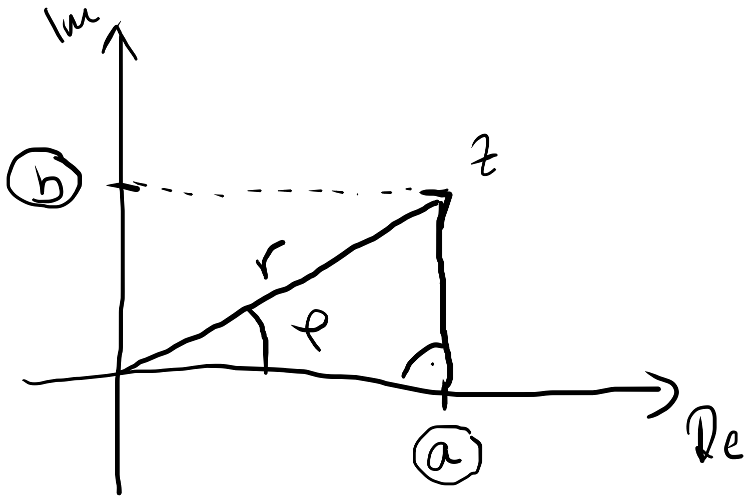
Komplex számok trigonometrikus alakja

$\mathbb{C} = \mathbb{R} \times \mathbb{R}$

$z = a + bi \sim (a, b)$

valós rész — képzetes rész



$|z| = \text{hossz} = \sqrt{a^2 + b^2} \quad (r)$

pl: $r = 2$

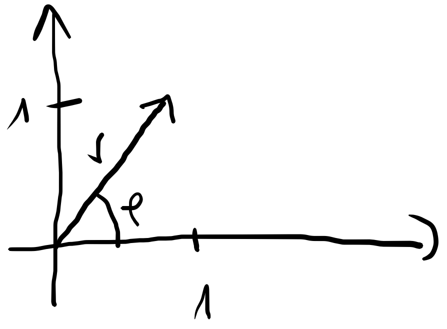
$\varphi = 30^\circ = \frac{\pi}{6}$

$a = r \cdot \cos\varphi$

$b = r \cdot \sin\varphi$

$$a + bi = \underbrace{r(\cos\varphi + i \cdot \sin\varphi)}_{\text{trigonometrikus alak}}$$

1) $z = 1 + i \sim (1, 1)$



$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

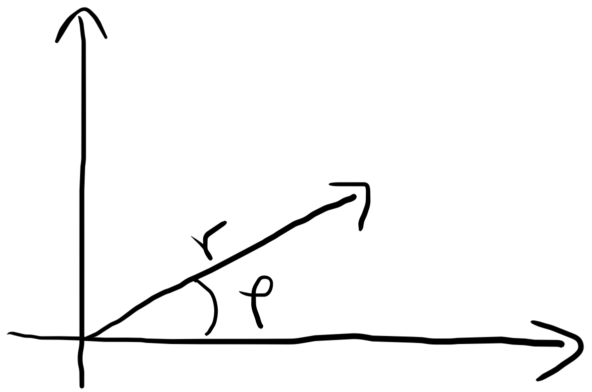
$z = 1 + i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)$

$\wr$

$r(\cos\varphi + \sin\varphi \cdot i)$

$\left.\begin{array}{l} \cos\varphi = \frac{1}{\sqrt{2}} \\ \sin\varphi = \frac{1}{\sqrt{2}} \end{array}\right\} \varphi = 45^\circ = \frac{\pi}{4}$

$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i \cdot \sin\frac{\pi}{4}\right)$



$\varphi, \varphi+2\pi, \varphi+4\pi$

jel: $\arg(z)$: valós tengellyel bezárt szög

$\arg(z) \in [0, 2\pi)$

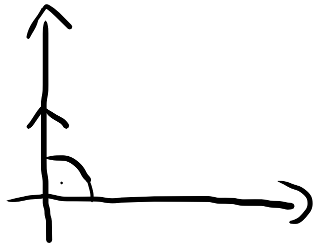
$r=5$

$\boxed{\varphi = \frac{\pi}{3}}$

$r=5$

$\varphi = \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$

2) $z = i \sim (0,1)$



$r = 1 \Rightarrow z = 0 + 1\cdot i = 1\cdot\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$\cos\varphi = 0$
$\sin\varphi = 1$ $\Rightarrow \varphi = \frac{\pi}{2}$

