# Leveraging Time-Series Information to Improve Small-Area Estimation

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July 19, 2024

# Analyzing Subnational Politics

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  - Conservatism in the American South (Key 1949)
  - Gender role attitudes and female representation (Arceneaux 2001)
  - Democratic backsliding (Giraudy 2015, Grumbach 2022)

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    - National surveys not representative at subnational level, usually too small for unbiased estimates in smaller units
  - Multilevel regression with poststratification (MRP) (Gelman and Little 1997, Park et al. 2004)
    - Surveys ask similar questions across years—**how to harness temporal variation in MRP?**

# Subnational Opinion over Time

- Benefits of incorporating time into small-area estimation:
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  - Assumes independence of opinion across years
- Others proposed (e.g. Claassen and Traunmüller 2020, Kastellec 2018), but little guidance on...
  1. ...how well dynamic MRP models perform
  2. ...which models are appropriate for which purposes
  3. ...under what conditions they can be employed

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2. Analysis of 29 time series from CES
    - **Wide variation** in model performance
3. Simulation evidence
    - Performance varies with time series characteristics: degree of over-time volatility, length of time series, sample size in target time period
    - Models with **demographic-year random intercepts** most versatile

# Multilevel Regression with Poststratification

**Stage one:**

- Survey responses modeled hierarchically as function of demographic and state-level covariates:

$$
\begin{aligned}
y_i &\sim \mathrm{Bernoulli}(\pi_i), \\
\pi_i &= \mathrm{logit}^{-1}(\beta_0 + \alpha^{\mathrm{gender}}_{g[i]} + \alpha^{\mathrm{race}}_{g[i]} + \alpha^{\mathrm{age}}_{g[i]} + \alpha^{\mathrm{educ}}_{g[i]} + \alpha^{\mathrm{state}}_{g[i]}), \qquad (1) \\
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**Stage two:**

- First-stage predictions calculated for each combination of demographic predictors, weighted by joint distribution of demographics to produce state-level estimate

# Approaches to Dynamic MRP

1. **No-pooling** (Enns and Koch 2013, Lewis and Jacobsmeier 2017)
   - Prohibits model from using information from other time periods, eliminating partial-pooling benefits and risking imprecise estimates (Caughey and Warshaw 2019)
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2. **Moving average** (Pacheco 2011, 2014)
    - No-pooling model with manually imposed partial-pooling determined by researcher
    - Pool data within 3-year moving intervals, fit model on each interval
    - Pooling impossible outside each interval, assumes no opinion change within each interval

# Approaches to Dynamic MRP

3. **Linear trend** (Shirley and Gelman 2015, Wiertz and Lim 2021)

$$\begin{aligned} \pi_i &= \text{logit}^{-1}(\beta_0 + \alpha^{\text{gender}}_{g[i]} + \alpha^{\text{race}}_{g[i]} + \alpha^{\text{age}}_{g[i]} + \alpha^{\text{educ}}_{g[i]} + \alpha^{\text{state}}_{g[i]} + \delta \cdot \text{year}) \\ \alpha^{\text{state}}_{g,\,t} &\sim \text{N}(\gamma \cdot \text{pres}_{g[i],\,t[i]},\ \sigma^2_{\text{state}}) \end{aligned} \tag{2}$$

- Assumes linear time trends
- More flexibility with higher-order polynomials (Kastellec 2018) or splines (Kołczynska et al. 2024)

# Approaches to Dynamic MRP

4. **Random intercepts by year** (Simonovits and Bor 2023, Smith et al. 2020)

$$\pi_i = \text{logit}^{-1}(\beta_0 + \alpha^{\text{gender}}_{g[i]} + \alpha^{\text{race}}_{g[i]} + \alpha^{\text{age}}_{g[i]} + \alpha^{\text{educ}}_{g[i]} + \alpha^{\text{state}}_{g[i]} + \alpha^{\text{year}}_{t[i]}) \tag{3}$$

