A Group-Based Approach to Measuring Polarization

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Abstract

Despite its growing importance in social scientific topics, the quantitative measurement of polarization has lagged behind its conceptual development. Political and social polarization are group-based phenomena characterized by intergroup heterogeneity and intragroup homogeneity, but existing measures capture only one of these features or make it difficult to compare across cases or over time. To bring the concept and measurement of polarization into closer alignment, I introduce the cluster-polarization coefficient (CPC), a measure of multimodality that allows scholars to incorporate multiple variables and compare across contexts with varying numbers of parties or social groups. Simulation exercises and three applications to ideological and affective polarization demonstrate that the CPC returns more substantively sensible results than other popular measures. An open-source software package implements the measure.

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Polarization has become a key concept in the social sciences, playing a role in important political outcomes ranging from representation (Ahler and Broockman 2018) and party-building (Lupu 2015) to policymaking (Sinclair 2008) and democracy (McCoy and Somer 2019). Yet despite the proliferation of research on polarization, analysts often arrive at different conclusions with respect to its effect on key variables of interest, and they enthusiastically debate whether polarization is occurring at all (Abramowitz and Saunders 2008; DiMaggio, Evans, and Bryson 1996; Fiorina 2011). Conceptually, polarization has two important features: intergroup heterogeneity and intragroup homogeneity. But this theoretical foundation is rarely translated into empirical application, as the most common measures of polarization—difference-in-means and variance—are ill-suited to capture both features.

I offer an alternative. The cluster-polarization coefficient (CPC) explicitly models intergroup heterogeneity and intragroup homogeneity, placing polarization's conceptual foundation front and center. I evaluate the CPC's ability to discern polarization relative to difference-in-means and variance—the two most widely used measures of polarization—by applying them to synthetic data and three datasets of elite ideology and mass party affect. This analysis highlights three advantages of the CPC, one theoretical and two empirical: It better captures the theoretical concept, it displays greater flexibility when considering multiple variables, and it facilitates measurement of and comparison across cases that include more than two groups. Given that most party systems have more than two parties and most political conflict occurs along more than one dimension, the CPC provides a very useful tool for scholars studying polarization.

To make the measurement procedure widely accessible to researchers and practitioners, I provide an open-source R package, `CPC`: Implementation of Cluster-Polarization Coefficient, which is freely available on the Comprehensive R Archive Network (https://cran.r-project.org/package=CPC). In addition to calculating the CPC with researcher-specified group memberships, this package contains support for a variety of clustering methods to assign observations to groups, making the measure easily applicable to a wide variety of data structures.

Features of Polarization

Polarization is a phenomenon that happens both between and within groups, emerging when group members disagree with members of other groups and agree with members of their own. It thus has two conceptual features: distance from opponents (intergroup heterogeneity) and concentration within groups (intragroup homogeneity). The challenge for measuring polarization in real-world data is that these two features can occur separately, simultaneously, or not at all, and each may increase or decrease independent of the other. This challenge is reflected in the diversity of measures used by scholars. A survey of eight political science journals reveals 322 articles published about polarization since 2000,[1] employing at least twenty distinct operationalizations. However, two approaches account for most uses: Measures of difference-in-means and variance were used in 57% and 14% of articles, respectively.[2]

Disagreement—intergroup heterogeneity—is one critical feature of polarization. For instance, as parties' ideal points gradually approach opposite ends of a policy space, that party system becomes more polarized. The most popular measurement approach—difference-in-means—approximates distance by subtracting the ideological or policy positions of one party from another (Abramowitz and Saunders 2008; Hetherington 2001; McCarty, Poole, and Rosenthal 2009). However, this has the effect of data reduction—extremists get rolled into their parties' overall mean, perhaps leading to an estimate of polarization which is biased downward. Moreover, polarization is not merely an increase in distance between the extremes; it also implies some level of concentration around the emerging poles. There can be a wide distance between the two most extreme parties in a given party system but, if voters are evenly dispersed throughout the policy space, that party system may not be polarized. Below, I demonstrate why Levendusky and Pope (2011) urge scholars to "go beyond the mean" when measuring polarization.

Group concentration—intragroup homogeneity—is the second critical feature of polarization. As the ideal points of party members cluster more tightly together, each party becomes more

[1] See Supplementary Information section S1 for details and results of this search.

[2] Throughout this paper, "variance" refers to total variance, as opposed to within-group variance.

internally homogeneous, parties share less in common with each other, and the party system becomes more polarized. Crucially, polarization can increase in this manner even in the *absence* of increasing intergroup heterogeneity. Party sorting is one process through which this happens; individuals do not become more extreme but the population nevertheless polarizes due to individuals sorting themselves into parties with positions similar to their own (Levendusky and Pope 2011). As groups become more homogeneous, the amount of overlap between them decreases. Scholars frequently point to this group overlap as an important diagnostic of polarization (e.g. Han and Brady 2007; Levendusky and Pope 2011) and attempt to capture it using variance (Hill and Tausanovitch 2015; Lupu 2016) because it is "the most common measure of dispersion" and "does not depend on whether [data points] are clustered in distinct groups" (Lindqvist and Östling 2010, p. 546). While true, this operationalization seems at odds with the conceptual understanding of polarization, which emphasizes distinct clusters of opinions as one of the phenomenon's defining characteristics. Ideally, a measure of polarization would model those clustering properties directly instead of smoothing over them.[3]

To accurately capture polarization, both features are necessary. Without intergroup heterogeneity, there are no differences in position to analyze. Without intragroup homogeneity, there are no meaningful group positions to compare.[4] In this conceptualization, polarization can increase as the positions of social or political groups grow farther apart from one another, as they become more internally cohesive, or both. Many scholars either explicitly or implicitly adopt this conceptualization of polarization (Baldassarri and Bearman 2007; Fortunato and Stevenson 2021; Hill and Tausanovitch 2015; Maoz and Somer-Topcu 2010; Rehm and Reilly 2010), and the combina-

[3]Many public opinion researchers attempt to capture the concentration of survey responses by calculating the correlation between ideology and policy preferences (e.g. Abramowitz and Saunders 2008; Hetherington, Long, and Rudolph 2016). By contrast, I see correlations across issue positions as representations of issue constraint (Converse 1964). This may be a fine approximation of within-group variation, but individuals lack coherent ideologies even in polarized societies (Kinder and Kalmoe 2017) and simply showing high issue constraint does not necessarily imply the distance associated with polarization.

[4]The measure I propose below is designed to capture these two features. It may not be suitable for analysis of party system polarization (e.g. Dalton 2008), which affords fewer data points and places less emphasis on intragroup homogeneity.

tion of interparty conflict and intraparty homogeneity lays at the root of theories that explain how polarization develops (e.g. Aldrich and Rohde 2000).

In addition, only considering one feature has an impact on the substantive inferences one can draw. Abramowitz and Saunders (2008) and Fiorina, Abrams, and Pope (2008) engage in a prominent debate on the very existence of polarization in the American mass public, with both sets of scholars presenting compelling evidence. Abramowitz and Saunders use measures designed to capture intergroup heterogeneity to argue that the American public has polarized in recent years, while Fiorina, Abrams, and Pope incorporate other measures designed to capture intragroup homogeneity and conclude that the public has *not* polarized but has instead sorted itself by party (see also Fiorina 2011; Levendusky and Pope 2011). Even if the public has not become more extreme, the party sorting process hypothesized by Fiorina, Levendusky, and others has nevertheless had a significant impact on American mass politics, resulting in polarization even in the supposed absence of increasing extremity.

To illustrate why both features are important and why difference-in-means and variance may paint an incomplete picture, consider the stylized distributions in Figure 1. They imitate four possible combinations of intergroup heterogeneity and intragroup homogeneity. The distribution in plot A has neither feature of polarization. Two clusters are faintly apparent, but they are close together and display little cohesion. Plot B has high intragroup homogeneity and low intergroup heterogeneity, and plot C has the reverse. The group means are the same in plot B as in plot A, but the groups are more concentrated. Intragroup homogeneity is the same in plot C as in plot A, but group means are further apart. Finally, plot D has high levels of both characteristics of polarization.

A measure correctly measuring polarization levels should indicate more polarization in plots B, C, and D relative to A and less polarization in plots A, B, and C, relative to D. Plot labels display the estimated level of polarization in each distribution as measured by difference-in-means, variance, and the CPC. In all cases, higher numbers indicate greater polarization.

