Leveraging Time-Series Information to Improve Small-Area Estimation

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Analyzing Subnational Politics

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  - Disaggregate sample (Brace et al. 2002, Erikson et al. 1993)
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  - Multilevel regression with poststratification (MRP) (Gelman and Little 1997, Park et al. 2004)
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    - Surveys ask similar questions across years—**how to harness temporal variation in MRP?**
- Benefits of incorporating time into small-area estimation:
  1. Ability to answer causal questions (Blackwell 2013)
  2. Enhance external validity, help assess scope conditions
  3. Improve cross-sectional estimates when data is scarce (Gelman et al. 2018)

Multilevel Regression with Poststratification

Stage one:

- Survey responses modeled hierarchically as function of demographic and state-level covariates:

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\begin{aligned}
y_i &\sim \text{Bernoulli}(\pi_i), \\
\pi_i &= \text{logit}^{-1}(\beta_0 + \alpha^{\text{gender}}_{g[i]} + \alpha^{\text{race}}_{g[i]} + \alpha^{\text{age}}_{g[i]} + \alpha^{\text{educ}}_{g[i]} + \alpha^{\text{state}}_{g[i]}), \\
\alpha^{\text{state}}_{g} &\sim \text{N}(\gamma \cdot \text{pres}_{g[i]},\ \sigma^2_{\text{state}}).
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\tag{1}
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Stage two:

- First-stage predictions calculated for each combination of demographic predictors, weighted by joint distribution of demographics to produce state-level estimate

Approaches to Dynamic MRP

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\end{aligned} \tag{2}
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4. **Random intercepts by year** (Simonovits and Bor 2023, Smith et al. 2020)

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5. **Random intercepts by demographic-year** (Gelman et al. 2018)

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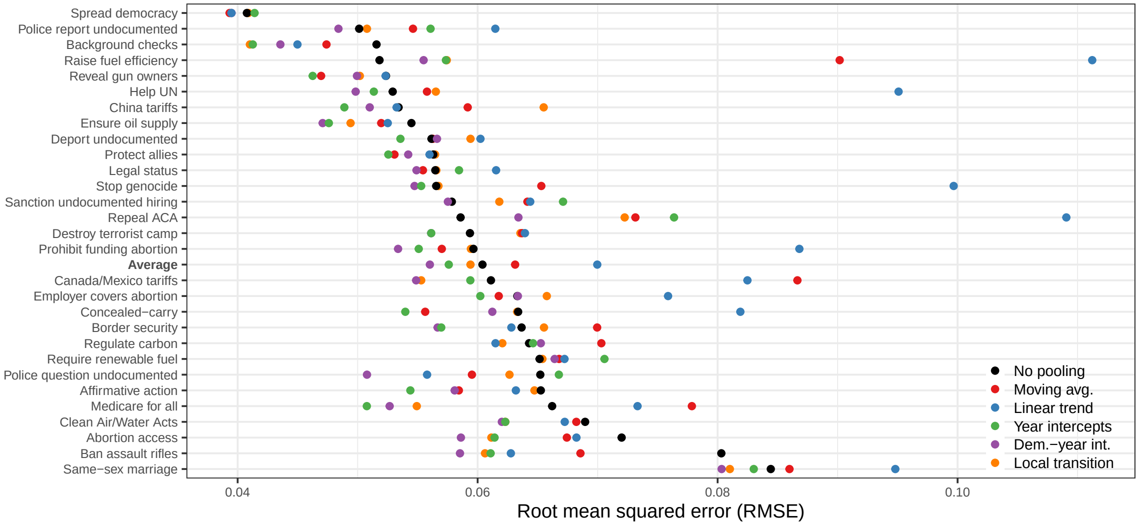
6. **Local transition model**

$$\begin{aligned}\alpha^{\text{year}}_t &\sim \text{N}(\alpha^{\text{year}}_{t-1}, \sigma^2_{\text{year}}), \\ \alpha^j_{g,\,t} &\sim \text{N}(\alpha^j_{g,\,t-1},\ \sigma^2_j)\ \forall\ g,\ t,\ j \in \{\text{gender},\ \text{race},\ \text{age},\ \text{educ}\}.\end{aligned} \tag{5}$$

