A Group-Based Approach to Measuring Polarization

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Abstract

Political and social polarization are key concerns in many important social scientific topics, with a rapidly expanding literature emphasizing two features: intergroup heterogeneity and intragroup homogeneity. The quantitative measurement of polarization, however, has not evolved alongside this refined conceptual understanding, as existing measures capture only one feature or complicate comparison over time and space. To bring the concept and measurement of polarization into closer alignment, I introduce the cluster-polarization coefficient (CPC), a measure of multimodality that allows scholars to incorporate numerous variables and compare across contexts with varying numbers of parties or social groups. Applying the CPC to two data sets of elite ideological ideal points demonstrates that different measures can lead to different substantive results. An open-source software package implements the measure.

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Polarization has become an indispensable concept in the social sciences, playing a role in important political outcomes ranging from representation (Ahler and Broockman 2018) and party-building (Lupu 2015) to policymaking (Sinclair 2008) and democracy (McCoy and Somer 2019). Researchers generally employ a conceptual understanding of polarization that emphasizes two features—intergroup heterogeneity and intragroup homogeneity—yet scholars rarely use tools that translate this theoretical foundation into empirical application. Analysts have employed a wide variety of measures but, at best, they capture only one of these features.

I offer an alternative. The cluster-polarization coefficient (CPC) explicitly models intergroup heterogeneity and intragroup homogeneity, placing polarization's conceptual foundation front and center. I apply the CPC to two data sets of elite ideological ideal points and compare it to three widely used but potentially flawed measures. Beyond providing a closer fit with the theoretical concept of interest, results make clear that the CPC is a better choice when considering multiple dimensions of contestation or examining multi-party systems. In both of these cases, current measures of polarization may provide spurious or inaccurate results.

To make the measurement procedure widely accessible to researchers and practitioners, I provide an open-source R package. In addition to calculating the CPC with researcher-specified group memberships, this package contains support for a variety of clustering methods to assign observations to groups, making the measure easily applicable to a wide variety of data structures.

Features of Polarization

The academic literature on political polarization is diverse, with important sub-literatures that address party system polarization on one hand and societal, ideological, or mass polarization on the other. Research on party system polarization primarily focuses on the distance between parties, party families, or coalitions on various programmatic dimensions (Mainwaring and Scully 1995; Sartori 1976). For scholars concerned with societal, ideological, or mass polarization, the picture gets more complicated. Not only is distance from political opponents (intergroup heterogeneity) conceptually important, so too is the degree of concentration within social or political groups (intragroup homogeneity). In this conceptualization, polarization can increase in two different ways: as the positions of social or political groups grow farther apart from one another or as they become more internally cohesive (Baldassarri and Bearman 2007; Fortunato and Stevenson 2021; Levendusky and Pope 2011).[1] Common measures of polarization can capture one of these features at a time, but not both.

