Komplex számok trigonometrikus alakja 2

$z = r(\cos\varphi + i\cdot\sin\varphi)$

gyökvonás: $w^3 = 1 \Rightarrow w = \sqrt[3]{1}$ $\quad w = 1$ megoldás

$w = r(\cos\alpha + i\cdot\sin\alpha) \Rightarrow w^3 = r^3(\cos 3\alpha + i\cdot\sin 3\alpha)$

$1 = 1\cdot(\cos 0 + i\cdot\sin 0)$

$w^3 = 1 \quad : \quad r^3(\cos 3\alpha + i\cdot\sin 3\alpha) = 1\cdot(\cos 0 + i\cdot\sin 0) \Rightarrow r^3 = 1 \Rightarrow r = 1$

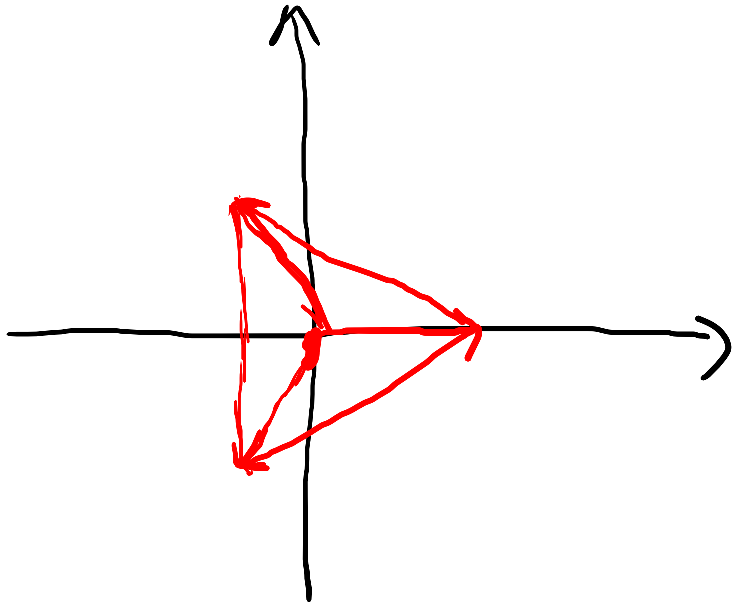
$3\alpha = 0 \Rightarrow \alpha = 0$

$w = 1\cdot(\cos 0 + i\cdot\sin 0) = 1$

$= 1\cdot(\cos 2\pi + i\cdot\sin 2\pi) \Rightarrow r^3 = 1 \Rightarrow r = 1$

$3\alpha = 2\pi \Rightarrow \alpha = \frac{2\pi}{3}$

$w = 1\cdot\left(\cos\frac{2\pi}{3} + i\cdot\sin\frac{2\pi}{3}\right)$



$= 1 \cdot (\cos 4\pi + i \cdot \sin 4\pi) \Rightarrow \quad r^3 = 1 \Rightarrow r = 1$

$3\alpha = 4\pi \Rightarrow \alpha = \frac{4\pi}{3}$

$w = 1 \cdot \left(\cos \frac{4\pi}{3} + i \cdot \sin \frac{4\pi}{3}\right)$

$= 1 (\cos 6\pi + i \cdot \sin 6\pi) \Rightarrow \quad r^3 = 1 \Rightarrow r = 1$

$3\alpha = 6\pi \Rightarrow \alpha = 2\pi$

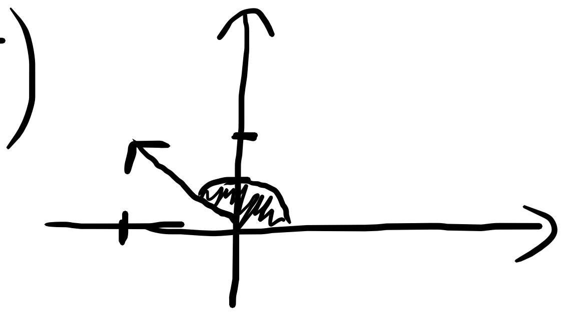
$\arg(z) \in [0, 2\pi)$

z komplex szám, akkor az $\sqrt[n]{z}$-nek n különböző gyököt fogunk kapni.

