
2. Iterate log(n) times; stride from 1 to n-1: double stride each iteration



ITERATION = 1

STRIDE = 1

- Active threads *stride* to n-1 (n-stride threads)
- Thread *j* adds elements *j* and *j-stride* from shared memory and writes result into element j in shared memory
- Requires barrier synchronization, once before read and once before write

A Better Parallel Scan Algorithm

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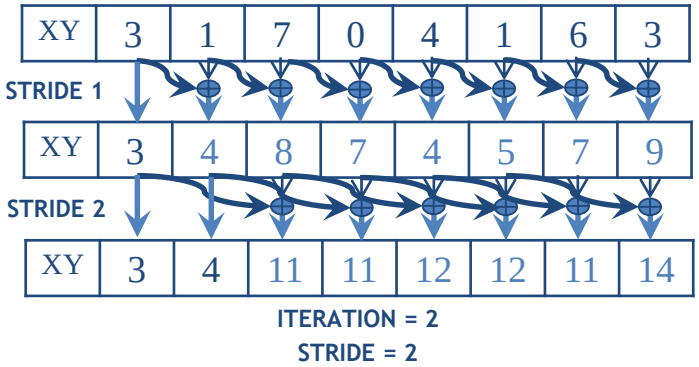
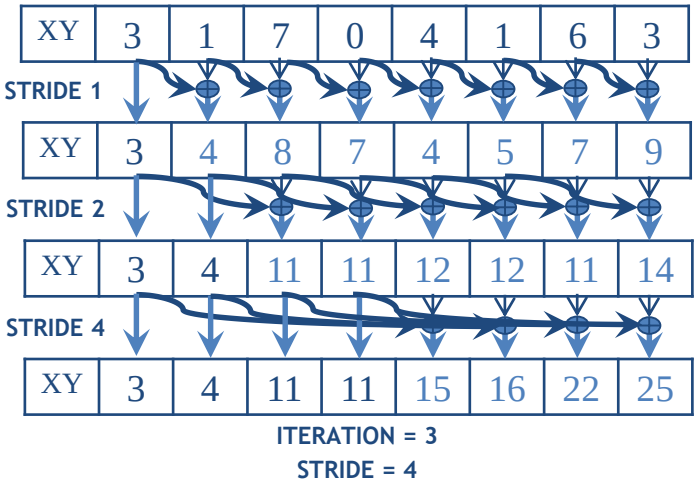


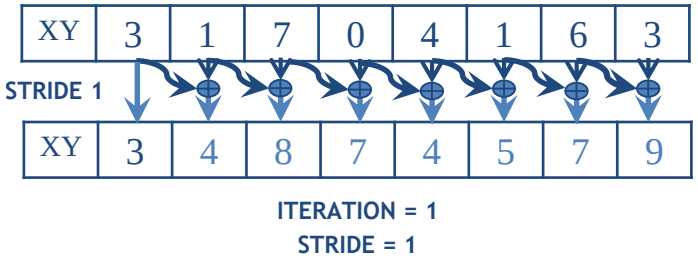
A Better Parallel Scan Algorithm

1. Read input from device to shared memory
2. Iterate log(n) times; stride from 1 to n-1: double stride each iteration
3. Write output from shared memory to device memory



Handling Dependencies

- During every iteration, each thread can overwrite the input of another thread
  - Barrier synchronization to ensure all inputs have been properly generated
  - All threads secure input operand that can be overwritten by another thread
  - Barrier synchronization is required to ensure that all threads have secured their inputs
  - All threads perform addition and write output



A Work-Inefficient Scan Kernel

```
__global__ void work_inefficient_scan_kernel(float *X, float *Y, int InputSize)
    {
    __shared__ float XY[SECTION_SIZE];
    int i = blockIdx.x * blockDim.x + threadIdx.x;
    if (i < InputSize) {XY[threadIdx.x] = X[i];}
        // the code below performs iterative scan on XY
        for (unsigned int stride = 1; stride <= threadIdx.x; stride *= 2) {
        __syncthreads();
            float in1 = XY[threadIdx.x - stride];
            __syncthreads();
            XY[threadIdx.x] += in1;
        }
      __ syncthreads();
      If (i < InputSize) {Y[i] = XY[threadIdx.x];}
}
```

Work Efficiency Considerations

- This Scan executes log(n) parallel iterations
  - The iterations do (n-1), (n-2), (n-4),..(n- n/2) adds each
  - Total adds: n * log(n) - (n-1) → O(n*log(n)) work

- This scan algorithm is not work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) can hurt: 10x for 1024 elements!