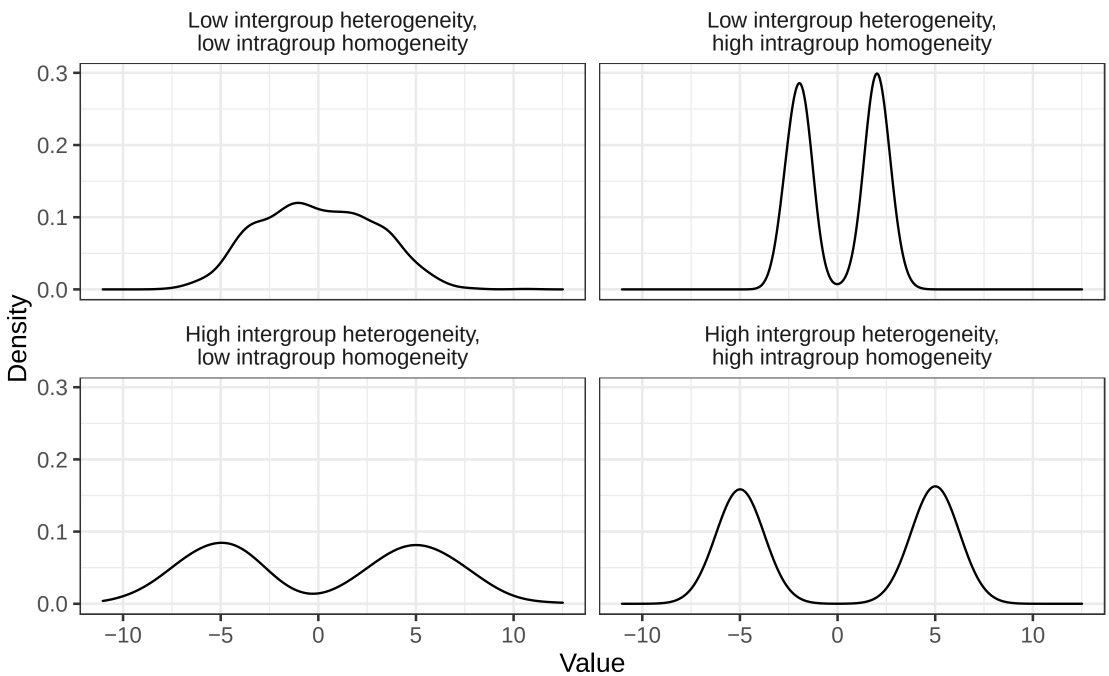


Figure 1: Stylized Distributions of Polarization Features. Simulated bimodal Gaussian mixture distributions all with $\mu_{global} = 0$.

To illustrate the importance of both features, Figure 1 displays stylized distributions designed to imitate them. The distribution at the top left would be characterized as not polarized. It does indeed have two groups, but they are so close to each other and display so little cohesion that

[1]This is the theoretical understanding of polarization with which I am primarily concerned. The measure I propose below is specifically designed to capture these two features, and it may not be suitable for analysis of party system polarization, which typically affords fewer data points and does not place theoretical emphasis on the concentration feature.

the distribution appears unimodal. The other three plots capture polarization with three different combinations of intergroup heterogeneity and intragroup homogeneity. In the bottom left, the groups have maintained the same degree of intragroup homogeneity as in the top left, but they have moved further apart. This is the most common way polarization is detected empirically, and current measures such as difference-in-means and variance tend to capture this feature.

But this is not the only way a population may become more polarized. The top right plot displays a distribution with the same group means as those in the top left plot, but the groups are more tightly concentrated, capturing increased intragroup homogeneity. The resulting distribution reflects polarization without any group divergence, yet common measures would struggle to capture this polarization. Indeed, analysts applying a difference-in-means measure to both plots in the top row would conclude that they exhibit the same degree of polarization—a result that does not reflect reality. Finally, the bottom right distribution is the most polarized of the four, featuring both intergroup heterogeneity and intragroup homogeneity. The challenge for measuring polarization in real-world data is that these two features can occur separately, simultaneously, or not at all. Moreover, each may increase or decrease independent of the other. To translate the conceptual understanding of polarization into quantitative terms, an ideal measure should incorporate both features.

Cluster-Polarization Coefficient

I offer a measure that captures both features, which I call the cluster-polarization coefficient (CPC). This measure is centrally concerned with groups and how features within and between those groups manifest in polarization. Citizens and political elites hold multifaceted social and political opinions, exist in a political climate that is similarly multifaceted, and cluster together with other citizens and elites who hold similar social and political opinions. I explicitly model these grouping patterns by decomposing the total variation in clustered data into components corresponding to the distance between clusters and the concentration of each cluster. I also develop this measure with an eye toward enabling comparison across space and time. This is critical because intra-state political features, the number and nature of important sociopolitical cleavages, and the size and number of political coalitions almost certainly vary across countries or within countries over time.