$z = r \cdot (\cos \varphi + i \cdot \sin \varphi)$

$\sqrt[n]{z} = \sqrt[n]{r} \left(\cos \frac{\varphi + 2k\pi}{n} + i \cdot \sin \frac{\varphi + 2k\pi}{n} \right) \qquad k = 0, 1, \dots, n-1$

$$z = -\frac{7}{2} + \frac{7}{2}i = \frac{7}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \frac{7}{\sqrt{2}}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$



$$\sqrt[5]{z} = \sqrt[5]{\frac{7}{\sqrt{2}}}\left(\cos\frac{\frac{3\pi}{4}+2k\pi}{5} + i\cdot\sin\frac{\frac{3\pi}{4}+2k\pi}{5}\right)$$

$k = 0, 1, 2, 3, 4$

$$k=0: \quad \sqrt[5]{\frac{7}{\sqrt{2}}}\left(\cos\frac{3\pi}{20} + i\cdot\sin\frac{3\pi}{20}\right)$$

$$k=1: \quad \sqrt[5]{\frac{7}{\sqrt{2}}}\left(\cos\frac{11\pi}{20} + i\cdot\sin\frac{11\pi}{20}\right)$$

$$k=2: \quad \sqrt[5]{\frac{7}{\sqrt{2}}}\left(\cos\frac{19\pi}{20} + i\cdot\sin\frac{19\pi}{20}\right)$$

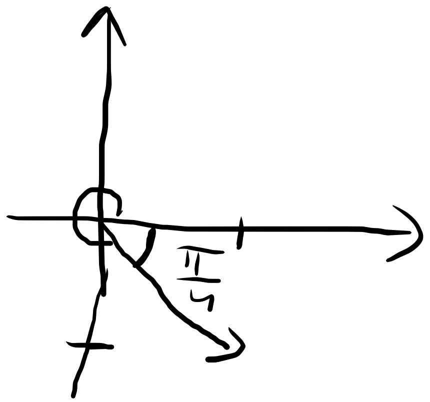
$$k=3: \quad \sqrt[5]{\frac{7}{\sqrt{2}}}\left(\cos\frac{27\pi}{20} + i\cdot\sin\frac{27\pi}{20}\right)$$

$$k=4: \quad \sqrt[5]{\frac{7}{\sqrt{2}}}\left(\cos\frac{35\pi}{20} + i\cdot\sin\frac{35\pi}{20}\right)$$

$$k=5: \quad \sqrt[5]{\frac{7}{\sqrt{2}}}\left(\cos\frac{43\pi}{20} + i\cdot\sin\frac{43\pi}{20}\right)$$

$$\frac{43\pi}{20} \wr \frac{3\pi}{20} \qquad \frac{43\pi}{20} \wr \frac{3\pi}{20}$$

$$z = 1 - i = \sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i\right) = \sqrt{2}\left(\cos\frac{7\pi}{4} + i \cdot \sin\frac{7\pi}{4}\right)$$



$$2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

$$\sqrt[3]{z} = \underbrace{\sqrt[3]{\sqrt{2}}}_{\sqrt[6]{2}}\left(\cos\frac{\frac{7\pi}{4} + 2k\pi}{3} + i \cdot \sin\frac{\frac{7\pi}{4} + 2k\pi}{3}\right)$$

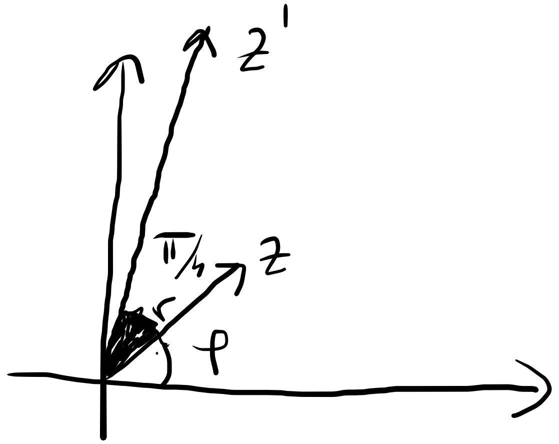
$k = 0$: $\sqrt[6]{2}\left(\cos\frac{7\pi}{12} + i \cdot \sin\frac{7\pi}{12}\right)$

$k = 1$: $\sqrt[6]{2}\left(\cos\frac{15\pi}{12} + i \cdot \sin\frac{15\pi}{12}\right)$