Difference-in-means successfully distinguishes between plots A and C and between plots B and D, assigning a higher polarization estimate to the latter plot in each pairing, but it cannot distinguish

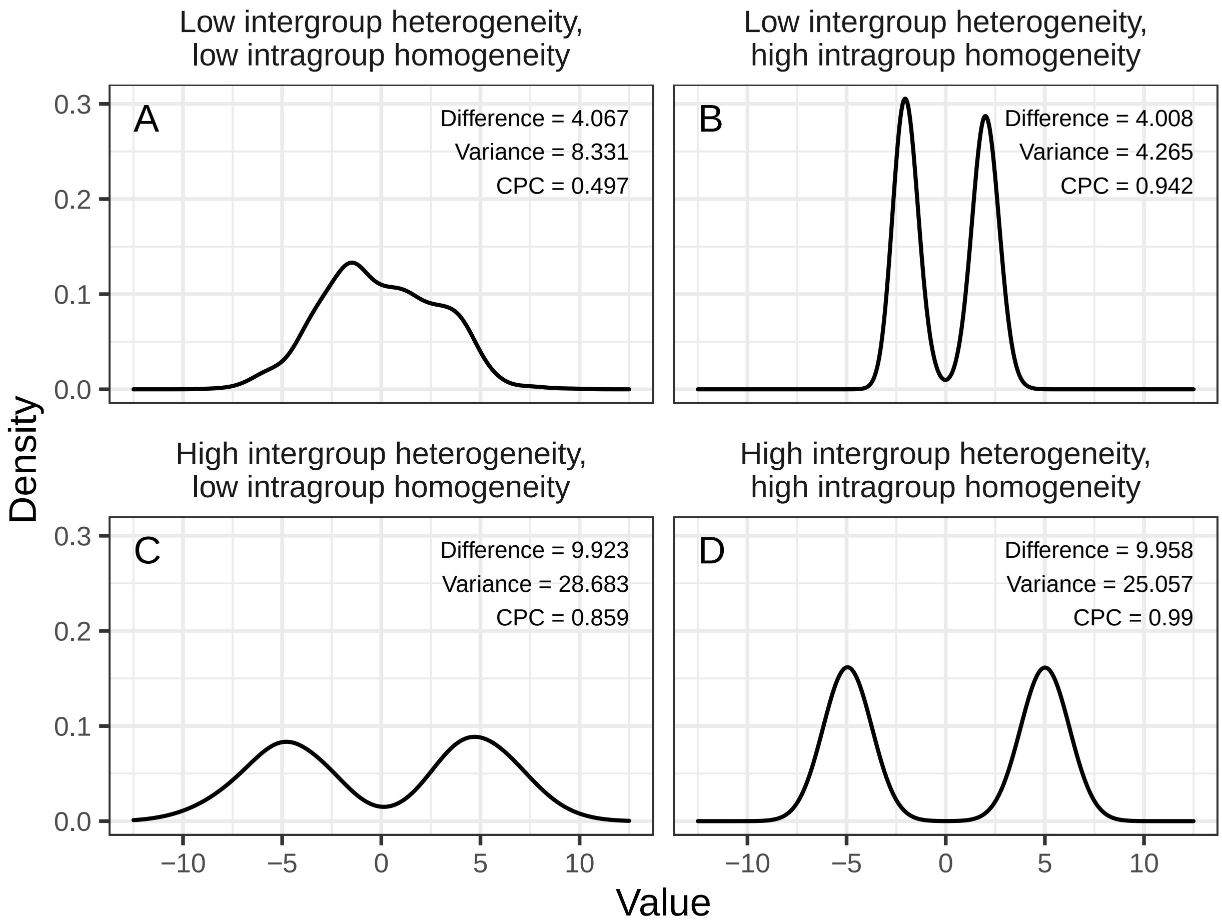


Figure 1: Stylized Distributions of Polarization Features. Simulated bimodal Gaussian mixture distributions all with $\mu_{global} = 0$. Labels show polarization levels according to difference-in-means, variance, and the CPC.

plot A from B nor plot C from D. It assigns identical polarization estimates to each distribution even though plots B and D are qualitatively more polarized than plots A and C, respectively. Variance performs even worse. It, too, distinguishes between plots A and C and between plots B and D, but cannot identify polarization vis-à-vis intragroup homogeneity. In fact, it assigns *lower* polarization estimates to plots B and D relative to plots A and C, a result opposite the conceptual understanding of polarization. This simple exercise suggests that using difference-in-means and variance may, in some cases, allow polarization to go undetected and, in others, identify it where it does not exist. The CPC is the only measure of the three that accurately mirrors the qualitative comparisons between all four distributions and captures both features of polarization.

Cluster-Polarization Coefficient

I begin from the premise that social scientific data—just like the distributions in Figure 1—are often comprised of distinct clusters of observations. In political contexts, such clusters are typically represented by parties or social groups. To derive the CPC, I decompose the total variance of this clustered data (TSS) in (1) into components corresponding to the two features of polarization: the variance accounted for between the clusters (BSS, corresponding to intergroup heterogeneity) and the variance accounted for within all clusters (WSS, corresponding to intragroup homogeneity). Dividing by TSS and solving for the BSS term in (1) gives an expression for the proportion of the total variance accounted for by the between-cluster variance—what I call the cluster-polarization coefficient. As it is a proportion, the value produced by this expression varies on the domain $[0,1]$.

$$\begin{aligned} TSS &= BSS + WSS, \\ \rightarrow CPC &= 1 - \frac{WSS}{TSS} = \frac{BSS}{TSS}. \end{aligned} \tag{1}$$

More formally, the CPC takes the expression in (2), where each individual i in cluster k holds a position on dimension j, and n_i, n_j, and n_k denote the number of observations, dimensions, and clusters, respectively. Similarly, x signifies a data point and μ gives the mean of those data points.[5]

$$CPC = 1 - \frac{\sum_{k=1}^{n_k}\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ikj}-\mu_{kj})^2}{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij}-\mu_j)^2} = \frac{\sum_{k=1}^{n_k}\sum_{j=1}^{n_j}(\mu_{kj}-\mu_j)^2}{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij}-\mu_j)^2}. \tag{2}$$

Expressed in this way, the CPC appears related to a one-way ANOVA F-statistic and the coefficient of determination (R^2). These are useful similarities for exploring properties of the measure, which I do below. But first, one additional modification is needed to make the CPC appropriate for comparison across contexts with varying numbers of observations, variables, and clusters. This is critical because intra-state political dynamics, the number and nature of sociopolitical cleavages,

[5] Full derivations are shown in Supplementary Information section S2.1.

and the size and number of political coalitions typically vary across countries or within countries over time. The expression in (2) will be biased upward in small samples, so I incorporate corrections for lost degrees of freedom and express the adjusted CPC in (3):

$$CPC_{adj} = 1 - \frac{\frac{\sum_{k=1}^{n_k}\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ikj}-\mu_{kj})^2}{n_i - n_j n_k}}{\frac{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij}-\mu_j)^2}{n_i - n_j}} = 1-(1-CPC)\frac{n_i - n_j}{n_i - n_j n_k}. \tag{3}$$

By explicitly modeling both BSS and WSS, the CPC takes into account both features of polarization. It increases when the distance between groups increases or when groups become more tightly concentrated around their collective ideal point, with the rate of those increases depending on the relative levels of BSS and WSS. All calculations in this paper use the adjusted CPC.

Distribution and Properties

Because the CPC is effectively a ratio of two variances, an F statistic can be calculated:[6]

$$F = \frac{\frac{BSS}{n_j n_k - n_j}}{\frac{WSS}{n_i - n_j n_k}} = \frac{n_i - n_j n_k}{n_j n_k - n_j}\frac{CPC}{1-CPC}, \tag{4}$$

where $BSS \sim \chi^2(n_j n_k - n_j)$ and $WSS \sim \chi^2(n_i - n_j n_k)$ under the null hypothesis that $BSS - WSS = 0$. F is therefore distributed as a central $F(n_j n_k - n_j, n_i - n_j n_k)$ random variable. Solving for the CPC term from (4) produces:

$$\begin{aligned} F &= \frac{n_i - n_j n_k}{n_j n_k - n_j}\frac{CPC}{1-CPC}, \\ \rightarrow CPC &= \frac{(n_j n_k - n_j)F}{(n_i - n_j n_k) + (n_j n_k - n_j)F}, \end{aligned} \tag{5}$$

[6]This set-up and the proofs which follow are similar to the derivation of a sampling distribution for R^2 (Magee 1990). Full derivations are shown in Supplementary Information section S2.2-S2.4.

implying that $CPC \sim \text{Beta}(\frac{n_j n_k - n_j}{2}, \frac{n_i - n_j n_k}{2})$ under the null—a sensible result given that the CPC and the Beta distribution both have continuous support on the [0, 1] interval. From this distribution, it is straightforward to recover the mean:

$$\text{E}(CPC) = \frac{\frac{n_j n_k - n_j}{2}}{\frac{n_j n_k - n_j}{2} + \frac{n_i - n_j n_k}{2}} = \frac{n_j n_k - n_j}{n_i - n_j}. \tag{6}$$