I begin from the premise that social scientific data—just like the distributions in Figure 1—often cluster into identifiable groups. To derive the CPC, I decompose the total variance of this clustered data (TSS) in (1) into components directly corresponding to the two features of polarization: the variance accounted for between the clusters (BSS, corresponding to intergroup heterogeneity) and the variance accounted for within all clusters (WSS, corresponding to intragroup homogeneity). Dividing by TSS and solving for the BSS term in (1) gives an expression for the proportion of the total variance accounted for by the between-cluster variance—what I call the cluster-polarization coefficient (CPC). As it is a proportion, the value produced by this expression will vary on the domain $[0,1]$.

$$\begin{aligned} TSS &= BSS + WSS, \\ \rightarrow CPC &= 1 - \frac{WSS}{TSS} = \frac{BSS}{TSS}. \end{aligned} \tag{1}$$

More formally, the CPC takes the expression in (2), where each individual i in cluster k holds a position on dimension j.[2] Expressed in this way, the CPC appears related to a one-way ANOVA F-statistic and the coefficient of determination (R^2). These are useful similarities for deriving properties of the measure, which I show in the Supplementary Materials.

$$CPC = 1 - \frac{\sum_{k=1}^{n_k}\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ikj}-\mu_{kj})^2}{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij}-\mu_j)^2} = \frac{\sum_{k=1}^{n_k}\sum_{j=1}^{n_j}(\mu_{kj}-\mu_j)^2}{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij}-\mu_j)^2}. \tag{2}$$

One additional modification is needed to make the CPC appropriate for comparison across contexts with varying numbers of observations, variables, and clusters, as the expression in (2) will be

[2] Full derivations are shown in the Supplementary Materials.

biased upward in small samples. I therefore incorporate corrections for lost degrees of freedom and express the adjusted CPC in (3):

$$CPC_{adj} = 1 - \frac{\frac{\sum_{k=1}^{n_k}\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ikj}-\mu_{kj})^2}{n_i - n_j n_k}}{\frac{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij}-\mu_j)^2}{n_i - n_j}} = 1 - (1 - CPC)\frac{n_i - n_j}{n_i - n_j n_k}. \tag{3}$$

In the Supplementary Materials, I derive a sampling distribution for the CPC and show that it is both unbiased and consistent after incorporating these corrections. All calculations in this paper use the adjusted CPC.

By explicitly modeling both *BSS* and *WSS*, the CPC takes into account both features of polarization. It increases when the distance between groups increases or when groups become more tightly concentrated around their collective ideal point, with the rate of those increases depending on the relative levels of *BSS* and *WSS*.

Applications: Ideology of Political Elites

In addition to a close fit with the conceptual understanding of polarization, the CPC proves particularly valuable when studying multiple dimensions of contestation or comparing across cases with more than two groups. I use two data sets of elite ideological ideal points to illustrate these benefits. First, I engage the well-researched connection between congressional polarization and income inequality in the United States, showing that the CPC is the only measure that recovers this connection in multivariate data. Next, I turn to a set of six industrially developed, consolidated democracies, taking on the methodological puzzle of how to measure polarization in multi-party systems and compare across systems with different numbers of parties or groups (Gidron, Adams, and Horne 2020; Wagner 2021). Throughout both applications, I compare the CPC to the three most popular strategies for measuring polarization in the political science literature: difference-in-means (e.g. McCarty, Poole, and Rosenthal 2006), variance (e.g. DiMaggio, Evans, and Bryson 1996), and kurtosis (e.g. Baldassarri and Bearman 2007).

Polarization in the United States Congress

I calculate congressional polarization using DW-NOMINATE data (Lewis et al. 2021), which uses a scaling procedure to estimate the ideology of individual legislators in a two-dimensional latent space. The first dimension generally captures economic issues and the second captures racial and other social issues. The presence of two dimensions can undermine the value of common measures of polarization, but it allows me to demonstrate a major advantage of the CPC.

To establish the CPC's construct validity, I show that it produces estimates consistent with well-known historical patterns. Historians and political scientists frequently point to three notable periods of ideological change in Congress. These periods are shaded on Figure 2. First, distinct partisan divides emerged as the Democratic Party made its comeback following the end of Reconstruction (Brady and Han 2006). Second, party convergence and high levels of bipartisanship characterized the "Textbook Congress" in the middle of the twentieth century (Shepsle 1989). Finally, consistently high levels of polarization are well-documented in the lead-up to the twenty-first century and extending to the present (Theriault 2008). Each measure should therefore indicate high polarization in the first time period, low polarization in the second period, and high polarization again in the third.

