A Group-Based Approach to Measuring Polarization

Abstract

Despite its growing importance in social scientific topics, the quantitative measurement of polarization has lagged behind its conceptual development. Political and social polarization are group-based phenomena characterized by intergroup heterogeneity and intragroup homogeneity, but existing measures capture only one of these features or make it difficult to compare across cases or over time. To bring the concept and measurement of polarization into closer alignment, I introduce the cluster-polarization coefficient (CPC), a measure of multimodality that allows scholars to incorporate multiple variables and compare across contexts with varying numbers of parties or social groups. Applying the CPC to two data sets of elite ideological ideal points demonstrates that the CPC returns more substantively sensible results than other popular measures. An open-source software package implements the measure.

Polarization has become a key concept in the social sciences, playing a role in important political outcomes ranging from representation (Ahler and Broockman 2018) and party-building (Lupu 2015) to policymaking (Sinclair 2008) and democracy (McCoy and Somer 2019). Conceptually, polarization has two important features: intergroup heterogeneity and intragroup homogeneity. Yet the most common measures of polarization—difference-in-means and variance—are ill-suited to capture both features.

I offer an alternative. The cluster-polarization coefficient (CPC) explicitly models intergroup heterogeneity and intragroup homogeneity, placing polarization's conceptual foundation front and center. I apply the CPC to two data sets of elite ideological ideal points and evaluate its ability to discern polarization relative to difference-in-means and variance—the two most widely used measures of polarization. This analysis highlights three advantages of the CPC, one theoretical and two empirical: It better captures the theoretical concept, it displays greater flexibility when considering multiple variables, and it facilitates measurement of and comparison across cases that include more than two groups. Given that most party systems have more than two parties and most political conflict occurs along more than one dimension, the CPC provides a very useful tool for scholars studying polarization.

To make the measurement procedure widely accessible to researchers and practitioners, I provide an open-source R package. In addition to calculating the CPC with researcher-specified group memberships, this package contains support for a variety of clustering methods to assign observations to groups, making the measure easily applicable to a wide variety of data structures.

Features of Polarization

Polarization is a phenomenon that happens both between and within groups, emerging when group members disagree with members of other groups and agree with members of their own. It thus has two conceptual features: distance from opponents (intergroup heterogeneity) and concentration within groups (intragroup homogeneity). Without intergroup heterogeneity, there are no differences in position to analyze. Without intragroup homogeneity, there are no meaningful group positions to compare.[1] In this conceptualization, polarization can increase as the positions of social or political groups grow farther apart from one another, as they become more internally cohesive, or both. Many scholars either explicitly or implicitly adopt this conceptualization of polarization (Baldassarri and Bearman 2007; Fortunato and Stevenson 2021; Hill and Tausanovitch 2015; Maoz and Somer-Topcu 2010; Rehm and Reilly 2010). In addition, the combination of interparty conflict and intraparty homogeneity lays at the root of theories that explain how polarization develops (e.g. Aldrich and Rohde 2000).

The challenge for measuring polarization in real-world data is that these two features can occur separately, simultaneously, or not at all, and each may increase or decrease independent of the other. This challenge is reflected in the diversity of measures used by scholars. A survey of eight political science journals reveals 322 articles published about polarization since 2000,[2] employing at least twenty distinct operationalizations. However, two approaches account for most uses: Measures of difference-in-means and variance were used in 57% and 14% of articles, respectively.[3] Below, I demonstrate why Levendusky and Pope (2011) urge scholars to "go beyond the mean" when measuring polarization. I show that it is useful to go beyond variance, as well. The CPC, the measure I present below, offers a method to do so.

To illustrate why both features are important and why difference-in-means and variance may paint an incomplete picture, consider the stylized distributions in Figure 1. They imitate four possible combinations of intergroup heterogeneity and intragroup homogeneity. The distribution in plot A has neither feature of polarization. Two clusters are faintly apparent, but they are close

[1]The measure I propose below is designed to capture these two features. It may not be suitable for analysis of party system polarization (Dalton 2008), which affords fewer data points and places less emphasis on intragroup homogeneity.

[2]See Supplementary Information section S1 for details and results of this search.

[3]For a brief critique of each measure, see Supplementary Information section S2.

together and display little cohesion. Plot B has high intragroup homogeneity and low intergroup heterogeneity,[4] and plot C has the reverse. The group means are the same in plot B as in plot A, but the groups are more concentrated. Intragroup homogeneity is the same in plot C as in plot A, but group means are further apart. Finally, plot D has high levels of both characteristics of polarization.