- A parallel algorithm can be slower than a sequential one when execution resources are saturated from low work efficiency

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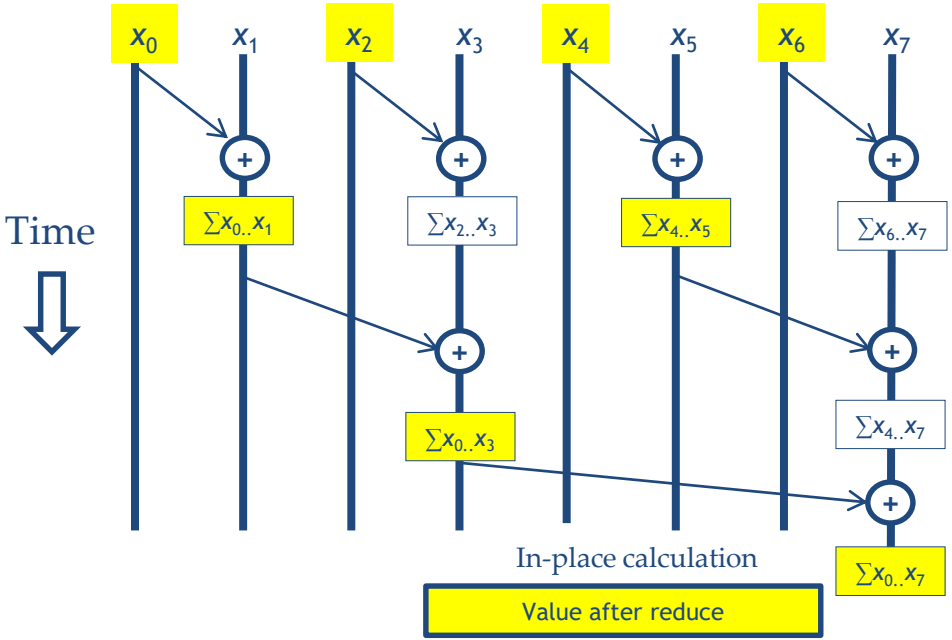
Objective

- To learn to write a work-efficient scan kernel
  - Two-phased balanced tree traversal
  - Aggressive re-use of intermediate results
  - Reducing control divergence with more complex thread index to data index mapping

Improving Efficiency

- *Balanced Trees*
  - Form a balanced binary tree on the input data and sweep it to and from the root
  - Tree is not an actual data structure, but a concept to determine what each thread does at each step
- For scan:
  - Traverse down from leaves to the root building partial sums at internal nodes in the tree
    - The root holds the sum of all leaves
  - Traverse back up the tree building the output from the partial sums