The unadjusted CPC is therefore asymptotically unbiased, again under the null hypothesis that $BSS - WSS = 0$, as $\lim_{n_i \to \infty} \text{E}(CPC) = 0$. However, it will be biased upward in finite samples, and the degree of that bias will depend on both the number of dimensions and number of clusters, underscoring the need for the degrees-of-freedom corrections shown above. Taking advantage of the expression in (6) reveals that the *adjusted* CPC is, in fact, an unbiased estimator under the null:

$$\text{E}(CPC_{adj}) = \text{E}[1 - (1 - CPC)\frac{n_i - n_j}{n_i - n_j n_k}] = 0. \tag{7}$$

The variance of the CPC can also be recovered from its sampling distribution:

$$\text{Var}(CPC) = \frac{\frac{n_j n_k - n_j}{2}\frac{n_i - n_j n_k}{2}}{(\frac{n_j n_k - n_j}{2} + \frac{n_i - n_j n_k}{2})^2(\frac{n_j n_k - n_j}{2} + \frac{n_i - n_j n_k}{2} + 1)} = \frac{2(n_j n_k - n_j)(n_i - n_j n_k)}{(n_i - n_j)^2(n_i - n_j + 1)}. \tag{8}$$

Using (8), the variance of the adjusted CPC is therefore:

$$\text{Var}(CPC_{adj}) = \text{Var}[1 - (1 - CPC)\frac{n_i - n_j}{n_i - n_j n_k}] = \frac{2(n_j n_k - n_j)}{(n_i - n_j n_k)(n_i - n_j + 1)}, \tag{9}$$

implying that the adjusted CPC is consistent, as $\lim_{n_i \to \infty} \text{Var}(CPC_{adj}) = 0$. Supplementary Information section S2.5 presents additional properties of the measure.

Simulation Evidence

Set-Up

In this section, I present evidence for the efficacy of the CPC by simulating both univariate and bivariate data using Gaussian mixture distributions. The purpose of this simulation exercise is to evaluate—in a controlled environment—the extent to which the CPC captures the two features of polarization, and whether it does so better than existing measures. Gaussian mixtures are uniquely suited for this purpose because they provide a straightforward method for mimicking distributional polarization. Each component of a Gaussian mixture is parameterized by a location parameter μ and scale parameter σ, which neatly correspond to the two features of polarization: intergroup heterogeneity and intragroup homogeneity, respectively.

Figure 2 presents a visualization of this intuition. This figure displays kernel density plots for simulated univariate data with two components. The plots are arranged such that the least polarized distributions fall at the top left and the most polarized distributions fall at the bottom right, and component parameters are provided by the plot labels along the top and right axes. Consider what happens to these component parameters as we move from a less polarized to a more polarized distribution. Moving from left to right across the rows of the facet plot, for example, the distributions become more polarized as the difference between component means increases and the components grow farther apart. Likewise, moving from top to bottom along the columns of the facet plot, the distributions become more polarized as component standard deviations decrease and the components grow more compact.

By manipulating these component parameters, therefore, I can generate mixture distributions with varying levels of polarization and estimate those levels using the CPC and other existing measures. I focus here on comparing the CPC to the two most popular strategies for measuring polarization in the political science literature: difference-in-means and variance. To identify the accuracy of each polarization measure, I examine the intergroup heterogeneity and intragroup

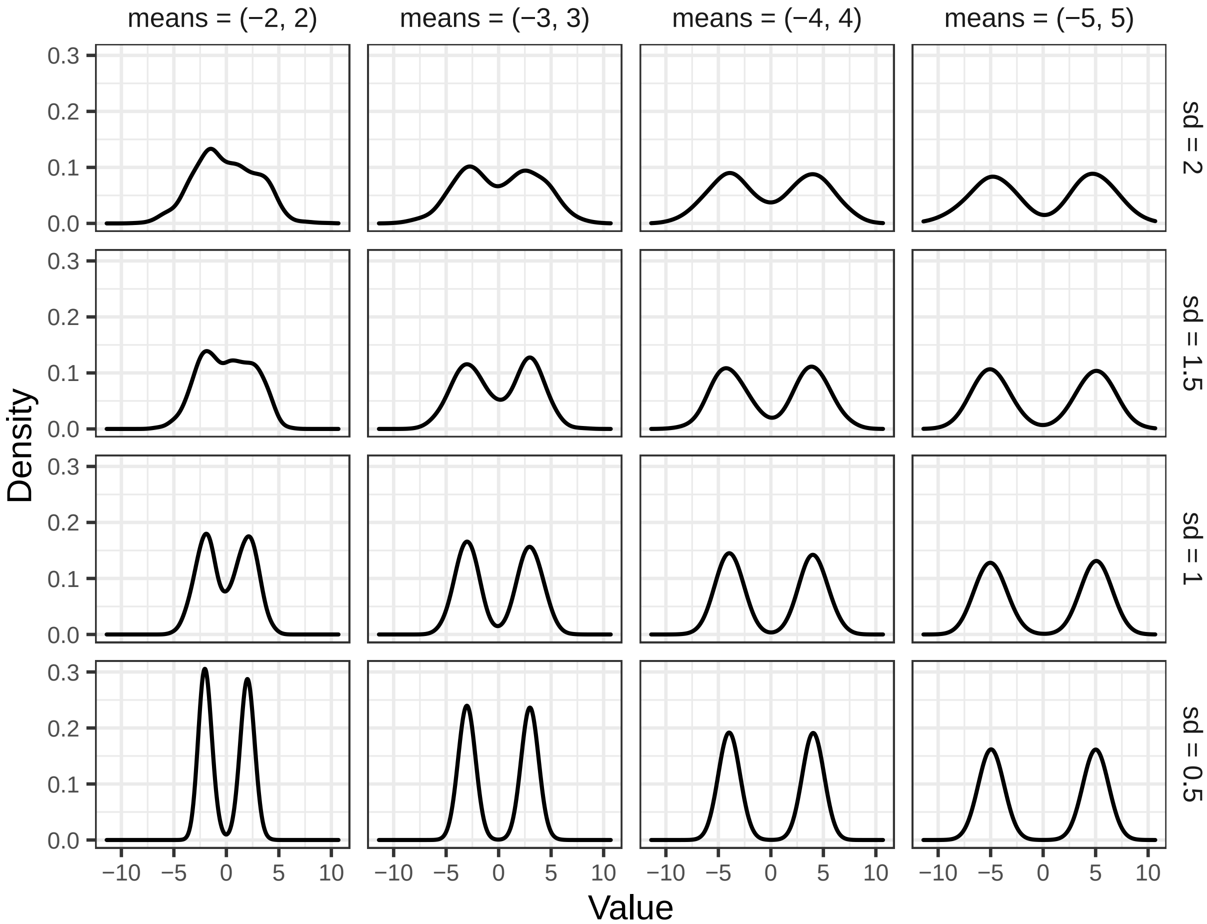


Figure 2: Visualization of Simulation Set-Up. Simulated Gaussian mixture distributions with $\mu_{global} = 0$; rows represent diverging means with standard deviations held constant and columns represent decreasing standard deviations with means held constant; thus, the least polarized distributions appear at top left and the most polarized distributions appear at bottom right.

homogeneity features separately. To simulate polarization as a result of increasing intergroup heterogeneity, I execute a four-step simulation exercise:

1. Fix component standard deviations at a range of values $\sigma \in \{0.5, 1, 1.5, 2\}$.[7] For identification, I use the same σ for each component and maintain a global mean of zero.

2. For each component standard deviation σ, select 1,000 values of μ as independent draws from $\mathrm{U}(2,5)$.

[7]As seen in Figure 2, even this relatively short range of values is sufficient to generate distributions ranging from unimodal to distinctly bimodal.

3. Take 1,000 independent draws from a Gaussian mixture parameterized by $\mathrm{N}(-\mu,\sigma;\mu,\sigma)$.

4. Apply each polarization measure to the resulting distribution.

The result of this procedure is 1,000 distributions, each with $N = 1000$, with which to evaluate the performance of each polarization measure. To simulate polarization as a result of increasing intragroup homogeneity, I execute a similar four-step simulation exercise:

1. Fix component means at a range of values $\mu \in \{2,3,4,5\}$.[8] For identification, I use the same absolute value of μ for each component and maintain a global mean of zero.

2. For each component mean μ, select 1,000 values of σ as independent draws from $\mathrm{U}(0.5,2)$.

3. Take 1,000 independent draws from a Gaussian mixture parameterized by $\mathrm{N}(-\mu,\sigma;\mu,\sigma)$.

4. Apply each polarization measure to the resulting distribution.

The result of this procedure is 1,000 distributions, each with $N = 1000$, with which to evaluate the performance of each polarization measure. For identification, I use equal component weights across all simulated distributions.[9]

Using these simulation frameworks, I evaluate the performance of the adjusted CPC relative to difference and variance in both univariate and bivariate contexts.[10] Pursuant to the two definitional characteristics of polarization, an appropriate measure should indicate higher polarization when the distance between component means increases or when the standard deviation of each component decreases. Because polarization can occur around more than two poles, especially in multiparty systems, I conduct these procedures for distributions with two, three, and four components.[11] For all simulations, I calculate the adjusted CPC using true group memberships, which are known from

[8] Again, as seen in Figure 2, even this relatively short range of values is sufficient to generate distributions ranging from unimodal to distinctly bimodal.