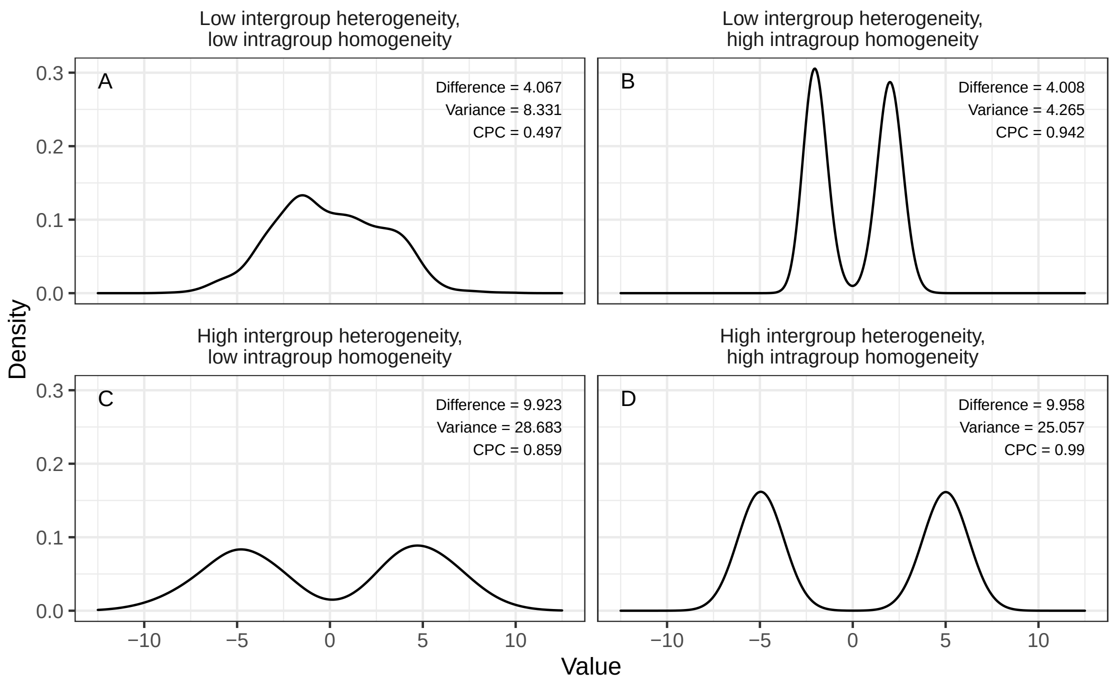

Figure 1: Stylized Distributions of Polarization Features. Simulated bimodal Gaussian mixture distributions all with $\mu_{global} = 0$. Labels show polarization levels according to difference-in-means, variance, and the CPC.

A measure correctly measuring polarization levels should indicate more polarization in plots B, C, and D relative to A and less polarization in plots A, B, and C, relative to D. Plot labels display the estimated level of polarization in each distribution as measured by difference-in-means, variance, and the CPC. In all cases, higher numbers indicate greater polarization.

[4]Party sorting is one substantive example of this, where individuals do not become more extreme but the population nevertheless polarizes due to individuals sorting themselves into parties with positions similar to their own (Levendusky and Pope 2011).

Difference-in-means successfully distinguishes between plots A and C and between plots B and D, assigning a higher polarization estimate to the latter plot in each pairing, but it cannot distinguish plot A from B or plot C from D. It assigns identical polarization estimates to each distribution even though plots B and D are qualitatively more polarized than plots A and C, respectively. Variance performs even worse. It, too, distinguishes between plots A and C and between plots B and D, but cannot identify polarization vis-à-vis intragroup homogeneity. In fact, it assigns *lower* polarization estimates to plots B and D relative to plots A and C, a result opposite the conceptual understanding of polarization. This simple exercise suggests that using difference-in-means and variance may, in some cases, allow polarization to go undetected and, in others, identify it where it does not exist. The CPC is the only measure of the three that accurately mirrors the qualitative comparisons between all four distributions and captures both features of polarization.[5]

Cluster-Polarization Coefficient

I begin from the premise that social scientific data—just like the distributions in Figure 1—are often comprised of distinct clusters of observations. In political contexts, such clusters are typically represented by parties or social groups. To derive the CPC, I decompose the total variance of this clustered data (TSS) in (1) into components corresponding to the two features of polarization: the variance accounted for between the clusters (BSS, corresponding to intergroup heterogeneity) and the variance accounted for within all clusters (WSS, corresponding to intragroup homogeneity). Dividing by TSS and solving for the BSS term in (1) gives an expression for the proportion of the total variance accounted for by the between-cluster variance—what I call the cluster-polarization coefficient. As it is a proportion, the value produced by this expression varies on the domain $[0,1]$.