$k = 2$: $\sqrt[6]{2}\left(\cos\frac{23\pi}{12} + i \cdot \sin\frac{23\pi}{12}\right)$

Kiadott feladat : V. / a, b, c, d

$$z = r(\cos\varphi + i\cdot\sin\varphi)$$



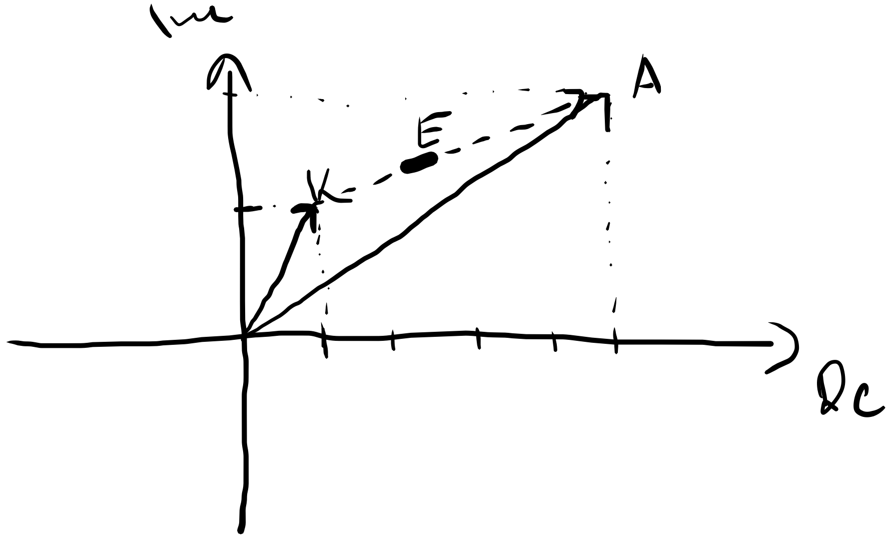
$$\underbrace{(1+i)}_{\sqrt{2}\left(\cos\frac{\pi}{4} + i\cdot\sin\frac{\pi}{4}\right)}\cdot z = z'$$

$$(1+i)z = \underbrace{\sqrt{2}\cdot r}\left(\cos\left(\varphi+\frac{\pi}{4}\right) + i\cdot\sin\left(\varphi+\frac{\pi}{4}\right)\right) = z'$$

i) $\sqrt{2}$ - szeres nyújtás

ii) $\frac{\pi}{4}$ - gyel való forgatás

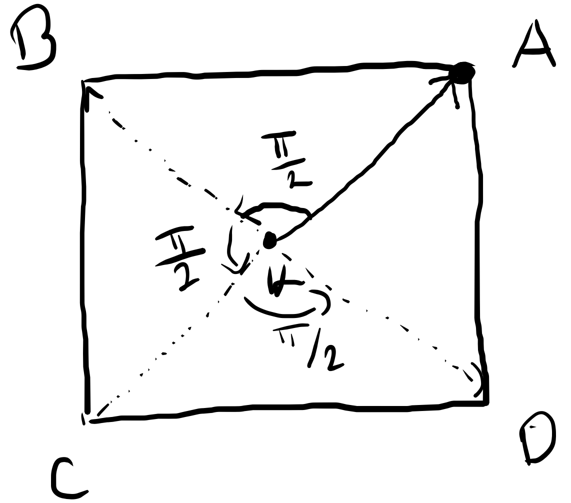
} forgatva nyújtás



$K = 1+2i$: négyzet középpontja

$A = 5+4i$: négyzet csúcsa

Hat. meg a négyzet többi csúcsát!