[9] Additional simulations in Supplementary Information section S3.3 investigate how the CPC changes in response to varying component weights.

[10] For bivariate data, I calculate difference by taking the average Euclidean distance between all component means, and I calculate variance as the trace of the covariance matrix.

[11] For three and four components, I calculate the difference score by taking the average distance between all component means.

the data randomization procedure. By using true group memberships instead of estimating them using a clustering algorithm, we can be sure that any advantages or disadvantages uncovered in the simulation results are attributable to the CPC itself and not to a clustering method being well- or ill-suited to this particular data structure.

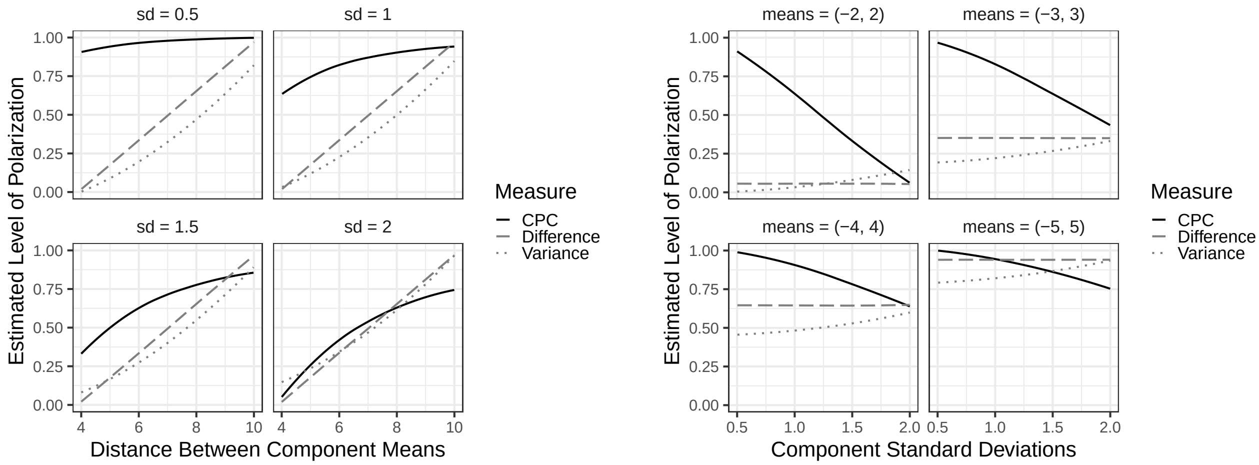
Results

I evaluate each simulated distribution using all three measures and present the results in two ways. First, Figures 3 and 4 present the raw polarization estimates as a function of the randomized parameters for two-component simulations with univariate and bivariate data, respectively.[12] All measures are scaled to $[0, 1]$ to enable comparison and plotted using locally estimated scatterplot smoothing (LOESS). A measure performing in line with theoretical expectations would register a positive slope in plot (a) and a negative slope in plot (b). However, the magnitude of those slopes and the absolute level of estimated polarization should differ depending on the fixed parameter. For example, the sets of distributions with fixed $\sigma = 0.5$ or fixed $\mu = (-5, 5)$ are more polarized on average than the distributions with a greater fixed σ or fixed μ parameters that are closer together. As a result, polarization estimates should generally be higher for those distributions and less sensitive to the value of the randomized parameter.

The results presented in Figures 3 and 4 generally align with expectations. Looking first at plot (a), slopes for difference, variance, and the adjusted CPC carry the expected sign. The magnitude of the difference and variance slopes, however, is relatively constant regardless of the fixed parameter, and the absolute level of estimated polarization appears similar. For example, with random component means of $(-5, 5)$ at the far right hand side of each facet, difference and variance output almost identical polarization estimates regardless of whether component standard deviations are 0.5, 2, or anywhere in between. The CPC, on the other hand, appears more sensitive to those fixed parameters and displays intercepts and slope magnitudes more in line with expectations.[13] The

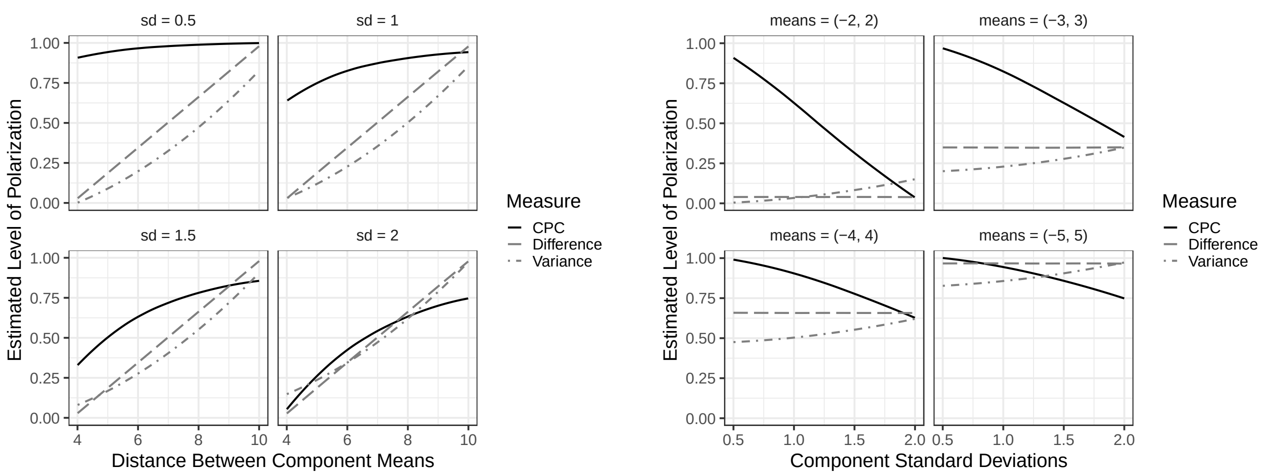
[12] Supplementary Information section S3.1 contains results of three- and four-component simulations.

[13] At extremely high levels of polarization (e.g. $\sigma = 0.5$), however, the CPC is likewise relatively insensitive to increasing the distance between component means. This may not be a desirable property if the intended use is to



(a) Random μ (intergroup heterogeneity) (b) Random σ (intragroup homogeneity)

Figure 3: Univariate Polarization Estimates with Two Components. Results from univariate simulations of polarization measures with two components, showing estimated level of polarization for a randomly varying distribution parameter, holding the other parameter constant. All measures scaled to $[0,1]$ to enable comparison and plotted using LOESS.



(a) Random μ (intergroup heterogeneity) (b) Random σ (intragroup homogeneity)

Figure 4: Bivariate Polarization Estimates with Two Components. Results from bivariate simulations of polarization measures with two components, showing estimated level of polarization for a randomly varying distribution parameter, holding the other parameter constant. All measures scaled to $[0,1]$ to enable comparison and plotted using LOESS.

insensitivity of difference and variance to component standard deviations can be seen more clearly in plot (b). While the adjusted CPC again performs as expected, difference and variance appear as nearly flat lines, although they do output higher polarization estimates when the difference between fixed component means grows larger.

Understanding how raw polarization estimates track with distributional characteristics is valuable, but it complicates a formal evaluation of a measure's effectiveness because the estimated level of polarization (the output of each measure) and the parameters controlling the simulated level of polarization (standard deviation or distance between means) are different quantities and are on different scales. Moreover, we do not have information about the "true" level of distributional polarization—estimating such quantities is the very goal of this measurement approach.[14]

By holding all other distributional characteristics constant and randomly varying only component means and standard deviations, however, we do have information about each distribution's level of polarization *relative* to every other distribution. For example, the simulation to assess the intergroup heterogeneity feature holds standard deviations constant and randomly varies component means. Randomly generated means that are further apart will generate a distribution that is, in theory, more polarized. The result of the simulation, then, is 1,000 distributions that randomly vary in their level of polarization, and those relative levels of polarization can be identified by the relative value of the random component means. I therefore follow the approach taken by Lupu, Selios, and Warner (2017) and use the estimated polarization from each measure to rank order the distributions and compare those rankings to the true rank order recovered from the randomized parameters, with a higher rank indicating a greater level of polarization.[15]

measure polarization when groups are extremely concentrated around their ideal point. However, Supplementary Information section S6.3 analyzes the relative weight each feature has on the CPC across a range of values plausible in real-world data, and results suggest this diminishing impact only occurs at very high levels of polarization.

[14]Supplementary Information section S4 pursues another strategy for procuring "ground-truth" polarization estimates: using human coders to evaluate relative polarization levels.

[15]One downside to this rank order approach is that it coerces all polarization measurements to an evenly spaced scale, thereby requiring an assumption of cardinality. However, when considered in conjunction with Figures 3 and 4, I believe the benefit from evaluating the magnitude of error introduced by each measure is greater than the cost imposed by this assumption.