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[5] Supplementary Information sections S4.1 and S4.2 use simulations to evaluate this more systematically.

$$
\begin{aligned}
&TSS = BSS + WSS,\\
&\rightarrow CPC = 1 - \frac{WSS}{TSS} = \frac{BSS}{TSS}.
\end{aligned}
\tag{1}
$$

More formally, the CPC takes the expression in (2), where each individual i in cluster k holds a position on dimension j.[6] Expressed in this way, the CPC appears related to a one-way ANOVA F-statistic and the coefficient of determination (R^2). These are useful similarities for deriving properties of the measure, which I show in Supplementary Information sections S3.2-S3.5.

$$
CPC = 1 - \frac{\sum_{k=1}^{n_k}\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ikj} - \mu_{kj})^2}{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij} - \mu_j)^2} = \frac{\sum_{k=1}^{n_k}\sum_{j=1}^{n_j}(\mu_{kj} - \mu_j)^2}{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij} - \mu_j)^2}. \tag{2}
$$

One additional modification is needed to make the CPC appropriate for comparison across contexts with varying numbers of observations, variables, and clusters. This is critical because intra-state political dynamics, the number and nature of sociopolitical cleavages, and the size and number of political coalitions typically vary across countries or within countries over time. The expression in (2) will be biased upward in small samples, so I incorporate corrections for lost degrees of freedom and express the adjusted CPC in (3):

$$
CPC_{adj} = 1 - \frac{\frac{\sum_{k=1}^{n_k}\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ikj}-\mu_{kj})^2}{n_i - n_j n_k}}{\frac{\sum_{i=1}^{n_i}\sum_{j=1}^{n_j}(x_{ij}-\mu_j)^2}{n_i - n_j}} = 1 - (1 - CPC)\frac{n_i - n_j}{n_i - n_j n_k}. \tag{3}
$$

By explicitly modeling both BSS and WSS, the CPC takes into account both features of polarization. It increases when the distance between groups increases or when groups become more tightly concentrated around their collective ideal point, with the rate of those increases depending on the relative levels of BSS and WSS. In Supplementary Information sections S3.2-S3.5, I derive a sam-

[6]Full derivations are shown in Supplementary Information section S3.1.

pling distribution for the CPC and show that it is both unbiased and consistent after incorporating the corrections in (3). All calculations in this paper use the adjusted CPC.

In addition to the substantive applications below, I pursue a variety of validation exercises in the Supplementary Information using synthetic data. Sections S4.1-S4.3 evaluate how well difference, variance, and the CPC capture both features of polarization in univariate and bivariate contexts, as well as in distributions with two, three, and four groups. Section S4.4 uses heavy-tailed log-normal distributions to evaluate sensitivity to outliers, and section S4.5 investigates how results change when group sizes vary. Finally, section S5 benchmarks each measure's performance against "ground-truth" data gathered from human annotators. The CPC consistently captures both features of polarization better than difference-in-means and variance, and it adheres more closely to intuitive identifications of polarization.

Applications: Ideology of Political Elites

In addition to a close fit with the conceptual understanding of polarization, the CPC proves valuable when studying multiple dimensions of contestation or comparing across cases with more than two groups. I use two data sets of elite ideological ideal points to illustrate these benefits. First, I engage the well-researched connection between congressional polarization and income inequality in the United States, showing that the CPC is the only measure of polarization that recovers this connection in multidimensional data. Next, I turn to a set of six industrially developed, consolidated democracies, taking on the methodological puzzle of how to measure polarization in multi-party systems and compare across systems with different numbers of parties or groups. Throughout both applications, I compare the CPC to difference-in-means and variance, the two most popular strategies for measuring polarization in the political science literature.

