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Bayesian analysis of dynamic linkages among gold price, stock prices, exchange rate and interest rate in Pakistan

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A B S T R A C T

Understanding of complex relationships among economic variables has much significance for investors, researchers and policy makers alike. This study is conducted to examine dynamic linkages among gold price, stock prices, exchange rate and interest rate nexus using monthly data of Pakistan economy ranging from 2001-1 to 2014–12. Search of the best model motivates the use of Bayesian inference. Hence, the study is also used to compare performance of classical VAR model and Bayesian VAR model under four types of priors. It may be considered another significant contribution in terms of methodological framework. All the four variables are nonstationary at level but stationary at first difference. However, no long-run relationship among the variables is found by employing three cointegration tests, i.e. JJ test without considering structural break, GH test considering one unknown structural break and HJ test allowing two unknown structural breaks. Hence short-run relationship is analyzed in the study. Bayesian VAR model under independent normal inverse Wishart priors is selected as the best model which is then used to conduct impulse response analysis. Inverse bilateral relationships between stock prices and gold price as well as between rupee value and gold price are determined where as positive bilateral relationship between stock prices and rupee value is explored. It implies that stock prices and rupee value move downward during recessionary periods but gold becomes more pretty and vice versa also hold. Hence, gold is not only considered as safe haven but it is also considered as an alternative investment during adverse fluctuations in stock and foreign exchange markets of Pakistan. Changes in stock prices and rupee value are negatively responded by monetary policy makers in Pakistan but changes in gold prices are not considered by monetary authorities. Results also reveal that monetary policy actions significantly and adversely affect the three markets under consideration. Direct relationship between nominal interest rate and exchange rate in Pakistan conform to the International fisher effect theory.

1. Introduction

For a long time, gold has been the foundation of monetary systems and monetary authorities held sufficient gold reserves to convert all of the money into a specified quantity of gold on demand. With the Jamaica Agreement, IMF accepted managed floating exchange rate system. Under this system, currencies are fiat money as money supply is infinitely expandable and the gold price is determined by market forces, i.e. supply and demand. There are two types of demand for gold, i.e. use demand and asset demand. The asset demand for gold is based upon the view of investors including governments, fund managers and individuals that gold is an effective hedge or a safe haven against inflation and other forms of uncertainty like exchange rate fluctuations (Ghosh et al., 2004). That's why, the Federal Reserve (Fed) of USA interpret rising gold prices as indicator of inflationary pressure (Lastrapes and Selgin, 1995). Investors have to maintain their purchasing power in inflationary period and in the periods of exchange rates fluctuations and hence, they move towards precious metals market and hedge their portfolios with the purpose of saving their purchasing power. It implies that investors consider gold as a commodity that provides cushion against declining purchasing power. Gold market may also serve as an alternative source of investment during volatility in stock market if gold and stock are considered as substitutes to each other and many studies conclude inverse relationship between gold price and stock prices. Gold is considered as a safe haven investment in emerging economies when the stock market crashes or when the dollar weakens (Gaur and Bansal, 2010). Hence, analysis of interdependence of gold market, stock market and foreign exchange market may have much significance for investor's expectations and portfolio managers in a developing economy like Pakistan.

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Since monetary policy tools are used to stabilize the economy and hence policy makers are interested to understand the changes that are considered sensitive with respect to their objectives. Interest rate is the important monetary policy tool that is decided by the monetary authorities while considering inflation, investment level and fluctuations of exchange rate which have direct links with domestic gold price. Monetary authorities have to respond these fluctuations by changes in interest rate. In a small open economy like Pakistan which follows floating exchange rate system and gold is traded in US Dollar, fluctuations in exchange rate affects gold prices. However, segregation of the effect of exchange rate variations and internal market factors upon gold prices is much important for gold market as well as for monetary authorities in Pakistan. Moreover, response of monetary authorities to these fluctuations in exchange rate and gold prices by changing level of interest rate may also have a strong impact on stock market. Hence, gold price, interest rate, exchange rate and stock prices can be considered an important nexus to be analyzed. The relationship of the nexus is getting attention due to their important role in financial markets. Current fluctuations in gold prices and exchange rates, investments in gold instead of other securities, the choice of gold in using savings in some countries and large fluctuations in stock markets have motivated the economic policy managers to develop their economic policies based upon the understanding of these variations. Hence, understanding the nexus of domestic gold price, stock prices, exchange rate and interest rate has much significance for policy makers in stabilizing the Pakistan economy as well as for portfolio managers.

To the best of our knowledge, no study exists in the literature that contains analysis of the nexus under consideration using the data of Pakistan economy. Moreover, there are a number of studies that have analyzed relationships among gold price, exchange rates and other macroeconomic variables for different economies. Literature shows that results of interrelationship among the variables of the nexus varied and mixed. VAR modeling under classical framework is used for forecasting and analysis of linkages among these variables. However, Bayesian VAR model may be considered superior to classical VAR model as it overcomes the problem of over fitting by incorporating prior information about parameters along with sample information. The priors act as restrictions on coefficients which cause shrinking and sharpening the estimates and hence, Bayesian VAR may give more accurate estimates as compare to classical VAR model (See Litterman (1986), Robertson and Tallman (1999); Sims, 2007). Application of Bayesian econometric inference has not been much common in the literature due to the reason that it requires a laborious work of simulations along with mathematical derivations of probability density functions and specification of priors. However, advent of latest computers have made easier to conduct MCMC simulations and hence, better results may be obtained by using Bayesian econometric techniques.

In light of the above discussion, this study is conducted to analyze dynamic relationships among variables of the nexus by employing cointegration analysis, classical VAR model and Bayesian VAR model. For estimation of Bayesian VAR model, the Minnesota Prior has been widely used for forecasting. However, in some cases, the Minnesota prior is not the best prior that can be used. Hence, four types of priors including Minnesota priors are studied in this study. Evaluation of comparative forecasting ability of the VAR model under classical and Bayesian econometric framework is one of the major objectives of this study. It may be interpreted as testing the truth affirmation of Granger (1986) "a good Bayesian will beat a non-Bayesian, who will do better than a bad Bayesian". Best VAR model selected on the basis of forecasting performance is then used to analyze dynamic relationships among the variables of the nexus. Hence, this study explores dynamic linkages of gold price, stock prices, exchange rate and interest rate in Pakistan using more sophisticated technique as compare to previous studies. The remainder parts of the study are divided into 4 sections. Section 2 presents review of literature regarding the topic. Section 3 describes material and methodology and section 4 presents empirical results. Finally, the study is concluded in Section 5.

2. Literature review

A number of studies have been conducted to analyze causal relationships among macroeconomic and financial variables including exchange rate, gold price, stock prices and interest rate for different economies. Dooley et al. (1995) proves long-run relationship between gold prices and exchange rates in different countries. Laughlin (1997) and Pravit (2009) prove that changes in different economic and financial variables affect gold prices but changes in gold prices have no effect upon other economic variables. Ghosh et al. (2004) estimates a long-run relationship between the nominal gold price and the US retail price index. Capie et al. (2005) finds negative relationship between gold price and Yen-Dollar exchange rates. Levin and Wright (2006) finds positive relationship among gold prices, US inflation and credit risk. Moreover, the study's findings prove that gold is considered as a long-term hedge against inflation and uncertainty in the major gold consuming countries such as China, India, Turkey and Indonesia. Sjaastad (2008) examines the theoretical and empirical relationships between the major exchange rates and price of gold and finds that floating exchange rate system has been a major source of price instability in the world gold market. These views are supported by Beckers and Soenen (1984), Sjaastad and Scacciavillani (1996), Capie et al. (2005) and Wang and Lee (2011). Han et al. (2008) findings indicate that Australian dollar, US dollar and gold prices have long-run as well as short-run relationship to each other. Accordingly, one might prefer holding gold to protect against a loss in the purchasing power of the paper currency. Sari et al. (2010) finds only short-run linkages between precious metals' prices, exchange rate and oil price. Ismail (2009) reveals that exchange rate, inflation, money supply and stock market index have significant impact upon gold price but gold price does not affect these variables. Sujit and Kumar (2011) prove that gold price significantly affect different exchange rates but exchange rates have no effect upon gold prices. Yahyazadehfar and Babaie (2012) demonstrates that house price, interest rate, gold price and stock market return in Iran have only short-run effects upon each other. Hussain et al. (2013) found bi-directional association among stock returns and oil price in Malaya. Jain and Ghosh (2013) shows casual linkages among global oil prices, precious metals and Indian rupee-US dollar. The study concluded that exchange rate does not granger cause gold price. Sinton (2014) finds that there is no long-run association among gold price, stock price and exchange rate in Indonesia. Bhunia and Pakira (2014) found no causal relationship among stock market and gold price as well as among exchange rate and stock-market whereas bidirectional association was found among gold price and exchange rate in India. Srinivasan and Prakasam (2015) investigated casual relationship among stock price, gold price and foreign exchange rate in India by applying ARDL approach. The study found long-run relationship of gold price and stock price with exchange rate but no indication of cointegration and causality among stock price and gold price. Tiwari and Gupta (2015) found no casual relationship among gold price and Bombay Stock Exchange in India. Raza et al. (2016) examined the impact of gold price and oil price volatilities on stock market and found that gold and oil price volatilities have negative impact on stock market of all emerging economies. Jain and Biswal (2016) explored the linkage among global gold price, oil price, exchange rate and stock market in India. The study found that fall in gold and oil price cause to fall in Indian rupee and stock market.