For each Congress, I estimate the level of polarization in DW-NOMINATE scores using difference-in-means, variance, kurtosis, and the CPC. These estimates appear in Figure 2. In general, all measures capture the expected trends in the first NOMINATE dimension. Adding the second dimension, however, leads current measures to return trends in polarization that fail to match the historical record. For example, difference-in-means estimates do detect increasing polarization in the modern period, but they also indicate that Congress experienced some of its most polarized years during the Textbook Congress. Variance and kurtosis estimates make similar indications, and they even find flat or declining polarization levels leading up to the present-day. In contrast, the CPC handles the addition of the second dimension well and recovers trends that reflect the historical record in each period.

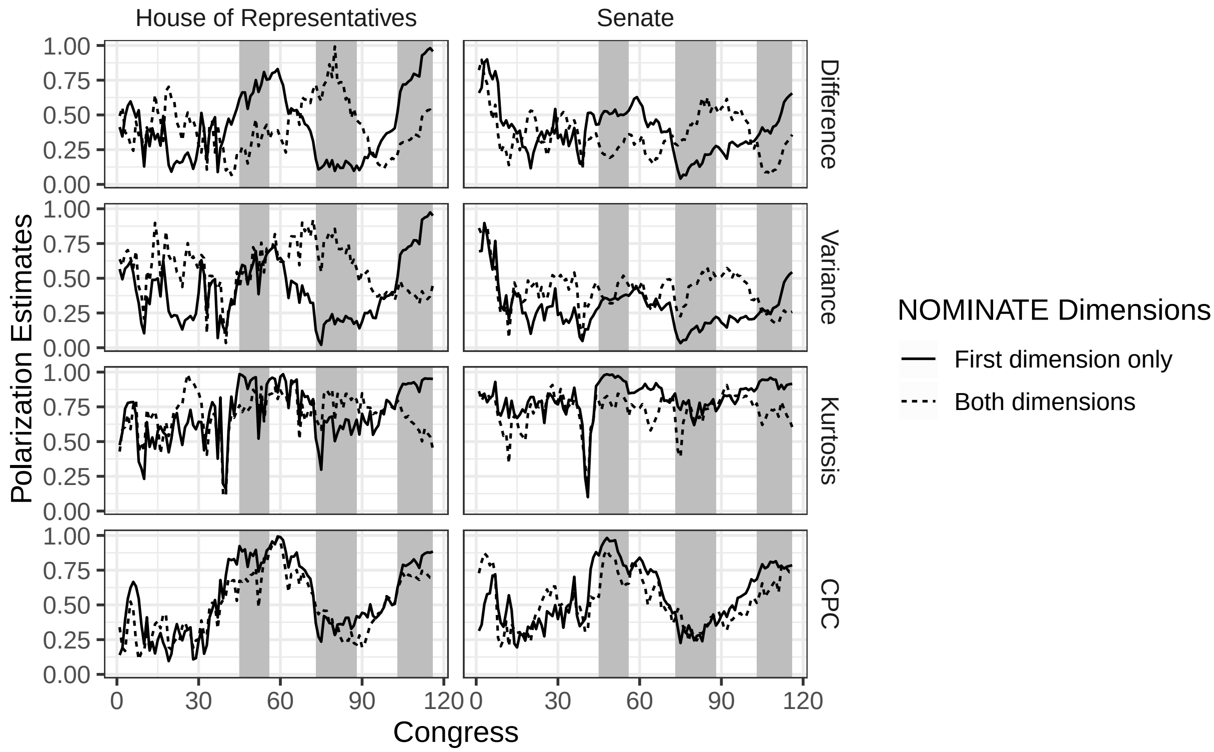


Figure 2: Estimates of Congressional Polarization. Calculated using NOMINATE ideological estimates; each measure scaled to $[0,1]$ to enable comparison.