Polarization in the United States Congress

I calculate congressional polarization using DW-NOMINATE data (Lewis et al. 2021), which uses a scaling procedure to estimate the ideology of individual legislators in a two-dimensional latent space. The first dimension generally captures economic issues and the second captures racial and other social issues. The presence of two dimensions can undermine the performance of difference-in-means and variance, but it allows me to demonstrate a major advantage of the CPC.

To establish the CPC's construct validity, I show that it produces estimates consistent with well-known historical patterns. Historians and political scientists frequently point to three periods of ideological change in Congress. These periods are shaded on Figure 2. First, distinct partisan divides emerged as the Democratic Party made its comeback following the end of Reconstruction (Brady and Han 2006). Second, party convergence and high levels of bipartisanship characterized the "Textbook Congress" in the mid-twentieth century (Shepsle 1989). Finally, high levels of polarization are well-documented in the lead-up to the twenty-first century and extending to the present (Theriault 2008). Each measure should therefore indicate high polarization in the first time period, low polarization in the second period, and high polarization again in the third.

For each Congress, I estimate the level of polarization in DW-NOMINATE scores using difference-in-means, variance, and the CPC. These estimates appear in Figure 2. All measures generally capture the expected trends in the first NOMINATE dimension. Adding the second dimension, however, leads difference-in-means and variance to return trends that do not match the historical record.[7] For example, difference-in-means estimates detect increasing polarization in the modern period, but they also indicate that Congress experienced some of its most polarized years during the Textbook Congress. Variance estimates make similar indications, and they even find flat or declining polarization levels leading up to the present-day. In contrast, the CPC handles the addition of the second dimension well and recovers trends that reflect the historical record in each period.

[7] I calculate difference-in-means by taking the average Euclidean distance between group means, and I calculate variance using the sum of squared Euclidean distances from the means.

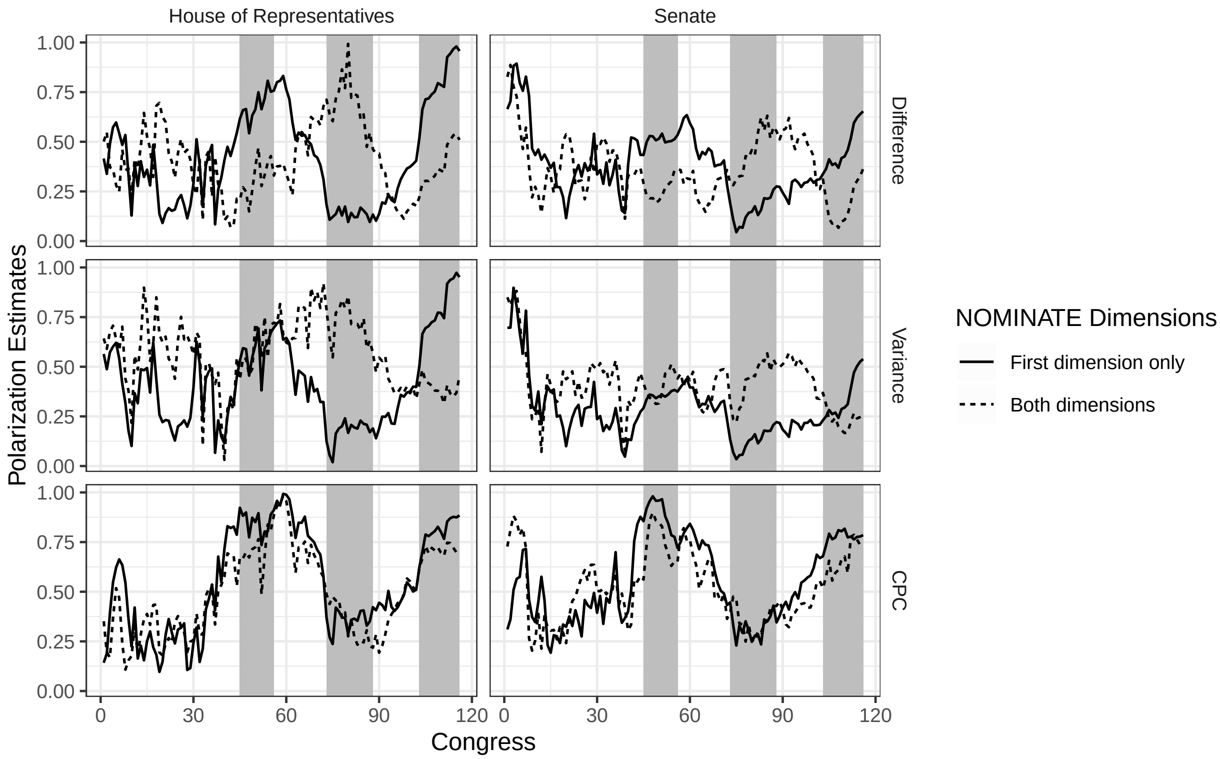


Figure 2: Estimates of Congressional Polarization. Calculated using NOMINATE ideological estimates; each measure scaled to $[0, 1]$ to enable comparison.