Some studies on the topic related to Pakistan economy are also available in the literature. Irshad et al. (2012) examined the relationship among Stock price, oil price and gold price for Pakistan and concluded that no causal relationship exists among the variables. Mushtaq et al. (2012) examined linkages between stock market instability and macroeconomic instability by using EGARCH and LA-VAR model with respect to Pakistan. The study found bidirectional relationship among foreign investment and stock market as well as among currency value and stock market. Aslam (2014) found bidirectional relationship between KSE 100 index and Exchange rate in Pakistan. Tabassum and Gulzar (2015) conducted an analysis of the relationship among Exchange rate, oil price, stock prices and foreign reserves in Pakistan. The study finds that oil price and stock price granger cause exchange rate, exchange rate cause foreign reserves, stock price cause oil price and foreign reserve cause stock price.

Review of literature shows that a number of studies have been conducted to explore short-run and long run equilibrium relationships among gold price, stock prices, exchange rates and interest rate for different countries and a mix conclusion about the relationships emerges. Econometric analysis of all these studies bases upon classical econometric framework. The present study tries to explore dynamic relationships among the variables of the nexus by using more sophisticated Bayesian econometric technique. Moreover, published literature does not contain such study that contains such analysis using data of Pakistan economy. Hence, the present study may have a significant contribution in terms of methodological framework as well as for analyzing dynamic linkages of the variables under consideration using data of a Pakistan economy.

3. Material and methods

The nexus under consideration consists of Gold Price, Stock Prices, Exchange rate and Interest rate. Objective of the study is to analyze dynamic inter-linkages of fluctuations among gold market, stock market and foreign exchange market and real sector of the economy like GDP etc. is not being considered in this study. Specific focus is to analyze hedging or safe haven status of gold market during periods of financial market instability on the basis of investors' point of view while considering gold as a financial asset. Usually, portfolio managers and investors make short-term investments in financial assets and trends of nominal prices of financial assets are taken into account during adverse fluctuations. Besides, nominal interest rate is considered in order to analyze linkages of fluctuations in the three markets with monetary policy actions because expected inflation is embedded within the nominal interest rate. Moreover, the variables selected for the study do not affect economic wellbeing directly rather the effect is through investment and therefore, nominal data is more appropriate. Hence, in line with Sujit and Kumar (2011), Chang et al. (2013), Beckmann et al. (2015), Raza et al. (2016), Aye et al. (2016), Jain and Biswal (2016), Zhu et al. (2018) and Bouri et al. (2017), the study uses data of nominal gold price in rupees per troy ounce, nominal exchange rate in rupees per US dollar, KSE 100 index of stock prices and nominal discount rate. Average Monthly data ranging from 2001-1 to 2016-8 are taken from different sources. The data of domestic gold price (G) in rupees are collected from Karachi gold market while data of exchange rate (E) and interest rate (R) are taken from State Bank of Pakistan. Monthly average stock price (S) data represented by KSE-100 index are taken from Market Statistics Historical indices.

Data from 2001-1 to 2014-12 are used for estimation purpose. Although the most recent data is available, yet the actual data from 2015-1 to 2016-8 are used to establish validity of the estimated model by evaluating out-of-estimation sample forecasting performance. It is because the most important diagnostic test of VAR framework is to make a comparison of out-of-sample forecasts with actual data.

Fig. 1 reflects the patterns of all series over the period of study. Stock prices show overall increasing trend with a sharp decline during 2008–09. Exchange rate also shows increasing trend with a jump during 2008–09 and a sharp spike in 2014. Similarly, interest rate shows both increasing and decreasing trends over the period of study. Level of interest rate remains at peak during 2009–10. Fluctuations can be observed in gold price. However, Gold price shows increasing trend up to 2012 and then declining trend after 2012. Financial year 2008–09 was a crucial year for Pakistan economy as graphical projection shows downturn in stock market, depreciation of exchange rate and sharp rise in nominal interest rate. However, gold price maintain its previous pattern during this period. It was the time when International financial crisis, war against terrorism and internal security threats badly affected the Pakistan economy.

For time series analysis, testing stationarity of the data is the first step. For this purpose, three unit root tests namely Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) are employed. Results of the three unit root tests presented in Table A (Appendix) show that all the four variables are non-stationary at level and stationary at first difference. Hence Cointegration is tested to determine long-run relationship among the variables of the nexus.

Johansen and Juselius (1990) cointegration test (JJ) is applied taking '1' as the optimum lag length on the basis of information criterion presented in Table C (Appendix). Results of JJ co-integration test presented in Table 1 found no evidence of cointegration among variables of the nexus. JJ cointegration test does not take in to account structural breaks which are most likely to occur in long time series data. Bai and Perron (1998) multiple structural break test is conducted on all the four series to check unknown structural breaks. Results of Bai-Perron tests presented in Table B (Appendix) show that there is no structural break in the series of stock prices and interest rate whereas at least one structural break persists in gold price as well as exchange rate. Hence threshold cointegration tests proposed by Gregory and Hansen (1996), and Hatemi-j (2008), i.e. (GH) and (HJ) tests are also employed to examine the long run relationship of the nexus. These tests incorporate regime changes endogenously with level and slope dummies. GH and HJ tests allow to incorporate one and two unknown structural breaks respectively. Taking gold price as the dependent variable, GH and HJ models are defined as eq. (1) and eq. (2) respectively.

$$\mathrm{G_t} = \alpha_{11} + \alpha_{12}\,\mathrm{D_{1t}} + \beta_{11}\mathrm{S_t} + \beta_{12}\mathrm{D_{1t}S_t} + \beta_{13}\mathrm{E_t} + \beta_{14}\mathrm{D_{1t}E_t} + \beta_{15}\mathrm{R_t} + \beta_{16}\mathrm{D_{1t}R_t} + \epsilon_{1t} \tag{1}$$

$$\mathrm{G_t} = \alpha_{21} + \alpha_{22}\mathrm{D_{1t}} + \alpha_{23}\mathrm{D_{2t}} + \beta_{21}\mathrm{S_t} + \beta_{22}\mathrm{D_{1t}S_t} + \beta_{23}\mathrm{D_{2t}S_t} + \beta_{24}\mathrm{E_t} + \beta_{25}\mathrm{D_{1t}E_t} + \beta_{26}\mathrm{D_{2t}E_t} + \beta_{27}\mathrm{R_t} + \beta_{28}\mathrm{D_{1t}R_t} + \beta_{29}\mathrm{D_{2t}R_t} + \epsilon_{2t} \tag{2}$$

