## Problem 1.

Replicate the following:

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This is the start of a paragraph. This is some text
This is a new paragraph.

**No more indentation.**

On 25 June, 2020, the SPDR S&P 500 ETF Trust closed at $307.35. Here is a table of its *most relevant* values:

| | | | |
|---|---|---|---|
| **Open** | 306.17 | **Div yield** | 1.91% |
| **High** | 306.39 | **Prev close** | 307.35 |
| **Low** | 299.43 | **Mkt cap** | 268.01B |

As an exercise, attempt to figure out the following:

(i) Try to add "Data from *Google Finance*." as a caption below the table above.

(ii) Attempt to re-align all the text above

- Left align the first column
- Center align the second column
- Left align the third column
- Right align the last column

**Problem 2.**

Replicate the following:

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a)

$$\begin{aligned} f'(x) &= \left(\frac{du}{dx}\right)\left(\frac{-\sin u}{\sqrt{1-\cos^2 u}}\right) \\ &= \frac{2}{1+x^2}\left(-\frac{2x}{1+x^2}\right)\left(\frac{1+x^2}{|2x|}\right) \qquad \text{(for } x \neq 0\text{)} \\ &= \sigma\left(\frac{2}{1+x^2}\right) \end{aligned}$$

where

$$\sigma = \begin{cases} -1 & \text{for } x > 0 \\ 1 & \text{for } x < 0 \end{cases}$$

Our domain will hence be $(-\infty, 0) \cup (0, \infty)$. Note that

$$y_x \in \mathbb{R} \iff y_x < \frac{\pi}{2}$$

b) The characteristic polynomial is

$$\begin{aligned} \det \begin{bmatrix} 3-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} &= (3-\lambda)(1-\lambda) - (-1)(1) \\ &= \lambda^2 - 4\lambda + 4 \\ &= (\lambda - 2)^2 \end{aligned}$$

As such,

$$\lambda_1 = \lambda_2 = 2$$

c) We have

**Theorem 1** (Wave Equation Solution)**.**

$$\begin{aligned} u(x,t) = \sum_{n=1}^{\infty} \Big[ & A_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi\alpha t}{L}\right) \\ & + B_n \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{n\pi\alpha t}{L}\right) \Big] \end{aligned} \tag{1}$$

*Where*

$$\begin{aligned} A_n &= \frac{2}{L}\int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx \\ B_n &= \frac{2}{n\pi\alpha}\int_0^L g(x) \sin\left(\frac{n\pi x}{L}\right) dx \end{aligned}$$

*Proof.* By observation. □

**Problem 3.**

Replicate the following:

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We want to plot the following function:

$$f(x) = \begin{cases} (x^8 + x^{16})/(1 + x^{24}) & \text{for } 0 \leq x < 1 \\ 0 & \text{for } x = 1 \end{cases}$$

First we use Desmos

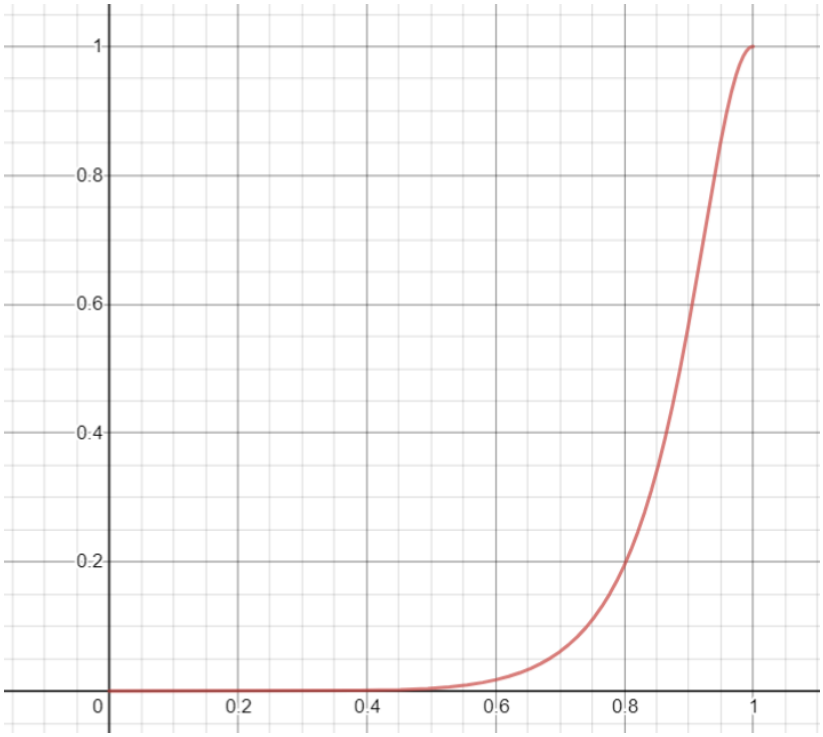


Figure 1: Demos graph of $f(x)$

Next, we use pgfplots:

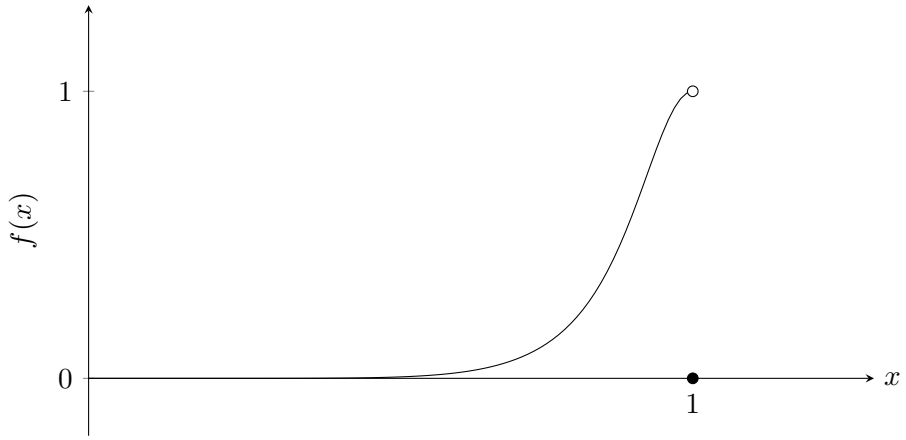


Figure 2: Pgfplots graph of $f(x)$