- Mirrors method of partial-pooling in static MRP (benefit comes from estimating $\sigma^2_{\text{year}}$)
- Assumes effect of each demographic category is constant over time (Ben-Shalom et al. 2021)
- Would struggle to capture, e.g., expanding gender gap in political attitudes

# Approaches to Dynamic MRP

5. **Random intercepts by demographic-year** (Gelman et al. 2018)

$$\pi_i = \text{logit}^{-1}(\beta_0 + \alpha^{\text{gender}}_{g[i]} + \alpha^{\text{race}}_{g[i]} + \alpha^{\text{age}}_{g[i]} + \alpha^{\text{educ}}_{g[i]} + \alpha^{\text{state}}_{g[i]} + \alpha^{\text{year}}_{t[i]} + \alpha^{\text{gender}}_{g[i],\,t[i]} + \alpha^{\text{race}}_{g[i],\,t[i]} + \alpha^{\text{age}}_{g[i],\,t[i]} + \alpha^{\text{educ}}_{g[i],\,t[i]} + \alpha^{\text{state}}_{g[i],\,t[i]}) \tag{4}$$

- Relaxes assumption of constant demographic effects
- Mirrors practice of allowing state random intercepts to vary by year (Ben-Shalom et al. 2021, Shirley and Gelman 2015)
- Still assumes time is a collection of independent years; in reality, $y_t = f(y_{t-1})$, $y_t \neq f(y_{t+1})$