Here D_{1t} is the dummy variable representing endogenous structural break at time t = 1,2,3, ….,n for eq. (1) but for eq. (2), D_{1t} represent first endogenous structural break at time t = 1,2,3, ….,n., D_{2t} is the dummy variable representing second endogenous structural break at time t = 1,2,3, ….,n for eq. (2). These dummy variables are defined as follows.

$$\mathrm{D_{1t}} = 0;\ \text{if}\ \ t \le [n\tau_1]\ \textit{and}\ \mathrm{D_{1t}} = 1;\ \text{if}\ \ t > [n\tau_1]$$

$$\mathrm{D_{2t}} = 0;\ \text{if}\ \ t \le [n\tau_2]\ \textit{and}\ \mathrm{D_{2t}} = 1;\ \text{if}\ \ t > [n\tau_2]$$

Here τ_1 and τ_2 are two known parameters which belong to the set (0,1) implying the relative timing of structural break points. The bracket denotes the integer part. In order to test null hypotheses of no cointegration, bias-corrected modified ADF^*, Z_t^* and Z_α^* tests proposed by GH and HJ are employed by estimating eqs. (1) and (2) for each possible structural breaks $\tau \in T = (0.15, 0.85)$ for GH test and $\tau_1 \in T_1 = (0.15, 0.70)$ and $\tau_2 \in T_2 = (0.15 + \tau_1, 0.85)$ for HJ test. Modified ADF^*, Z_t^* and Z_α^* tests statistics are defined as follows.

$$ADF^* = \inf_{\tau \in T} ADF(\tau) \tag{3}$$

$$Z_t^* = \inf_{\tau \in T} Z_t^*(\tau) \tag{4}$$

$$Z_\alpha^* = \inf_{\tau \in T} Z_\alpha^*(\tau) \tag{5}$$

Eqs. (3)–(5) are then used for residuals of each possible structural break. Finally, the smallest values of (3), (4) and (5) are compared with the critical values given by Gregory and Hansen (1996) and Hatemi-j (2008) to test null hypotheses of no cointegration among the variables under study. Results of threshold cointegration tests presented in Table 2 show that the null hypothesis of no cointegration cannot be

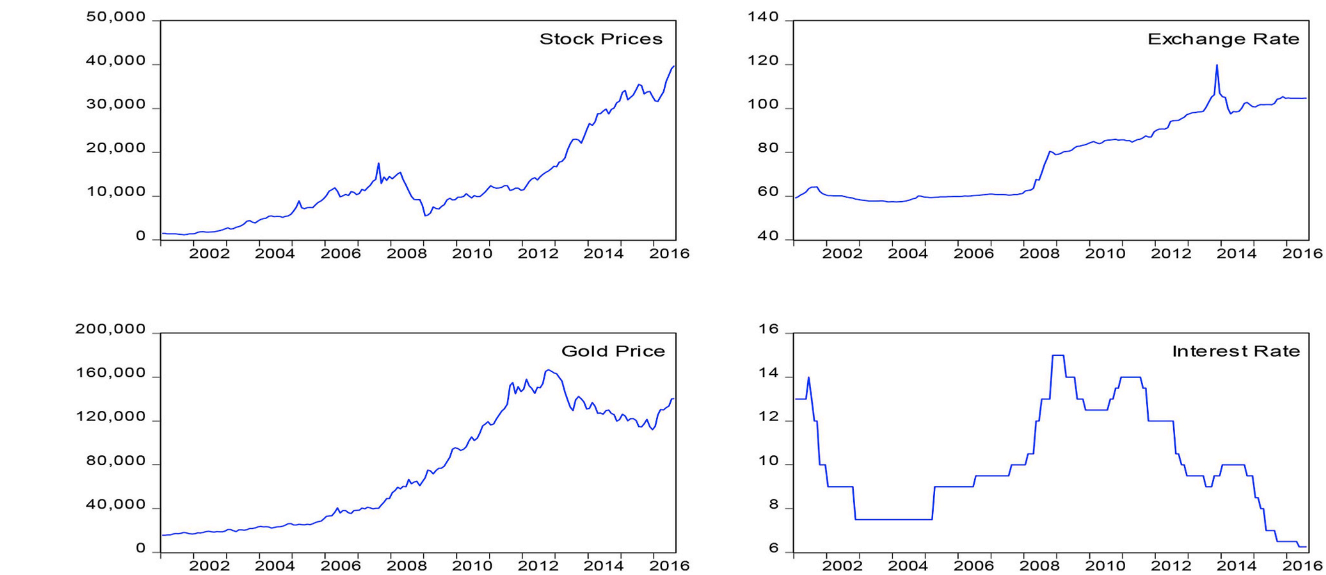

Fig. 1. Graphical Projection of the variables.

Table 1
Results of JJ Cointegration test.

Hypothesized No. of CE	λ_{Trace}	$P-Value$	$\lambda_{Max.Eign}$	$P-Value$
$r = 0$	34.90414	0.4531	17.80150	0.5117
$r = 1$	17.10264	0.6325	11.25882	0.6215
$r = 2$	5.843819	0.7139	5.163458	0.7211
r = 3	0.680361	0.4095	0.680361	0.4095

Table 2
Results of threshold cointegration test.[a]

Test Statistics	GH test	Critical values at 1% & 5%	HJ test	Critical values at 1% & 5%
ADF^*	−3.13	−6.89 & −6.32	−4.25	−7.83 & −7.35
Z_t^*	−3.65	−6.89 & −6.32	−5.11	−7.83 & −7.35
Z_α^*	−35.45	−90.84 & −78.87	−55.21	−118.58 & −104.86

[a] The critical values for GH and HJ tests are taken from Gregory and Hansen (1996) and Hatemi-j (2008) for m = 3.

rejected on the basis of modified ADF^*, Z_t^* and Z_α^* test statistics for GH as well as HJ test at 1% or 5% level of significance.[1] Hence, VAR model is estimated to explore short-run dynamic relationships among variables of the nexus.

Under classical inference of VAR model, each equation is estimated separately by OLS. VAR(p) model for Gold Price, Stock Price, Exchange rate and Interest rate can be specified as follows.

$$Y_t = \beta_0 + \sum_{j=1}^{p} \beta_j Y_{t-j} + U_t \tag{6}$$

Where Y_t is 4×1 vector of endogenous variables, β_0 is 4×1 vector of intercepts, β_j j = 1,2,3, …,p are 4×4 matrices of coefficients and U_t is 4×1 vector of disturbance terms where $U_t \sim N_4(0,\Sigma)$.Optimum lag length P = 1 is taken on the basis of various information criterion and hence, the four equations of VAR(1) model can be written as follows.

$$S_t = \beta_{01} + \beta_{11}S_{t-1} + \theta_{11}G_{t-1} + \delta_{11}E_{t-1} + \varphi_{11}R_{t-1} + \mu_{1t} \tag{7}$$

$$G_t = \beta_{02} + \beta_{21}S_{t-1} + \theta_{21}G_{t-1} + \delta_{21}E_{t-1} + \varphi_{21}R_{t-1} + \mu_{2t} \tag{8}$$

$$E_t = \beta_{03} + \beta_{31}S_{t-1} + \theta_{31}G_{t-1} + \delta_{31}E_{t-1} + \varphi_{31}R_{t-1} + \mu_{3t} \tag{9}$$

$$R_t = \beta_{04} + \beta_{41}S_{t-1} + \theta_{41}G_{t-1} + \delta_{41}E_{t-1} + \varphi_{41}R_{t-1} + \mu_{4t} \tag{10}$$

Alternatively, the above four equations can be written into matrix form as follows.

$$[S \quad G \quad E \quad R] = Z\begin{bmatrix}\beta_1 & \beta_2 & \beta_3 & \beta_4\end{bmatrix} + [\mu_1 \quad \mu_2 \quad \mu_3 \quad \mu_t]$$

$$Y = Z\Gamma + U \tag{11}$$

Here "Y" is $T \times 4$ matrix of observation on endogenous variables, "Z" is $T \times 5$ matrix of right hand side variables including unity column for constants, "Γ" is 5×4 matrix of model's coefficients and "U" is $T \times 4$ matrix of residuals where $U \sim N_4(0, \Sigma_{4\times4})$. Then, Least Square estimates of the parameters can be obtained by $\hat{\Gamma} = (Z'Z)^{-1}Z'Y$. Portmanteau test for residual autocorrelation, BGLM test for testing the serial correlation and stability test is applied to establish validity of the estimated VAR model. The estimated model is then used to make out-of sample forecasts from 2015-1 to 2016-8.

