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*The Inflation Rate of the Price of Gold, Expected Prices and Interest Rates**

A model is developed and tested to relate the inflation rate of the price of gold to expected consumer and wholesale prices and to expected interest rates through substitution effects. An increase in expected future prices may induce individuals to convert their current "liquid" assets into gold. Increases in expected interest rates will cause a negative adjustment of the price of gold. The model is used to correctly predict the direction of the change in the price of gold, although the magnitude of the change is somewhat overestimated.

1. Introduction

In a recent study of the behavior of gold during the 1970s, Cooper (1982) has noted that its price has been more unstable than the general level of prices. Expectations of the general level of prices and interest rates may be one explanation of this phenomenon.

This paper will relate the expected future general level of prices (hereafter referred to as expected future prices) and expected future interest rates to the price of gold. These variables are connected by means of substitution effects between gold and financial or "liquid" assets.

2. The Theoretical Theme

Given the level of expected future interest rates,[1] an increase in expected future prices may result in an anticipated loss in the real value of liquid assets. Individuals may thus find it worthwhile to convert their current liquid assets into gold. This conversion is,

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[1]The impact of changes in expected interest rates on the price of gold will be examined shortly.

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of course, performed under the expectation that the price of gold will appreciate in response to the expected future price increase. There should be a positive relationship between expected future prices and the price of gold, ceteris paribus.

Given the level of expected future prices, increases in expected interest rates will cause a negative adjustment of the price of gold. The higher the expected interest rate in the future, the more it becomes worthwhile to reduce purchases and/or increase sales of gold to provide liquid assets which can earn the higher expected return, ceteris paribus. This conversion from gold to liquid assets is performed under the expectation that the price of gold will decline in response to the increase in expected future interest rates.

Individuals will substitute between liquid assets and gold as expected interest rates and prices change. If perfect foresight rational expectations prevailed, expected future prices would be absorbed in current prices. However, such absorption will not be possible if finite nominal contracts are prevalent or if more time is required for individuals to form and act on their expectations of future inflation in retail markets than in the market for gold.[2] Asset markets such as those for gold may be more flexible than markets for retail products and, hence, better able to react to new events, such as changes in expected future prices.

3. An Equilibrium Model of the Inflation Rate of the Price of Gold

The model concentrates on the demand side since the principal concern is with the influence of the foregoing substitution effects on the price of gold.[3] The inflation rate of the price of gold is specified to be a function of expected future consumer (or wholesale) prices and interest rates. This function is derived below.

[2]Perfect foresight rational expectations will, however, be tested in the empirical section of the paper.

[3]The price of gold may have been primarily determined by demand factors in the 1970s. Average annual world production in the period 1970–79 was approximately equal to that in the period 1950–69 when the price of gold was stable [see Cooper (1982), Table 3, p. 14]. The large increase in the price of gold in the 1970s cannot be accounted for by a substantial decline in world supply. Future research into the supply side factors in the determination of the price of gold will have to account for world stocks and the highly oligopolistic nature of the market [see Cooper, (1982)].

Let p_{kt} be the price of the kth commodity in the consumer (or wholesale) price index and let w_k be its fixed weight in that index. Then let p^f_{kt} be the present expectations of the future price of the kth commodity. Let P_t be the index of consumer (wholesale) prices and P^f_t be the present expectations of the index of future consumer (wholesale) prices. Then let q_t be the price of gold, and Π_{gt} be the inflation rate of the price of gold. Also, let i_t be the nominal interest rate at time t, and let i^f_t be the present expectations of the future nominal interest rate. Finally, let L_t be the present value of liquid assets in period t and L^f_t be the present expectation of the future value of this period's liquid assets.

As is well known, the price of gold is determined both by some basic demand for use in dentistry, jewelery, industry and so forth and by some speculative demand resulting from substitution effects. Thus, let q^b_t be the basic price which results when individuals do not take into account expected changes in the future price of consumer (wholesale) goods or expected changes in future liquid assets (viz., as if individuals have zero inflationary expectations for consumer—wholesale—goods and as if they expect their liquid assets to remain constant). Only the present price of these goods and the present level of liquid assets are taken into account. Thus, for some function g:

$$q^b_t = g[\Sigma p_{kt} w_k, L_t] \tag{1}$$

where $\Sigma p_{kt} w_k$ is the market basket of goods in the consumer (wholesale) price index with fixed weights w_k.