3) $-\sqrt{3} + i \sim (-\sqrt{3}, 1)$

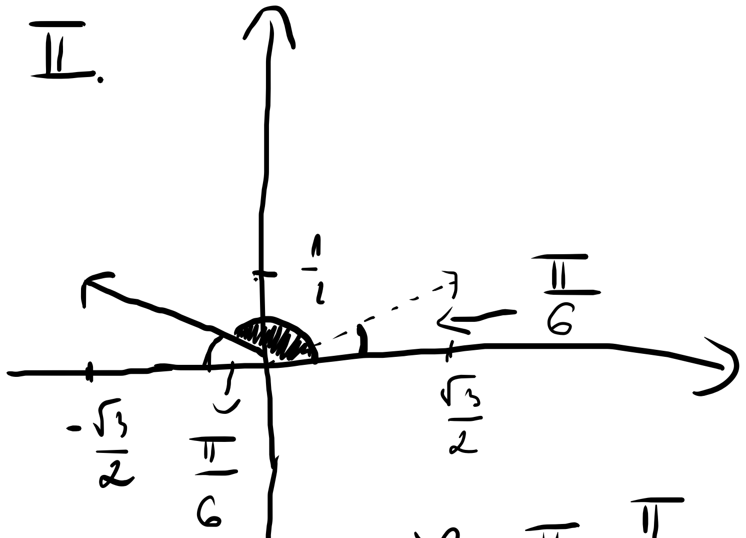
$r = \sqrt{(-\sqrt{3})^2 + 1^2} = 2 \Rightarrow 2\Big(\underbrace{-\frac{\sqrt{3}}{2}}_{\cos\varphi} + \underbrace{\frac{1}{2}}_{\sin\varphi} i\Big)$

$= 2\left(\cos\frac{5\pi}{6} + i\cdot\sin\frac{5\pi}{6}\right)$

$\cos\varphi = -\frac{\sqrt{3}}{2}$

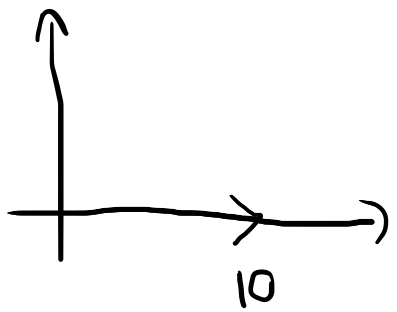
$\sin\varphi = \frac{1}{2}$

II.



$\varphi = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$z = 10 \sim (10,0) \qquad 10 = 10(\cos 0 + i \cdot \sin 0)$



$z_1 = r_1(\cos\varphi_1 + i \cdot \sin\varphi_1)$

$z_2 = r_2(\cos\varphi_2 + i \cdot \sin\varphi_2)$

i) $z_1 \cdot z_2 = r_1 \cdot r_2 \left(\cos(\varphi_1 + \varphi_2) + i \cdot \sin(\varphi_1 + \varphi_2)\right)$

ii) $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \cdot \left(\cos(\varphi_1 - \varphi_2) + i \cdot \sin(\varphi_1 - \varphi_2)\right)$

iii) $z_1^n = r_1^n \left(\cos(n \cdot \varphi_1) + i \cdot \sin(n \cdot \varphi_1)\right)$

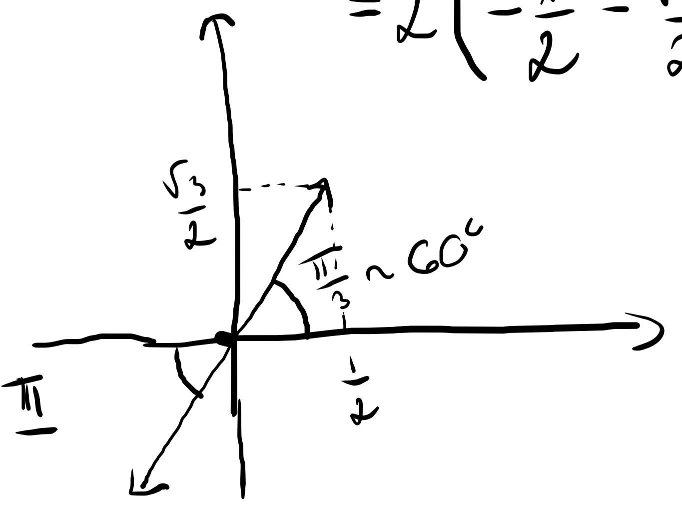
$$\underbrace{\left(-\frac{3\sqrt{3}}{2}-\frac{3}{2}i\right)}_{z_1}\underbrace{\left(\frac{\sqrt{3}}{3}+\frac{1}{3}i\right)}_{z_2}$$