To examine whether the accuracy of different measures might lead researchers to different substantive conclusions, I turn to the connection between congressional polarization and economic inequality. As inequality and polarization in the United States have coevolved, scholars have contributed to an influential literature positing a link between the two (Garand 2010; McCarty, Poole, and Rosenthal 2016; Stewart, McCarty, and Bryson 2020). Assuming these previous works have accurately identified an association between economic inequality and polarization, measures of congressional polarization should correlate strongly with measures of inequality.

The CPC performs well on this test, while the other measures lag behind. Figure 3 plots each measure's estimates of congressional polarization against the Gini coefficient of household income during the corresponding Congress (Atkinson et al. 2017). The first column of plots uses difference-in-means, the second column variance, and the third the CPC. A steep, positive slope

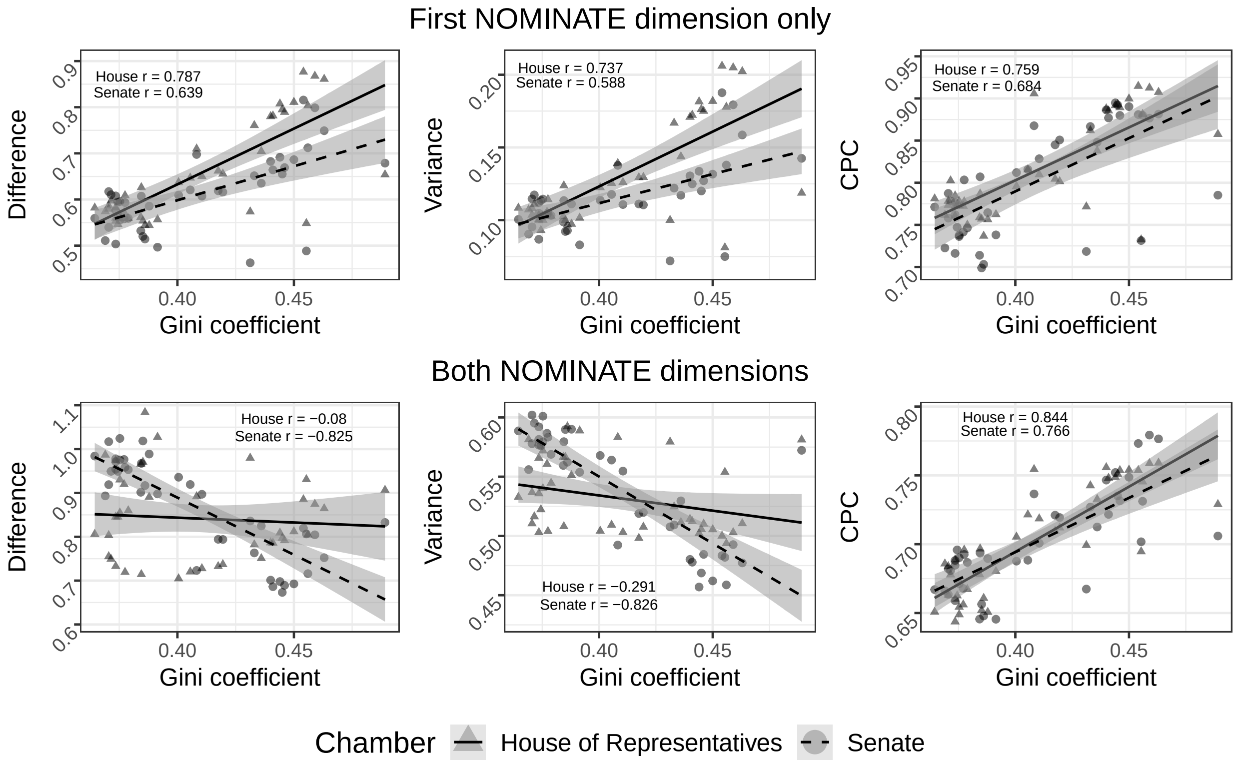


Figure 3: Correlations Between Adjusted CPC and Economic Inequality Within Chambers and Across Congresses 65-114. Polarization calculated using NOMINATE ideological estimates. Shaded areas represent 95% confidence intervals. Results of models generating lines-of-best-fit shown in Supplementary Information section S6.

for both the House of Representatives and Senate would indicate that a polarization measure is performing well.