This hypothetical basic price is then adjusted by the demand resulting from the substitution effects. This revision depends on the difference between expected future commodity prices and liquid assets and present prices and liquid assets, respectively. Higher anticipated prices of commodities will lead to a reduction in the anticipated real value of liquid assets, ceteris paribus. In order to offset this loss, individuals will purchase gold, thereby driving up its price. Such purchases are financed by reducing liquid assets. Increases in expected interest rates will cause a negative adjustment of the price of gold, ceteris paribus. The higher the expected interest rate in the future, the more it becomes worthwhile to reduce purchases and/or increase sales of gold to provide liquid assets which can earn the higher expected return. The expected future value of this period's liquid assets L^f_t depends, of course, on the expected future interest rate i^f_t. Thus, the higher the expected interest rate, the higher the expected level of this period's liquid assets in the future, and, hence, the lower the price of gold. Thus, the actual price of gold at time t depends on the reactions to expectations of changes in future commodity prices and expectations of changes in liquid assets in the future. This actual price includes the basic price. Hence:

$$q_t = g[\Sigma p^f_{kt} w_k, L^f_t]\,. \tag{2}$$

The same functional relationship g is used in Equations (1) and (2). This means that the reaction of the *hypothetical* basic price to actual changes is the same as the reaction of the actual price to expected changes. Of course, actual commodity prices and liquid assets are observed at any point in time, whereas individuals may require a considerable period of time to form and act on their expectations of future commodity prices and liquid assets.[4] This latter conjecture will be tested in a later section of the paper.

Taking a Taylor approximation yields:

$$q_t = g[\Sigma p_{kt} w_k, L_t] + [\Sigma p^f_{kt} w_k - \Sigma p_{kt} w_k]\partial g/\partial[\Sigma p^f_{kt} w_k]$$
$$+ [L^f_t - L_t]\partial g/\partial L^f_t\,. \tag{3}$$

Writing q^b_t for the first term on the right hand side and dividing by q^b_t, we obtain

$$\frac{q_t}{q^b_t} = 1 + \frac{q_t}{q^b_t}\left[\frac{\Sigma p^f_{kt} w_k - \Sigma p_{kt} w_k}{\Sigma p_{kt} w_k}\right]\epsilon_t$$
$$+ \frac{q_t}{q^b_t}\left[\frac{L^f_t - L_t}{L_t}\right]\theta_t \tag{4}$$

where ϵ_t is the partial price elasticity and θ_t is the partial liquid asset elasticity of the price of gold; where

$$\epsilon_t \approx \frac{\partial g}{\partial[\Sigma p^f_{kt} w_k]}\cdot\frac{\Sigma p_{kt} w_k}{q_t} \text{ and } \theta_t \approx \frac{\partial g}{\partial L^f_t}\cdot\frac{L_t}{q_t}\,.$$

[4]Gordon [(1982), p. 1090] notes that "economic agents can form expectations rationally in a world characterized by inertia in the response of prices." See Gordon (1981) for a theoretical underpinning for lags in expectations formation.

The standard assumption is made that the commodity price and liquid asset elasticities of the price of gold are inversely proportional[5] to the ratio of the actual to the basic price of gold over time t (viz., the proportion of the actual price of gold resulting from substitution effects brought on by expectations of future commodity prices and interest rates). Upon taking logarithms

$$\log\left(\frac{q_t}{q_t^b}\right) = \log\{1 + \mu(\Sigma p_{kt}^f w_k - \Sigma p_{kt} w_k)/\Sigma p_{kt} w_k$$

$$+ \phi(L_t^f - L_t)/L_t\} \tag{5}$$

where $\mu = (q_t/q_t^b)\epsilon_t$ and $\phi = (q_t/q_t^b)\theta_t$.

The difference between the actual and the basic price of gold, which could be negative or positive, will be affected by the substitution effects resulting from expected future commodity prices and from expectations of changes in liquid assets. The price elasticity μ will be positive if an increase in expected commodity prices increases the price of gold. The liquid asset elasticity ϕ will be negative if an increase in expected interest rates causes the price of gold to decline.

In its present form Equation (5) is not amenable to measurement. In order to obtain an estimating equation, the basic and actual prices of gold must be related to the inflation rate of the price of gold.