Dynamic MRP on 29 Policy Issues



Explaining Variation in Dynamic MRP Performance

- Sources of variation in **static** MRP performance:
  - Sample size (Lax and Phillips 2009)
  - First-stage model complexity (Warshaw and Rodden 2012)
  - Importance of individual- and state-level covariates (Buttice and Highton 2013)

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  - First-stage model complexity (Warshaw and Rodden 2012)
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- Focus on **characteristics of time series** likely to affect performance:
  - Volatility of opinion over time
  - Length of time series
  - Sample size in each year

Simulation Set-Up

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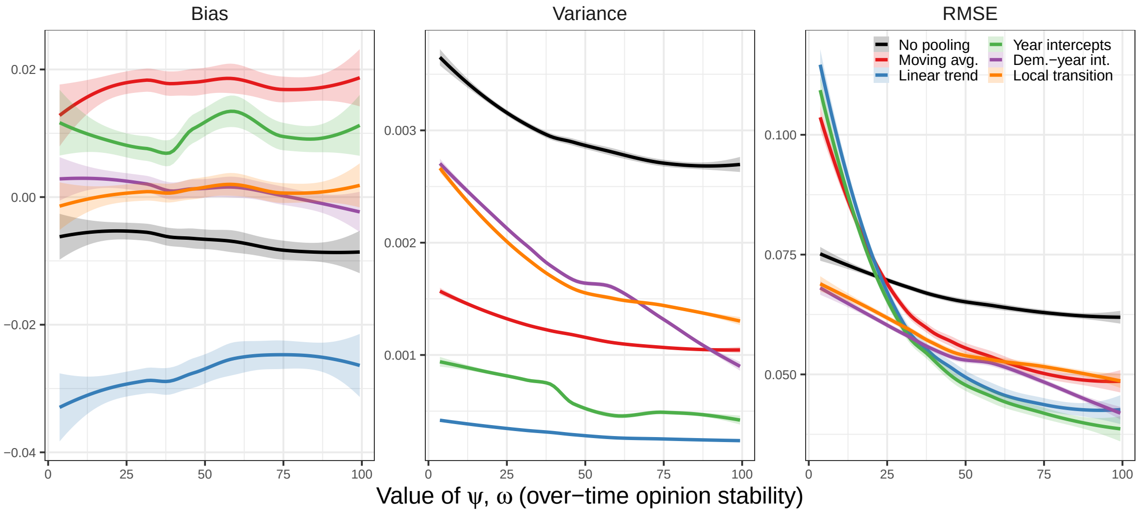
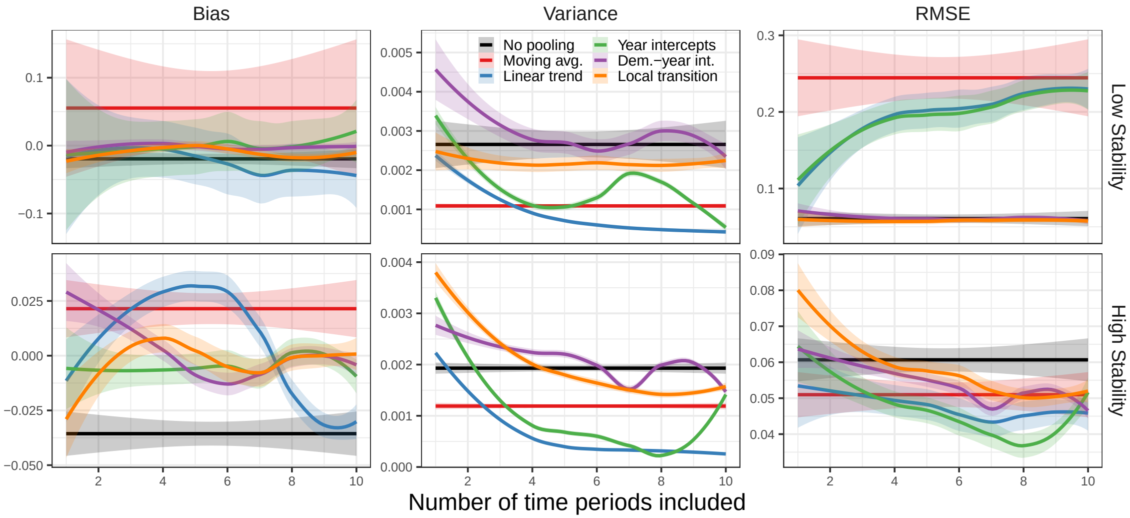
- 300 iterations, fit models on randomly sampled 10% of data in each year