Classical inference is followed by Bayesian inference of the VAR model. Bayesian econometric framework consists of derivation of likelihood function, construction of prior distributions, derivation of posterior distribution, simulation of parameters' estimates, derivation of posterior predictive density and conduct of simulations for forecasting and for impulse responses. In order to derive likelihood function, we consider the m dimensional VAR(P) model where m = 4, i.e.

$$Y_{T\times m} = Z_{T\times k}\Gamma_{k\times m} + U_{T\times m} \tag{12}$$

Where $k = mp+1$ be the number of parameters estimated in each equation with p lag terms and "T" is the number of observations. By stacking the VAR coefficients matrix into a vector, the model in equation (12) may be written as follows.

$$y_{mT\times 1} = (I_m \otimes Z)\gamma_{q\times 1} + u_{mT\times 1} \tag{13}$$

Where $q = mk$. Assuming that $u \sim N(0, \Sigma \otimes I_T)$ then the likelihood function can be written as follows.

[1] GAUSS 10.0 software is used to apply GH and HJ threshold cointegration tests. The code for GH test is taken from http://www.ssc.wisc.edu/~bhansen/ and the code for HJ test is taken from Hatemi J A.

$$P\left({}^{y}/_{\gamma,\ \Sigma}\right) \propto |\Sigma \otimes I_T|^{-\frac{1}{2}} \exp\left[-\frac{1}{2}\left\{(y-(I_m \otimes Z)\gamma)'(\Sigma \otimes I_T)^{-1}(y - (I_m \otimes Z)\gamma)\right\}\right]$$

After solving the above function for γ and Σ the final form of likelihood function is as follows

$$P\left({}^{y}/_{\gamma,\ \Sigma}\right) \propto |\Sigma|^{-\frac{k}{2}} \exp\left[-\frac{1}{2}\{(\gamma-\hat{\gamma})'(\Sigma^{-1} \otimes Z'Z)(\gamma-\hat{\gamma})\}\right] \times |\Sigma|^{-\frac{(T-k-m-1)+m+1}{2}} \exp\left[-\frac{1}{2}\mathrm{tr}(\Sigma^{-1}S)\right]$$

$$P\left({}^{Y}/_{\gamma,\ \Sigma}\right) \propto MN(\hat{\gamma},\ \Sigma \otimes (Z'Z)^{-1}) \times IW(S,\ T-k-m-1) \tag{14}$$

Where $\otimes$ is known as kronecker product[2] and $\hat{\gamma} = vec(\hat{\Gamma})$.

After derivation of likelihood function, prior distribution of model's parameters is specified in Bayesian framework. Four types of priors are taken in this study. These include Minnesota Prior, Non-Informative Normal-Inverse-Wishart (natural conjugate) Prior, Informative Normal-Inverse-Wishart (natural conjugate) Prior and Independent Normal-Inverse -Wishart Prior. Hence, four posterior distributions are derived by multiplying likelihood function with each of the prior distribution.

Minnesota prior firstly used by researchers at the University of Minnesota (see Doan et al., 1984; Litterman, 1986) allows variance-covariance matrix of residuals to be fixed and diagonal calculated from univariate autoregressive model for each equation of the model. Model's parameters are assumed to follow multivariate normal distribution, i.e. $\gamma \sim MN(\gamma_0, \Psi_0)$ and hence the following posterior distribution under Minnesota prior is derived.

$$P\left({}^{\gamma}/_{\Sigma,\ y,\ Z}\right) \propto \exp\left[-\frac{1}{2}(\gamma-\bar{\gamma})'\bar{\Psi}^{-1}(\gamma-\bar{\gamma})\right]$$

$$P\left({}^{\gamma}/_{\Sigma,\ y,\ Z}\right) \sim MN(\bar{\gamma},\ \bar{\Psi}) \tag{15}$$

Where $\bar{\gamma}_{(q\times 1)} = \bar{\Psi}[\Psi_0^{-1}\gamma_0 + \Sigma^{-1} \otimes Z'Z\hat{\gamma}]$ and $\bar{\Psi}_{(q\times q)} = [\Psi_0^{-1} + \Sigma^{-1} \otimes Z'Z]^{-1}$ Here Σ is assumed to be fixed whose diagonal elements, S1, S2, …., Sm, are taken as the variance of residuals by estimating autoregressive models for each series.γ_0 in Minnesota prior is taken as zero vector in this study because our VAR(1) model is estimated by using stationary variables. According to Litterman (1986), if the model has to be estimated by using nonstationary variables then γ_0 is taken so that the prior means for the parameters on own lag is set to unity and the prior means for other lag is set to zero. The idea is that the own lags contribute more in the explanation than the other lags. If we have to estimate the model by using stationary data then γ_0 is set as a null vector (see detail in Moreira et al. (2015), Koop and Korobilis (2010) and Blake and Mumtaz (2012)). The prior variance (Ψ_0) of the parameters in equation 'i' is specified as follows.

$$\mathrm{Var}(\gamma_{ij}) = \begin{Bmatrix} \frac{\pi_1}{p} & \text{i= j for parameters on own lag} \\ \frac{\pi_2\sigma_i^2}{p\sigma_j^2} & \text{i}\neq\text{ j for parameters of other lag} \\ \pi_3\sigma_i^2 & \text{for constant} \end{Bmatrix}$$

Where p denote the lag length of the VAR model, σ_i^2 is the residual variance of the p-lag univariate autoregressive model of the variable 'i', π_1 represents the whole shrinkage on the variance of first own variable lag and importance of the data and previous information, π_2 represents the tightness of the variance of other variable and π_3 tightness on the variance of intercepts. The experiments is performed with various values for π_1, π_2 and π_3 and choose the values that provides better forecasts[3].

Second posterior distribution is derived by using Independent Normal-Inverse -Wishart Prior that allows the parameters of vector autoregressive model and matrix of residual's covariance term to be independent of each other i.e. $\gamma \sim MN(\gamma_0, \Psi_0)$ and $\Sigma \sim IW(S_0, v_0)$. Considering the likelihood function from eq. (14), posterior distribution is derived as follows.

$$P\left({}^{\gamma,\ \Sigma}/_{Z,\ y}\right) \propto \exp\left[-\frac{1}{2}(\gamma-\hat{\gamma})'(\Sigma^{-1} \otimes Z'Z)(\gamma-\hat{\gamma})\right] \times \exp\left[-\frac{1}{2}(\gamma-\gamma_0)'\Psi_0^{-1}(\gamma-\gamma_0)\right] \times |\Sigma|^{-\frac{T}{2}} \exp\left[-\frac{1}{2}\mathrm{tr}(\Sigma^{-1}S)\right] \times |\Sigma|^{-\frac{(v_0+m+1)}{2}} \exp\left[-\frac{1}{2}\mathrm{tr}(\Sigma^{-1}S_0)\right] \tag{16}$$

Simplifying the above function, final form of posterior distribution under independent Normal-Inverse Wishart prior is obtained as follows.

$$P\left({}^{\gamma,\ \Sigma}/_{Z,\ y}\right) \propto \exp\left[-\frac{1}{2}(\gamma-\bar{\gamma})'\bar{\Psi}(\gamma-\bar{\gamma})\right] \times |\Sigma|^{-\frac{T+v_0+m+1}{2}} \exp\left[-\frac{1}{2}\mathrm{tr}(\Sigma^{-1}\bar{S})\right]$$

$$P\left({}^{\gamma,\ \Sigma}/_{Z,\ y}\right) \sim MN(\bar{\gamma},\ \bar{\Psi}) \times IW(\bar{S},\ T+v_0) \tag{17}$$

Where $\bar{\Psi} = (\Sigma^{-1} \otimes Z'Z + \Psi_0^{-1})^{-1}$, $\bar{\gamma} = \bar{\Psi}(\gamma_0\Psi_0^{-1} + \Sigma^{-1} \otimes Z'Z\hat{\gamma})$, $\bar{S} = S_0 + S$ and $S = (Y-\hat{Y})'(Y-\hat{Y})$.