The top row of Figure 3 shows that all measures recover positive correlations when using one dimension. The CPC, in particular, is strongly correlated with inequality. However, the bottom row of Figure 3 clearly indicates that difference-in-means and variance struggle after adding the second ideological dimension. All correlations between polarization and inequality are now negative. In the case of difference-in-means and variance, results differ drastically across chambers. The CPC is the only measure of the three that maintains a positive correlation after adding the second dimension of ideology, and the correlations in both chambers even increase in magnitude compared to the one-dimensional case.[8]

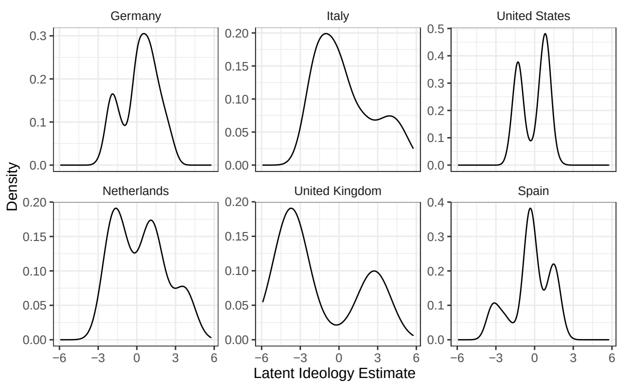
This underscores an important benefit of the CPC. Aldrich, Montgomery, and Sparks (2014) show that high-dimensional estimates are crucial for accurate ideal point estimation, especially in polarized contexts. Researchers using current measures may need to ignore higher-dimensional information as a pragmatic matter, but the CPC allows it to be preserved, resulting in more accurate polarization estimates.

Polarization in Multi-Party Systems

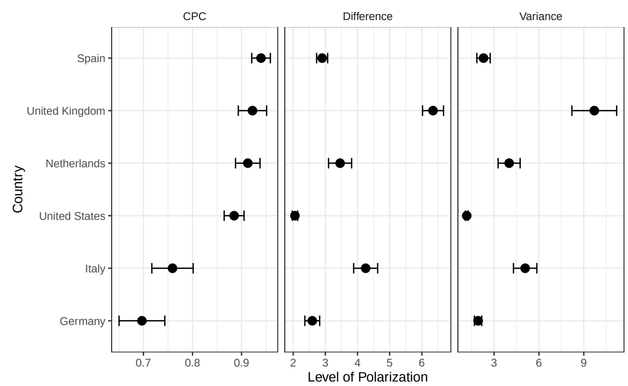
One common challenge confronted by scholars is how to measure polarization in multi-party systems (Gidron, Adams, and Horne 2020; Ward and Tavits 2019)—an obstacle that applies to most modern democracies. To illustrate, I turn to elite ideological ideal points from six countries in Western Europe and North America (Barberá 2015). This application demonstrates how the CPC captures polarization across country contexts, as each differs in the extent to which its latent elite groups are internally homogeneous and externally heterogeneous, and even in how many groups exist within each country. These differences can be seen in Figure 4, plot (a). For instance, some countries—like the United States and Germany—exhibit two parties or ideological coalitions, while Spain and the Netherlands have three. Even comparing across countries with the same number of coalitions reveals substantial variation in the size and strength of each. In the United States, the two parties are of roughly equal size and are tightly concentrated around their midpoints. In contrast, Germany's left-most coalition is small but homogeneous while its right-most coalition

[8]Including the second dimension should still result in high correlations for two reasons. First, immigration is an important mechanism linking polarization to inequality (McCarty, Poole, and Rosenthal 2016). Second, redistributive issues in the United States are often racialized (Gilens 1999). If the second dimension captures attitudes toward race and immigration, polarization on that dimension should still track closely with inequality.

is large but more heterogeneous. Taking all this variation into account and enabling comparison across diverse cases is a challenging task for most measures of polarization, but the CPC offers a solution.