$$z_1 \cdot z_2 = \underbrace{3\cdot\frac{2}{3}}_{2}\left(\cos\underbrace{\left(\frac{7\pi}{6}+\frac{\pi}{6}\right)}_{\frac{8\pi}{6}} + i\cdot\sin\left(\frac{7\pi}{6}+\frac{\pi}{6}\right)\right)$$

$$= 2\left(\cos\frac{4\pi}{3} + i\cdot\sin\frac{4\pi}{3}\right)$$

$$= 2\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) =$$

$$= \boxed{-1-\sqrt{3}\,i}$$



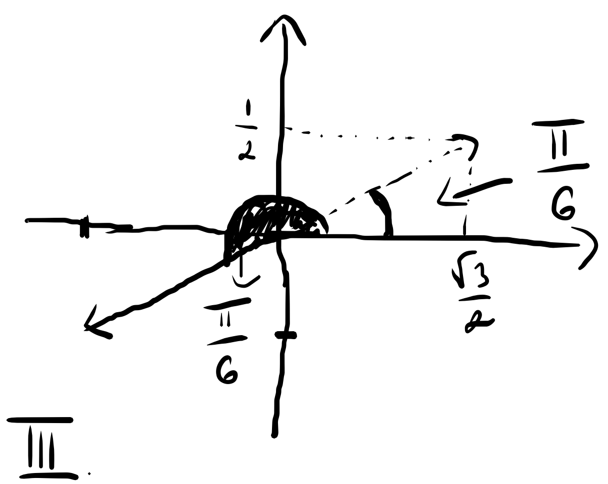
$$z_1: \quad r_1 = \sqrt{\left(\frac{-3\sqrt{3}}{2}\right)^2+\left(-\frac{3}{2}\right)^2} = 3$$

$$z_1 = 3\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)$$

$$\cos\varphi = -\frac{\sqrt{3}}{2}$$

$$\sin\varphi = -\frac{1}{2}$$

$$\varphi = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$



$$z_1 = 3\left(\cos\frac{7\pi}{6} + i\cdot\sin\frac{7\pi}{6}\right)$$

$$z_2: \quad r_2 = \sqrt{\left(\frac{\sqrt{3}}{3}\right)^2+\left(\frac{1}{3}\right)^2} = \frac{2}{3}$$

$$z_2 = \frac{2}{3}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\cdot i\right)$$

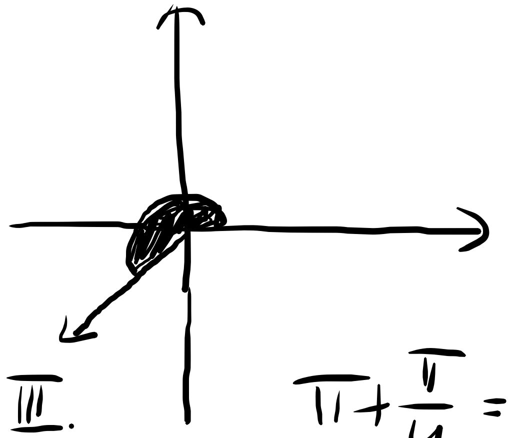
$$\cos\varphi = \frac{\sqrt{3}}{2}$$

$$\sin\varphi = \frac{1}{2}$$

$$\varphi = \frac{\pi}{6}$$

$$z_2 = \frac{2}{3}\left(\cos\frac{\pi}{6} + i\cdot\sin\frac{\pi}{6}\right)$$

$$z = -\frac{\sqrt{10}}{2} - \frac{\sqrt{10}}{2} i = \frac{\sqrt{10}}{\sqrt{2}} \Big(\underbrace{-\frac{1}{\sqrt{2}}}_{\cos\varphi} \underbrace{-\frac{1}{\sqrt{2}} i}_{\sin\varphi} \Big) = \underbrace{\frac{\sqrt{10}}{\sqrt{2}}}_{\sqrt{\frac{10}{2}} = \sqrt{5}} \left(\cos \frac{5\pi}{4} + i \cdot \sin \frac{5\pi}{4} \right)$$