The prior mean and the prior variance-covariance matrix for the model parameters in case of independent Normal-Inverse-Wishart prior are specified same as in the Minnesota prior. Any choice about covariance matrix of parameters may be adopted such that out of sample forecasting performance of the model may be best. Prior scale matrix of the inverse Wishart prior is taken as $S_0 = (v_0 - m - 1)\mathrm{diag}(S_1, S_2, S_3, S_4)$, Where $v_0 = m + 2$ and S_1, S_2, S_3, S_4 are the variances that are assumed fixed in the Minnesota Prior (for detail see Carriero et al. (2015), Nalban (2015), Kadiyala and Karlsson (1997) and Koop and Korobilis (2010)). Specification of hyper parameters in independent Normal-Inverse-Wishart prior is same as in Minnesota prior except that $\pi_2 = 1$ (specified based on forecasting performance of BVAR model under independent NIW prior). In this study, prior degree of freedom is set as 6 and the prior scale matrix for Inverse Wishart prior is set to diagonal whose diagonal elements are taken as $S_1 = 641541.3$, $S_2 = 11309841$, $S_3 = 3.036732$, $S_4 = 0.1809513$. These estimates of variances are obtained by estimating AR (1) model for each of the endogenous variable of the nexus.

Third posterior distribution is derived by using Normal-Inverse-Wishart (natural conjugate) prior. Kadiyala, and Karlsson, (1997) criticizes the specification of Litterman and allow the variance-covariance matrix Σ to be unknown and treat all the equations symmetrically. Normal-Inverse-Wishart prior is the conjugate prior which is the generalization of the Minnesota prior. For this type of priors, we assume Γ follows matrix variate normal distribution, i.e. $\Gamma \sim MN(\Gamma_0,\ \Sigma_{(m\times m)} \otimes \Psi_{0(k\times k)})$ and $\Sigma \sim IW(S_0, v_0)$. Considering the likelihood function of eq. (14), posterior distribution is derived as follows.

$$P\left({}^{\Gamma,\ \Sigma}/_{y,\ Z}\right) \propto |\Sigma|^{-\frac{k}{2}} \exp\left[-\frac{1}{2}\mathrm{tr}\{\Sigma^{-1}(\Gamma-\hat{\Gamma})'Z'Z(\Gamma-\hat{\Gamma}) + \Sigma^{-1}(\Gamma-\Gamma_0)'\Psi_0^{-1}(\Gamma-\Gamma_0)\}\right] \times |\Sigma|^{-\frac{(T+v_0+m+1)}{2}} \exp\left[-\frac{1}{2}\mathrm{tr}(\Sigma^{-1}S_0 + \Sigma^{-1}S)\right] \tag{18}$$

Simplifying the above function, final form of posterior distribution under Normal-Inverse-Wishart (natural conjugate) prior is derived as follows.

[2] Kronecker product is used for multiplication of matrix having different dimensions (see Lütkepohl (2005)).

[3] $\pi_1 = 0.05$, $\pi_2 = 0.99$, $\pi_3 = 10^4$ (diffuse prior for constant).

$$P\left(\Gamma, \Sigma/_{y,\,Z}\right) \propto |\Sigma|^{-\frac{k}{2}}\exp\left[-\frac{1}{2}\mathrm{tr}\{\Sigma^{-1}(\Gamma-\bar{\Gamma})'\bar{\Psi}^{-1}(\Gamma-\bar{\Gamma})\}\right]$$
$$\times|\Sigma|^{-\frac{T+v_0+m+1}{2}}\exp\left[-\frac{1}{2}\mathrm{tr}(\Sigma^{-1}\bar{S})\right]$$

$$P\left(\Gamma, \Sigma/_{y,\,Z}\right) \sim \mathrm{MN}(\bar{\Gamma},\ \Sigma\otimes\ \bar{\Psi})\times \mathrm{IW}(\bar{S},\ T+\ v_0) \tag{19}$$

Where $\bar{\Gamma} = \bar{\Psi}(\Psi_0^{-1}\Gamma_0 + Z'Z\hat{\Gamma})$, $\bar{\Psi} = (Z'\ Z+\ \Psi_0^{-1})^{-1}$,

$\bar{S} = S_0 + S + \hat{\Gamma}'Z'Z\hat{\Gamma} + (\Gamma_0'\Psi_0^{-1}\Gamma_0 - \bar{\Gamma}'\bar{\Psi}^{-1}\bar{\Gamma})'$ and $S = (\ Y-\ \hat{Y})'(\ Y-\ \hat{Y})$

Prior means of parameters are taken as same as in Minnesota prior and prior variances of parameters set the diagonal elements of Ψ_0 by setting π_2 equal to unity (as a thumb rule for natural conjugate NIW prior). Then the elements of prior variance-covariance matrix ($\Sigma_{(m\times m)} \otimes\ \ \Psi_{0(k\times k)}$) are taken as follows.

$$\mathrm{Var}(\gamma_i)\left\{\begin{array}{l}\left(\frac{\pi_1}{p}\right)^2\left(\frac{1}{\sigma_i^2}\right);\ \text{lag p for ith variable}\\ (\pi_1\pi_3)^2;\ \text{For constants}\end{array}\right\}$$

The prior scale parameters and degrees of freedom for the Inverse-Wishart distribution are taken as same as in independent Normal-Inverse-Wishart prior (see detail in Karlsson (2012) and Dieppe et al. (2016)).

Fourth type of priors are considered Non-Informative Normal-Inverse-Wishart (natural conjugate) Prior where eq. (19) can be considered as posterior distribution and the prior parameters are set as $\pi_1 = \pi_2 = \pi_3 = 10^4$ where as $v_0 = 0$ and S_0 is fixed diagonal variance covariance matrix i.e if the prior variance for the model coefficients is set to large then the posterior means approach to OLS estimates.

Specified values of hyper-parameters in all of the above cases are given in Table 3. After derivation of posterior distribution and specification of hyper-parameters, the next step in Bayesian econometric inference is to simulate parameters of the model. For this purpose, MCMC simulations using Gibbs sampling algorithm are conducted to estimate parameters of the model specified as eq. (13). For this purpose, simulations are conducted using derived posterior distributions as eqs. (15), (17) and (19). As we have closed form posterior distributions, there is no need to check convergence of simulated estimates.

After estimation of the model, out-of sample forecasts are obtained for the periods from 2015-1 to 2016-8 in order to evaluate forecasting performance of the estimated models. In order to obtain h-step forecasts from the model, posterior predictive distribution is required. The posterior predictive distribution is the integrated product of posterior distribution and distribution of future observations conditional on data and parameters. Consider the model from eq. (13), the posterior predictive in the form of integrated product is represented as follows.

$$f(y_{T+1:T+h}/y_t) = \iint f(y_{T+1:T+h}/y_t,\ \gamma,\ \Sigma)f(\gamma,\ \Sigma/y_t)d\gamma d\Sigma$$

Out-of-sample forecasts are obtained from posterior predictive distribution conducting MCMC simulations using algorithm explained by Karlsson (2012). Hence, out-of-sample forecast estimates of the four variables from 2015-1 to 2016-8 are obtained by using the four estimated Bayesian VAR models as well as classical VAR model. Then, Root Mean Square Errors are calculated to compare forecasting performance of the estimated models. Best model is selected on the basis of minimum MSE. The selected best model is then used to analyze dynamic relationship among the variables of the nexus by conducting impulse responses on the basis of Posterior Predictive distribution.