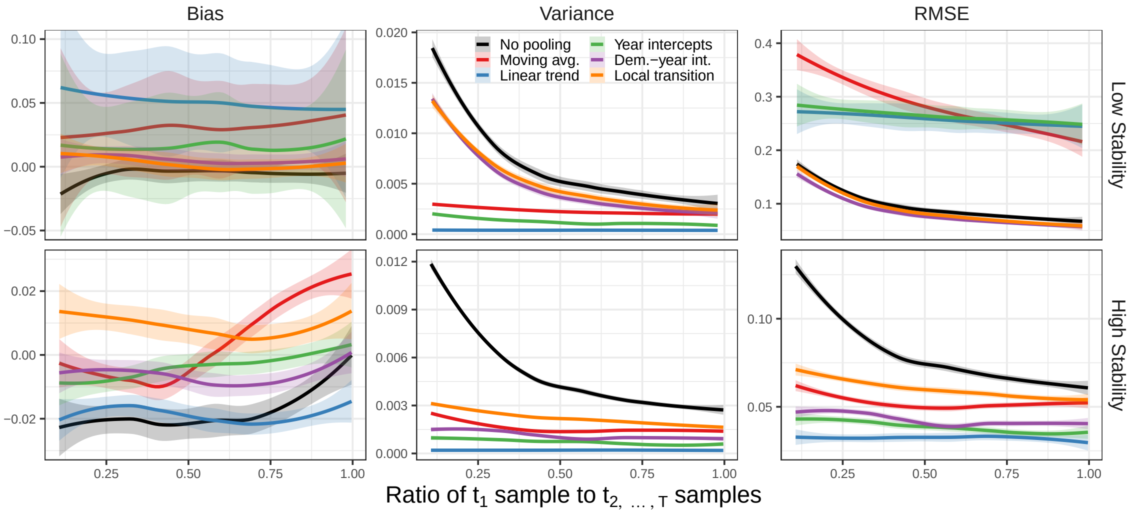
Can we recover time trends in state-level opinion?


Bias
Variance
RMSE
No pooling
Moving avg.
Linear trend
Year intercepts
Dem.−year int.
Local transition
0.02
0.00
−0.02
−0.04
0.003
0.002
0.001
0.100
0.075
0.050
0
25
50
75
100
Value of ψ, ω (over−time opinion stability)

Can we improve cross-sectional estimates by increasing T?


Bias
Variance
RMSE
No pooling
Moving avg.
Linear trend
Year intercepts
Dem.−year int.
Local transition
Low Stability
High Stability
Number of time periods included

Can we recover cross-sectional opinion when data is scarce?


Bias
Variance
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Year intercepts
Dem.−year int.
Local transition
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High Stability
0.10
0.05
0.00
−0.05
0.02
0.00
−0.02
0.020
0.015
0.010
0.005
0.000
0.012
0.009
0.006
0.003
0.000
0.4
0.3
0.2
0.1
0.10
0.05
0.25
0.50
0.75
1.00
Ratio of t1 sample to t2, ..., T samples

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6. **Most versatile**: random intercepts by demographic-year

Future Work

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- MRP most often applied to public opinion
  - Assess appropriateness for other applications of small-area estimation (e.g. urban planning, agriculture), relative to other dynamic approaches (Rao and Yu 1994, Singh et al. 2005)