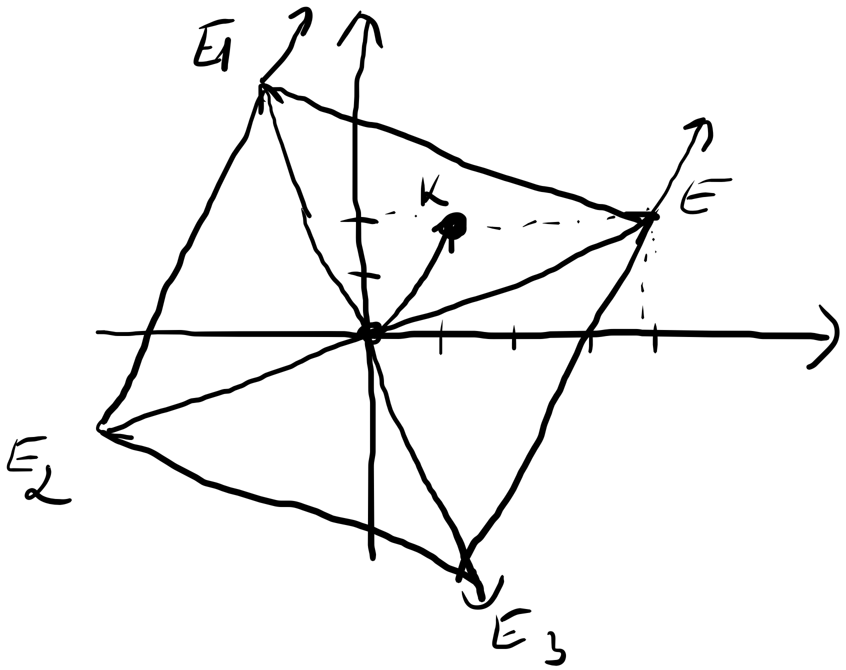
Forgatás, mely nem nyújt: $\left.\begin{array}{l} r = 1 \\ \varphi = \frac{\pi}{2} \end{array}\right\} \; 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = i$

$E = A - K = (5+4i) - (1+2i) = 4+2i$

$E_1 = i\cdot(4+2i) = -2+4i$

$E_2 = i\cdot(-2+4i) = -4-2i$

$E_3 = i\cdot(-4-2i) = 2-4i$



$A = E+K = 5+4i$

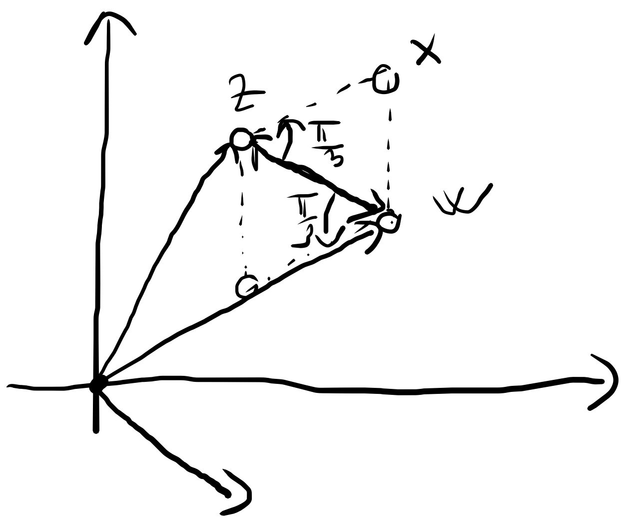
$\Rightarrow$

$B = E_1 + K = -1+6i$

$C = E_2 + K = -3$

$D = E_3 + K = 3-2i$

z, w : szabályos $\triangle$ két csúcsa, mi lesz a harmadik csúcs?



$$\text{i,}\ (w - z)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) + z = x$$

$$\varepsilon = 1\cdot\left(\cos\frac{\pi}{3} + i\cdot\sin\frac{\pi}{3}\right) = \frac{1}{2} + i\cdot\frac{\sqrt{3}}{2}$$

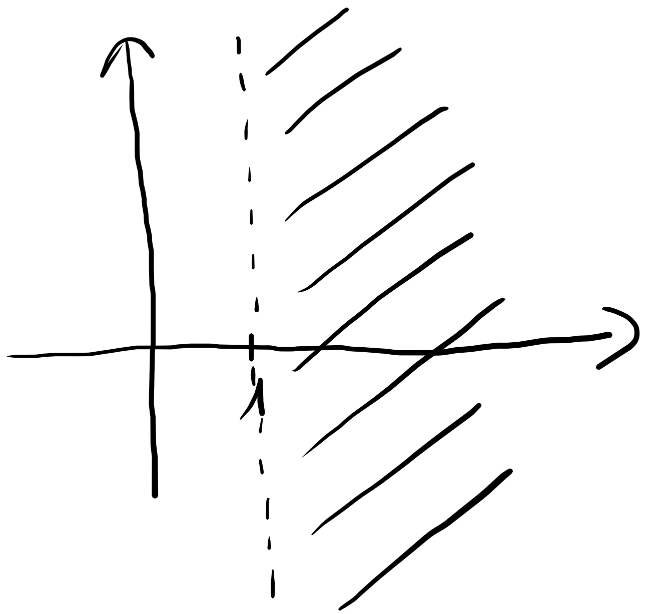
$$\text{ii,}\ (z - w)\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) + w = y$$