		Univariate			Bivariate		
		Difference	Variance	CPC	Difference	Variance	CPC
				$k=2$			
RMSE	Intergroup Heterogeneity	**14.67**	15.38	30.73	**10.59**	11.2	22
	Intragroup Homogeneity	406.65	572.36	**24.01**	406.15	574.71	**16.79**
	Overall	287.73	404.87	**27.58**	287.29	406.46	**19.57**
MAE	Intergroup Heterogeneity	**10.73**	11.2	23.84	**7.68**	8	17.16
	Intragroup Homogeneity	335.05	496.49	**17.54**	334.47	498.37	**12.28**
	Overall	172.89	253.84	**20.69**	171.07	253.18	**14.72**
				$k=3$			
RMSE	Intergroup Heterogeneity	17.7	**19.03**	32.33	16.39	**13.29**	21.91
	Intragroup Homogeneity	400.4	573.84	**25.41**	409.82	575.34	**18.08**
	Overall	283.4	405.99	**29.07**	290.02	406.93	**20.08**
MAE	Intergroup Heterogeneity	**12.88**	13.64	25.13	11.99	**9.32**	17.12
	Intragroup Homogeneity	329.82	497.46	**18.41**	338.37	498.92	**13.27**
	Overall	171.35	255.55	**21.77**	175.18	254.12	**15.2**
				$k=4$			
RMSE	Intergroup Heterogeneity	**10.22**	10.39	103.02	15.17	**11.57**	21.81
	Intragroup Homogeneity	403.64	567.64	**21.5**	433.29	574.87	**17.35**
	Overall	285.51	401.45	**25.5**	306.57	406.58	**19.71**
MAE	Intergroup Heterogeneity	**7.44**	7.54	22.38	11	**8.27**	16.91
	Intragroup Homogeneity	332.94	493.01	**15.91**	361.31	498.28	**12.73**
	Overall	170.19	250.27	**19.15**	186.16	253.28	**14.82**

Table 1: Error of Distribution Rankings in Simulation. Root mean squared error and mean absolute error calculated for univariate and bivariate simulations with two, three, and four components; bolded values denote measure with lowest error in each category.

Table 1 reports the root mean squared error and mean absolute error for all three measures across all simulations. Bolded values represent the best-performing measure in each category. Examining these results, a clear pattern emerges. Difference and variance register the lowest error rates on the intergroup heterogeneity feature, regardless of the number of variables or components. This makes intuitive sense; holding the standard deviations of a mixture distribution constant and pulling the component means in opposite directions should, by definition, result in wider differences between means and higher variance for the distribution as a whole. The CPC performs as expected on both features and often improves slightly in higher dimensions and multimodal distributions, which enables comparison across a wider variety of contexts.

Finally, although it is necessary to examine each feature in isolation, real-world data rarely holds one feature constant while manipulating the other. When looking at each measure's respective overall performance, the CPC appears to perform best regardless of the number of variables or components, and often by a drastic margin.[16] As seen in Table 1, this performance gain appears to come from the CPC performing only slightly worse on the intergroup heterogeneity feature but dramatically better on the intragroup homogeneity feature.

The Supplementary Information contains three additional validation exercises using synthetic data. In section S3.2, I use heavy-tailed log-normal distributions to show that the CPC can be sensitive to outliers, but that scaling the data before applying the measure can ameliorate this problem. In section S3.3, I show that the CPC is relatively insensitive to changes in polarization when the difference in component weights is very high. However, it still performs better than difference-in-means and variance. Finally, in section S4, I benchmark each measure's performance against "ground-truth" data gathered from human annotators. The CPC consistently captures both features of polarization better than difference-in-means and variance, and it adheres more closely to intuitive identifications of polarization.

Applications

In addition to a close fit with the conceptual understanding of polarization, the CPC proves valuable when studying multiple dimensions of contestation or comparing across cases with more than two groups. I use two datasets of elite ideal points and one dataset of party affect in mass publics to illustrate these benefits. First, I investigate the well-established rise in congressional polarization in the United States, showing that the CPC is the only measure of polarization that recovers this increase in multidimensional data. Next, I take on the methodological puzzle of how to measure polarization in multi-party systems and compare across systems with different numbers of parties or groups. To do so, I turn to a set of elite ideal points in six industrially developed, con-

[16]Overall performance is calculated by simply combining results from the intergroup heterogeneity and intragroup homogeneity simulations and calculating error statistics over the entire body of evidence.

solidated democracies. Finally, I engage the growing research program on affective polarization in comparative perspective, showing how the CPC produces sensible results when *both* the number of dimensions and number of groups vary across cases. Throughout all applications, I compare the CPC to difference-in-means and variance, the two most popular strategies for measuring polarization in the political science literature.

Polarization in the United States Congress

I calculate congressional polarization using DW-NOMINATE data (Lewis et al. 2021), which uses a scaling procedure to estimate the ideal points of individual legislators in a two-dimensional latent space. The first dimension generally captures economic issues and the second captures racial and other social issues. The presence of two dimensions enables me to show how the CPC can extract theoretically sensible polarization estimates from multiple variables at once, an exercise that can undermine the performance of difference-in-means and variance.

A steady increase in party polarization has been one of the defining features of congressional politics in the late twentieth and early twenty-first centuries (Hill and Tausanovitch 2015; Theriault 2008), with the partisan chasm in the House of Representatives especially widening upon Newt Gingrich's rise through the leadership ranks in the 1990s (Mason 2018; Rosenfeld 2018). Focusing on the period from 1975 to the present,[17] each measure should indicate increasing polarization whether they are examining only the first dimension or incorporating both dimensions. Many polarizing events in this time period had social issues at their root, such as the Southern strategy that pushed the Republican Party to the right following the civil rights movement. Even redistributive issues—which are captured by the first dimension—are often racialized in the United States (Gilens 1999), implying that the two dimensions are often linked in practice.

For each Congress, I estimate the level of polarization in NOMINATE scores using difference-in-means, variance, and the CPC. These estimates appear in Figure 5. All measures generally capture the expected trend in the first NOMINATE dimension. Adding the second dimension,

[17]Voting reforms changed the composition of the roll-call record beginning in the early 1970s. To ensure results are not affected by this structural change in the underlying data (Roberts and Smith 2003), I begin the analysis in 1975.

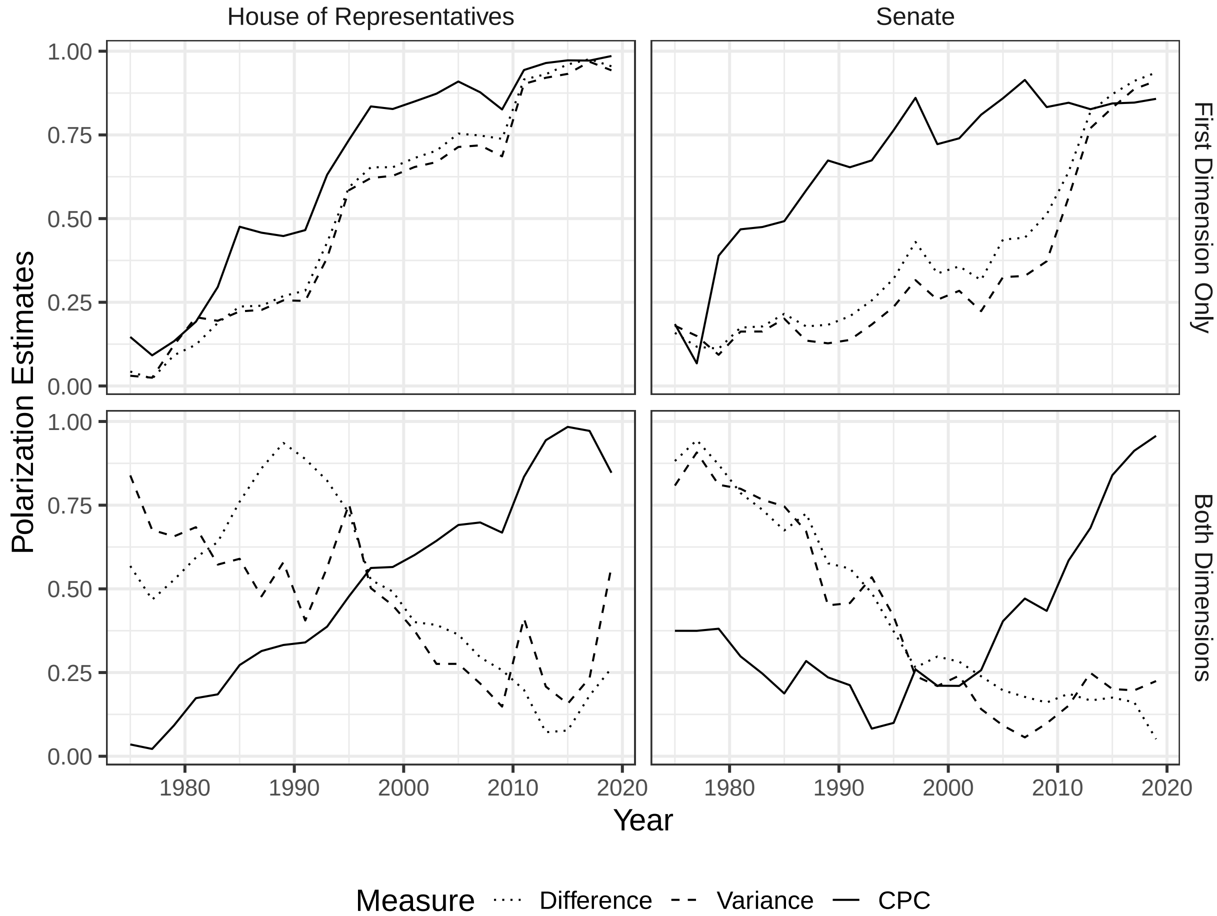


Figure 5: Estimates of Congressional Polarization. Calculated using NOMINATE ideal point estimates; each measure scaled to $[0, 1]$ to enable comparison.