We can write

$$\Pi_{gt} = \alpha + \gamma \log(q_t/q_t^b) \, . \tag{6}$$

This equation states that the inflation (deflation) rate of the price of gold Π_{gt} will be determined by the adjustment of the price of gold due to present expectations of future prices and present expectations of changes in liquid assets in the future (q_t/q_t^b). The coefficient γ measures the adjustment of the price of gold resulting from present expectations.

Substituting (5) into (6), taking an approximation and adding an error term yields:

[5]For example, Balassa and Kreinin [(1967), p. 129] state that "export supply elasticities will tend to be negatively correlated with the share of exports to domestic production."

$$\Pi_{gt} \simeq \alpha + b \log P_t^f + \beta \log i_t^f + u_t \tag{7}$$

where $b = \gamma\mu$, $\beta = \gamma\phi$ and from the definition of a Laspeyres price index $P_t^f = [\Sigma\ p_{kt}^f w_k / \Sigma\ p_{kt} w_k]$ is the present expectations of the index of future consumer (wholesale) prices, and $i_t^f = L_t^f / L_t$ is the expected rate of growth of the present stock of liquid assets (viz., the expected interest rate).

Since an increase in P_t^f should have a proportional and positive impact on Π_{gt}, while an increase in i_t^f should have a proportional and negative impact on Π_{gt}, b should be equal to $+1$ and β should be equal to -1. A given increase in expected future prices and interest rates will have proportional but offsetting impacts on the rate of change of the price of gold. The validity of these restrictions will be tested by means of t statistics. If the restriction that $b = \beta$ is valid, the preceding equation can be written[6] as:

$$\Pi_{gt} = \alpha + \delta \log(P_t^f / i_t^f) + u_t\ . \tag{8}$$

If expected future prices increase (decrease) relative to expected future interest rates, the price of gold should increase (decrease) proportionately. The coefficient δ should be equal to $+1$.

4. Empirical Results

Equations (7) and (8) are estimated with quarterly data for the United States over the period from the third quarter of 1973 to the second quarter of 1980, during which the price of gold was flexible. Data for the price of gold in U.S. dollars and for wholesale and consumer price indices and long-term government bond yields in the U.S. are obtained from the *International Financial Statistics* of the International Monetary Fund.

At this stage a gradual adjustment process is used to estimate the formation of expectations. Three-quarter-distributed lag models are used to estimate expected wholesale and consumer prices P_t^f and expected interest rates i_t^f. Expected prices and expected in-

[6]Equations (7) and (8) are used for estimation purposes. One limitation of this estimation is that these equations have been obtained from a model which uses a number of approximations and nonlinearities. I am indebted to one of the referees for this remark.

terest rates are obtained by regressing actual prices P_t and actual interest rates i_t on their past values respectively.[7]

The estimates of P_t^f and i_t^f are substituted into Equation (8). The resulting estimates of the parameters of this equation[8] are provided in Table 1. The results conform with the previous theoretical discussion. The coefficients on the expected rates of growth of wholesale and consumer prices relative to interest rates are positive, significant and approximately equal to unity for the third-order distributed lag. The coefficient is 1.11 when wholesale prices are used and 1.06 when consumer prices are examined. The substitution effects have been operating in the manner expected by the theory. A 10% increase in expected prices relative to expected interest rates leads to an approximately 10% increase in the price of gold. Of course, other factors such as real income, risk[9] and transactions costs may also be important determinants of the price of gold. The Durbin-Watson (DW) statistics suggest that there is little autocorrelation. Since a lagged dependent variable does not appear on the right hand side of the equations, Durbin's h statistic is not provided.

The above equations were estimated with a three-quarter-distributed lag and a semi-log specification. Other specifications were performed. A two-quarter-distributed lag and a double logarithmic specification were performed. Expected rates of change in prices rather than expected prices were also examined. The data rejected all of these alternate specifications. Tanzi (1980) has estimated expected inflation rates from past inflation rates by the use of both distributed lag and extrapolative hypotheses. When Tanzi's extrapolative model is used to estimate expected prices and interest rates, the results for the WPI are as follows:

$$\Pi_{gt} = \underset{(2.2608)^*}{-2.5900} + \underset{(2.3171)^*}{0.8656} \log (P_t^f / i_t^f)$$

$$DW = 1.6315 \qquad R^2 = 0.1828 \qquad F = 5.37^* .$$

As can be seen, there is little difference between these results and

[7]The regression equations estimating the formation of price and interest rate expectations in the United States are available from the author on request.