Parallel Scan - Reduction Phase

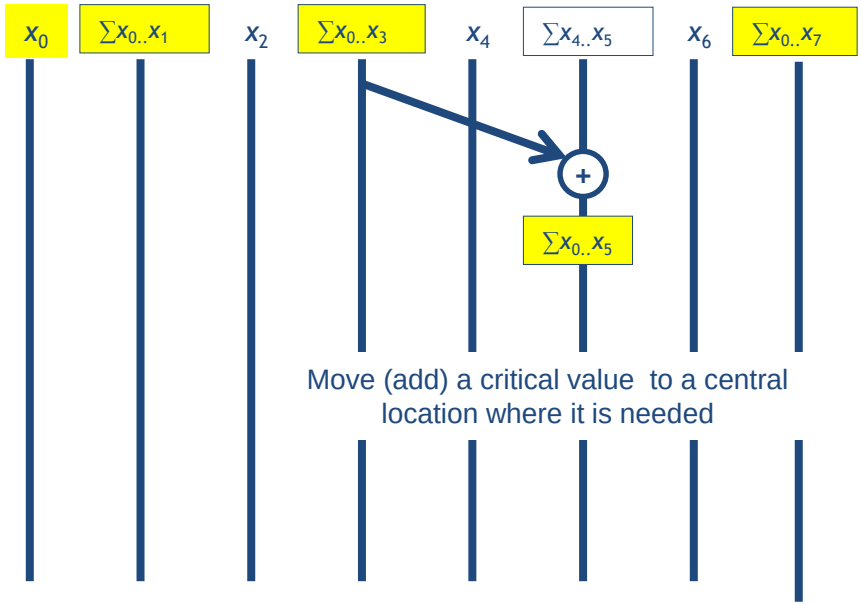


Reduction Phase Kernel Code

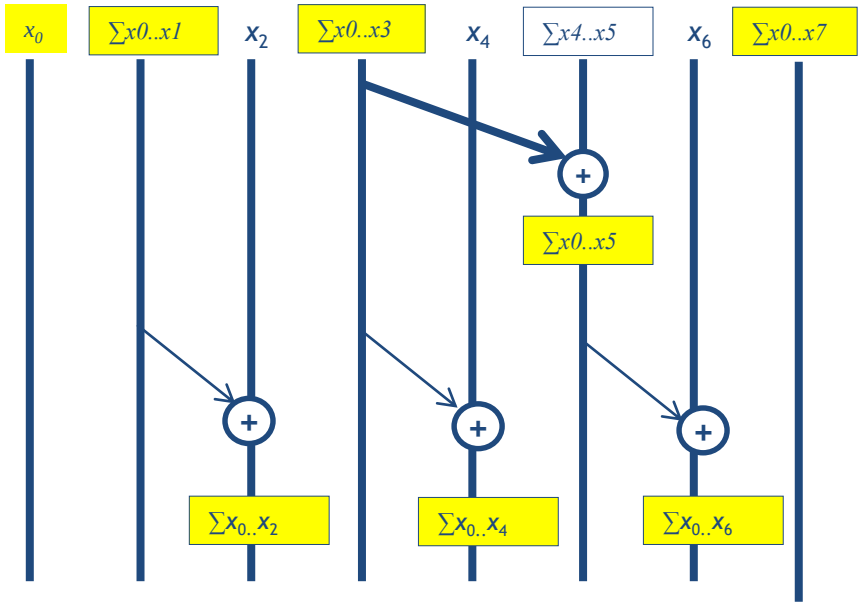
```
// XY[2*BLOCK_SIZE] is in shared memory

for (unsigned int stride = 1;stride <= BLOCK_SIZE;
stride *= 2) {
    int index = (threadIdx.x+1)*stride*2 - 1;
    if(index < 2*BLOCK_SIZE)
        XY[index] += XY[index-stride];
    __syncthreads();
}
```

threadIdx.x+1 = 1, 2, 3, 4….
stride = 1,
index = 1, 3, 5, 7, …

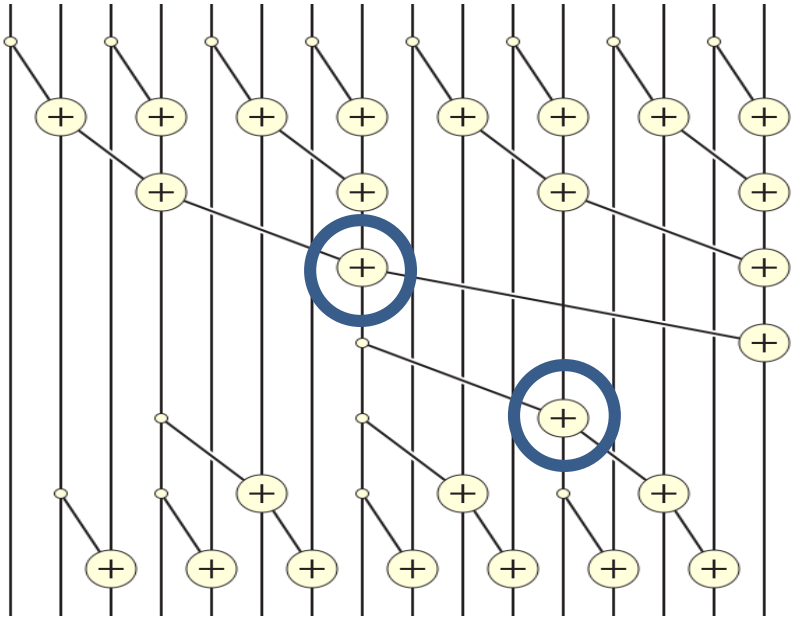
Parallel Scan - Post Reduction Reverse Phase



Parallel Scan - Post Reduction Reverse Phase



Putting it Together



Post Reduction Reverse Phase Kernel Code

```
for (unsigned int stride = BLOCK_SIZE/2; stride > 0; stride /= 2) {
      __syncthreads();
      int index = (threadIdx.x+1)*stride*2 - 1;
      if(index+stride < 2*BLOCK_SIZE) {
        XY[index + stride] += XY[index];
      }
}
  __syncthreads();
if (i < InputSize) Y[i] = XY[threadIdx.x];
```

First iteration for 16-element section
threadIdx.x = 0
stride = BLOCK_SIZE/2 = 8/2 = 4
index = 8-1 = 7

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Objective

- To learn more about parallel scan
  - Analysis of the work efficient kernel
  - Exclusive scan
  - Handling very large input vectors