(a) Kernel density plots of ideal points



(b) Ideological polarization estimates

Figure 4: Estimates of Polarization in Elite Twitter Data. Plot (a) presents kernel density estimates for each country, with facets ordered by level of polarization. Plot (b) presents polarization estimates with 95% confidence intervals, calculated using a case-resampling bootstrap and again ordered by level of polarization estimated by the CPC.

Figure 4, plot (b) displays the degree to which each measure suggests the party systems are polarized.[9] The CPC is shown in the leftmost facet, and countries are ordered by their estimated level of polarization. CPC estimates indicate that Spain, the United Kingdom, the Netherlands, and the United States have similar, relatively high levels of polarization, while Germany and Italy are significantly less polarized. The distributions in Figure 4, plot (a) seem to support such a conclusion if one considers polarization as a function of both intergroup heterogeneity and intragroup homogeneity. The United Kingdom and the Netherlands have high intergroup heterogeneity, the United States has high intragroup homogeneity, and Spain has a fair amount of both. Italy and Germany both have two identifiable modes, but their intragroup homogeneity is so low and their

[9]The CPC and difference-in-means measures require knowledge of the cluster to which each observation belongs, so I perform k-means clustering with the appropriate number of clusters in each country.

minor-party coalitions so small in relation to their major-party coalitions that their consequent levels of polarization are not appreciable.

Difference-in-means and variance—shown in the center and right facets in Figure 4, plot (b)—tell very different stories that are often at odds with the visual representation of the data. They suggest that the United States is the least polarized, despite a distribution that displays perhaps the highest intragroup homogeneity of the six countries. Italy is the second-most polarized according to these measures, despite a nearly unimodal distribution of ideal points. In sum, current measures of polarization frequently fail to return sensible estimates because they cannot account for both intergroup heterogeneity and intragroup homogeneity, they struggle in more than one dimension, or they are not comparable across distributions with varying numbers of groups.

Conclusion

Polarization is a critical concept in a wide variety of social scientific phenomena, yet efforts to quantify it fail to capture its two core conceptual elements: intergroup heterogeneity and intragroup homogeneity. The cluster-polarization coefficient takes both features into account, more closely aligning concept and measure and producing more sensible estimates of polarization in important contexts.

Critically, the CPC enables comparison over time and space. Political systems and opinions—and thus the polarization of those systems and opinions—are multidimensional (Bermeo 2003), yet scholars often restrict their analysis to a single left-right dimension. However, the CPC can accommodate numerous variables, allowing polarization estimates to accurately reflect the character of contestation in a political system. Equally important is the CPC's ability to compare across systems with different numbers of parties and groups. Multi-party systems are common outside the United States, but scholars often ignore this contextual feature either by calculating the variance of the entire distribution (Ward and Tavits 2019) or by converting party systems into a series of dyads and measuring polarization as if they were two-party systems (Gidron, Adams, and Horne 2020). The CPC, on the other hand, is explicitly designed to measure polarization in such contexts.

I employ two sets of elite ideological ideal points to demonstrate how the CPC overcomes these challenges. Across both applications, the CPC consistently returns estimates in line with theoretical expectations, and it demonstrates its advantages over current measures in cases where more than one dimension is included or more than two groups are present. By introducing the CPC and providing an open-source software package to simplify its implementation, I aim to advance the critical study of polarization's causes and consequences.

References

Ahler, Douglas J. and David E. Broockman (Oct. 2018). "The Delegate Paradox: Why Polarized Politicians Can Represent Citizens Best". In: *The Journal of Politics* 80.4, pp. 1117–1133.

Aldrich, John H., Jacob M. Montgomery, and David B. Sparks (Oct. 2014). "Polarization and Ideology: Partisan Sources of Low Dimensionality in Scaled Roll Call Analyses". In: *Political Analysis* 22.4, pp. 435–456.

Aldrich, John H. and David W. Rohde (Feb. 2000). "The Republican Revolution and the House Appropriations Committee". In: *The Journal of Politics* 62.1, pp. 1–33.

Atkinson, Anthony B. et al. (2017). *The Chartbook of Economic Inequality*. URL: `https://www.chartbookofeconomicinequality.com`.

Baldassarri, Delia and Peter Bearman (Oct. 2007). "Dynamics of Political Polarization". In: *American Sociological Review* 72.5, pp. 784–811.