however, leads difference-in-means and variance to return trends that do not match the historical record.[18] In the House of Representatives, difference-in-means suggests polarization reached its peak in 1989 and has been declining ever since, and variance indicates a mostly flat trend, with present-day polarization no more severe than in the mid-1970s. In the Senate, both measures indicate steadily decreasing polarization. None of these results reflect scholars' qualitative understanding of congressional polarization. In contrast, the CPC handles the addition of the second dimension well and recovers more sensible trends: increasing polarization since the start of the time series in the House of Representatives and since the turn of the century in the Senate.

[18] As before, I calculate difference-in-means by taking the average Euclidean distance between group means, and I calculate variance using the trace of the covariance matrix.

Polarization in Multi-Party Systems

One common challenge confronted by scholars is how to measure polarization in multi-party systems (Gidron, Adams, and Horne 2020; Reiljan 2020; Ward and Tavits 2019)—an obstacle that applies to most modern democracies. To demonstrate how the CPC captures polarization across contexts with different numbers of groups, I present two examples of polarization in comparative perspective, one using a small dataset of elite ideological ideal points and another investigating affective polarization in mass publics.

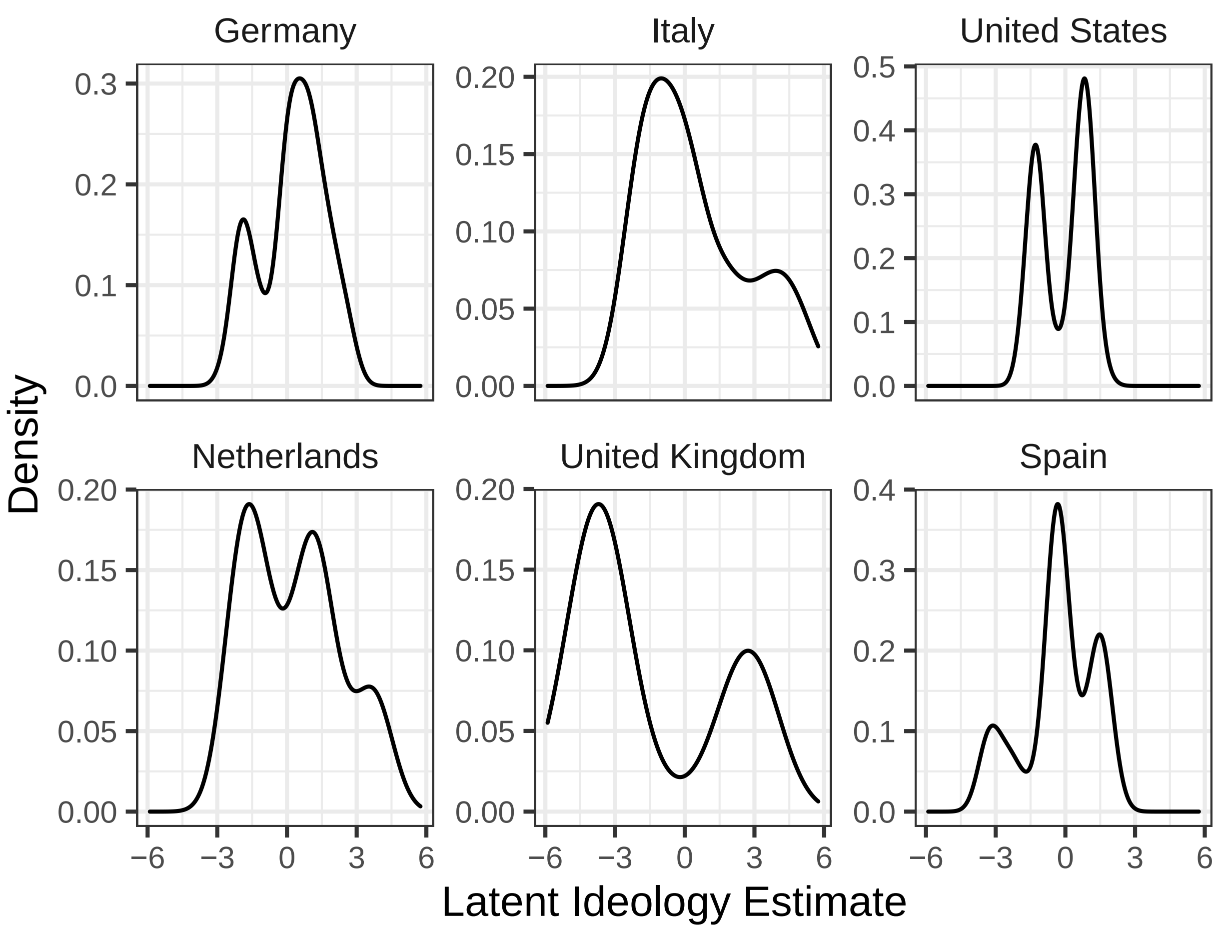


Figure 6: Kernel Density Estimates of Elite Ideal Points in Each Country

I turn first to a set of elite ideological ideal points from six countries in Western Europe and North America (Barberá 2015). These countries differ in the extent to which their latent elite groups are internally homogeneous and externally heterogeneous, and even in how many groups exist within each country. These differences can be seen in Figure 6. For instance, some countries—like the United States and Germany—exhibit two parties or ideological coalitions, while Spain and the Netherlands have three. Even comparing across countries with the same number of coalitions reveals substantial variation in the size and strength of each. In the United States, the two parties are of roughly equal size and are tightly concentrated around their midpoints. In contrast, Germany's left-most coalition is small but homogeneous while its right-most coalition is large but more heterogeneous.

With all this variation, it is difficult to determine an exact ordering of countries by their level of polarization simply by inspecting density plots. However, examining contemporaneous political events occurring in these countries can help establish which ones should reveal higher levels of polarization and which ones should reveal lower levels. At the time of data collection (2012-2013), Spain was enduring vigorous debate over austerity measures in the wake of the Euro crisis (Miley 2017), the United Kingdom was beginning down the path toward Brexit (Hobolt, Leeper, and Tilley 2021), and the Netherlands was navigating growing tensions surrounding immigration and ethnic integration (Oosterwaal and Torenvlied 2010). These countries were experiencing notable elite conflict. They should therefore be among the most polarized countries in this small sample. Germany, on the other hand, was enjoying its third grand coalition since the mid-2000s (Voigt 2019). This elite cooperation should be reflected in a much lower level of polarization.

Figure 7 displays the degree to which each measure suggests the party systems are polarized.[19] The CPC is shown in the leftmost facet and countries are ordered by their estimated level of polarization. CPC estimates indicate that Spain, the United Kingdom, the Netherlands, and the United States have similar, relatively high levels of polarization, while Germany and Italy are significantly less polarized. This grouping of more- and less-polarized countries corresponds to the historical events occurring in each case. The distributions in Figure 6 also seem to support this conclu-

[19]The CPC and difference-in-means measures require knowledge of the cluster to which each observation belongs, so I perform k-means clustering with the number of clusters in each country implied by Figure 6. For example, Germany has two clusters, Spain has three, and so on. Analyses of silhouette scores in Supplementary Information section S5.1 corroborate these choices of cluster numbers.