Work Analysis of the Work Efficient Kernel

- The work efficient kernel executes log(n) parallel iterations in the reduction step
  - The iterations do n/2, n/4,..1 adds
  - Total adds: (n-1) → O(n) work
- It executes log(n)-1 parallel iterations in the post-reduction reverse step
  - The iterations do 2-1, 4-1, …. n/2-1 adds
  - Total adds: (n-2) – (log(n)-1) → O(n) work
- Both phases perform up to no more than 2x(n-1) adds
- The total number of adds is no more than twice of that done in the efficient sequential algorithm
  - The benefit of parallelism can easily overcome the 2X work when there is sufficient hardware

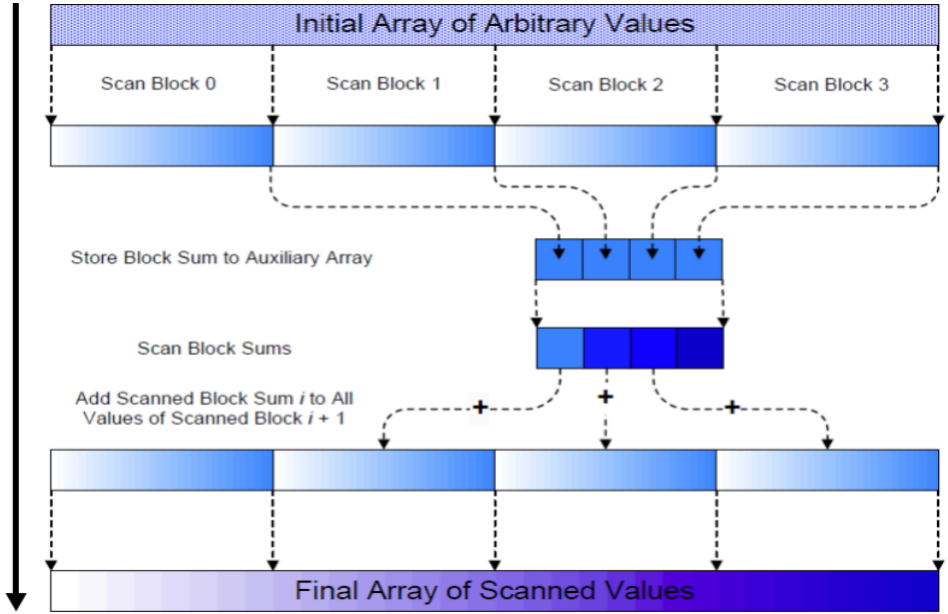
Some Tradeoffs

- The work efficient scan kernel is normally more desirable
  - Better Energy efficiency
  - Less execution resource requirement
- However, the work inefficient kernel could be better for absolute performance due to its single-phase nature (forward phase only)
  - There is sufficient execution resource

Handling Large Input Vectors

- Build on the work efficient scan kernel
- Have each section of 2*blockDim.x elements assigned to a block
  - Perform parallel scan on each section
- Have each block write the sum of its section into a Sum[] array indexed by blockIdx.x
- Run the scan kernel on the Sum[] array
- Add the scanned Sum[] array values to all the elements of corresponding sections
- Adaptation of work inefficient kernel is similar.

Overall Flow of Complete Scan



Exclusive Scan Definition

Definition: *The* exclusive scan *operation takes a binary associative operator* $\oplus$, *and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}]$$

and returns the array

$$[0, x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-2})].$$

Example: If $\oplus$ is addition, then the exclusive scan operation
on the array [3 1 7 0 4 1 6 3],
would return [0 3 4 11 11 15 16 22].

Why Use Exclusive Scan?

- To find the beginning address of allocated buffers
- Inclusive and exclusive scans can be easily derived from each other; it is a matter of convenience

	[3 1 7 0 4 1 6 3]
Exclusive	[0 3 4 11 11 15 16 22]
Inclusive	[3 4 11 11 15 16 22 25]

A Simple Exclusive Scan Kernel

- Adapt an inclusive, work inefficient scan kernel
- Block 0:
  - Thread 0 loads 0 into XY[0]
  - Other threads load X[threadIdx.x-1] into XY[threadIdx.x]
- All other blocks:
  - All thread load X[blockIdx.x*blockDim.x+threadIdx.x-1] into XY[threadIdex.x]
- Similar adaption for work efficient scan kernel but ensure that each thread loads two elements
  - Only one zero should be loaded
  - All elements should be shifted to the right by only one position

Read the Harris article (Parallel Prefix Sum with CUDA) for a more intellectually interesting approach to exclusive scan kernel implementation.