Barberá, Pablo (2015). "Birds of the Same Feather Tweet Together: Bayesian Ideal Point Estimation Using Twitter Data". In: *Political Analysis* 23.1, pp. 76–91.

Bermeo, Nancy (2003). *Ordinary People in Extraordinary Times: The Citizenry and the Breakdown of Democracy*. Princeton, NJ: Princeton University Press.

Brady, David W. and Hahrie C. Han (2006). “Polarization Then and Now: A Historical Perspective”. In: *Red and Blue Nation? Characteristics and Causes of America’s Polarized Politics*. Ed. by Pietro S. Nivola and David W. Brady. Vol. 1. Baltimore: Brookings Institution Press, pp. 119–174.

Dalton, Russell J. (July 2008). “The Quantity and the Quality of Party Systems: Party System Polarization, Its Measurement, and Its Consequences”. In: *Comparative Political Studies* 41.7, pp. 899–920.

Fortunato, David and Randolph T. Stevenson (May 2021). “Party Government and Political Information”. In: *Legislative Studies Quarterly* 46.2, pp. 251–295.

Garand, James C. (Oct. 2010). “Income Inequality, Party Polarization, and Roll-Call Voting in the U.S. Senate”. In: *The Journal of Politics* 72.4, pp. 1109–1128.

Gidron, Noam, James Adams, and Will Horne (2020). *American Affective Polarization in Comparative Perspective*. New York: Cambridge University Press.

Gilens, Martin (1999). *Why Americans Hate Welfare: Race, Media, and the Politics of Antipoverty Policy*. Chicago: The University of Chicago Press.

Hill, Seth J. and Chris Tausanovitch (Oct. 2015). “A Disconnect in Representation? Comparison of Trends in Congressional and Public Polarization”. In: *The Journal of Politics* 77.4, pp. 1058–1075.

Levendusky, Matthew S. and Jeremy C. Pope (Sum. 2011). “Red States vs. Blue States: Going Beyond the Mean”. In: *Public Opinion Quarterly* 75.2, pp. 227–248.

Lewis, Jeffrey B. et al. (2021). *Voteview: Congressional Roll-Call Votes*. URL: `https://voteview.com/`.

Lupu, Noam (June 2015). “Party Polarization and Mass Partisanship: A Comparative Perspective”. In: *Political Behavior* 37.2, pp. 331–356.

Maoz, Zeev and Zeynep Somer-Topcu (Oct. 2010). “Political Polarization and Cabinet Stability in Multiparty Systems: A Social Networks Analysis of European Parliaments, 1945–98”. In: *British Journal of Political Science* 40.4, pp. 805–833.

McCarty, Nolan, Keith T. Poole, and Howard Rosenthal (2016). *Polarized America: The Dance of Ideology and Unequal Riches*. 2nd ed. Cambridge, MA: The MIT Press.

McCoy, Jennifer and Murat Somer (Jan. 2019). "Toward a Theory of Pernicious Polarization and How It Harms Democracies: Comparative Evidence and Possible Remedies". In: *The ANNALS of the American Academy of Political and Social Science* 681.1, pp. 234–271.

Rehm, Philipp and Timothy Reilly (Mar. 2010). "United We Stand: Constituency Homogeneity and Comparative Party Polarization". In: *Electoral Studies* 29.1, pp. 40–53.

Shepsle, Kenneth A. (1989). "The Changing Textbook Congress". In: *Can the Government Govern?* Ed. by John E. Chubb and Paul E. Peterson. Washington, DC: Brookings Institution Press.

Sinclair, Barbara (2008). "Spoiling the Sausages? How a Polarized Congress Deliberates and Legislates". In: *Red and Blue Nation? Characteristics and Causes of America's Polarized Politics*. Ed. by Pietro S. Nivola and David W. Brady. Vol. 2. Washington, DC: Brookings Institution Press, pp. 55–87.

Stewart, Alexander J., Nolan McCarty, and Joanna J. Bryson (Dec. 2020). "Polarization under Rising Inequality and Economic Decline". In: *Science Advances* 6.

Theriault, Sean M. (2008). *Party Polarization in Congress*. New York: Cambridge University Press.

Ward, Dalston G. and Margit Tavits (Aug. 2019). "How Partisan Affect Shapes Citizens' Perception of the Political World". In: *Electoral Studies* 60, pp. 1–9.