To examine whether the accuracy of different measures might lead researchers to different substantive conclusions, I turn to the connection between congressional polarization and economic inequality. As inequality and polarization in the United States have coevolved over the last few decades, scholars have contributed to an influential literature positing a link between the two (Bonica et al. 2015; Stewart, McCarty, and Bryson 2020). McCarty, Poole, and Rosenthal (2006) argue that rising inequality since the 1970s has tracked closely with polarization at both the elite and mass levels, and several comparative studies argue that inequality is strongly associated with polarization cross-nationally (Gunderson forthcoming; Pontusson and Rueda 2008). Assuming these previous works have accurately identified an association between economic inequality and polarization, measures of congressional polarization should correlate strongly with measures of inequality.

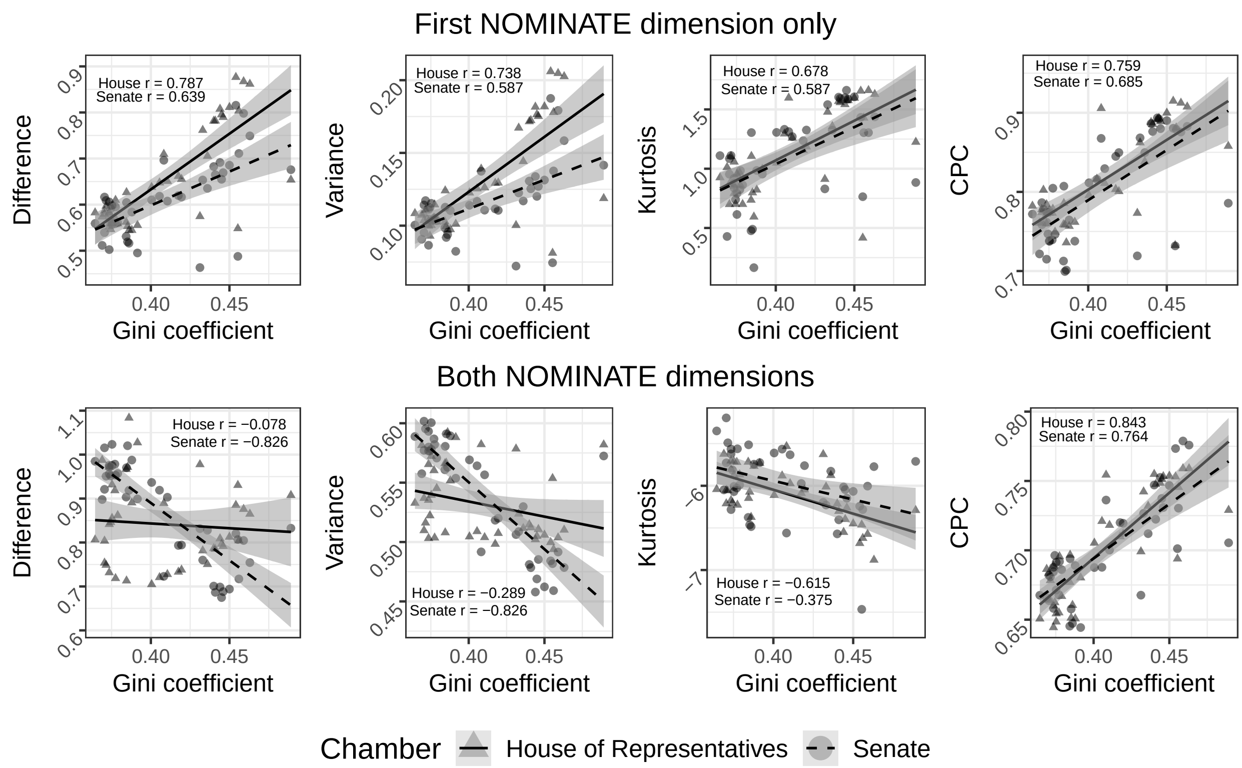


Figure 3: Correlations Between Adjusted CPC and Economic Inequality Within Chambers and Across Congresses 65-114. Polarization calculated using NOMINATE ideological estimates. Shaded areas represent 95% confidence intervals.