Komplex számsíkon való ábrázolás

$$A = \{ z \in \mathbb{C} : \operatorname{Re}(z) > 1 \}$$

$z = a + bi \Rightarrow \operatorname{Re}(z) > 1$

$a > 1$



$$B = \{ z \in \mathbb{C} : |z-2| = 3 \}$$

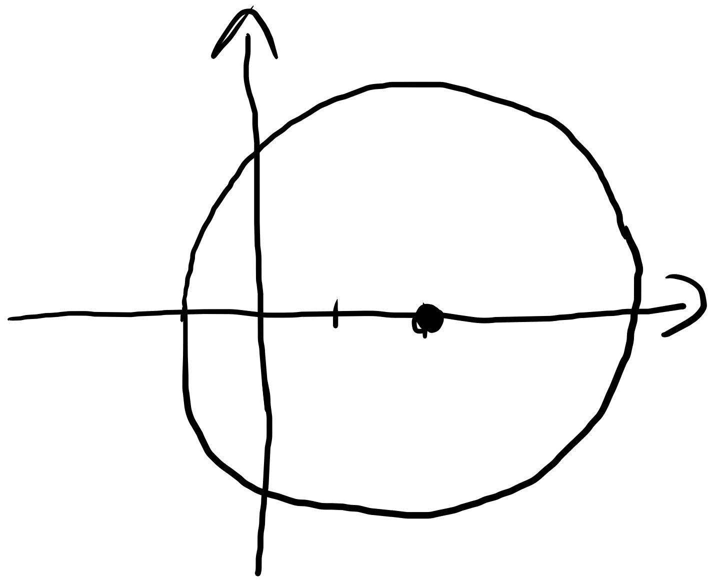
$z = a+bi$: $|z-2| = |a+bi-2| = |(a-2)+bi| =$

$$= \sqrt{(a-2)^2+b^2} = 3 \qquad /^2$$

$$\boxed{(a-2)^2+b^2 = 9}$$

kp : $(2,0)$

sugár $= 3$



$$\boxed{\begin{array}{l} (x-u)^2+(y-v)^2 = r^2 \\ \text{kp} : (u,v) \\ \text{sugár} : r \end{array}}$$

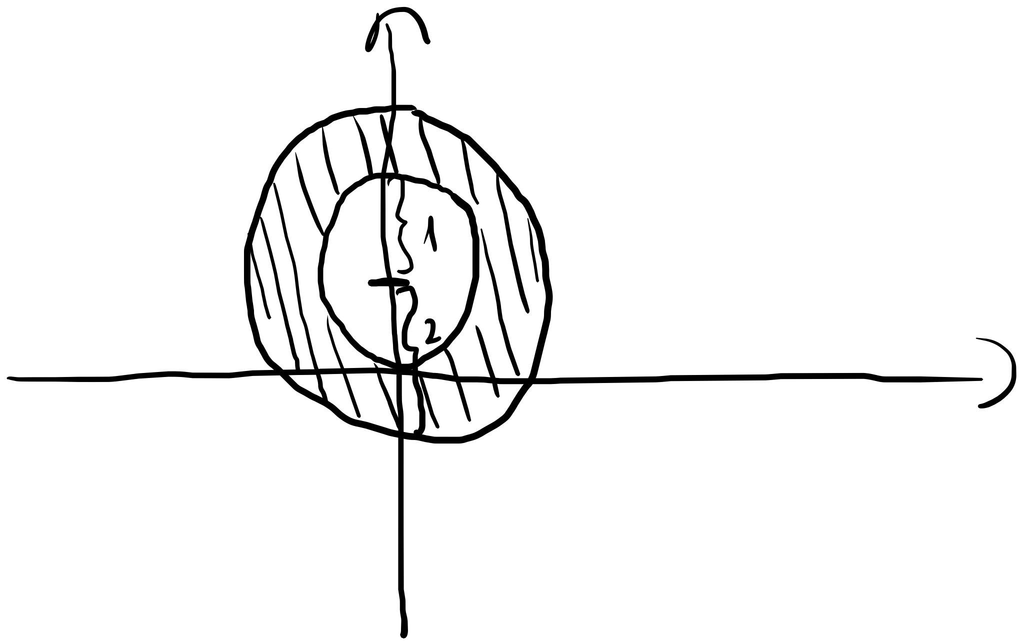
$C = \{ z \in \mathbb{C} : 1 \leq |z - i| \leq 2 \}$ $\quad z = a + bi$