4. Results and discussion

VAR(1) model is estimated using classical approach as well as Bayesian framework under different priors. VAR model may be estimated using stationary as well as non-stationary data. As our purpose is to analyze dynamic relationships among the variables of the nexus on the basis of impulse responses, we use first difference data series due to two advantages. Firstly, it tells about increasing or decreasing trend rather than the actual change in variables by the impulse response function. Secondly, it focuses more on the shocks as first difference data represent changes in the past two time periods. Five types of VAR(1) model are estimated in this study. Estimation sample consists of the period from 2001:01 to 2014:12. Under classical framework, VAR(1) is estimated by using OLS. CUSUM test presented in Fig. A (Appendix) shows stability of the estimates as the lines are not crossing the confidence limits. It also implies that there is no structural break in data used for analysis. VAR stability test presented in Fig. B (Appendix) shows that all roots lie within unit circle and hence the estimated VAR model is stable. Other diagnostic tests of classical VAR model presented in Table D (Appendix) also establish validity of the model. Estimated results are not presented here as estimates have no structural interpretation. Out-of-sample forecasts from 2015-1 to 2016-8 are made from the estimated VAR model under classical framework.

Bayesian VAR model of the nexus is estimated using four types of priors, i.e. Minnesota prior, Independent Normal Inverse Wishart prior, Normal Inverse Wishart prior and Non-informative Normal Inverse Wishart prior. These four models are estimated by using specification of hyper parameters given in Table 3. Posterior distributions derived as eq. (15), eq. (17) and eq. (19) are used to estimate Bayesian VAR model under Minnesota priors, Independent Normal Inverse Wishart prior and Natural Conjugate Normal Inverse Wishart priors respectively. Posterior distribution derived as eq. (19) is also used to estimate BVAR under Non-informative Normal Inverse Wishart prior. MCMC simulations are conducted using Gibbs sampling algorithm to estimate parameters. For Minnesota priors, 20000 MCMC simulations are conducted while 10000 simulations are burned. For other three models, 15000 MCMC simulations are conducted and first 1500 simulations are burned. The four estimated Bayesian VAR models are used to make 20 step ahead forecasts, i.e. from 2015-1 to 2016-8. Forecasts from each of the BVAR model are obtained from the posterior predictive distributions by algorithm explained in section-(3). The use of Minnesota prior simplifies the computational process as the researchers only specify prior for model coefficients. This prior has analytical closed form for posterior and predictive for one step. From the predictive distribution

Table 3
Specification of hyper parameters.

	Priors	Hyper parameters
Minnesota	$\gamma \sim \mathrm{MN}(\gamma_0\ ,\ \Psi_0)$	γ_0 = Null Vector $\pi_1 = 0.05,\ \pi_2 = 0.99,\ \pi_3 = 10^4$
Independent NIW	$\gamma \sim\ \mathrm{MN}(\gamma_0\ ,\ \Psi_0)\Sigma \sim \mathrm{IW}(S_0\ ,\ v_0)$	γ_0 = Null Vector $\pi_1 = 0.05,\ \pi_2 = 1,\ \pi_3 = 10^4,\ v_0 = 6,\ S_1^2 = 641541.3,\ S_2^2 = 11309841,\ S_3^2 = 3.036732,$ $S_4^2 = 0.1809513$
Natural Conjugate NIW	$\Gamma/_{\Sigma}\ \sim\ \mathrm{MN}(\Gamma_0\ ,\ \Sigma\otimes\ \Psi_0)\Sigma \sim \mathrm{IW}(S_0\ ,\ v_0)$	Same as for Independent NIW
Non-informative Natural Conjugate	$\Gamma/_{\Sigma}\ \sim\ \mathrm{MN}(\Gamma_0\ ,\ \Sigma\otimes\ \Psi_0)\Sigma \sim \mathrm{IW}(S_0\ ,\ v_0)$	Γ_0 = Null matrix, $\pi_1 = \pi_2 = \pi_3 = 10^4 v_0 = 0,\ S_1{}^2 = 641541.3,\ S_2{}^2 = 11309841,\ S_3^2 = 3.036732,\ S_4^2 = 0.1809513$

Table 4
Results of root mean square errors and mean absolute percentage errors.[a]

Model/Variable	S	G	E	R
OLS –VAR	2054.469 [0.046454]	8106.243 [0.05312]	0.774183 [0.006234]	2.457597 [0.356459]
BVAR-1 (Minnesota prior)	1909.914 [0.046002]	7581.062 [0.050837]	0.768622 [0.005923]	2.724376 [0.393809]
BVAR-2 (independent NIW)	1909.165 [0.04538]	7414.453 [0.050168]	0.760328 [0.00622]	1.680956 [0.243295]
BVAR-3 (natural conjugate NIW)	2029.670 [0.045806]	7921.226 [0.051609]	0.763733 [0.005892]	2.452306 [0.355714]
BVAR-4 (non-informative prior)	2051.136 [0.046394]	8055.165 [0.052741]	0.765135 [0.006077]	2.434787 [0.353169]

[a] MAPE are given in brackets.

20000 samples are drawn from which first 10000 are discarded as burn in. The BVAR model based on independent Normal-Inverse-Wishart prior is computationally demanding as described earlier. We therefore use Gibbs sampling to obtain the posterior distribution for this prior. The Gibbs sampler is used to obtain 15000 draws from the predictive density in which 1500 are discarded as burn in. For the last two priors the family of prior distribution is same but the values of prior parameters are different. The natural conjugate prior has the analytical closed form of posterior and predictive distribution. 15000 samples are drawn from the predictive density by using Gibbs sampling in which first 1500 are discarded as burn in. Same numbers of iterations are used for both informative and non-informative natural conjugate prior. After obtaining forecasts from each of the estimated model, root mean square errors and mean absolute percentage errors are calculated by comparing actual and forecasted values for the period 2015-1 to 2016-8 and the results are presented in Table 4. Comparison of RMSE and MAPE shows that the four Bayesian VAR models perform better than the estimated VAR model under classical framework. Moreover, Bayesian VAR model under independent Normal Inverse Wishart prior is selected as the best estimated model because values of RMSE as well as MAPE under this model are minimum as compare to other competing models.

The best selected model is used to calculate impulse responses in order to examine the dynamic relationships of the nexus. Impulse response functions trace out the dynamic responses of all the variables in the system to a unit positive shock in each variable. Impulse responses are generated for 20 periods on the basis of posterior predictive density using algorithm explained in section 3 of the study.

Graphical presentation of impulse responses given in Fig. 2 reveals that one unit positive shock in stock prices is negatively responded by gold price, exchange rate and interest rate. These responses start in first month and end at 3rd or 4th month. One unit positive shock in gold price causes adverse fluctuations in stock prices during 2nd month but it ends after 3rd month. Exchange rate depreciation occurs in response to one unit shock in gold price. Hence, a rise in gold prices causes devaluation of rupee and adverse fluctuations in exchange rate can be observed up to 5th month. The response of interest rate to shock in gold price is insignificant. A unit shock in exchange rate represents devaluation of rupee value. One unit depreciation of exchange rate causes fluctuations in stock prices as impulse responses show ups and downs in 2nd, 3rd and 4th months coming to normal point in 5th month. Gold price tend to decline in response to depreciation of exchange rate converging to the normal value after 04 months. Moreover, one unit shock in exchange rate causes an increase in interest rate as monetary authorities respond to currency devaluation by raising level of interest rate in the economy. It implies that exchange rate devaluation negatively affects stock market as well as gold market where as monetary authorities seem to respond exchange rate shock by raising level of interest rate in Pakistan. Hence, we may verify that State Bank of Pakistan practices a basic interest rate above its normal value in response to devaluation immediately converging to the normal value after 04 months. One unit positive shock in interest rate causes decline in stock prices and gold prices whereas response of exchange rate to a shock in interest rate shows devaluation. These effects of one unit rise in interest rate upon stock market, gold market and foreign exchange market remains up to 4th month. Short-run dynamic relationships of the nexus can be summarized as follows.