The CPC performs well on this test, while current measures lag behind. Figure 3 plots each measure's estimates of congressional polarization against the Gini coefficient of household income during the corresponding Congress (Atkinson et al. 2017). The first column of plots uses difference-in-means, the second column variance, the third kurtosis, and the fourth the CPC. A steep, positive slope for both the House of Representatives and Senate would indicate that a polarization measure is performing well.

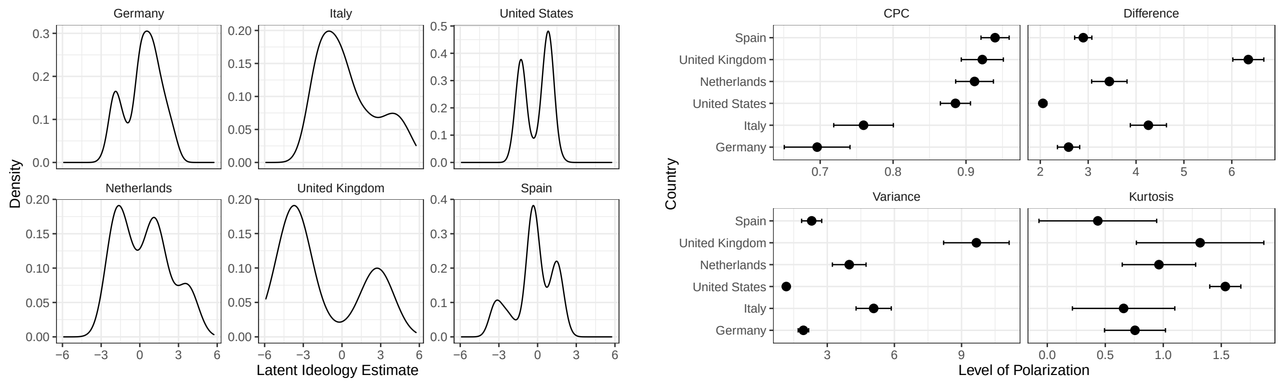
The top row of Figure 3 shows that all measures recover positive correlations when using one dimension. The CPC, in particular, is strongly correlated with inequality. The bottom row of Figure 3, however, clearly indicates that current measures struggle after adding the second ideological dimension. All correlations between polarization and inequality are now negative. In the case of difference-in-means and variance, results differ drastically across chambers. The CPC is the only measure of the four which maintains a positive correlation after adding the second dimension of ideology, and the correlations in both chambers even increase in magnitude compared to the one-dimensional case.

Polarization in Multi-Party Systems

Another advantage the CPC holds over other measures is its ability to measure polarization around more than two poles and to compare across such distributions. To illustrate, I turn to elite ideological ideal points from six countries in Western Europe and North America (Barberá 2015). This application provides a straightforward demonstration of how the CPC captures polarization across countries, as each differs in the extent to which its latent elite groups are internally homogeneous and externally heterogeneous, and even in how many groups exist within each country. These differences can be seen clearly in Figure 4, plot (a). For instance, some countries—like the United States and Germany—exhibit two parties or ideological coalitions, while Spain and the Netherlands have three. Even comparing across countries with the same number of coalitions reveals substantial variation in the size and strength of each. In the United States, the two parties are of roughly equal size and are tightly concentrated around their midpoints. In contrast, Germany's left-most coalition is small but homogeneous while its right-most coalition is large but more heterogeneous. Taking all this variation into account and enabling comparison across diverse cases is a challenging task for most measures of polarization, but the CPC offers a solution.

Figure 4, plot (b) displays the degree to which each measure suggests the party systems are polarized. The CPC is shown in the upper left plot, and countries are ordered according to their level of estimated polarization. The three other plots show estimates of difference-in-means, variance, and kurtosis, but they retain the country ordering derived from the CPC estimates, making it immediately evident that different measures give drastically different results.[3]

[3] The CPC and difference-in-means measures require knowledge of the cluster to which each observation belongs, so I perform k-means clustering with the appropriate number of clusters in each country.



