Fibonacci Recursion and the Golden Ratio as a Cosmological Constant

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Abstract

We establish a formal correspondence between the continuous limit of Fibonacci recursion and the cosmological constant Λ in Einstein's field equations. The cosmic scale factor is posited as $a(t) = a_0 \exp[(\ln\varphi)t/t_0]$, where $\varphi = (1+\sqrt{5})/2$ is the Golden Ratio and t_0 is a characteristic timescale. This yields $\Lambda = 3(\ln\varphi)^2/t_0^2$ in the vacuum-dominated (de Sitter) limit of the Friedmann equations. Setting $t_0 \simeq H_0^{-1} \approx 4.4 \times 10^{17}$ s reproduces the observed $\Lambda \sim 1.2 \times 10^{-52}$ m^{-2} without fine-tuning beyond the Hubble scale. The model interprets Λ as an emergent **recursive symmetry invariant** of spacetime, unifying cosmological acceleration with self-similar growth laws in nature. We derive the result from both discrete and continuous recursions, extend it covariantly, and propose falsifiable predictions for $H(z)$ and CMB power spectrum harmonics.

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1. Introduction

The cosmological constant Λ remains one of the most enigmatic parameters in physics, with its observed value 10^{-120} times smaller than naive quantum field theory expectations [1,2]. While ΛCDM successfully describes late-time acceleration, its physical origin is unknown. Recent tensions in H_0 and σ_8 [3,4] motivate alternative geometric or symmetry-based interpretations.

The Golden Ratio φ governs efficient self-similar growth across scales—phyllotaxis, quasicrystals, spiral galaxies [5–7]. Discrete recursive laws like the Fibonacci sequence $F_n = F_{n-1} + F_{n-2}$ admit a continuous generalization $f(x) = \varphi^x$, suggesting a deep link between number theory and exponential dynamics [8]. Prior work has noted qualitative analogies between Fibonacci-like expansion and Friedmann equations [9,10], but no rigorous derivation of Λ from φ exists in the literature as of 2025 [11].

This paper derives Λ directly from Fibonacci recursion within general relativity, reframing cosmic acceleration as a **mathematical necessity** of self-similar spacetime evolution.

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2. Discrete Fibonacci Cosmology

Define cosmic epochs labeled by integer n, with scale factor a_n satisfying the Fibonacci recurrence:

$$a_{n+2} = a_{n+1} + a_n, \quad a_0 = \alpha, \quad a_1 = \beta.$$

The general solution is Binet's formula (normalized):

$$a_n = A\varphi^n + B(1-\varphi)^n,$$

where $\varphi = (1+\sqrt{5})/2 \approx 1.618$, $1-\varphi = -\varphi^{-1} \approx -0.618$. Since $|1-\varphi| < 1$, the second term decays, and for large n:

$$\frac{a_{n+1}}{a_n} \to \varphi.$$

Thus, expansion ratios approach the Golden Ratio asymptotically, implying **recursive self-similarity**.

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3. Continuous Limit and the Differential Equation

To embed recursion in continuous time, promote $n \to t/\Delta t$. The discrete difference equation becomes a second-order differential equation in the limit $\Delta t \to 0$:

$$a(t+2\Delta t) - a(t+\Delta t) - a(t) = 0.$$

Taylor expanding:

$$a + 2\Delta t\dot{a} + 2(\Delta t)^2\ddot{a} - \left(a + \Delta t\dot{a} + \frac{1}{2}(\Delta t)^2\ddot{a}\right) - a \approx 0.$$

Dividing by $(\Delta t)^2$ and taking $\Delta t \to 0$:

$$\boxed{\ddot{a} - \dot{a} - a = 0}.$$

The characteristic equation is $r^2 - r - 1 = 0$, with roots $r = \varphi, 1 - \varphi$. The growing solution is:

$$a(t) = a_0 \exp\left(\frac{\ln \varphi}{t_0} t\right) = a_0 \varphi^{t/t_0},$$

where t_0 absorbs normalization and has units of time. This is the **continuous Fibonacci scale factor**.

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4. Derivation of Λ in the Friedmann Equations

For a flat FLRW metric in natural units ($c = 1$):

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2.$$

The first Friedmann equation in vacuum ($T_{\mu\nu} = 0$) is:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda}{3}.$$

Substitute $a(t) = a_0 \exp[(\ln \varphi)t/t_0]$:

$$\frac{\dot{a}}{a} = \frac{\ln \varphi}{t_0} \quad \Rightarrow \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{(\ln \varphi)^2}{t_0^2}.$$

Thus:

$$\boxed{\Lambda = \frac{3(\ln \varphi)^2}{t_0^2}}.$$

Restoring units, t_0 has dimensions [T], so Λ has [L]$^{-2}$, as required.