$|z - i| = \sqrt{a^2 + (b-1)^2}$

kp: $(0,1)$

$1 \leq r \leq 2$

$1 \leq \sqrt{a^2 + (b-1)^2} \leq 2 \quad \Rightarrow \quad \underset{①}{1} \leq \overbrace{a^2 + (b-1)^2} \underset{②}{\leq 4}$



$$D = \{ z \in \mathbb{C} : |z-2| \leq |z+3| \} \qquad z = a+bi$$

$$z-2 = a-2+bi$$
$$z+3 = a+3+bi$$

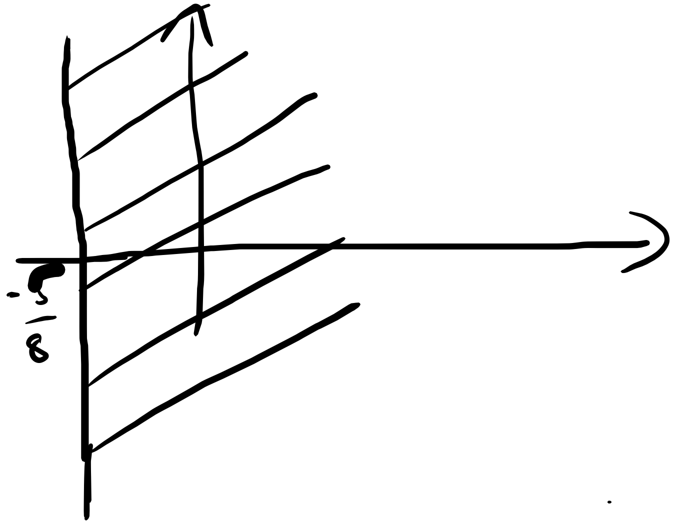
$$\sqrt{(a-2)^2+b^2} \leq \sqrt{(a+3)^2+b^2} \qquad /\hat{\ }2$$

$$(a-2)^2+b^2 \leq (a+3)^2+b^2$$

$$\cancel{a^2}-2a+4+\cancel{b^2} \leq \cancel{a^2}+6a+9+\cancel{b^2}$$

$$-5 \leq 8a$$

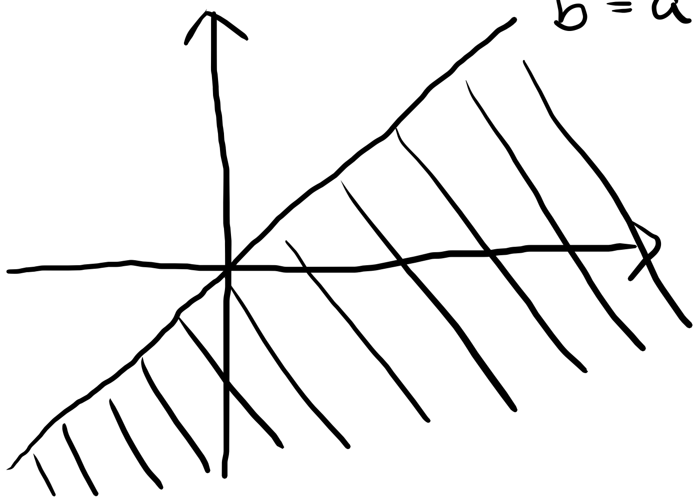
$$-\frac{5}{8} \leq a$$



$$E = \{ z \in \mathbb{C} : \mathrm{Re}(z) \geq \mathrm{Im}(z+1) \} \qquad z = a+bi$$

$$a \geq b$$

$b = a$



Kiadott feladatok:

VI./6. feladat