Stock prices and gold prices seem to have inverse bilateral relationship. It is because gold prices had no portfolio risk aversion value and stock investments emerge as risk deterrent during stock market uncertainty. It shows that investors in Pakistan consider stock market and gold market as substitute for their investments. Moreover, the study shows negative bidirectional relationship between stock market and exchange rate in Pakistan. It is because stock prices as well as exchange rate reflect health of a country's economy. When stock market is performing well, international investors make their investments and convert their domestic currency into rupee in order to participate in equity market rising. It causes appreciation of rupee value in terms of dollar. The opposite also holds true. Moreover, depreciation of local currency depresses the stock market and appreciating exchange rate boosts the stock market. As to the negative currency effect on the stock market, exchange rate depreciation suggests higher inflation in the future, which makes investors skeptical about the future performance of companies. As a result, the stock prices tend to decline. It implies that stock prices and rupee value move downward during recessionary periods but gold becomes more pretty and its value increases. Where as stock prices and rupee value move upward during boom periods in the economy and hence, gold prices start decline.

Relationships of gold market and exchange rate seem to be more critical. Appreciation of exchange rate causes a rise in gold prices and depreciation of exchange rate causes a decline in gold prices. It is because when devaluation of foreign currency starts, investors holding foreign assets move their investments to gold market which raises gold price in Pakistan. When depreciation of local currency occurs in Pakistan, it attracts investors of stock and gold markets to shift their investments for holding foreign currency and hence stock prices and gold prices start to decline. These findings may also help the portfolio managers, who would be willing to include major commodities into their investment portfolios because commodities can be used as hedging instruments against unexpected inflation. However, a rise in gold price causes devaluation of local currency due to rise of gold demand as compare to local currency and gold price decline appreciates rupee in terms of US dollar due to decline of gold demand.

Impulse responses reveals that changes in stock prices and rupee value are negatively responded by monetary policy makers in Pakistan but changes in gold prices are not considered by monetary authorities. It is because improvement in stock market is accompanied with appreciation of exchange rate which is responded by easy monetary policy. On the other hand, devaluation of local currency causes decline in stock prices which is responded by tight monetary policy in Pakistan. Results also reveal that monetary policy actions significantly affect the three markets under consideration. Impulse responses show that tight monetary policy action adversely affects stock prices, gold price as well as rupee value in terms of foreign currency. It implies that a rise in interest rate negatively affects investments and investors respond it by shifting their investments to bank deposits or to other economies. Hence gold price as well as stock prices show declining trend in response to a unit rise of interest rate. Moreover, tight monetary policy action causes devaluation of local currency. According to Frenkel (1976, 1979), changes in the nominal interest rate reflect changes in the expected inflation rate if prices are assumed perfectly flexible. When

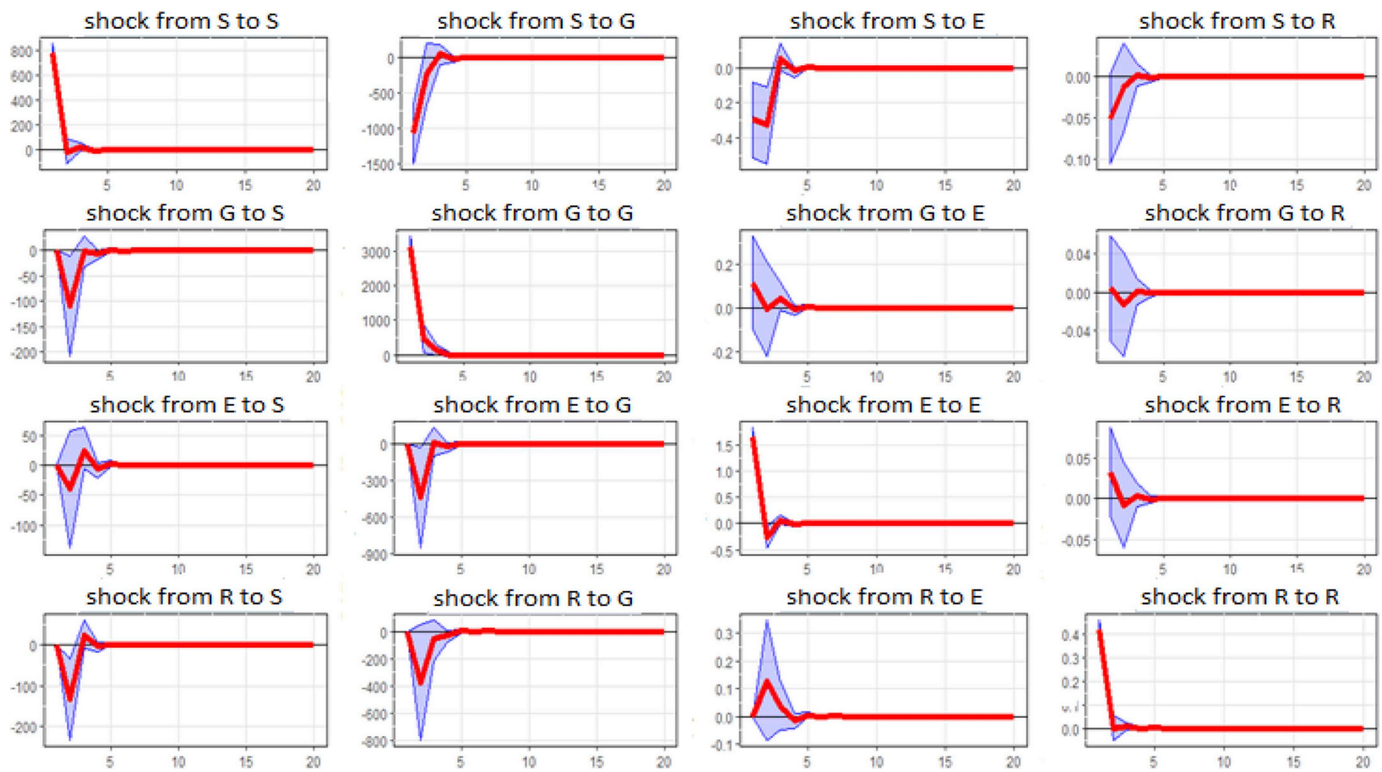

Fig. 2. Impulse responses function (IRF).

the domestic interest rate rises relative to the foreign interest rate, demand for the domestic currency falls which causes depreciation instantly. Hence positive relationship between nominal interest rate differential and nominal exchange rate changes conforms to the International fisher effect theory.

Establishing the relationships among the variables of the nexus is important for a few reasons. Firstly, the established linkages among gold price, stock price and exchange rate may be helpful to predict the path of the exchange rate that may benefit multinational corporations and other foreign investors to foresee exchange rate risk for stabilizing their earnings in Pakistan. Secondly, currency and gold are included as an asset in investment funds' portfolios. Knowledge about the link between currency rates and other assets in a portfolio is vital for the performance of the fund. According to a number of studies including Khalid and Kawai (2003) and Ito and Hashimoto (2004), links between the stock and currency markets helped to propagate the Asian Financial Crisis in 1997. Hence, stock price-exchange rate relationship may be helpful to foresee a crisis and the established relationships between the two markets may be helpful for policy makers to take preventive measures before the spread of a crisis. Thirdly, the linkages are helpful for the conduct monetary policy in Pakistan.

5. Conclusion

Stock and foreign exchange markets are considered to be more sensitive to commodity prices due to interdependence of global markets and international financial integration. Therefore, investors are interested about the risk of equity investing due to unexpected fluctuations which motivates them to make investments in gold market. It is because gold is considered not only as safe haven to hedge during financial market instability but it is also considered as an alternative investment. Hence, analysis of interdependence of gold market, stock market and foreign exchange market may have much significance for investor's expectations and portfolio managers in a developing economy like Pakistan. Moreover, economic policy managers are also interested to understand these linkages to decide monetary policy steps. Hence, domestic gold price, stock prices, exchange rate and interest rate nexus has much significance for investors, portfolio funds managers and monetary policy makers. Review of literature shows a mix conclusion about the relationships of the nexus for different economies. All the previous studies base upon classical econometric framework. We motivate this research to explore dynamic linkages of gold price, stock prices, exchange rate and interest rate in Pakistan using Bayesian econometric technique.