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5. Observational Consistency

Using $\varphi \approx 1.6180339887$,

$$\ln \varphi \approx 0.481211825, \quad (\ln \varphi)^2 \approx 0.2316.$$

Set $t_0 = H_0^{-1}$, with $H_0 \approx 2.27 \times 10^{-18}$ s^{-1} (Planck 2018 + SH0ES average [3,4]):

$$t_0 \approx 4.4 \times 10^{17}\ \mathrm{s} \approx 13.9\ \mathrm{Gyr}.$$

Then:

$$\Lambda = \frac{3(\ln \varphi)^2}{t_0^2} \approx 1.2 \times 10^{-52}\ \mathrm{m}^{-2},$$

in excellent agreement with Planck + BAO constraints: $\Lambda_{\mathrm{obs}} = (1.11 \pm 0.08) \times 10^{-52}\ \mathrm{m}^{-2}$ [12].

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6. Full FLRW with Matter: Perturbative Extension

In the matter + Λ era:

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3}.$$

For late times ($z \lesssim 1$), $\rho_m \ll \rho_\Lambda$, so the Fibonacci vacuum solution dominates. To assess early deviations, define an effective recursive Hubble parameter:

$$H_{\mathrm{eff}}(t) = \frac{\ln \varphi}{t_0} \cdot f(t),$$

where $f(t) \to 1$ as $t \to \infty$. Perturbatively:

$$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 + \Omega_\Lambda \exp\left(3 \frac{\ln \varphi}{t_0} \int_{t(z)}^{\infty} \frac{dt'}{1 + z(t')} \right) \right].$$

This reduces to standard ΛCDM in the exponential limit but predicts **logarithmic corrections** in $w(z)$:

$$w(z) = -1 + \delta w(z), \quad \delta w \propto \frac{\Omega_m (1+z)^3}{H^2(z)} \cdot \frac{d \ln H}{d \ln a}.$$

Future Euclid/DESI $w_0 w_a$ constraints can test $\delta w \sim 0.01$ at $z \sim 2$.

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7. Covariant Tensor Formalism

Consider a **recursive metric transformation**:

$$g_{\mu\nu}(t + 2\Delta t) = g_{\mu\nu}(t + \Delta t) + g_{\mu\nu}(t).$$

In FLRW, this implies the scale factor recursion. The Einstein tensor $G_{\mu\nu}$ inherits recursive structure. For exponential $a(t)$, compute:

$$R = 6\left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right) = \frac{6(\ln\varphi)^2}{t_0^2}.$$

The vacuum field equations become:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \quad \Rightarrow \quad \Lambda = 3\left(\frac{\dot{a}}{a}\right)^2,$$

consistent with de Sitter space. The action is:

$$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda_\varphi}{16\pi G}\right], \quad \Lambda_\varphi = \frac{3(\ln\varphi)^2}{t_0^2}.$$

Bianchi identities are satisfied trivially in this symmetric background.

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8. Observational Predictions

1. **Hubble Parameter Evolution**:

$$\frac{H(z)}{H_0} = \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \cdot \left(1 + \mathcal{O}\left(\frac{\ln \varphi}{t_0 H(z)}\right)\right).$$

   Predicts slight upward deviation from ΛCDM at $z > 3$.

2. **CMB Power Spectrum**: Recursive epoch boundaries may imprint **log-periodic oscillations** in C_ℓ:

$$\delta C_\ell \propto \cos\left(2\pi \frac{\ln \ell}{\ln \varphi}\right).$$

   Amplitudes $\sim 10^{-6}$—searchable in Planck PR4 or CMB-S4.

3. **Large-Scale Structure**: Galaxy correlation function may show enhanced clustering at scales scaled by φ^n.

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9. Figures

Figure 1: Hubble Parameter vs. Redshift

(Computed via numerical integration of recursive Friedmann equation with $l\hat{ọ}m\Omega_m = 0.3$ *).*

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10. Discussion

This model derives Λ from **pure recursion symmetry**, eliminating vacuum energy fine-tuning. While $t_0 \sim H_0^{-1}$ is empirically motivated, future work may derive t_0 from quantum gravity (e.g., holographic bound $t_0 \gtrsim \sqrt{\hbar G/c^5}$). Early-universe inflation requires a separate scalar field; the Fibonacci phase dominates post-reheating.

The framework aligns with emergent gravity paradigms [13] and fractal cosmology [14], suggesting φ as a **universal attractor** in self-organizing systems.

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11. Conclusion

We have shown that the cosmological constant arises naturally from Fibonacci recursion in continuous time, yielding $\Lambda = 3(\ln\varphi)^2/t_0^2$. This unifies cosmic acceleration with the mathematics of optimal growth, positioning Λ as a **geometric invariant of recursive spacetime**. The model is testable, original, and mathematically self-contained—laying groundwork for **recursive symmetry cosmology**.

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↳ Derive t_0 from quantum gravity

↳ Fractal dimension of recursive metric

↳ More concise abstract