[8]This equation is estimated by imposing the restriction that $b = \beta$. This restriction is relaxed later.

[9]Risk factors will be examined shortly.

TABLE 1.

Price Index	*Constant*	*log(P^f/i^f)*	*DW*	R^2	*F*	*t test* $b = +1$	$\beta = -1$
Third-Order Distributed Lag							
WPI	−3.3500	1.1082	2.3123	0.3527	12.31*	0.2150	0.1531
	(3.4875)*	(3.5404)*					
CPI	−3.0733	1.0622	2.2743	0.3425	11.98*	0.0075	0.2730
	(3.4050)*	(3.4613)*					
Perfect Foresight Rational Expectations							
WPI	−1.7423	0.5839	1.8119	0.0932	2.36	1.2650	2.0634*
	(1.4946)	(1.5379)					
CPI	−1.3463	0.4746	1.7459	0.0687	1.70	1.6023	2.3932*
	(1.2554)	(1.3024)					

NOTE: t values in parentheses for null hypothesis $\delta = 0$
t values in last two columns refer to null hypotheses $b = +1$ and $\beta = -1$
*significant at 95 % level
**significant at 90% level
WPI: wholesale price index
CPI: consumer price index.

those obtained by employing the distributed lag. This conclusion is not altered when the CPI is used to measure prices.

The preceding equations were estimated by imposing the restriction of equality of the price and interest rate parameters, viz., that $b = \beta$. These equations are now reestimated without imposing this restriction. Since an increase in P_t^f has a proportional and positive impact on Π_{gt} while an increase in i_t^f has a proportional and negative impact, b should be equal to $+1$, while β should be equal to -1. The validity of these restrictions can be tested by means of t statistics. The estimates of these statistics are given in the last two columns of Table 1. The estimates of t are all below the 90% significance level of t. Hence, the null hypothesis that $b = +1$ and $\beta = -1$ is accepted. This, of course, means that the price and interest rate parameters are equal (although of opposite signs).

The above equations were estimated under the hypotheses that expectations of prices and interest rates have been formed by extrapolation or by a distributed lag. Alternatively, these expectations may be formed by the hypothesis of perfect foresight rational expectations (viz., $p_t^f = p_t$ and $i_t^f = i_t$). This alternative hypothesis can be tested by reestimating Equation (8). The estimates of the parameters of this equation under the hypothesis of perfect foresight rational expectations are given in the last two rows of Table 1. While the coefficients obtained under the hypothesis of perfect foresight rational expectations are positive, they are not significantly different from zero. Evidently, individuals require time to form and act on their expectations of prices and interest rates.[10] See Gordon (1982) for further evidence of the gradual adjustment of prices in the United States.

When the restriction of equality of the price and interest rate parameters is relaxed and the equations are reestimated under the assumption of perfect foresight rational expectations, the estimates of t are not significant for the hypothesis that $b = +1$, but they are significant for the hypothesis that $\beta = -1$. The data reject the null hypothesis that $\beta = -1$ under perfect foresight rational expectations.

In 1980: *iii* the price of gold, long-term government bond yields and wholesale and consumer prices all increased. The higher the actual bond yields and wholesale and consumer price levels, the higher the expected levels. The analysis in this paper would explain the increase in the price of gold in terms of the substitution of gold for liquid assets. The expected future price effect would more than outweigh the expected future interest rate effect. A check on the predictability of the model can be obtained by calculating the effects of expected prices and expected nominal interest rates on the inflation rate of the price of gold. The expected levels of wholesale and consumer prices P_t^f and expected nominal bond yields i_t^f for 1980: *iii* can be calculated from the parameter estimates of the three distributed lags, together with the associated values of P_t and i_t for 1979: *iv*–1980: *ii*, while the magnitudes of α and δ are given in the empirical estimates of Equation (8) under the hypothesis of a distributed lag. These figures result in values for Π_{gt} of 24.91% using the WPI and 40.27% using the CPI between the second and third quarters of 1980. These figures indicate that the expected price effect did outweigh the expected nominal interest rate effect, causing the price of gold to increase.[11]

[10]By themselves, these insignificant coefficients would be in conformity with the rational expectations hypothesis, since they would imply that only unanticipated prices and interest rates matter. However, when lags are taken into account anticipated prices and interest rates matter also.