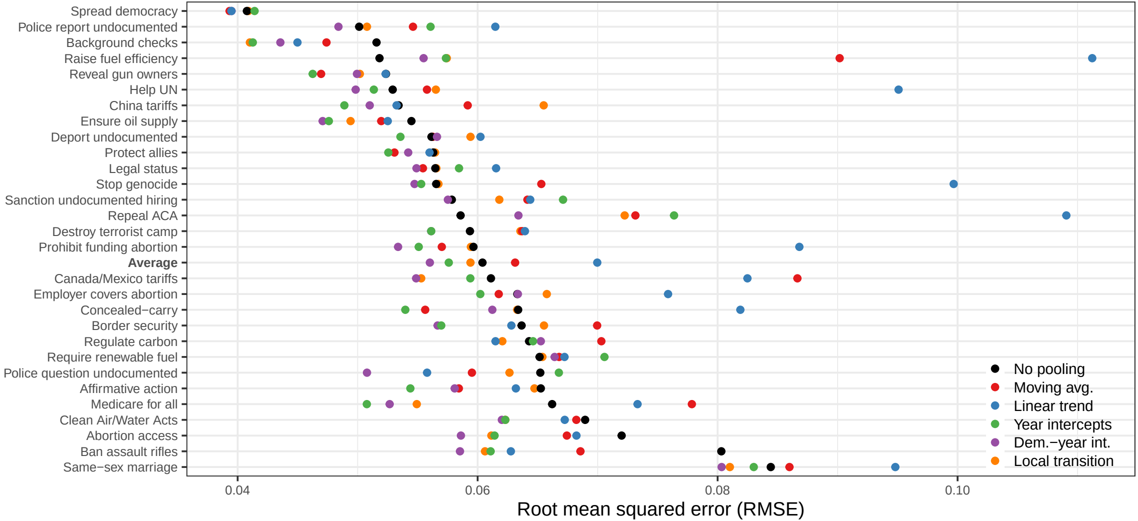
# Approaches to Dynamic MRP

6. **Local transition model**

$$
\begin{aligned}
\pi_i = \text{logit}^{-1}(\beta_0 &+ \alpha^{\text{gender}}_{g[i]} + \alpha^{\text{race}}_{g[i]} + \alpha^{\text{age}}_{g[i]} + \alpha^{\text{educ}}_{g[i]} + \alpha^{\text{state}}_{g[i]} + \alpha^{\text{year}}_{t[i]} \\
&+ \alpha^{\text{gender}}_{g[i],\,t[i]} + \alpha^{\text{race}}_{g[i],\,t[i]} + \alpha^{\text{age}}_{g[i],\,t[i]} + \alpha^{\text{educ}}_{g[i],\,t[i]} + \alpha^{\text{state}}_{g[i],\,t[i]}), \\
\alpha^{\text{year}}_t &\sim \text{N}(\alpha^{\text{year}}_{t-1}, \sigma^2_{\text{year}}), \\
\alpha^j_{g,\,t} &\sim \text{N}(\alpha^j_{g,\,t-1},\ \sigma^2_j)\ \forall\ \ g,\ t,\ j \in \{\text{gender},\ \text{race},\ \text{age},\ \text{educ}\}.
\end{aligned} \tag{5}
$$

- Directed structured priors lead to first-stage bias, variance reduction (Gao et al. 2021)
- Complete info-sharing among years, assuming individuals have similar opinions to individuals with same demographics in previous years

# Dynamic MRP on 29 Policy Issues



# Explaining Variation in Dynamic MRP Performance

- Sources of variation in **static** MRP performance:
  - Sample size (Lax and Phillips 2009)
  - First-stage model complexity (Warshaw and Rodden 2012)
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- Focus on **characteristics of time series** likely to affect performance:
  - Volatility of opinion over time
  - Length of time series
  - Sample size in each year

# Simulation Set-Up

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- Allow **over-time volatility** of state-level opinion to vary:
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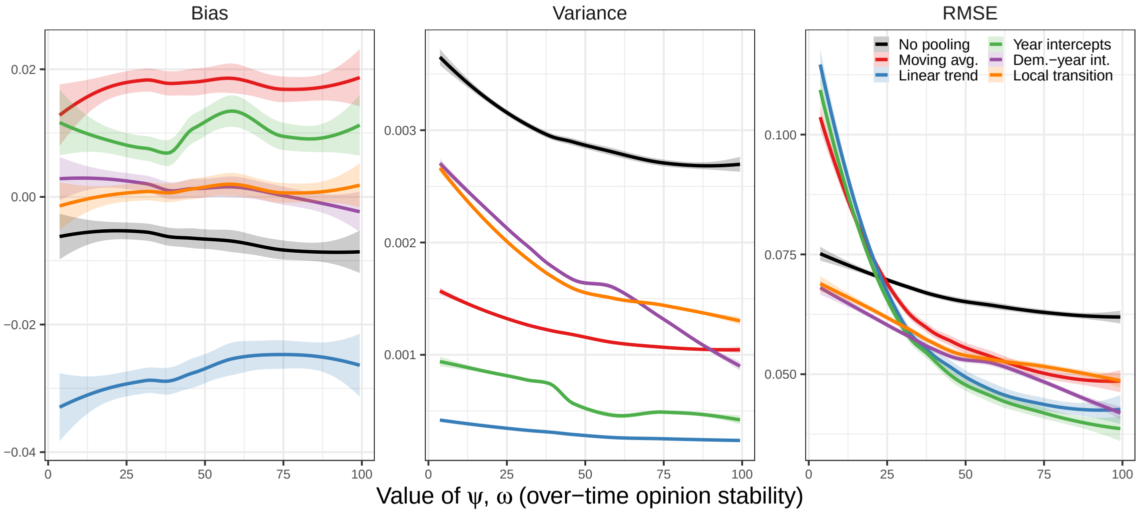
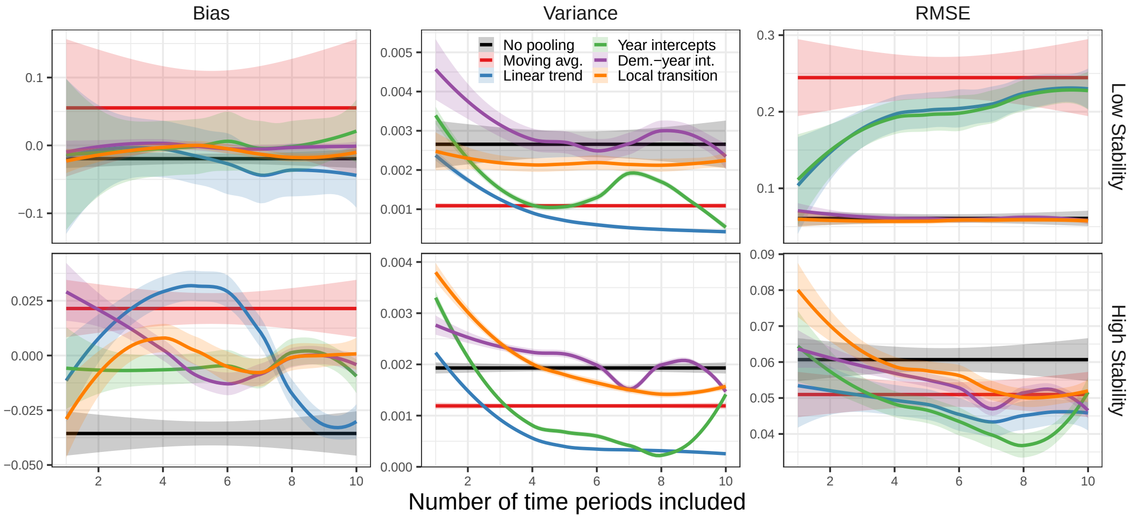
- 300 iterations, fit models on randomly sampled 10% of data in each year