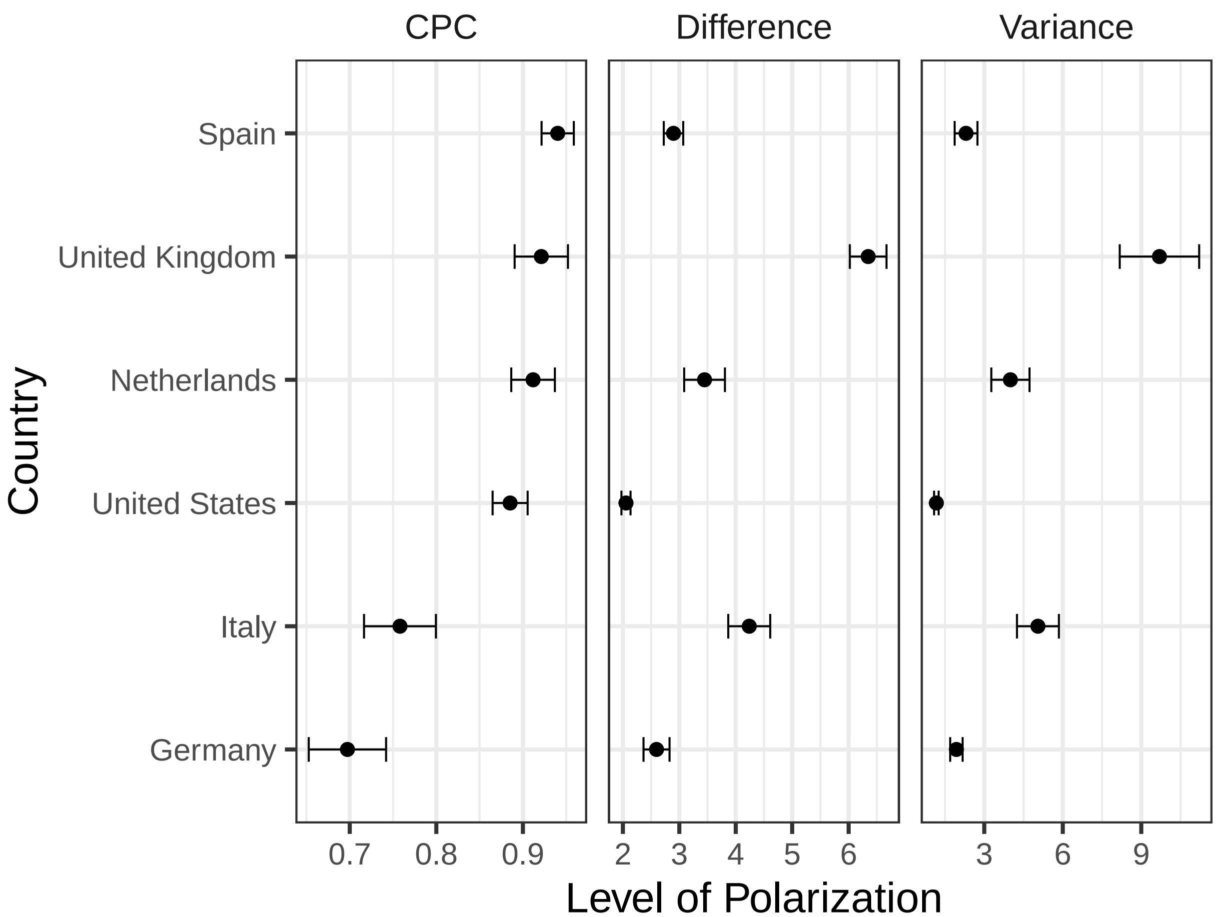


Figure 7: Estimates of Elite Polarization Across Countries. Error bars give 95% confidence intervals, calculated using a case-resampling bootstrap. Countries ordered by level of polarization estimated by the CPC. Supplementary Information section S5.2 presents polarization estimates and standard errors.

sion if one considers polarization as a function of both intergroup heterogeneity and intragroup homogeneity. The United Kingdom and the Netherlands have high intergroup heterogeneity, the United States has high intragroup homogeneity, and Spain has a fair amount of both. Italy and Germany both have two identifiable modes, but their intragroup homogeneity is so low and their minority coalitions so small in relation to their majority coalitions that their consequent levels of polarization are not appreciable.

Difference-in-means and variance—shown in the center and right facets in Figure 7—tell very different stories that are often at odds with the visual representation of the data. They suggest that the United States is the least polarized, despite a distribution that displays perhaps the highest intragroup homogeneity of the six countries. Italy is the second-most polarized according to these measures, despite a nearly unimodal distribution of ideal points. Even in only one dimension, existing measures struggle to produce polarization estimates that are comparable across cases with different numbers of groups.

Affective Polarization Across Contexts

I move now from elite ideological polarization to mass affective polarization. This topic has received substantial scholarly attention in the United States (Iyengar, Sood, and Lelkes 2012; Mason 2015) and, more recently, a robust research program has sought to understand affective polarization by taking a comparative perspective (Gidron, Adams, and Horne 2020; Wagner 2021). I use party feeling thermometers—a common operationalization among scholars of affective polarization (Reiljan 2020; Ward and Tavits 2019)—from module four of the Comparative Study of Electoral Systems (CSES, fielded 2011-2016), the most recent module that has been completed and finalized.[20]

In the previous two applications, I grappled with the problems of multi-dimensional and multi-group data *separately*, but party affect data present both challenges *together*. Party systems differ in size across countries, so the number of parties represented in the sample (groups) and the number of parties respondents are asked to rate (dimensions) both vary from one case to the next.[21] Taking all this variation into account and enabling comparison across diverse cases is a challenging task for most measures of polarization, but the CPC offers a solution. Figure 8 displays estimated levels of affective polarization across all twenty-eight countries in the sample, according to each measure. As in Figure 7, the CPC is shown in the leftmost facet and countries are ordered by their estimated level of polarization.

[20]Following Wagner (2021), I exclude countries with a Polity score less than 8.

[21]To avoid small, insignificant parties having an outsize influence on results, I only retain ratings of parties which won at least 10 percent vote share in the most recent election.

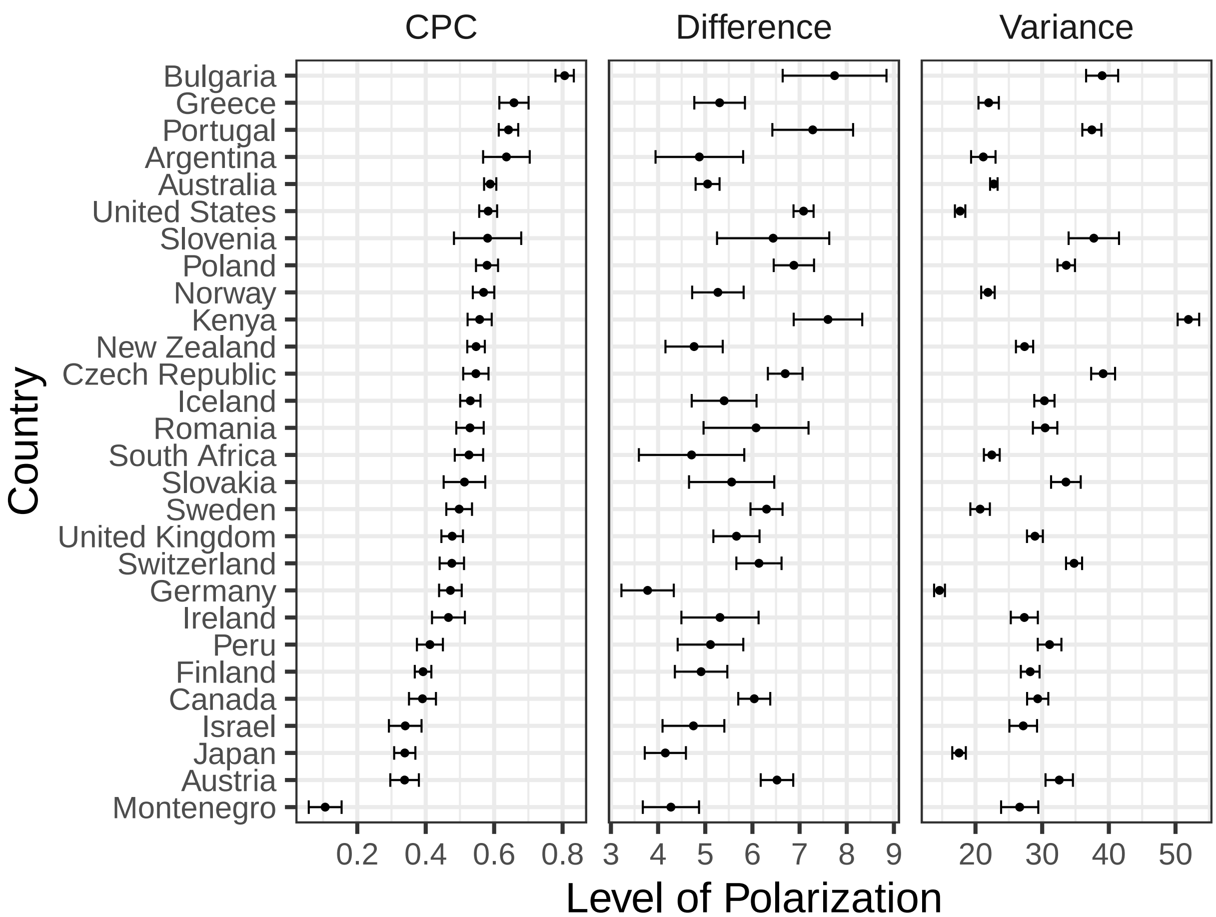


Figure 8: Affective Polarization Levels Across Countries. Error bars give 95% confidence intervals, calculated using a case-resampling bootstrap. Supplementary Information section S6.1 presents polarization estimates and standard errors.