(a) Kernel density plots of ideal points

(b) Ideological polarization estimates

Figure 4: Estimates of Polarization in Elite Twitter Data. Plot (a) presents kernel density estimates for each country, with facets ordered by level of polarization. Plot (b) presents polarization estimates with 95% confidence intervals, calculated using a case-resampling bootstrap and again ordered by the level of polarization estimated by the CPC.

CPC estimates indicate that Spain, the United Kingdom, the Netherlands, and the United States have similar, relatively high levels of polarization, while Germany and Italy are significantly less polarized. The distributions in Figure 4, plot (a) seems to support such a conclusion if one considers polarization as a function of both intergroup heterogeneity and intragroup homogeneity. The United Kingdom and the Netherlands have high intergroup heterogeneity, the United States has high intragroup homogeneity, and Spain has a fair amount of both. Italy and Germany both have two identifiable modes, but their intragroup homogeneity is so low and their minor-party coalitions so small in relation to their major-party coalitions that their consequent levels of polarization are not appreciable.

Other measures of polarization tell very different stories that are often at odds with the visual representation of the data. Difference-in-means and variance suggest, oddly, that the United States is the least polarized, despite a distribution that displays perhaps the highest intragroup homogeneity of the six countries. Italy, equally oddly, is the second-most polarized according to these measures, despite a nearly unimodal distribution of ideal points. Kurtosis does seem to capture intragroup homogeneity in bimodal United States, but it does not seem to capture it in trimodal Spain, which ranks as the least polarized country by this measure. In sum, current measures of polarization frequently fail to return sensible estimates because they cannot account for both intergroup heterogeneity and intragroup homogeneity, they struggle in more than one dimension, or they are not comparable across distributions with varying numbers of groups.

Conclusion

Polarization is a critical concept in a wide variety of social scientific phenomena, yet common efforts at quantitative measurement fall short of fully capturing its core conceptual elements. Working from an understanding of polarization that emphasizes both intergroup heterogeneity and intragroup homogeneity, I offer a solution. The cluster-polarization coefficient (CPC) takes both group features into account, resulting in closer alignment between concept and measure.

Critically, the CPC also enables comparison over time and space. Political systems and opinions—and thus the polarization of those systems and opinions—are multidimensional (Bermeo 2003; Gross and Sigelman 1984), yet scholars often restrict their analysis to a single left-right dimension or use various methods to recover a single dimension from multivariate data. The CPC, by contrast, can take into account numerous variables, allowing polarization estimates to accurately reflect the character of contestation in a political system. Equally important for scholars of comparative politics is the CPC's ability to compare across systems with different numbers of parties and groups. Multi-party systems are common outside the United States, yet scholars have repeatedly run into the challenge of how to measure polarization in such systems (Gidron, Adams, and Horne 2020; Wagner 2021).

I employ two sets of elite ideological ideal points to demonstrate how the CPC overcomes these challenges. Across both applications, the CPC consistently returns estimates in line with theoretical expectations, and it especially demonstrates its advantages over current measures in cases where more than one dimension is included or more than two groups are present. By introducing the CPC and providing an open-source software package to simplify its implementation, I aim to advance the critical study of polarization's causes and consequences.

References

Ahler, Douglas J. and David E. Broockman (Oct. 2018). “The Delegate Paradox: Why Polarized Politicians Can Represent Citizens Best”. In: *The Journal of Politics* 80.4, pp. 1117–1133.

Atkinson, Anthony B. et al. (2017). *The Chartbook of Economic Inequality*. URL: `https://www.chartbookofeconomicinequality.com`.

Baldassarri, Delia and Peter Bearman (Oct. 2007). “Dynamics of Political Polarization”. In: *American Sociological Review* 72.5, pp. 784–811.

Barberá, Pablo (2015). “Birds of the Same Feather Tweet Together: Bayesian Ideal Point Estimation Using Twitter Data”. In: *Political Analysis* 23.1, pp. 76–91.

Bermeo, Nancy (2003). *Ordinary People in Extraordinary Times: The Citizenry and the Breakdown of Democracy*. Princeton, NJ: Princeton University Press.