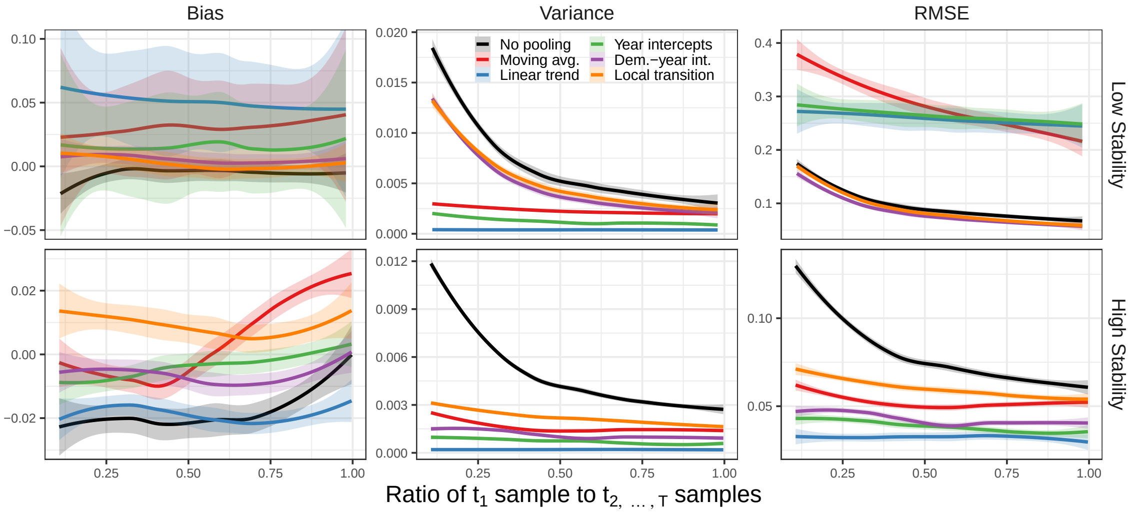
# Estimating Time Trends in State-Level Opinion



# Increasing Time Series Length



# Recovering State-Level Opinion with Scarce Data



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6. **Most versatile**: random intercepts by demographic-year

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- MRP most often applied to public opinion
  - Assess appropriateness for other applications of small-area estimation (e.g. urban planning, agriculture), relative to other dynamic approaches (Rao and Yu 1994, Singh et al. 2005)