Three unit root tests, i.e. ADF, PP and KPSS show that all the four variables are nonstationary at level and stationary at first difference. Bai-Perron test shows that there is one structural break in gold price and exchange rate but there is no structural break in stock prices and interest rate. Three cointegration tests are applied, i.e. JJ test, GH test allowing one unknown structural break and HJ test allowing two unknown structural breaks. No evidence of long-run relationship among the variables of the nexus is found. Hence, short-run dynamic relationships are explored by estimating VAR model using monthly data of Pakistan economy in differenced form ranging from 2001-1 to 2014–12. VAR model is estimated by using classical as well as Bayesian Econometric framework in order to compare performance of the two approaches. BVAR models are estimated using four types of priors. The study concludes that an inclusion of subjective prior shrinkage to the parameters of VAR model provide best forecasting for the variables of the nexus than classical approach. Another important finding is that Bayesian VAR model with independent Normal-Inverse-Wishart prior provide better forecasting than Bayesian VAR model under others priors including Minnesota priors. Short-run dynamic relationships among the nexus in Pakistan are explored on the basis of impulse response analysis. Impulse responses reveals that variations in stock market, gold market and foreign exchange market are inter connected to each other in Pakistan. Adverse fluctuations in exchange rate causes a decline in stock prices and gold price and vice versa also holds. Decline in stock market causes boom in gold market but it causes depreciation of rupee value and vice versa also holds. Decline in gold price exerts positive impact upon stock market as well as upon rupee value in terms of dollars. It is because some investments in gold market are shifted to stock market but some parts of investments are converted into local currency which results into appreciation of exchange rate. It may be concluded that gold is not only considered as safe haven to hedge against economic and financial risk in Pakistan but it is also considered as an alternative investment with greater sense of certainty during periods of financial market instability. Fluctuations in exchange rate and stock prices are responded by monetary policy actions which affect gold prices also but variations in gold prices are not considered by monetary policy makers in Pakistan. However, tight monetary policy exerts adverse impact on stock market, gold market as well as foreign exchange market in Pakistan. Hence, macroeconomic policy managers may consider changes of gold price as indicators of fluctuations and uncertainty in Pakistan economy.

Appendix

Table A
Results of Unit root tests.

Variables	ADF		Phillips-Perron		KPSS	
	Level [P-Value]	1st Difference [P-Value]	Level [P-Value]	1st Difference [P-Value]	Level [P-Value]	1st Difference [P-Value]
S	1.714332 [0.9997]	−7.796410* [0.0000]	1.752302 [0.9997]	−13.31532* [0.0000]	4.1179 [0.01]	0.48667* [0.09]
G	0.012849 [0.6853]	−2.-24921* [0.0414]	−0.570883 [0.8726]	−10.83060* [0.0000]	5.2906 [0.01]	0.26742* [0.1]
E	−2.429637 [0.3629]	−6.388832* [0.0000]	−2.095861 [0.5439]	−14.73999* [0.0000]	5.1063 [0.01]	0.1973* [0.1]
R	−2.025620 [0.2575]	−4.723794* [0.0001]	−1.750405 [0.4041]	−13.17826* [0.0000]	1.5892 [0.01]	0.24921* [0.1]

Table B
Bai-Perron Sequential multiple break point test.

Variables	F-statistic	Critical value	Decision
S	7.4724	8.58	No break point
G	49.064	8.58*	One break point at 2009:10
G at 1st difference	7.8174	8.58	No break point
E	11.8756	8.58*	One break point at 2008:09
E at 1st difference	4.0000	8.58	No break point
R	3.2958	8.58	No break point

* indicate the rejection of null hypothesis of no structural break.

Table C
Results of Lag Length Criteria.

	Akaik Information Criterion	Hannan Quinn Criterion	Swartz Criterion	Final Prediction Error test
1	2.894693e + 01*	2.910369e + 01*	2.933295e + 01*	3.72843e + 12*
2	2.899729e + 01	2.927946e + 01	2.969214e + 01	3.922597e + 12
3	2.903858e + 01	2.944616e + 01	3.004225e + 01	4.091953e + 12

Table D
Residual's Diagnostic Tests of Classical VAR model.

Residual's Tests	χ^2-statistic	d.f	p-value
Portmanteau test	252.06	240	0.2837
Brauch Godfrey LM test	92.923	80	0.1531

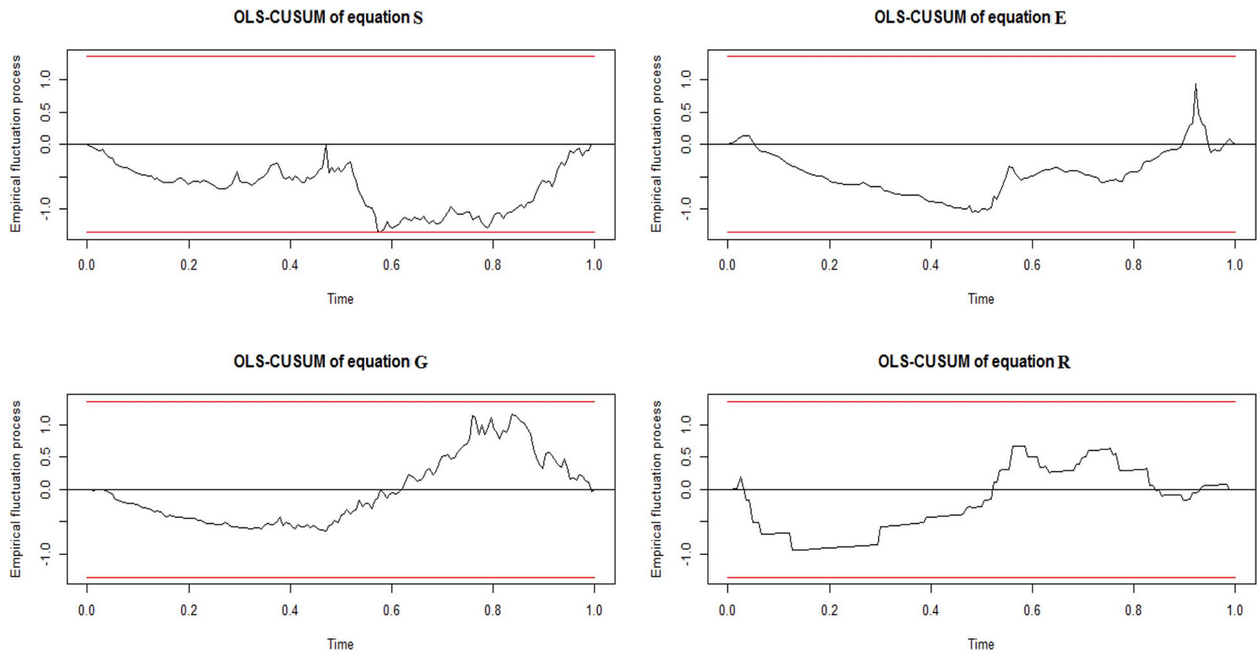

Fig. A. CUSUM Structural Stability plot.

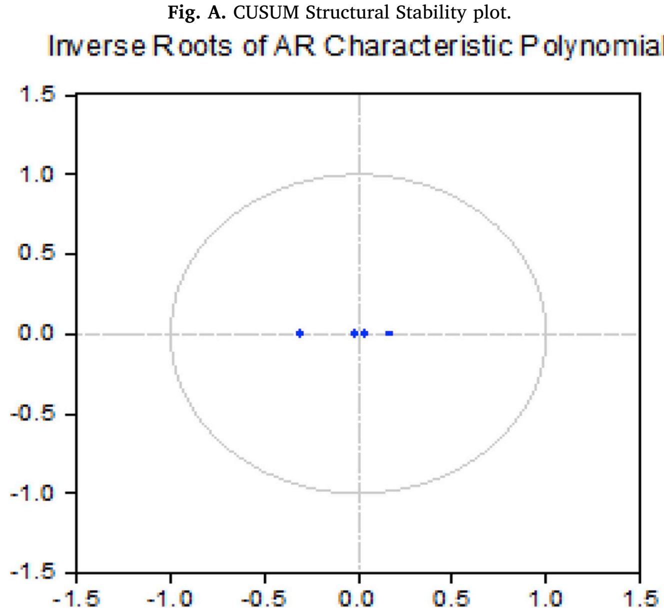

Fig. B. VAR Stability test.

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