analízis-1 / 5. gyak. (2020. tavasz)

(1a)
$$\frac{n^3-3n^2+n-1}{1-2n^3+n} = \frac{\frac{n^3-3n^2+n-1}{n^3}}{\frac{1-2n^3+n}{n^3}} =$$

$$= \frac{1-\frac{3}{n}+\frac{1}{n^2}-\frac{1}{n^3}}{\frac{1}{n^3}-2+\frac{1}{n^2}} \xrightarrow{(n\to\infty)} \frac{1-0+0-0}{0-2+0} = \underline{\underline{-\frac{1}{2}}}$$

(1b)
$$\frac{(2-n)^7+(2+n)^7}{(n^2+n+1)\cdot(2n+1)^5} = \frac{\frac{(2-n)^7+(2+n)^7}{n^7}}{\frac{(n^2+n+1)\cdot(2n+1)^5}{n^2\cdot n^5}} =$$

$$= \frac{\left(\frac{2}{n}-1\right)^7+\left(\frac{2}{n}+1\right)^7}{\left(1+\frac{1}{n}+\frac{1}{n^2}\right)\cdot\left(2+\frac{1}{n}\right)^5} \xrightarrow{(n\to\infty)} \frac{(0-1)^7+(0+1)^7}{(1+0+0)\cdot(2+0)^5} = \underline{\underline{0}}$$

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(2) <u>áll.</u>: (x_n) konv., $\lim(x_n)=\alpha \implies \lim(|x_n|)=|\alpha|$

<u>biz.</u>: háromszög-egyenlőtlenség miatt

$$\big||x_n|-|\alpha|\big| \le \underbrace{|x_n-\alpha|}_{\text{nullsorozat, mivel } \lim(x_n)=\alpha}$$

$$\Downarrow$$

$(|x_n|-|\alpha|)$ nullsorozat

$$\Downarrow$$

$$\lim(|x_n|)=|\alpha|$$

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(3) áll.: Legyen $x_n \geq 0 \; (n \in \mathbb{N})$, (x_n) konv., $\lim(x_n) = \alpha$.

Ekkor: a) $\alpha \geq 0$ b) $\lim(\sqrt{x_n}) = \sqrt{\alpha}$

biz.: a) $x_n \geq 0 \; (n \in \mathbb{N}) \Rightarrow \lim(x_n) \geq 0 \Rightarrow \alpha \geq 0$

↑ határérték és rendezés

b) $$\left|\sqrt{x_n} - \sqrt{\alpha}\right| = \left|\frac{(\sqrt{x_n} - \sqrt{\alpha}) \cdot (\sqrt{x_n} + \sqrt{\alpha})}{\sqrt{x_n} + \sqrt{\alpha}}\right| =$$

$$= \frac{|x_n - \alpha|}{\sqrt{x_n} + \sqrt{\alpha}} \leq \frac{|x_n - \alpha|}{\sqrt{\alpha}} \xrightarrow{(n \to \infty)} 0$$

$$\Downarrow$$

$(\sqrt{x_n} - \sqrt{\alpha})$ nullsorozat $\Rightarrow \lim(\sqrt{x_n}) = \sqrt{\alpha}$

A fenti levezetésben feltesszük, hogy $\alpha > 0$.

$\alpha = 0$ eset: Legyen $\varepsilon > 0$. $\lim(x_n) = 0$ miatt

$$\exists N \in \mathbb{N} \; \forall n \geq N: \quad |x_n - 0| < \varepsilon^2$$

$$x_n < \varepsilon^2$$

$$\sqrt{x_n} < \varepsilon$$

$$|\sqrt{x_n} - 0| < \varepsilon$$

Így a határérték def. alapján: $\lim(\sqrt{x_n}) = 0$.

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(4)

$$\sqrt{n^2+2n+3}-\sqrt{n^2-n+1}=$$

$$=\frac{\left(\sqrt{n^2+2n+3}-\sqrt{n^2-n+1}\right)\cdot\left(\sqrt{n^2+2n+3}+\sqrt{n^2-n+1}\right)}{\sqrt{n^2+2n+3}+\sqrt{n^2-n+1}}=$$

$$=\frac{(n^2+2n+3)-(n^2-n+1)}{\sqrt{n^2+2n+3}+\sqrt{n^2-n+1}}=\frac{3n+2}{\sqrt{n^2+2n+3}+\sqrt{n^2-n+1}}=$$

$$=\frac{\frac{3n+2}{n}}{\frac{\sqrt{n^2+2n+3}+\sqrt{n^2-n+1}}{n}}=\frac{3+\frac{2}{n}}{\sqrt{1+\frac{2}{n}+\frac{3}{n^2}}+\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}}$$

$$\xrightarrow{(n\to\infty)}\frac{3+0}{\sqrt{1+0+0}+\sqrt{1-0+0}}=\underline{\underline{\frac{3}{2}}}$$

(5d)

$$\frac{(-2)^n+n}{n!+3^n}=\frac{\frac{(-2)^n+n}{n!}}{\frac{n!+3^n}{n!}}=\frac{\frac{(-2)^n}{n!}+\frac{n}{n!}}{1+\frac{3^n}{n!}}=$$

$$=\frac{\frac{(-2)^n}{n!}+\frac{n}{2^n}\cdot\frac{2^n}{n!}}{1+\frac{3^n}{n!}}\xrightarrow{(n\to\infty)}\frac{0+0\cdot 0}{1+0}=\underline{\underline{0}}$$

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5a

$$\frac{5^{n+1}+2^n}{3\cdot 5^n-5^{-n}} = \frac{\dfrac{5^n\cdot 5+2^n}{5^n}}{\dfrac{3\cdot 5^n-5^{-n}}{5^n}} =$$

$$= \frac{5+\left(\frac{2}{5}\right)^n}{3-\left(\frac{1}{25}\right)^n} \xrightarrow{(n\to\infty)} \frac{5+0}{3-0} = \underline{\underline{\frac{5}{3}}}$$

5b

$$\frac{n^2\cdot 3^n+2^{2n}}{4^{n+1}+2^n} = \frac{\dfrac{n^2\cdot 3^n+4^n}{4^n}}{\dfrac{4^n\cdot 4+2^n}{4^n}} =$$

$$= \frac{n^2\cdot\left(\frac{3}{4}\right)^n+1}{4+\left(\frac{2}{4}\right)^n} \xrightarrow{(n\to\infty)} \frac{0+1}{4+0} = \underline{\underline{\frac{1}{4}}}$$

5c

$$\sqrt{\frac{(-5)^n+7^n}{7^{n+1}+n^7}} = \sqrt{\frac{\dfrac{(-5)^n+7^n}{7^n}}{\dfrac{7^n\cdot 7+n^7}{7^n}}} = \sqrt{\frac{\left(-\frac{5}{7}\right)^n+1}{7+\dfrac{n^7}{7^n}}} \to$$

$$\xrightarrow{(n\to\infty)} \sqrt{\frac{0+1}{7+0}} = \underline{\underline{\frac{1}{\sqrt{7}}}}$$