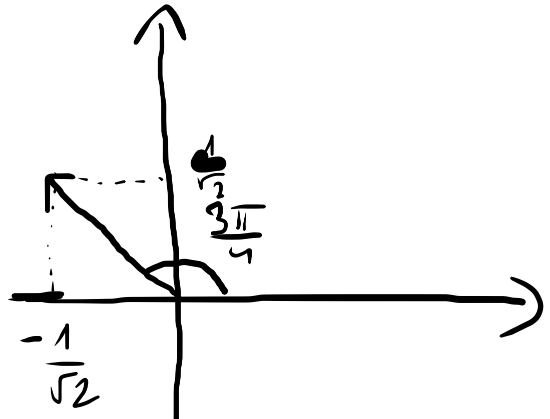


$$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

$$z^{15} = \left(\sqrt{5} \left(\cos \frac{5\pi}{4} + i \cdot \sin \frac{5\pi}{4} \right) \right)^{15} = (\sqrt{5})^{15} \cdot \left(\cos \overbrace{\frac{15 \cdot 5\pi}{4}}^{\frac{75\pi}{4}} + i \cdot \sin \frac{15 \cdot 5\pi}{4} \right) =$$

$$= 5^{\frac{15}{2}} \left(\cos \frac{3\pi}{4} + i \cdot \sin \frac{3\pi}{4} \right) = 5^{\frac{15}{2}} \cdot \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \right)$$

$$\cos \frac{75\pi}{4} = \cos \Big(\frac{75\pi}{4} - \underbrace{2\pi}_{\frac{8\pi}{4}} \Big) = \cos \frac{67\pi}{4} = \ldots\ldots = \cos \frac{11\pi}{4} = \cos \frac{3\pi}{4}$$

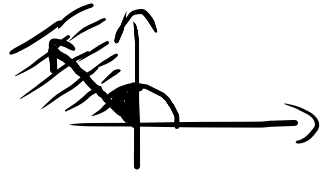


$$\frac{3\pi}{4} = \frac{2\pi}{4} + \frac{\pi}{4}$$

$$z = \frac{(2+2\sqrt{3}i)^{10}}{(-1+i)^{83}}$$

$$z_1 = 2+2\sqrt{3}i = 4\left(\underbrace{\frac{1}{2}}_{\cos\varphi} + \underbrace{\frac{\sqrt{3}}{2}}_{\sin\varphi} i\right) = 4\left(\cos\frac{\pi}{3} + i\cdot\sin\frac{\pi}{3}\right)$$

$$z_2 = -1+i = \sqrt{2}\left(\underbrace{-\frac{1}{\sqrt{2}}}_{\cos\varphi} + \underbrace{\frac{1}{\sqrt{2}}}_{\sin\varphi} i\right) = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\cdot\sin\frac{3\pi}{4}\right)$$



$$z = \frac{4^{10}}{(\sqrt{2})^{83}}\left(\cos\left(\underbrace{\frac{10\pi}{3} - \frac{83\cdot 3\pi}{4}}\right) + i\cdot\sin\left(\frac{10\pi}{3} - \frac{83\cdot 3\pi}{4}\right)\right) = \frac{2^{20}}{2^{\frac{83}{2}}}\left(\cos -\frac{707\pi}{12} + i\cdot\sin -\frac{707\pi}{12}\right) =$$

$$\frac{40\pi}{12} - \frac{249\cdot 3\cdot\pi}{12} = -\frac{707\pi}{12} \overset{\text{8crg}}{\rightsquigarrow} -\frac{707\pi}{12} + \underbrace{2k\pi}_{\frac{24}{12}\cdot k\cdot\pi} \rightsquigarrow -\frac{47\pi}{12} \to \frac{\pi}{12}$$

$$= 2^{20-\frac{83}{2}}\left(\cos\frac{\pi}{12} + i\cdot\sin\frac{\pi}{12}\right).$$

Kiadott feladatok: 8/d, g

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5. feladatsorról