Bonica, Adam et al. (2015). “Congressional Polarization and Its Connection to Income Inequality: An Update”. In: *American Gridlock: The Sources, Character, and Impact of Political Polarization*. Ed. by James A. Thurber and Antoine Yoshinaka. New York: Cambridge University Press, pp. 357–377.

Brady, David W. and Hahrie C. Han (2006). “Polarization Then and Now: A Historical Perspective”. In: *Red and Blue Nation? Characteristics and Causes of America’s Polarized Politics*. Ed. by Pietro S. Nivola and David W. Brady. Vol. 1. Baltimore: Brookings Institution Press, pp. 119–174.

DiMaggio, Paul, John Evans, and Bethany Bryson (Nov. 1996). “Have American’s Social Attitudes Become More Polarized?” In: *American Journal of Sociology* 102.3, pp. 690–755.

Fortunato, David and Randolph T. Stevenson (May 2021). “Party Government and Political Information”. In: *Legislative Studies Quarterly* 46.2, pp. 251–295.

Gidron, Noam, James Adams, and Will Horne (2020). *American Affective Polarization in Comparative Perspective*. New York: Cambridge University Press.

Gross, Donald A. and Lee Sigelman (July 1984). “Comparing Party Systems: A Multidimensional Approach”. In: *Comparative Politics* 16.4, p. 463.

Gunderson, Jacob R. (forthcoming). “When Does Income Inequality Cause Polarization?” In: *British Journal of Political Science*.

Levendusky, Matthew S. and Jeremy C. Pope (Sum. 2011). “Red States vs. Blue States: Going Beyond the Mean”. In: *Public Opinion Quarterly* 75.2, pp. 227–248.

Lewis, Jeffrey B. et al. (2021). *Voteview: Congressional Roll-Call Votes*. URL: `https://voteview.com/`.

Lupu, Noam (June 2015). “Party Polarization and Mass Partisanship: A Comparative Perspective”. In: *Political Behavior* 37.2, pp. 331–356.

Mainwaring, Scott and Timothy R. Scully, eds. (1995). *Building Democratic Institutions: Party Systems in Latin America*. Stanford, CA: Stanford University Press.

McCarty, Nolan, Keith T. Poole, and Howard Rosenthal (2006). *Polarized America: The Dance of Ideology and Unequal Riches*. 2nd ed. Cambridge, MA: The MIT Press.

McCoy, Jennifer and Murat Somer (Jan. 2019). “Toward a Theory of Pernicious Polarization and How It Harms Democracies: Comparative Evidence and Possible Remedies”. In: *The ANNALS of the American Academy of Political and Social Science* 681.1, pp. 234–271.

Pontusson, Jonas and David Rueda (2008). "Inequality as a Source of Political Polarization: A Comparative Analysis of Twelve OECD Countries". In: *Democracy, Inequality, and Representation in Comparative Perspective*. Ed. by Pablo Beramendi and Christopher J. Anderson. New York: Russell Sage Foundation, pp. 312–353.

Sartori, Giovanni (1976). *Parties and Party Systems: A Framework for Analysis*. Cambridge, UK: Cambridge University Press.

Shepsle, Kenneth A. (1989). "The Changing Textbook Congress". In: *Can the Government Govern?* Ed. by John E. Chubb and Paul E. Peterson. Washington, DC: Brookings Institution Press.

Sinclair, Barbara (2008). "Spoiling the Sausages? How a Polarized Congress Deliberates and Legislates". In: *Red and Blue Nation? Characteristics and Causes of America's Polarized Politics*. Ed. by Pietro S. Nivola and David W. Brady. Vol. 2. Washington, DC: Brookings Institution Press, pp. 55–87.

Stewart, Alexander J., Nolan McCarty, and Joanna J. Bryson (Dec. 2020). "Polarization under Rising Inequality and Economic Decline". In: *Science Advances* 6.

Theriault, Sean M. (2008). *Party Polarization in Congress*. New York: Cambridge University Press.

Wagner, Markus (Feb. 2021). "Affective Polarization in Multiparty Systems". In: *Electoral Studies* 69.