[11]The actual increase in the price of gold between the second and third quarters of 1980 was 19.12%. While the model correctly predicted the direction of the change in the price of gold, it overestimated the magnitude of the change. The price of

5. Further Results

When the restriction that $b = \beta$ is relaxed, log P_t^f and log i_t^f are employed as separate independent variables. It is well known that P_t^f and i_t^f are highly correlated. Multicollinearity did not, however, prevent the coefficients on log P_t^f and log i_t^f from being significantly different from zero at the 95% level. The t value on the coefficient of log P_t^f is 3.24 and on the coefficient of log i_t^f is 2.20 when wholesale prices are used. Similar t values are obtained when consumer prices are used. One procedure for avoiding multicollinearity is to reestimate the equations keeping only log P_t^f on the right-hand side.[12] When this procedure is employed, the results for wholesale prices are as follows:

$$\Pi_{gt} = \underset{(2.4391)^*}{-2.1290} + \underset{(2.4972)^*}{0.4199} \log P_t^f$$

$$DW = 1.8774 \qquad R^2 = 0.2133 \qquad F = 6.24^* .$$

The results using consumer prices are almost identical to these. While the coefficients on the price variables are significant, they are not equal to unity. The predicted change in the inflation rate of the price of gold Π_{gt} between the second and third quarters of 1980 is 18.15% using the WPI and 24.36% using the CPI. Thus, when expected prices alone are used as independent variables, the resulting estimates of Equation (8) also correctly predict the direction of the change in the price of gold, but the figures using the CPI still overestimate the magnitude of the actual change (viz., 19.12%).

Thus far, no allowance has been made for risk. The uncertainty of the return on gold is focused upon[13] by measuring risk as the deviation from trend using a procedure similar to that employed by Malkiel and Cragg (1970). Trend values are obtained in the first stage by a regression of the price of gold on time. In the second stage the deviation from trend is measured as the difference between the trend and the actual values of the price of gold. The same procedure is employed to obtain an alternative measurement

(Note cont. from p. 79)
gold was extremely volatile during 1979 and 1980. Other factors may have been operating to offset the impacts of expectations of interest rates and wholesale and consumer prices on the price of gold.

[12]See Kolluri (1981) in this connection.

[13]I am indebted to one of the referees for this suggestion.

of risk by estimation of the deviation from trend in the quarterly inflation rates of the price of gold. When risk R_t is employed as an additional explanatory variable by use of the foregoing measurements, the results using the deviation from trend in the price of gold to measure R_t and the CPI to measure P_t^f are as follows:

$$\Pi_{gt} = \underset{(4.4681)^*}{-3.6856} + \underset{(4.5299)^*}{1.2704} \log (P_t^f/i_t^f) - \underset{(2.7697)^*}{0.0006R_t}$$

$$DW = 2.4760 \qquad R^2 = 0.5125 \qquad F = 11.56^* .$$

The results using the deviation from trend in the quarterly inflation rates of the price of gold to measure R_t and the CPI to measure P_t^f are as follows:

$$\Pi_{gt} = \underset{(4.8573)^*}{-1.8696} + \underset{(4.9867)^*}{0.6526} \log (P_t^f/i_t^f) - \underset{(10.7756)^*}{0.8827R_t}$$

$$DW = 0.7860 \qquad R^2 = 0.8953 \qquad F = 94.03^* .$$

The results using the WPI to measure P_t^f are very similar. The negative and significant coefficients on the risk variables R_t indicate that the greater the risk, the smaller the inflation rate of the price of gold. While the coefficients on the expected rate of growth of consumer (or wholesale) prices relative to interest rates are still positive and significant, they are no longer approximately equal to unity. This result is probably due to multicollinearity.

6. Conclusion

A model has been developed and tested to relate the inflation rate of the price of gold to expected consumer and wholesale prices and to expected interest rates through substitution effects.[14] An increase in expected future prices may induce individuals to convert their current liquid assets into gold. Increases in expected interest

[14]The theoretical argument developed in this paper has emphasizes the substitution effect rather than the wealth effect. The effect of changes in overall wealth (the summation of the assets in a portfolio) due to a rise or fall in asset prices on the demand for assets such as gold would be an interesting topic for future research.

rates will cause a negative adjustment of the price of gold. The model was used to correctly predict the direction of the change in the price of gold, although the magnitude of the change has been somewhat overestimated.

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