To evaluate the validity of these estimates, I calculate their correlation with four survey items that were also included on CSES module 4.[22] The first two correlates—the proportion of respondents indicating "it makes a big difference who is in power" and "who people vote for can make a big difference"—capture the degree to which respondents see their political system as comprised of starkly differentiated options in the voting booth, a characteristic of affectively polarized societies (Ward and Tavits 2019). Next, the proportion of respondents expressing that they "feel very close" to their political party measures the intensity with which respondents hold their party iden-

[22]Wagner (2021) also examines three of these variables, but he combines all CSES modules and estimates multivariate models.

tity. This sentiment is likely to be heightened in an affectively polarized party system (Bankert, Huddy, and Rosema 2017). The final correlate—the proportion of respondents indicating that they hold positions at the extreme ends of a left-right self-placement scale—should also be related to affective polarization (Westwood et al. 2018).

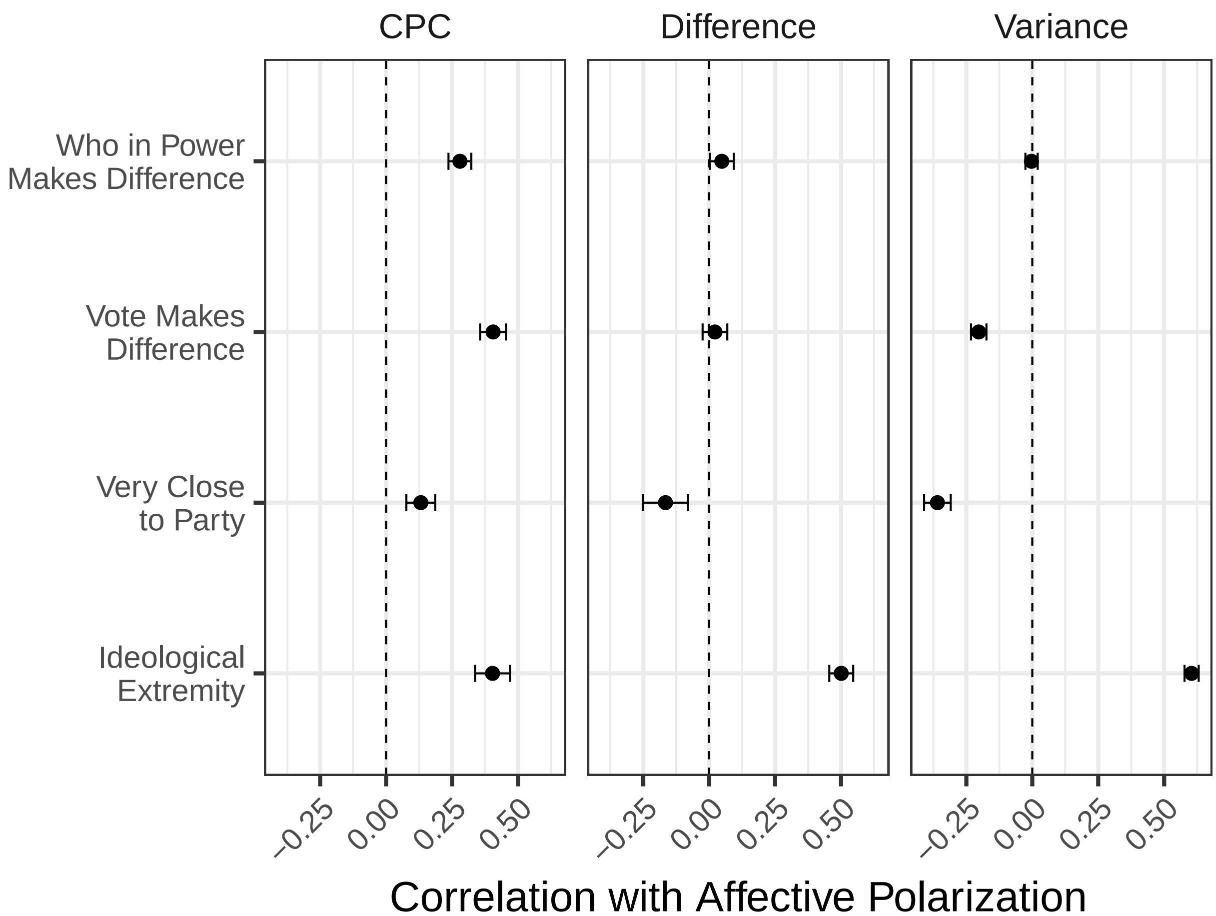


Figure 9: Correlates of Affective Polarization. Error bars give 95% confidence intervals, calculated using a case-resampling bootstrap. Supplementary Information section S6.2 presents correlations and standard errors.

Figure 9 illustrates how affective polarization correlates with each of these items. CPC estimates of affective polarization display positive, statistically significant correlations with all four variables. In the case of holding an extreme ideology and believing one's vote makes a difference, these estimates are greater than 0.4, a substantial correlation for survey data. Difference-in-means and variance also uncover a strong, positive relationship between affective polarization and ideological extremity, but their relationship with other variables is either statistically indistinguishable from zero or in the opposite direction of what would be theoretically expected. Difference-in-means uncovers positive correlations between polarization and beliefs about whether it matters who one votes for and who is in power, but these correlations are substantively small and not statistically significant. More concerning, both difference-in-means and variance suggest that polarization is associated with fewer respondents feeling “very close” to their party. This result would seem to be at odds with the very definition of affective polarization.

Across all three applications, current measures of polarization frequently fail to return sensible results because they cannot account for both intergroup heterogeneity and intragroup homogeneity, they struggle in more than one dimension, or they are not comparable across distributions with varying numbers of groups.

Conclusion

Polarization is a critical concept in a wide variety of social scientific phenomena, yet efforts to quantify it fail to capture its two core conceptual elements: intergroup heterogeneity and intragroup homogeneity. The cluster-polarization coefficient takes both features into account, more closely aligning concept and measure and producing more sensible estimates of polarization in important contexts.

Critically, the CPC enables comparison over time and space. Political systems and attitudes—and thus the polarization of those systems and attitudes—are multidimensional (Bermeo 2003). Aldrich, Montgomery, and Sparks (2014) show that high-dimensional estimates are crucial for accurate ideal point estimation, especially in polarized contexts. Researchers using current measures may need to ignore higher-dimensional information as a pragmatic matter, but the CPC can accommodate numerous variables, allowing polarization estimates to accurately reflect the character of contestation in a political system. Equally important is the CPC’s ability to compare across systems with different numbers of parties and groups. Multi-party systems are common outside the

United States, but scholars often ignore this contextual feature either by calculating the variance of the entire distribution (Ward and Tavits 2019) or by converting party systems into a series of dyads and measuring polarization as if they were two-party systems (Gidron, Adams, and Horne 2020). The CPC, on the other hand, is explicitly designed to measure polarization in such contexts.

Though difference-in-means and variance are the most widely used measures in political science and related fields, the measure perhaps most similar to the CPC is that of Esteban and Ray (1994), which has been used most frequently in economics and ethnic studies. However, Clark (2009) shows that this measure is not much different from variance, with correlations as high as 0.933. Maoz and Somer-Topcu (2010) further argue that it is based on arbitrary assumptions, such as a small number of groups, and is very highly correlated with the number of groups and the size of each group. The CPC is conceptually simpler than Esteban and Ray's measure, and the adjusted CPC, in particular, is explicitly developed to correct for these types of issues.

I focused my demonstrations on elite ideological ideal points, but elites are not the only political actors who display polarization. The CPC can be used to aid measurement in a wide variety of substantive applications. Existing work that could benefit from employing the CPC includes studies of polarization in mass attitudes (DiMaggio, Evans, and Bryson 1996), racial polarization (Jardina and Ollerenshaw 2022), economic polarization (Dwyer 2013), and religious polarization (Ribberink, Achterberg, and Houtman 2018). Because the CPC is ultimately a measure of multimodality, it can also be used to gain information about other quantities of interest. One example is conditional party government, which mirrors the two-feature definition of polarization by emphasizing the degree of preference homogeneity *within* parties and the degree of preference conflict *between* parties (Aldrich 2011; Rohde 1991). Studies building on the conditional party government theory have utilized a wide range of measures to capture a holistic picture of its implications, but a summary measure similar to what the CPC provides has remained elusive. By introducing the CPC and providing an open-source software package to simplify its implementation, I aim to advance the critical study of polarization's causes and consequences.

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